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Transmittivity through straight and stublike waveguides in a two-dimensional phononic crystal

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We study theoretically the propagation of acoustic waves through a linear waveguide, created inside a two-dimensional phononic crystal, along which a side branch (or stub) is attached. The primary effect of this resonator is to induce zeros of transmission in the transmission spectrum of the perfect waveguide. The transmittivity exhibits very narrow dips whose frequencies depend upon the width and the length of the stub. When a gap exists in the transmittivity of the perfect waveguide, the stub may also permit selective frequency transmission in this gap. We have considered phononic crystals constituted by either fluid or solid constituents. The calculations of the band structure and transmittivity are performed by a combination of finite-difference time domain and plane-wave expansion methods.

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Photonic crystals, made of a periodic repetition of dielectric or metallic inclusions in a homogeneous dielectric background, have attracted much attention during the last decade, in relation with their ability to control the propagation of light and the possibility of new optical devices.[1] Recent studies have been concerned with manipulating the propagation of light through channels (or waveguides) created inside the photonic crystal[2] that can be considered as photonic circuits.[3] These waveguides can have interesting engineering applications such as a sharp bending of light,[2] filtering, and wavelength division multiplexing,[4] to name a few. So far, such applications have been proposed and realized in the frame of planar waveguide structures where the properties of bandpass filtering or channel drop filtering are obtained by coupling the waveguides to microring or side branch resonators.[5] It is expected that ultrasmall optical integrated circuits can be realized by utilizing photonic crystals.[6]

In analogy to photonic band-gap materials, several works have been devoted to study the propagation of acoustic waves in the so-called phononic crystals, made of two- or three-dimensional periodic repetition of two different solid or fluid constituents, which exhibit large contrast betwen their elastic constants and/or mass densities. The existence of absolute band gaps has been investigated both theoretically[7,8] and experimentally.[9–12] Surface waves[13] and localization phenomena in linear and point defects were also considered.[14] Such materials can have potential applications as elastic-acoustic filters, mirrors, or transducers.

In a recent paper, Kafesaki *et al.*[15] calculated the transmission of elastic waves through a straight waveguide created in a two-dimensional (2D) phononic crystal by removing a row of cylinders. They emphasized the oscillations of the transmittivity as a function of frequency. The guidance of the waves is due to the existence of extended linear defect modes falling in the band gap of the phononic crystal. However, the dispersion curves of these defect modes can themselves exhibit a gap and hence, in the frequency range of this gap, the waveguide cannot transmit any wave.

In this paper, we are interested in studying the effect of a waveguide side branch (or stub) on the transmission coefficient. Indeed, a fundamental question is would any wave traveling along the guide ignore the presence of the stub or, on the contrary, the transmission be significantly altered by the stub due to interference phenomena. From the point of view of applications, such a resonator may serve as a building element for the design of specific functions such as filtering or add-drop multiplexing. One can point out that, in the frame of model calculations[16–18] (in particular, assuming perfect confinement of the waves or strict boundary conditions), several papers have calculated and discussed the transmission of waves through a guide with grafted side branches. These works show the occurrence of zeros of transmission associated with the resonators, the opening of band gaps if a set of stubs is periodically grafted along the waveguide, and then the possibility of filtering or wavelength multiplexing.[19] The object of this paper is to discuss the effect of a side branch resonator on the propagation along a waveguide in a phononic crystal, taking full account of the boundary conditions that are imposed by the crystal constituents. It will be shown that the transmittivity T is not affected by the stub over large domain of frequencies while T drops to vanishing values in some very narrow frequency ranges. These results are discussed and compared to those obtained by model calculations. It would be interesting to perform similar calculations in photonic crystals containing stubbed waveguides in view of potential device applications that can be expected.

Our calculation of the transmission coefficient is performed for 2D phononic crystals of square symmetry composed of fluid-fluid (air cylinders in water) and solid-fluid (steel cylinders in water) constituents. For the sake of simplicity, we mostly emphasize the results of the first case since it has been shown[20] that, for an appropriate choice of the volume filling fraction f, this system exhibits large band gaps and very narrow first few passbands. Of course, in practice, the air within the cylinders would be contained by means of some latex or polymer material. However, it has been

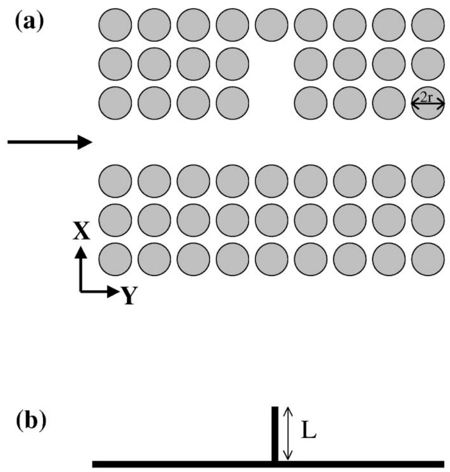

FIG. 1. (a) Two-dimensional cross section of the stubbed waveguide created inside the 2D phononic crystal. The system is periodically repeated in the *x* direction. The arrow indicates the direction of propagation of an incident wave packet. (b) Schematic of the stub attached to the waveguide in the model calculation (Ref. 16).

shown[21] that if the envelope is made of a soft material and is thin enough, then the first few band gaps remain still relatively large. Straight waveguides of different width are created in the phononic crystal by removing one row, or two parallel rows, of cylinders along the y axis of the square arrangement. Stubs of varying lengths are obtained by removing one, two, or three cylinders in the direction perpendicular to the waveguide (see Fig. 1). The width of the stub can also be varied. The transmission through the straight or stubbed waveguide is calculated by using the finite-difference time domain (FDTD) method, first applied to phononic crystals by Sigalas and Garcia.[22] Dispersion curves of the perfect crystal and defect modes associated with the straight guide are calculated using either the plane-wave expansion (PWE) method[7,8] or the FDTD method.

First we consider an air-water phononic crystal with a period $a=9$ mm and an air volume fraction of $f=0.349$. Figure 2(a) displays the band structure of the phononic crystal containing a straight waveguide of nominal width equal to one period, in the ΓX direction of the Brillouin zone. The calculation is performed with the PWE method using a supercell of seven periods (which means the waveguide is repeated every seven periods in the x direction). For the sake of clarity, let us notice that a frequency of 10 kHz corresponds to a reduced frequency $\omega a/c_{water}=0.379$ where c_{water} is the velocity of sound in water. The flatbands are the passbands of the perfect crystal where the modes are essentially localized inside air cylinders. The branches of parabolic shape are the defect modes associated with the straight guide. The defect branches interact weakly with the bulk modes at their crossing points and undergo a folding at the edge of the Brillouin zone with the opening of a small secondary gap around 102 kHz. Except in the vicinity of the Brillouin zone edge, the dispersion of the lowest defect branch can be described with a good accuracy by the same relation as in a guide with perfectly reflecting walls, namely,

$$\omega^2=\omega_0^2+c_{water}^2k^2, \tag{1}$$

where k is the wave vector and ω_0 can be considered as a cutoff frequency given by

$$\omega_0=\pi c_{water}/d, \tag{2}$$

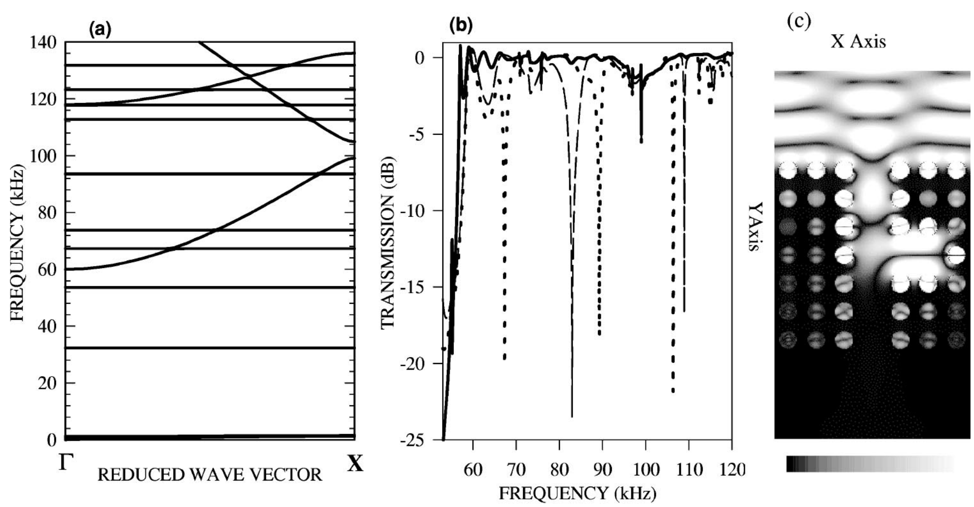

FIG. 2. (a) PWE dispersion curves for a supercell waveguide sample (without a stub). The phononic crystal consists of air cylinders in water with a square arrangement, with cylinder radius over lattice constant $r/a=1/3$. The supercell contains 1×7 unit cells. As a matter of comparison, 10 kHz on the vertical axis corresponds to the dimensionless frequency $\omega a/c_{water}=0.379$. (b) Solid line: the FDTD transmission coefficient as a function of frequency for acoustic waves propagating in the guide without the stub. Dashed line: the transmission coefficient for the guide with a one unit long stub. Dotted line: same as the dashed line, but the length of the stub is equal to two unit cells. (c) The square of the field amplitude averaged over one period of oscillation. The incident wave is a longitudinal monochromatic plane wave with frequency 67.35 kHz ($\omega a/c_{water}=2.54$) which corresponds to a dip in the transmission. The figure demonstrates that the transmission towards the end of the guide remains negligible at this frequency.

with d being the effective width of the guide. In the example of Fig. 2, this width turns out to be $d=12.7$ mm, i.e., a value slightly greater than the separation between two air cylinders on both sides of the guide. In analogy with a perfect guide where higher defect branches (corresponding to $\omega_0 = n\pi c_{water}/d$ where n is an integer) are expected to exist, one can notice in Fig. 2(a) the defect branch starting at the frequency $2\omega_0$.

In Fig. 2(b), we give the transmittivity through a waveguide without a stub and with a stub of nominal length equal to one or two periods inserted in the middle of the waveguide. The transmission coefficient is calculated by averaging the fluid displacement at different points on a line normal to the guide and by normalizing to the same quantity calculated for the incoming wave packet in the absence of the phononic crystal.[21] The incident wave is a longitudinal pulse, with a Gaussian envelope along the y axis and uniform along the x direction; the pulse is centered at the frequency $\omega a/c_{water}=2\pi$. The wave packet has a spatial extension of the order of half a wavelength, on both sides of its maximum. Therefore, the incident medium in front of the sample needs to have a thickness of the order of the wavelength in order to contain the initial wave packet. Periodic boundary conditions are applied in the x direction. In the y direction, the so-called Mur conditions are used at the free ends of the incoming and outgoing media. At the end of the outgoing medium, for instance, one imposes that the elastic wave is propagating in the forward direction. In other words, the wave is leaving the medium without reflection. In practice, the lengths of the incoming and outgoing media (water in our case) are taken to be equal to the length of the phononic crystal. More details about the calculation are given in Ref. 21.

The transmission T through the straight waveguide [Fig. 2(b)] jumps from 0 to 1 at the frequency of 59 kHz where the linear defect mode starts to exist and then remains close to unity for higher frequencies. The transmission spectrum exhibits a small dip around 100 kHz, in correspondence with the secondary gap at 100–105 kHz appearing in the dispersion curves [Fig. 2(a)] at the Brillouin zone boundary; the small magnitude of the dip in the transmission spectrum is due to the finite size of the sample and can be increased by increasing the length of the waveguide along the y direction. When a stub of nominal length and width equal to one period is attached to the guide, the transmission remains almost unchanged except for two very narrow dips occurring at the frequencies of 83 and 108.5 kHz where T becomes very small. For a stub of length two periods, three zeros of transmission occur at 67.35, 89.3, and 106.6 kHz, whereas a stub of the length of three periods exhibits four zeros of transmission (not shown in the figure). The frequencies of the dips are summarized in Table I. Therefore, it clearly appears that in the presence of a stub (or more generally of a resonator) the transmission through the waveguide can be significantly altered due to the interference phenomena; in particular, it is not true that any wave will travel along the guide without feeling the presence of the stub. To be more specific, we have calculated for different monochromatic incident waves the transmission through a guide containing a stub of length equal to two periods. We have investigated the time evolution of the wave for different frequencies of the incident wave. At the frequency of the dip, i.e., 67.35 kHz, the wave entering the guide penetrates into the stub [see Fig. 2(c)], is reflected from the end of the stub, and then returns back to the entrance of the guide while the transmission towards the end of the guide remains negligible. At a very close incident frequency of 67.53 kHz, the wave still penetrates into the stub. However, after being reflected by the end of the stub, a significant part of it is transmitted towards the end of the guide. Finally, at a frequency of 75 kHz, far from a dip, the wave travels straightly towards the end of the guide without being perturbed by the stub. To illustrate more clearly the total reflection of the incident wave at the frequency of 67.35 kHz, we have sketched in Fig. 2(c) the square of the field amplitude, averaged over one period of oscillation. This quantity gives a qualitative picture of average energy distribution along the guide and in the stub. One can notice the penetration of the wave into the stub, while no energy is transmitted towards the end of the guide.

TABLE I. Frequencies of the dips in the transmission spectrum of the stubbed waveguide.

Nominal length of the stub	Zero-transmission frequencies (kHz)	Effective length of the stub (mm)
One period	83.0	12.7
	108.5	16.2
Two periods	67.35	23.0
	89.3	22.2
	106.6	25.2
Three periods	62.3	37.2
	75.45	31.7
	92.4	31.4
	105.0	34.3

The above results can be qualitatively explained in the frame of a model calculation [see Fig. 1(b)], showing at the same time the interest and limitations of such a model. A calculation of the transmission coefficient along a slender tube[16] shows that, for a stub of length L, the zeros of transmission occur at frequencies such that $k=n\pi/L$ (where n is an integer) if the end of the stub is open, which means that the boundary condition at the end of the stub corresponds to the vanishing of the pressure. Similarly, the zeros of transmission occur at $k=(n+1/2)\pi/L$ for a closed stub for which the displacement vanishes at the end of the stub. The air-water phononic crystal considered in this paper resembles the case of open slender tube, since one can see in Fig. 2(c) that the wave penetrates over the first air cylinder at the end of the stub, which is equivalent to an almost zero pressure at the end of a slender tube. Then, using Eqs. (1) and (2) with $k=n\pi/L$ yields the effective lengths of the stubs which are summarized in Table I. In spite of the relatively good agreement in the calculated effective stub length, one notices a slight frequency dependence. However, the interesting point to mention is that, around a given frequency, the effective

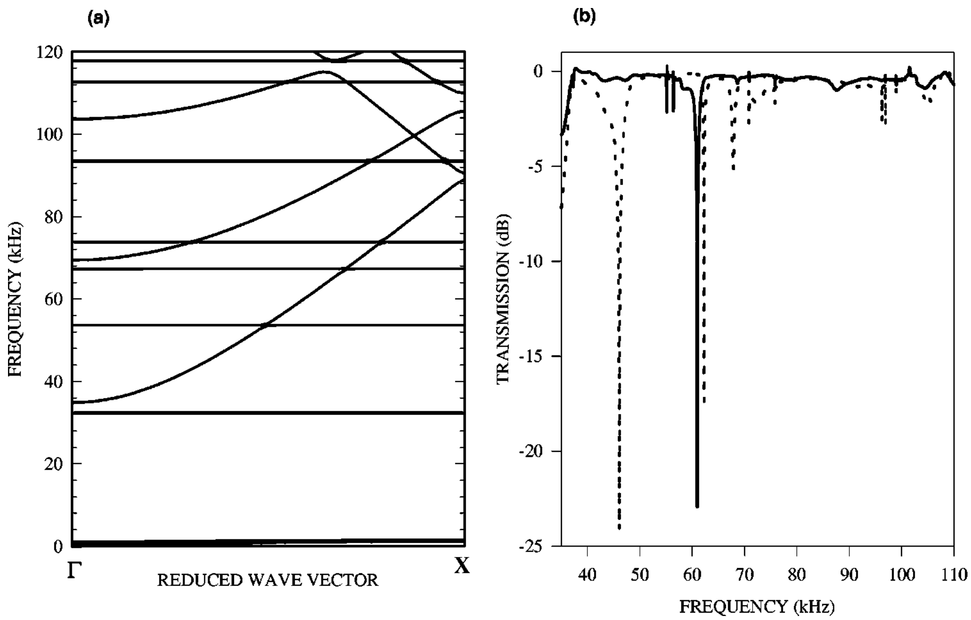

FIG. 3. (a) PWE Dispersion curves for a wide guide (two unit cells) in the air-water phononic crystal. (b) Solid line: the FDTD transmission coefficient as a function of frequency for acoustic waves propagating in the wide guide with a two unit long stub. Dashed line: same as the solid line but for a wider stub of width equal to two unit cells.

lengths of the different stubs, with nominal lengths of one, two, and three periods differ from each other by almost exactly 9 or 18 mm, i.e., one or two periods of the phononic structure. Of course, the application of the simple model becomes less (or not at all) reliable in the vicinity of the Brillouin zone boundary due to the bending of the dispersion curves associated with the guide.

We have also considered the propagation through a larger guide of nominal width equal to two periods in the air-water phononic crystal. The dispersion curves [Fig. 3(a)] display now several parabolic shape branches associated with the linear guide; these curves are closer to each other than in Fig. 2(a), and the lowest branch starts at a lower frequency. These behaviors are qualitatively similar to those of a guide with perfectly reflecting walls where the number of guided modes increases by widening the guide. We have checked that, except in the vicinity of the Brillouin zone boundary, the two lowest branches can accurately be described by Eq. (1) with cutoff frequencies given by 34.5 kHz and 69 kHz, respectively. Equation (2) yields the corresponding effective width of the guide to be $d=21.7$ mm, i.e., exactly 9 mm larger than in the case of Fig. 2(a). The waveguide is in the monomode regime between 34.5 and 69 kHz while it can support more than one mode above 69 kHz. In Fig. 3(b) we have presented the transmission through a stubbed waveguide, the stub being of nominal length two periods and nominal width equal to one or two periods. First, it is worth noting that zeros of transmission only occur in the frequency range where the linear guide remains monomode. Indeed, it is expected that a totally destructive interference phenomenon is more difficult to perform in the case of a multimode channel. The stub of larger width displays two zeros of transmission occurring at 46.3 and 62.4 kHz, respectively. However, since the narrower stub can only support waves above the cutoff frequency of 59 kHz (see Fig. 2), it only displays a single dip appearing at 60.98 kHz. The model calculation can again give a qualitative explanation of these results.

We have also studied the case of a phononic crystal constituted by steel cylinders immersed in water, with a period $a=9$ mm and a filling fraction $f=0.475$. The results are presented in Fig. 4 in analogy to those reported earlier in Fig. 2 for the air-water system. Here, the calculations of both band structures and transmission coefficients are performed on the basis of the FDTD method in order to take fully account of both transverse and longitudinal velocities of sound in solid materials. For propagation parallel to the xy plane, this structure exhibits an absolute band gap extending from 75 to 100 kHz, delimited by horizontal lines in Fig. 4(a). In Fig. 4(a), we give the dispersion curves of the crystal

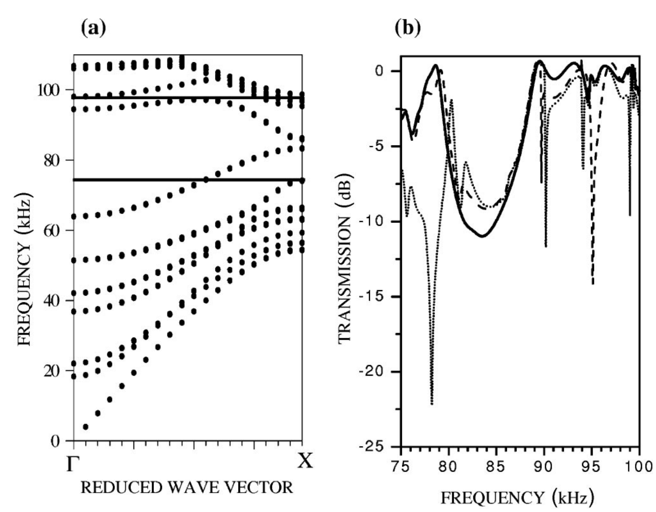

FIG. 4. (a) FDTD dispersion curves for a supercell waveguide sample (without a stub). The phononic crystal consists of steel cylinders in water with a square arrangement, with cylinder radius over lattice constant $r/a=1/3$. The supercell contains 1×7 unit cells. (b) Solid line: the FDTD transmission coefficient as a function of frequency for acoustic waves propagating in the guide without the stub. Dashed line: the transmission coefficient for the guide with a one unit long stub. Dotted line: same as the dashed line, but the length of the stub is equal to two unit cells.

containing a straight waveguide, with a supercell of seven periods in the direction x. Among all these branches, only the two falling inside and in the vicinity of the band gap are defect modes induced by the linear guide; the other branches are bulk modes resulting from the folding of the perfect crystal band structure, in relation with the superperiodicity along the x direction. The two defect branches bend in the vicinity of the Brillouin zone edge, giving rise to the opening of the secondary gap (83–85.8 kHz) where the propagation of any wave in the guide is forbidden. Figure 4(b) presents the transmittivity through the linear guide without stub or with a stub of length equal to one and two periods. Inside the main gap of the phononic crystal, the transmission through the linear guide oscillates around 1, except for a large dip (from 80 to 88 kHz) corresponding to the secondary gap between the two defect branches obtained in Fig. 4(a). The presence of the stub again induces very narrow dips where the transmission becomes very small; these occur at 89.72 and 95.11 kHz (respectively, 78.17, 90.19, 94.06, and 99.00) for the stub with length 1 (respectively, length 2). The frequencies of the dips can also be understood on the basis of our qualitative model.[16] However, the suitable boundary condition at the end of the stub appears here to correspond to the vanishing of the displacement, i.e., the case of a closed tube. Indeed, we have checked that, at the frequency of the dip, the wave does not penetrate into the steel cylinder at the end of the stub. Another interesting feature in Fig. 4(b) is the fact that, inside the secondary gap, where the transmission is not allowed in the perfect linear guide, the stub can induce peaks where selective frequencies can be transmitted through the phononic crystal. Finally, let us mention that we have obtained qualitatively similar behaviors for phononic crystal constituted by solid materials.

In conclusion, we have studied the propagation of acoustic waves through a guide in a phononic crystal along which a side branch is grafted. The effect of the stub is to induce zeros of transmission that can have potential applications in filtering and wavelength demultiplexing phenomena. Moreover, the stub can permit selective transmission of frequency inside a secondary gap in the spectrum of the defect modes associated with the rectilinear guide. Although some of the behaviors can be expected from modelistic calculations, the quantitative characteristics of the transmission spectrum requires to take full account of the geometry and elastic parameters of the phononic crystal. We have obtained qualitatively similar results for phononic crystals constituted by either fluid or solid constituents. Finally, we have checked that if two or three side branches are attached to the guide, the dips start to widen: this mimicks the opening of gaps when a periodic set of stubs are grafted along a guide. Such gaps may also be used for selective transmission of frequency if a defect is inserted among the stubs.

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