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Nonadiabatic detection of the geometric phase of the macroscopic quantum state with a symmetric SQUID

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We propose a type of nonadiabatic scheme to detect the geometric phase for the macroscopic state of a Josephson-junction system. After extending the scheme to the two-qubit cases, we provide a type of nonadiabatic geometric C-NOT gate that can play an important role in quantum computation.

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Testing the laws of quantum mechanics at the macroscopic scale is an important and interesting opic.[1,2] The geometric phase[3–5] plays an important role in quantum interferometry and many other disciplines. It is of potential interest to detect the geometric phase of a macroscopic quantum state. Recently, after a proposal of a geometric C-NOT gate[6] with nuclear magnetic resonance (NMR) in another work,[7] it was shown how to adiabatically detect the Berry phase in the Josephson junction of an asymmetric superconducting quantum interference device (SQUID), and, also, how to make a fault tolerant C-NOT gate through a conditional Berry phase shift.[7] However, one should overcome two drawbacks in such suggestions for performing the geometric quantum computation or detecting the Berry phase only. One is the adiabatic condition that makes such a gate impractical with current technology, the other is the extra operations needed to eliminate the dynamic phase. These drawbacks may seriously weaken the fault tolerant property. The experimental results with systematic errors were obtained on NMR.[6] In this paper, we give a simple scheme to detect the geometric phase or realize the C-NOT gate nonadiabatically on the zero dynamic phase evolution curve with a symmetric SQUID. By this scheme, both the detection and the realization can be done faster and more easily. The above-mentioned drawbacks of previous suggestions are removed. We believe our scheme can help to make the idea of a geometric C-NOT gate more practical than before.

For a two-level system, the geometric phase is equal to half of the solid angle subtended by the area in the Bloch sphere enclosed by the closed evolution loop of an eigenstate. Recently,[6] it was shown to make a fault tolerate C-NOT gate using Berry's phase, i.e., an adiabatic and cyclic geometric phase. It is well known[8] that, together with the single qubit rotation operations, a C-NOT gate can be used to carry out all quantum computation tasks. Very recently, there was a new proposal to test the geometric phase and the idea of the conditional geometric phase shift with an asymmetric SQUID. This is rather significant because testing the geometric phase on a macroscopic quantum state itself is important in quantum mechanics, and also, a C-NOT gate through a Josephson junction is undoubtedly a real *quantum* C-NOT gate. However, both proposals[6,7] strongly rely on the adiabatic process. This is a bit unrealistic because the macroscopic state of Josephson junction dephases fast.[1] The dephasing effects may distort the laboratory observation of the Berry phase seriously. On the other hand, for the purpose of quantum computation, any quantum gate should be run as fast as possible. Besides the adiabatic condition, both of the previous proposals take an extra operation to eliminate the dynamic phase. This extra operation is unwanted for a fault tolerate C-NOT gate because if we cannot eliminate the dynamic phase exactly, the fault tolerance property is weakened. For these reasons one is tempted to setup a new scheme that does not rely on the adiabatic condition and which does not involve any dynamic phase in the whole process.

The geometric phase also exists in nonadiabatic process. It was shown by Aharonov and Anandan[5] that the geometric phase is only dependent on the area enclosed by the loop of the state on the Bloch sphere. In nonadiabatic case, the path of the state evolution in general is different from the path of the parameters in the Hamiltonian. The external field need not always follow the evolution path of the state like that in the adiabatic case. So it is possible to let the external field be instantaneously perpendicular to the evolution path so that there is no dynamic phase involved in the whole process.

Consider a superconducting electron box formed by a symmetric SQUID (see Fig. 1), pierced by a magnetic flux Φ and with an applied gate voltage V_x. The device is operated in the charging regime, i.e., the Josephson couplings E_{J0} are much smaller than the charging energy E_{ch}. Also, a temperature much lower than the Josephson coupling is assumed. The Hamiltonian for this system is[7,9,10]

$$H = E_{\mathrm{ch}}(n-n_x)^2 - E_J(\Phi)\cos\chi,$$

where $E_J(\Phi) = 2E_{J0}\cos[\pi(\Phi/\Phi_0)]$, E_{ch} is the charging energy, and n_x can be tuned by the applied voltage V_x through

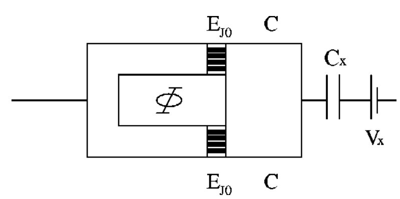

FIG. 1. SQUID with symmetric Josephson junctions. It consists of a superconducting box formed by a symmetric SQUID pierced by the magnetic voltage Φ. V_x is the applied voltage, which determines the offset charge n_x. The device operates in the charge regime, i.e., $E_{J0} \ll E_{\mathrm{ch}}$.

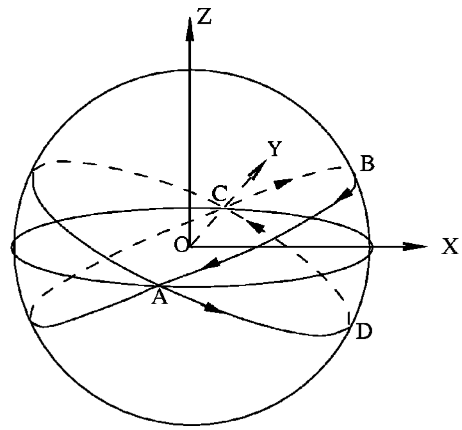

FIG. 2. A scheme for geometric phase detection in a symmetric SQUID. By suddenly changing the parameters n_x and Φ, we can get a sudden fictitious field in the x-z plane in directions perpendicular to the geodesic planes CBA and ADC, respectively. The fictitious field will determine the evolution path of CBADC. The angle between the geodesic plane ABC and the equator is θ. The solid angle subtended by the area of ABCDA is $4\pi\theta$.

$V_x=2en_x/C_x$ (see Fig. 1). The phase difference across the junction is χ and θ is canonically conjugated to the Cooper-pair number n, i.e., $[\theta,n]=i$. $\Phi_0=h/2e$ is the quantum of flux. So here n_x and Φ can be tuned externally. As it was pointed out earlier,[7,9,10] when n_x is around $\frac{1}{2}$ only two charge eigenstates $|0\rangle$ and $|1\rangle$ are relevant. The effective Hamiltonian in the computational two-dimensional Hilbert space is $H=-\frac{1}{2}\mathbf{B}\cdot\sigma$, where we have introduced the fictitious field $\mathbf{B}=[E_J,0,E_{\rm ch}(1-2n_x)]$ and σ are the Pauli matrices. In this picture, the states of zero Cooper pair ($|0\rangle$) or one Cooper pair ($|1\rangle$) are expressed by $|\downarrow\rangle$ or $|\uparrow\rangle$, respectively, where the states $|\downarrow\rangle$ and $|\uparrow\rangle$ are eigenstates of the Pauli matrix σ_z. Suppose that initially $\Phi=\pi/2$ and $n_x=0$, so that the initial state is $|\psi_0\rangle=|0\rangle=|\downarrow\rangle$. To avoid the dynamic phase, we select the geodesic curves in the time evolution. We can use the following scheme (see Fig. 2).

(1) We suddenly change Φ from $\pi/2$ to 0 and n_x (note that n_x can be tuned through V_x) from 0 to $n_0=\frac{1}{2}(1-\delta)$, $\delta>0$.

(2) Wait for a time

$$\tau=\frac{\pi}{\sqrt{(E_{\rm ch}\delta)^2+(2E_{J0})^2}}. \tag{1}$$

After this the state is rotated along the geodesic curve CBA by an angle π (See Fig. 2).

(3) We suddenly change n_x to $\frac{1}{2}+\delta$, and wait for a time τ again. The state is rotated along the geodesic curve ADC by an angle π (See Fig. 2).

(4) Do the measurement to detect the interference effects.

(5) Deduce the value γ from the interference pattern.

In the above operations, the time evolution curve will enclose an area on the Bloch sphere. The eigenstate basis $|\pm\rangle$, which are eigenstates of the Pauli matrix σ_y will acquire a geometric phase of

$$\pm\gamma=\pm 2\theta, \tag{2}$$

respectively. If the initial state is $|\psi(0)\rangle=|0\rangle$, at time 2τ it is

$$|\psi(2\tau)\rangle=U(2\tau)|0\rangle,$$

where $U(2\tau)$ is the time evolution operator from time 0 to 2τ. It has the property that

$$U(2\tau)|\pm\rangle=e^{\pm i\gamma}|\pm\rangle.$$

Using the relation

$$|0\rangle=-\frac{i}{\sqrt{2}}(|+\rangle-|-\rangle),$$

we have

$$|\psi(2\tau)\rangle=-\frac{i}{\sqrt{2}}U(2\tau)(|+\rangle+|-\rangle)=-\sin\gamma|\uparrow\rangle-\cos\gamma|\downarrow\rangle. \tag{3}$$

In general, suppose that the initial state is $|\Psi(0)\rangle=\alpha|\uparrow\rangle+\beta|\downarrow\rangle$, with the constraint of $|\alpha|^2+|\beta|^2=1$, the evolved state is given as

$$|\Psi(2\tau)\rangle=(\alpha\cos\gamma-\beta\sin\gamma)|\uparrow\rangle-(\alpha\sin\gamma+\beta\cos\gamma)|\downarrow\rangle. \tag{4}$$

We can see that the nonzero γ phase here will cause detectable interference effects. Specifically, if we choose an appropriate δ value so that $\gamma=\pi/2$, we can observe the flip of the initial state. This property can be potentially useful to realize the fault tolerant NOT gate in geometric quantum computation.[6,7] Equivalently, we can rotate a state on Bloch sphere around the x axis for the same angle θ instead of changing n_x in the scheme. We do so in the following for the conditional geometric phase shift.

So far we have a simple scheme to detect the nonadiabatic geometric phase for the macroscopic quantum state of a symmetric SQUID. Since it is operated nonadiabatically, the total time needed here should be comparable to that of one cycle in the experiment of Ref. 1, that is to say, much shorter than the decoherence time. We believe that the experimental realization of our scheme can be done with the same setup used in Ref. 1. Since we have chosen the zero dynamic phase path[11], we need not make an additional evolution loop to cancel the dynamic phase and preserve the geometric phase.

The above scheme for the nonadiabatic detection of a geometric phase in a single qubit system can be easily extended to the two-qubit system with a symmetric SQUID. Suppose we have two capacitively coupled symmetric SQUIDs (see Fig. 3) 1 and 2 with the same E_{J0} and $E_{\rm ch}$. For simplicity, we call them as qubit 1 and qubit 2, respectively, and use the subscripts 1 and 2 to indicate the corresponding qubits. For qubit 1 (the control qubit) we set $\Phi_1=\pi/2$ and $n_{x,1}=0$ during the whole process. For qubit 2 (the target bit), we set $\Phi_2=\Phi_0/2$ and $n_{x_2}=\frac{1}{2}$ initially. Qubit 1 or 2 can be either in the state $|0\rangle=|\uparrow\rangle$ or in the state $|1\rangle=|\downarrow\rangle$. The *weak* interacting Hamiltonian is given by

$$H_I=\Delta(n_1-n_{x,1})(n_2-n_{x,2}), \tag{5}$$

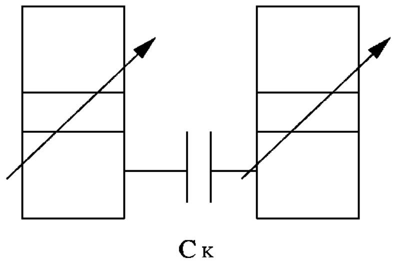

FIG. 3. Two capacitively coupled SQUIDs.

where the constant Δ is the charging energy that derives from the capacitive coupling. We assume that Δ is much smaller than E_{ch}. In our setting, the Hamiltonian also reads

$$H_I = \frac{1}{2}\Delta(\sigma_{1z} + \tfrac{1}{2})\sigma_{2z}. \tag{6}$$

That is to say, to qubit 2 the interacting Hamiltonian looks as an effective conditional Hamiltonian dependent on the specific state of qubit 1. Explicitly, it is $\frac{1}{2}\Delta\sigma_z$ if the state of qubit 1 is $|1\rangle = |\uparrow\rangle$, and it is 0 if the state of qubit 1 is $|0\rangle = |\downarrow\rangle$.

We first rotate qubit 2 around the x axis by an angle $-\theta$ (see Fig. 4). Note that E_{ch} is much larger than Δ, so the state of qubit 1 is (almost) not affected by any operation on qubit 2 during the whole process. The interaction Hamiltonian will thus determine an evolution path of the geodesic circle ABC on the Bloch sphere (see Fig. 4). After time $\tau = \pi/\Delta$, we rotate qubit 2 around the x axis by another angle $-(\pi - 2\theta)$. Again we wait for a time τ. Then we rotate qubit 2 around the x axis by an angle $\pi - \theta$ to let the state on the Bloch sphere go back to the original position. After the above operations, if qubit 1 was in the state $|\uparrow\rangle$, the evolution path of ABCDA on the Bloch sphere is produced; if qubit 1 was in the state $|\downarrow\rangle$ instead, nothing happens to qubit 2.

In our scheme we have performed a rotation operation to qubit 2 around the x axis. This can be done by suddenly changing Φ_1 to 0 and $n_{x,1} = \frac{1}{2}$ to qubit 2. Everytime after the rotation around the x axis is completed, we always set the parameters Φ_1 back to $\pi/2$. Note that we need not worry about the case of the $|\downarrow\rangle$ state for qubit 1. Since in this case the interaction is 0, qubit 2 is just rotated around an axis that is slightly different from the x axis and is then rotated back around the same axis, thus producing no net effect on qubit 2 in the whole process.

Summarizing, for qubit 2 the final state is changed by the unitary transformation $U(2\tau)$ in the following way:

$$U(2\tau)\begin{pmatrix} |\downarrow\rangle|\downarrow\rangle \\ |\downarrow\rangle|\uparrow\rangle \\ |\uparrow\rangle|\downarrow\rangle \\ |\uparrow\rangle|\uparrow\rangle \end{pmatrix} = \begin{pmatrix} I & 0 \\ 0 & M \end{pmatrix}\begin{pmatrix} |\downarrow\rangle|\downarrow\rangle \\ |\downarrow\rangle|\uparrow\rangle \\ |\uparrow\rangle|\downarrow\rangle \\ |\uparrow\rangle|\uparrow\rangle \end{pmatrix}, \tag{7}$$

where

$$M = \begin{pmatrix} \cos\gamma(\theta) & i\sin\gamma(\theta) \\ i\sin\gamma(\theta) & \cos\gamma(\theta) \end{pmatrix},$$

$\gamma(\theta)$ is the geometric phase acquired for the initial state $|X,+\rangle$ (point A in Fig. 4), and $\gamma(\theta) = -2\theta$. We see that the choice $|\gamma(\theta)| = \pi/4$ corresponds to $\theta = \pi/8$ (see Fig. 4) performs a C-NOT gate, which is fault tolerant to certain types of errors[6,12]. Since the adiabatic requirement is removed in our scheme, the total operation time needed here should be comparable to that of a normal C-NOT gate. Therefore we believe that our scheme has made the idea of geometric quantum computation much closer to the practical application. The various parameters of a normal C-NOT gate for an inductively coupled system are listed in Refs. 8, 9, 13, where it has been estimated that the operation time can be much shorter than the decoherence time. Instead of an inductively coupled system, here we have used a capacitively coupled-system, however, this should not be an essential modifica-

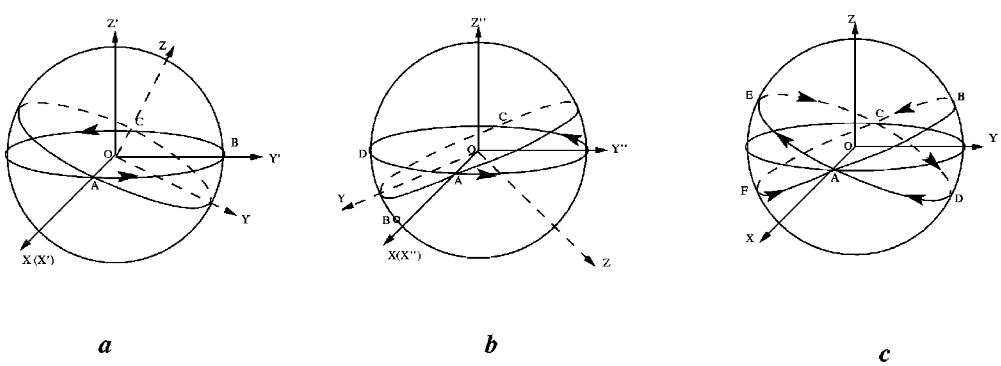

FIG. 4. Nonadiabatic conditional geometric phase shift acquired through 0 dynamic phase evolution path. These are pictures for the time evolution on the Bloch sphere of qubit 2 *only* in the case that qubit 1 is $|\uparrow\rangle$. If qubit 1 is $|\downarrow\rangle$, there is no net change to qubit 2. Picture (a) shows that after the Bloch sphere is rotated around the x axis by $-\theta$ angle, the interaction Hamiltonian will rotate the Bloch sphere around the z' axis. At the time it completes a π rotation, i.e., $\tau = \pi/\Delta$ we rotate the bloch sphere around the x axis again by an angle of $-(\pi - 2\theta)$, then we get picture (b). In picture (b) the sphere is rotated around the z'' axis by the interaction Hamiltonian. Note that the point B in picture (b) has changed its position now. The geodesic curve CBA is not drawn in picture (b). After time τ we rotate the qubit 2 around the x axis by an angle of $\pi - \theta$. Picture (c) shows the whole evolution path on the Bloch sphere. Point A evolves along the closed curve ABCDA, a geometric phase $\gamma = -2\theta$ is acquired. Point C evolves along the loop CFAEC, a geometric phase $-\gamma = 2\theta$ is acquired.

tion. In a real experiment, one also needs certain readout device. The single electron transistor could be a good candidate (for detailed analysis see Ref. 13). The interference patterns have been successfully observed for the quantum state of the Cooper-pair box.[1] In a real experiment for detecting the geometric phase presented by this paper, one can first detect the interference pattern between the state in Eq. (3) and the initial state, and then deduce the value γ from the observed results. In particular, the value $\gamma = \pi/2$ will cause the sharpest observation result, since this value flips the initial state. One can directly borrow the measurement technique used in Ref. 1 for the detection. However, in our case the measured interference pattern is totally determined by the geometric phase γ. The detailed conditions for the measurement experiment can be found in Ref. 14

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