Superconductor coupled to two Luttinger liquids as an entangler for electron spins

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(Received 17 December 2001; published 10 April 2002)

We consider an *s*-wave superconductor (SC) which is tunnel coupled to two spatially separated Luttinger-liquid (LL) leads. We demonstrate that such a setup acts as an entangler, i.e., it creates spin singlets of two electrons which are spatially separated, thereby providing a source of electronic Einstein-Podolsky-Rosen pairs. We show that in the presence of a bias voltage, which is smaller than the energy gap in the SC, a stationary current of spin-entangled electrons can flow from the SC to the LL leads due to Andreev tunneling events. We discuss two competing transport channels for Cooper pairs to tunnel from the SC into the LL leads. On the one hand, the coherent tunneling of two electrons into the same LL lead is shown to be suppressed by strong LL correlations compared to single-electron tunneling into a LL. On the other hand, the tunneling of two spin-entangled electrons into different leads is suppressed by the initial spatial separation of the two electrons coming from the same Cooper pair. We show that the latter suppression depends crucially on the effective dimensionality of the SC. We identify a regime of experimental interest in which the separation of two spin-entangled electrons is favored. We determine the decay of the singlet state of two electrons injected into different leads caused by the LL correlations. Although the electron is not a proper quasiparticle of the LL, the spin information can still be transported via the spin-density fluctuations produced by the injected spin-entangled electrons.

DOI: 10.1103/PhysRevB.65.165327 PACS number(s): 73.40.Gk, 71.10.Pm

I. INTRODUCTION

Pairwise and nonlocal entangled quantum states, so-called Einstein-Podolsky-Rosen (EPR) pairs,[1] represent the fundamental resource for quantum communication[2] schemes like dense coding, quantum teleportation, or quantum key distribution;[3] more fundamentally, they can be used to test Bell's inequalities.[4] Experiments tested Bell's inequalities,[5] dense coding,[6] and quantum teleportation[7,8] using photons, but to date no experiments for *massive* particles like electrons in a solid-state environment exist. This is so because it is difficult, first, to produce entangled electrons and also to detect them afterward in a controlled way due to other electrons interacting with the entangled pair. On the other hand, the spin of an electron was pointed out to be a most natural candidate for a quantum bit (qubit).[9,10] This idea was also supported by experiments which showed unusually long dephasing times for electron spins in semiconductors (approaching microseconds) and phase-coherent transport up to 100 μm.[11–13] In addition, the electron also possesses a charge which makes it well suited for transporting the spin information.[14,15] Further, the Coulomb interaction between the electron charges can be exploited to spatially separate the spin-entangled electrons, resulting in electronic EPR pairs. As first pointed out in Refs. 14 and 15, such electronic EPR pairs can be used for testing Bell inequalities and for quantum communication schemes in the solid state. The first step toward this goal is to have a scheme by which the electrons can be reliably entangled. One possibility is to use coupled quantum dots.[14,15] Alternatively, we recently proposed an entangler device[16] which creates mobile and nonlocal spin-entangled electrons, consisting of an *s*-wave superconductor (SC), where the electrons are correlated in Cooper pairs with spin-singlet wave functions.[17] The SC is tunnel coupled via two quantum dots in the Coulomb blockage regime[18] to two spatially separated Fermi-liquid leads. By applying a bias, a stationary current of spin-entangled electrons can flow from the SC to the leads. The quantum dots are used to mediate the necessary interaction between the two electrons initially forming a Cooper pair in the SC, so that the two electrons tunnel preferably not into the same lead but instead into different leads. This entangler then satisfies all requirements to detect entanglement via the current noise in a beam splitter setup.[15] It is straightforward to formulate spin measurements for testing Bell inequalities (it is most promising to measure spin via charge[9,19,20]). We refer to related work,[21–23] which also made use of Andreev tunneling, but in a regime opposite to the one considered in Ref. 16 and here, where the superconductor/normal interface is transparent and no Coulomb blockade nor strong correlations are present.

In the present work we propose and discuss an alternative realization of an entangler which is based on strongly interacting one-dimensional wires which show a Luttinger-liquid (LL) behavior. In comparison to our earlier proposal with quantum dots,[16] we now replace the Coulomb blockade behavior of the dots by strong correlations of the LL. Well-known examples for LL candidates are carbon nanotubes.[24] The low-energy excitations of these LL's are collective charge and spin modes rather than quasiparticles, which resemble free electrons as they exist in a Fermi liquid. As a consequence, the single-electron tunneling into a LL is suppressed by strong correlations. The question then arises quite naturally of whether these strong correlations can even further suppress the coherent tunneling of *two* electrons into the same LL, as provided by a correlated two-particle tunneling event (Andreev tunneling), so that the two electrons preferably separate and tunnel into different LL leads. It turns out that the answer is positive. To address this question we introduce a setup consisting of an *s*-wave SC which is weakly tunnel coupled to the center (bulk) of two spatially separated

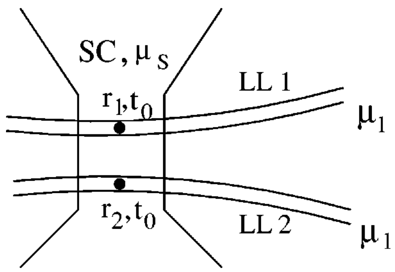

FIG. 1. A possible implementation of the entangler setup: Two quantum wires 1 and 2, described as infinitely long Luttinger liquids (LL), are deposited on top of an s-wave superconductor (SC) with a chemical potential μ_S. The electrons of a Cooper pair can tunnel by means of an Andreev process from two points $\mathbf{r}_1$ and $\mathbf{r}_2$ on the SC to the center (bulk) of the two quantum wires 1 and 2, respectively with a tunneling amplitude t_0. The interaction between the leads is assumed to be negligible.

one-dimensional wires 1 and 2 described as Luttinger liquids, see Fig. 1 and 2. In this model we calculate the stationary current generated by the tunneling of a singlet (spin-entangled electrons), transferred from the SC into two separate leads (nonlocal process) or into the same lead (local process), 1 or 2. We show that the ratio of these two competing current channels depends on the system parameters, and that it can be made large in order to have the desired injection of the two electrons in two separate leads, where, again, the two spins, forming a singlet, are entangled in spin space while separated in orbital space, and therefore represent an electronic EPR pair. It is well known that the tunneling of single electrons into LL's is suppressed compared to that in Fermi liquids due to strong many-body correlations. In addition, we now find that subsequent tunneling of a second electron into the same LL is further suppressed, again in a characteristic interaction-dependent power law, provided the applied voltage bias between the SC and the LL is much smaller than the energy gap Δ in the SC so that single-electron tunneling is suppressed. The two-particle tunneling event is strongly correlated within the uncertainty time $\hbar/\Delta$, characterizing the time delay between subsequent tunneling events of the two electrons of the same Cooper pair. In other words, the second electron of a Cooper pair is inclucenced by the existence of its preceding partner electron already present in the LL. This effect can also be interpreted as a Coulomb blockade effect, similar to what occurs in quantum dots attached to a SC.[25,16] Similar Coulomb blockade effects also occur in a mesoscopic chiral LL within a quantum dot coupled to macroscopic chiral LL edge states in the fractional quantum Hall regime.[26] There the Coulomb-blockade-like energy gap is quantized in units of the noninteracting energy-level spacing of the quantum dot, and its existence is therefore a finite-size effect, whereas in the present case we will see that the suppression comes from strong correlations in a two-particle tunneling event which is present even in an infinitely long LL as considered here. On the other hand, if the two electrons of a Cooper pair tunnel to different leads, they will preferably tunnel from different points $\mathbf{r}_1$ and $\mathbf{r}_2$ of the SC, with a distance $\delta\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$ due to the spatial separation of the leads; see Figs. 1 and 2. We find that the current is exponentially suppressed if the distance δr exceeds the coherence length ξ of a Cooper pair on the SC. This limitation poses no severe experimental restriction, since ξ is on the order of micrometers for typical s-wave materials, and δr can be assumed to be on the order of nanometers. Still, a power-law suppression $\propto 1/(k_F\delta r)^2$, with k_F being the Fermi wave vector in the SC, remains and is more relevant. We show, however, that in lower dimensions of the SC this suppression is less pronounced (with smaller powers). Further, we then discuss the decay in time of a spin-singlet state injected into two LL's, one electron in each lead, due to the interaction present in the LL, and find a characteristic power-law decay in time at zero temperature. Despite this decay of the singlet state, spin information can still be transported through the LL wires via the spin-density fluctuations created by the injected electrons.

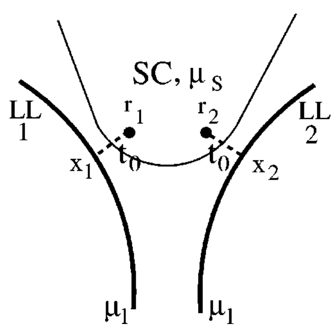

FIG. 2. An alternative implementation of the proposed entangler setup: two quantum wires 1 and 2, with a chemical potential μ_l, described as infinitely long Luttinger liquids (LL), are tunnel coupled with an amplitude t_0 from two points x_1 and x_2 to two points $\mathbf{r}_1$ and $\mathbf{r}_2$ of a superconducting (SC) tip with a chemical potential μ_S.

II. MODEL AND HAMILTONIAN

We consider an s-wave SC which is weakly tunnel coupled to the center (bulk) of two spatially separated LL leads (see Figs. 1 and 2). The Hamiltonian of the whole system is represented as $H=H_0+H_T$ with $H_0=H_S+\Sigma_{n=1,2}H_{Ln}$ describing the isolated SC and LL leads 1 and 2, respectively. Tunneling between the SC and the leads is governed by the tunneling Hamiltonian H_T. Each part of the system will be described in the following.

The s-wave SC, with a chemical potential μ_S, is described by the BCS Hamiltonian[27]

$$H_S-\mu_S N_S=\sum_{\mathbf{k},s} E_{\mathbf{k}}\gamma^\dagger_{\mathbf{k}s}\gamma_{\mathbf{k}s}, \tag{1}$$

where $s=(\uparrow,\downarrow)$ and $N_S=\Sigma_{\mathbf{k}s}c^\dagger_{\mathbf{k}s}c_{\mathbf{k}s}$ is the number operator for electrons in the SC. The quasiparticle operators $\gamma_{\mathbf{k}s}$ describe excitations out of the BCS ground state $|0\rangle_S$ defined by $\gamma_{\mathbf{k}s}|0\rangle_S=0$. They are related to the electron annihilation and creation operators $c_{\mathbf{k}s}$ and $c^\dagger_{\mathbf{k}s}$ through the Bogoliubov transformation[27]

$$\begin{aligned} c_{\mathbf{k}\uparrow}&=u_{\mathbf{k}}\gamma_{\mathbf{k}\uparrow}+v_{\mathbf{k}}\gamma^\dagger_{-\mathbf{k}\downarrow} \\ c_{-\mathbf{k}\downarrow}&=u_{\mathbf{k}}\gamma_{-\mathbf{k}\downarrow}-v_{\mathbf{k}}\gamma^\dagger_{\mathbf{k}\uparrow}, \end{aligned} \tag{2}$$

where $u_{\mathbf{k}}=(1/\sqrt{2})(1+\xi_{\mathbf{k}}/E_{\mathbf{k}})^{1/2}$ and $v_{\mathbf{k}}=(1/\sqrt{2})(1-\xi_{\mathbf{k}}/E_{\mathbf{k}})^{1/2}$ are the usual BCS coherence factors,[27] $\xi_{\mathbf{k}}=\epsilon_{\mathbf{k}}-\mu_S$ is the normal-state single-electron energy counted from the Fermi level μ_S, and $E_{\mathbf{k}}=\sqrt{\xi_{\mathbf{k}}^2+\Delta^2}$ is the quasiparticle energy. The field operator for an electron with spin s is $\Psi_s(\mathbf{r})=V^{-1/2}\Sigma_{\mathbf{k}}e^{i\mathbf{kr}}c_{\mathbf{k}s}$, where V is the volume of the SC.

The two leads 1 and 2 are supposed to be infinite one-dimensional interacting electron systems described by LL theory. We only include forward-scattering processes which describe scattering events in which electrons stay in the sme branch (left or right movers). We neglect backscattering interactions which involve large momentum transfers of order $2p_F$, where p_F is the Fermi wave vector in the LL.[28] The LL Hamiltonian for the low-energy excitations of lead $n=1,2$ can then be written in a bosonized form as[29]

$$H_{Ln}-\mu_l N_n=\sum_{\nu=\rho,\sigma}\int_{-L/2}^{+L/2}dx \times\left(\frac{\pi u_\nu K_\nu}{2}\Pi_{n\nu}^2+\frac{u_\nu}{2\pi K_\nu}(\partial_x\phi_{n\nu})^2\right), \quad (3)$$

where the fields $\Pi_n(x)$ and $\phi_n(x)$ satisfy bosonic commutation relations $[\phi_{n\nu}(x),\Pi_{m\mu}(x')]=i\delta_{nm}\delta_{\nu\mu}\delta(x-x')$, and μ_l is the chemical potential of the LL leads (assumed to be identical for both leads), and $N_n=\Sigma_s\int dx\,\psi_{ns}^\dagger(x)\psi_{ns}(x)$ is the number operator for electrons in LL n. Hamiltonian (3) describes long-wavelength charge ($\nu=\rho$) and spin ($\nu=\sigma$) density oscillations in the LL, propagating with velocities u_ρ and u_σ, respectively. The velocities u_ν and the stiffness parameters K_ν depend on the interactions between the electrons in the LL. In the limit of vanishing backscattering, we have $u_\sigma=v_F$ and $K_\sigma=1$, and the LL is described by only two parameters $K_\rho<1$ and u_ρ. In a system with full translational invariance we have $u_\rho=v_F/K_\rho$. We decompose the field operator describing electrons with spin s into right- and left-moving parts, $\psi_{ns}(x)=e^{ip_Fx}\psi_{ns+}(x)+e^{-ip_Fx}\psi_{ns-}(x)$. The right (left) moving field operator $\psi_{ns+}(x)$ $[\psi_{ns-}(x)]$ is then expressed as an exponential of bosonic fields as[30,31]

$$\psi_{ns\pm}(x)=\lim_{\alpha\to 0}\frac{\eta_{\pm,ns}}{\sqrt{2\pi\alpha}}\exp\left\{\pm\frac{i}{\sqrt{2}}\{\phi_{n\rho}(x)+s\,\phi_{n\sigma}(x)\mp[\theta_{n\rho}(x)+s\,\theta_{n\sigma}(x)]\}\right\}, \quad (4)$$

where $[\phi_{n\nu}(x),\theta_{m\mu}(x')]=-i(\pi/2)\delta_{nm}\delta_{\nu\mu}\mathrm{sgn}(x-x')$, and therefore $\partial_x\theta_{n\nu}(x)=\pi\Pi_{n\nu}(x)$. The operators $\eta_{\pm,ns}$ are needed to ensure the correct fermionic anticommutation relations. In the thermodynamic (TD) limit $(L\to\infty)$, $\eta_{\pm,ns}$ can be presented by Hermitian operators satisfying the anticommutation relation[29] $\{\eta_r,\eta_{r'}\}=2\delta_{rr'}$, with $r=\pm,ns$. We adopt the convention throughout the paper that $s=+1$ for $s=\uparrow$, and $s=-1$ for $s=\downarrow$, if s does not have the meaning of an operator index.

Transfer of electrons from the SC to the LL leads is described by the tunneling Hamiltonian $H_T=\Sigma_n H_{Tn}+\mathrm{H.c.}$, where H_{Tn} is defined as $H_{Tn}=t_0\Sigma_s\psi_{ns}^\dagger\Psi_s(\mathbf{r}_n)$. The field operator $\Psi_s(\mathbf{r}_n)$ annihilates an electron with spin s at point $\mathbf{r}_n$ on the SC, and $\psi_{ns}^\dagger$ creates it again with an amplitude t_0 at point x_n in the LL n which is nearest to $\mathbf{r}_n$, see Figs. 1 and 2. We assume that the spin is conserved during the tunneling process, and thus the tunneling amplitudes t_0 do not depend on spin, and, for simplicity, are the same for both leads $n=1$ and 2. We remark that our point-contact approach for describing the electron transfer from the SC to the LL is the simplest possible description, but it presumably captures the relevant features of a real device. The scheme shown in Fig. 2 has a geometry which suggests that electrons tunnel from point $\mathbf{r}_n\to x_n$ which are closest to each other, due to the fact that t_0 depends exponentially on the tunneling distance. In the setup shown in Fig. 1, a pointlike tunnel contact between the SC and the LL might be induced by slightly bending the quantum wires (e.g., nanotubes). If the contact area has a finite extension, we note that the two electrons preferably tunnel from the same point on the SC, when they tunnel into the same lead, since the two-particle tunneling event is coherent and already shows a suppression in the probability on a length scale given by $1/k_F$, as we discuss in detail below.

III. STATIONARY CURRENT FROM THE SC TO THE LL LEADS

We now calculate the current of singlets, i.e., pairwise spin-entangled electrons (Cooper pairs), from the SC to the LL leads due to Andreev tunneling[32,33] in first nonvanishing order, starting from a general T-matrix approach.[34] We thereby distinguish two transport channels. First we calculate the current when two electrons tunnel from different points $\mathbf{r}_1$ and $\mathbf{r}_2$ of the SC into *different* interacting LL leads, which are separated in space such that there is no interlead interaction. In this case the only correlation in the tunneling process is due to the superconducting pairing of electrons which results in a coherent two-electron tunneling process of opposite spins from different points $\mathbf{r}_1$ and $\mathbf{r}_2$ of the SC, and with a delay time $\sim\hbar/\Delta$ between the two tunneling events. Since the total spin is a conserved quantity $[H,\mathbf{S}^2]=0$, the spin entanglement of a Cooper pair is transported to *different* LL leads, thus leading to nonlocal spin entanglement. On the other hand, if two electrons tunnel from the same point of the SC into the *same* LL lead, there is an additional correlation in the LL lead itself due to the intralead interaction. It is the goal of this work to investigate how the transport current for tunneling of two electrons from the SC into the *same* LL lead is affected by this additional correlation. In the following we will use units such that $\hbar=1$.

IV. *T*-MATRIX

We apply a T-matrix (transmission matrix) approach[34] to calculate the current.[16] The stationary current of *two* electrons passing from the SC to the leads is then given by

$$I=2e\sum_{f,i}W_{fi}\rho_i. \quad (5)$$

Here W_{fi} is the transition rate from the superconductor to the leads, given by $W_{fi}=2\pi|\langle f|T(\varepsilon_i)|i\rangle|^2\delta(\varepsilon_f-\varepsilon_i)$. Here

$$T(\varepsilon_i)=H_T\frac{1}{\varepsilon_i+i\eta-H}(\varepsilon_i-H_0)$$

is the on-shell transmission or T matrix, with η being a positive infinitesimal which we set to zero at the end of the calculation. The T matrix can be expanded in a power series in the tunneling Hamiltonian H_T,

$$T(\varepsilon_i)=H_T+H_T\sum_{n=1}^{\infty}\left(\frac{1}{\varepsilon_i+i\eta-H_0}H_T\right)^n, \tag{6}$$

where ε_i is the energy of the initial state $|i\rangle$, which, in our case, is the energy of a Cooper pair at the Fermi surface of the SC, $\varepsilon_i=2\mu_S$. Finally, $\rho_i=\langle i|\rho|i\rangle$ is the stationary occupation probability for the entire system to be in the state $|i\rangle$. We work in the regime $\Delta>\mu>k_BT$, where $\mu=\mu_S-\mu_l$ is the applied voltage bias between the SC and the leads, and T the temperature with k_B the Boltzmann constant. The regime $\Delta>\mu$ ensures that single-electron tunneling from the SC to the leads is excluded and only tunneling of *two* coherent electrons of opposite spins is allowed. In the regime $\mu>k_BT$ we only have transport from the SC to the leads, and not in the opposite direction. Since the temperature is assumed to be the smallest energy scale in the system, we assume $k_BT=0$ in the calculation. The set of initial states $|i\rangle$, virtual states $|v\rangle$ and final states $|f\rangle$ consists of the BCS ground state (GS) $|0\rangle_S$ and excitations $\gamma^\dagger_{\mathbf{k}s}|0\rangle_S$ for the SC and a complete set[30,31] of energy eigenstates $|N_{nr\nu},\{b_{n\nu}\}\rangle$ of the LL Hamiltonian H_{Ln} given in Eq. (3). $N_{nr\nu}$ is the number of excess spin ($\nu=\sigma$) and charge ($\nu=\rho$) in branch r relative to the GS. The Bose operators $b_{n\nu}$ form a continuous spectrum describing collective spin and charge modes and will be introduced in Eqs. (10) and (11). The GS of the LL is then $|0,0\rangle$, which means that we have no integral excess charge and spin and no bosonic excitations. The energy contribution of the excess charge and spin is included in the so-called zero mode ($k=0$) terms in the diagonalized Hamiltonian K_{Ln} [Eq. (12)] and are of no importance in the TD limit ($L\to\infty$) considered here, since the contribution of these terms due to an additional electron on top of the GS is $O(1/L)$ and is neglected in Eq. (12). For a detailed description of the LL Hamiltonian [Eq. (3)] including the zero modes, see Appendix A. Since we want to calculate the transition rate for transport of a Cooper pair to the leads, the final states $|f\rangle$ of interest contain two additional electrons of opposite spins in the leads compared to the initial state $|i\rangle$.

V. CURRENT I_1 FOR TUNNELING OF TWO ELECTRONS INTO DIFFERENT LEADS

We first calculate the current for tunneling of two spin-entangled electrons into different leads. We expand the T-matrix to second order in H_T, and go over to the interaction representation by using $\delta(\epsilon)=(1/2\pi)\int_{-\infty}^{+\infty}dte^{i\epsilon t}$, and $\langle v|(\epsilon_i-H_0+i\eta)^{-1}|v\rangle=-i\int_0^\infty dte^{i(\epsilon_i-\epsilon_v+i\eta)t}$. By transforming the time-dependent phases into a time dependence of the tunneling Hamiltonian we can integrate out all final and virtual states. The forward current I_1 for tunneling of two electrons into different leads can then be written as

$$I_1=2e\lim_{\eta\to 0}\sum_{\substack{n\neq n'\\ m\neq m'}}\int_{-\infty}^{\infty}dt\int_0^\infty dt'\int_0^\infty dt''e^{-\eta(t'+t'')+i(2t-t'-t'')\mu}$$
$$\times\langle H^\dagger_{Tm}(t-t'')H^\dagger_{Tm'}(t)H_{Tn}(t')H_{Tn'}\rangle, \tag{7}$$

where $\langle\cdots\rangle$ denotes $\mathrm{Tr}\rho\{\cdots\}$. The bias has been introduced in a standard way,[35] and the time dependence of the operators in Eq. (7) is then governed by $H_{Tn}(t)=e^{i(K_{Ln}+K_S)t}H_{Tn}e^{-i(K_{Ln}+K_S)t}$, with $K_{Ln}+K_S=H_{Ln}+H_S-\mu_lN_n-\mu_SN_S$. The transport process involves two electrons of *different* spins which suggests that the average in Eq. (7) is of the form (suppressing time variables), $\langle\cdots\rangle=\Sigma_{ss'}\langle H^\dagger_{Tm-s'}H^\dagger_{Tm's'}H_{Tn-s}H_{Tn's}\rangle$, where H_{Tns} describes the tunneling of spin s governed by H_{Tn}. The time sequence in Eq. (7) contains the dynamics of the hopping of a Cooper pair from the SC to the LL leads (one electron per lead) and back. The times t' and t'' are delay times between subsequent hoppings of two electrons from the *same* Cooper pair, whereas t is the time between injecting and taking out a Cooper pair. We evaluate the thermal average in Eq. (7) at zero temperature where the expectation value is to be taken in the ground state of $K_0=\Sigma_nK_{Ln}+K_S$ which is the BCS ground state of the SC and the bosonic vacuum of the LL leads. We remark that since the interaction between the different subsystems (SC,L_1,L_2) is included in the tunneling-perturbation, the expectation value factorizes into a SC part times a LL part. In addition, the LL correlation function factorizes into two single-particle correlation functions due to the negligible interaction between LL leads 1 and 2 (this will be not the case if two electrons tunnel into the *same* lead). Note that in the TD limit the time dynamics of all LL correlation functions will be goverened by a Hamiltonian that depends only on Bose operators [see Eq. (12)]. The operators $\eta_{\pm,rs}$ commute with all Bose operators, and as a consequence $\eta_{\pm,rs}$ are time independent. Therefore, interaction terms of the LL of the form $\psi_\alpha\psi_\beta\psi^\dagger_\gamma\psi^\dagger_\delta$ can be written as $\eta_\alpha\eta_\beta\eta_\gamma\eta_\delta\times$(Bose operators), where $\alpha,\beta,\gamma,$ and δ are composite indices containing $r=\pm,ns$. The correlation function in Eq. (7) is then of the form

$$\sum_{\substack{n\neq n'\\ m\neq m'}}\langle H^\dagger_{Tm}(t-t'')H^\dagger_{Tm'}(t)H_{Tn}(t')H_{Tn'}\rangle$$
$$=|t_0|^4\sum_{\substack{s\\ n\neq m}}\langle\psi_{ns}(t-t'')\psi^\dagger_{ns}\rangle\langle\psi_{m-s}(t-t')\psi^\dagger_{m-s}\rangle$$
$$\times\langle\Psi^\dagger_s(\mathbf{r}_n,t-t'')\Psi^\dagger_{-s}(\mathbf{r}_m,t)\Psi_{-s}(\mathbf{r}_m,t')\Psi_s(\mathbf{r}_n)\rangle$$
$$-|t_0|^4\sum_{\substack{s\\ n\neq m}}\langle\psi_{m-s}(t-t'-t'')\psi^\dagger_{m-s}\rangle\langle\psi_{ns}(t)\psi^\dagger_{ns}\rangle$$
$$\times\langle\Psi^\dagger_{-s}(\mathbf{r}_m,t-t'')\Psi^\dagger_s(\mathbf{r}_n,t)\Psi_{-s}(\mathbf{r}_m,t')\Psi_s(\mathbf{r}_n)\rangle. \tag{8}$$

The four-point correlation functions of the SC can be calculated by Fourier decomposing $\Psi_s(\mathbf{r}_n,t) = V^{-1/2}\Sigma_{\mathbf{k}}(u_{\mathbf{k}s}\gamma_{\mathbf{k}s}e^{-iE_{\mathbf{k}}t}+v_{\mathbf{k}s}\gamma^{\dagger}_{-\mathbf{k}-s}e^{iE_{\mathbf{k}}t})e^{i\mathbf{k}\mathbf{r}_n}$, with $u_{\mathbf{k}s} = u_{\mathbf{k}}$, and $v_{\mathbf{k}\uparrow} = -v_{\mathbf{k}\downarrow} = v_{\mathbf{k}}$. For the first correlation function in Eq. (8), we then obtain

$$V^2\langle\Psi^{\dagger}_s(\mathbf{r}_n,t-t'')\Psi^{\dagger}_{-s}(\mathbf{r}_m,t)\Psi_{-s}(\mathbf{r}_m,t')\Psi_s(\mathbf{r}_n)\rangle = \sum_{\mathbf{k}\mathbf{k}'} u_{\mathbf{k}}v_{\mathbf{k}}u_{\mathbf{k}'}v_{\mathbf{k}'}e^{-i[E_{\mathbf{k}}t'-E_{\mathbf{k}'}t''-(\mathbf{k}+\mathbf{k}')\delta\mathbf{r}]} + \sum_{\mathbf{k}\mathbf{k}'}(v_{\mathbf{k}}v_{\mathbf{k}'})^2e^{-i[E_{\mathbf{k}}(t-t'')+E_{\mathbf{k}'}(t-t')]}, \tag{9}$$

where $\delta\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ is the distance vector between the two tunneling points in the SC. The first sum in Eq. (9) describes the (time-dependent) correlation of creating and annihilating a quasiparticle (with same spin), whereas the second term in Eq. (9) describes the correlation of creating *two* quasiparticles (with different spins). It is obvious that the second term describes processes which involve final states $|f\rangle$ in the T-matrix element $\langle f|T(\varepsilon_i)|i\rangle$ that contain two excitations in the SC and, therefore, does not describe an Andreev process. In the regime $\Delta > \mu$ such a process is not allowed by energy conservation. We will see this explicitly by calculating the integral over t which originates from the Fourier representation of the δ function present in the rate W_{fi}. Similarly, for the correlator $\langle\Psi^{\dagger}_{-s}(\mathbf{r}_m,t-t'')\Psi^{\dagger}_s(\mathbf{r}_n,t)\Psi_{-s}(\mathbf{r}_m,t')\Psi_s(\mathbf{r}_n)\rangle$ in Eq. (8), we obtain Eq. (9) with a minus sign, and we have to replace $t-t''$ by $t-t'-t''$, and $t-t'$ by t, in the second term of Eq. (9).

To evaluate the LL correlation functions in Eq. (8) we decompose the phase fields $\phi_{n\nu}(x,t)$ and $\theta_{n\nu}(x,t)$ into a sum over the spin and charge bosons (also see Appendix A):

$$\theta_{n\nu}(x,t) = -\sum_p \operatorname{sgn}(p)\sqrt{\frac{\pi}{2LK_\nu|p|}}e^{ipx}e^{-\alpha|p|/2} \times(b_{n\nu p}e^{-iu_\nu|p|t} - b^{\dagger}_{n\nu-p}e^{iu_\nu|p|t}) \tag{10}$$

and

$$\phi_{n\nu}(x,t) = \sum_p \sqrt{\frac{\pi K_\nu}{2L|p|}}e^{ipx}e^{-\alpha|p|/2} \times(b_{n\nu p}e^{-iu_\nu|p|t} + b^{\dagger}_{n\nu-p}e^{iu_\nu|p|t}). \tag{11}$$

The spin and charge bosons satisfy Bose commutation relations, in particular $[b_{n\nu p}, b^{\dagger}_{n'\nu'p'}] = \delta_{rr'}$, where $r \equiv n\nu p$, and the LL ground state is defined as $b_{n\nu p}|0\rangle_{LL} = 0$. Hamiltonian (3) can then be written in terms of the b operators as (see Appendix A)

$$K_{Ln} = \sum_{\nu p} u_\nu|p|b^{\dagger}_{n\nu p}b_{n\nu p}, \tag{12}$$

where we have subtracted the zero-point energy coming from the filled Dirac sea of negative-energy particle states. In all p sums we will explicitly exclude $p=0$, as discussed in Sec. IV, and explained in more detail in Appendix A. To account for the p dependence of the interaction, we apply a high momentum-transfer cutoff Λ on the order of $1/p_F$, so that $K_\nu(p) = K_\nu$, $u_\nu(p) = u_\nu$ for $|p| < 1/\Lambda$ and $K_\nu(p) = 1, u_\nu(p) = v_F$ for $|p| > 1/\Lambda$. By writing $\psi_{nsr}(x,t) = (2\pi\alpha)^{-1/2}\eta_{r,ns}e^{i\Phi_{nsr}(x,t)}$ with $r = \pm$ and Φ_{nsr} defined according to Eq. (4), we can represent the single-particle LL correlation function as

$$\langle\psi_{nsr}(x,t)\psi^{\dagger}_{nsr}\rangle = (2\pi\alpha)^{-1}\exp\{\langle\Phi_{nsr}(x,t)\Phi_{nsr} - (\Phi^2_{nsr}(x,t)+\Phi^2_{nrs})/2\rangle\},$$

with the well-known result[36–39]

$$G^1_{nrs}(x,t) \equiv \langle\psi_{nsr}(x,t)\psi^{\dagger}_{nsr}\rangle = \frac{1}{2\pi}\lim_{\alpha\to 0}\frac{\Lambda + i(v_Ft-rx)}{\alpha + i(v_Ft-rx)}\prod_{\nu=\rho,\sigma}\frac{1}{\sqrt{\Lambda + i(u_\nu t - rx)}} \times\left[\frac{\Lambda^2}{(\Lambda + iu_\nu t)^2 + x^2}\right]^{\gamma_\nu/2}, \tag{13}$$

where $\gamma_\nu = (K_\nu + K^{-1}_\nu)/4 - 1/2 > 0$ is an interaction-dependent parameter which describes the power-law decay of the long-time and long-distance correlations. The factor containing the Fermi velocity v_F in Eq. (13) is only important if one is interested in x,t satisfying $|v_Ft - rx| < \Lambda$, and is a result of including the p-dependence of the interaction parameters K_ν and u_ν. The LL correlation function [Eq. (13)] has singular points as a function of time in the upper complex plane. It is now clear that the integration over t is only nonzero if the phase of $e^{i\omega t}$ in Eq. (7) is positive, i.e., $\omega > 0$. Since the phases of the terms containing $(v_{\mathbf{k}})^2$ in Eq. (9) also depend on t, the requirement for a nonzero contribution to the current from these terms requires $2\mu - E_{\mathbf{k}} - E_{\mathbf{k}'} > 0$. However, this is excluded in our regime of interest, since $E_{\mathbf{k}} + E_{\mathbf{k}'} \geqslant 2\Delta$; therefore, we will not consider these terms any further in what follows. We remark that for $r \neq r'$, the correlation function $\langle\psi_{nsr}(x,t)\psi^{\dagger}_{nsr'}\rangle$ gives a negligible contribution in the TD limit. This statement is true also for a finite-size LL as long as the interaction preserves the total number of right and left movers. In our model the electrons tunnel into the same point x_n in LL n, i.e., $x=0$ in Eq. (13). In addition, LL's 1 and 2 are assumed to be similar. In this case the single-particle correlation function does not depend on n and r, i.e., $G^1_{nrs}(x,t) \equiv G^1(t)$, and the current I_1 can then be written as

$$I_1 = (32e|t_0|^4/V^2) \times\lim_{\eta\to 0}\int_{-\infty}^{\infty}dt\int_0^{\infty}dt'\int_0^{\infty}dt''e^{-\eta(t'+t'')+i(2t-t'-t'')\mu} \times\sum_{\mathbf{k}\mathbf{k}'}u_{\mathbf{k}}v_{\mathbf{k}}u_{\mathbf{k}'}v_{\mathbf{k}'}e^{-i[E_{\mathbf{k}}t'-E_{\mathbf{k}'}t''-(\mathbf{k}+\mathbf{k}')\delta\mathbf{r}]} \times\{G^1(t-t'')G^1(t-t') + G^1(t-t'-t'')G^1(t)\}. \tag{14}$$

We evaluate Eq. (14) in leading order in the small parameter μ/Δ, and remark that the delay times t' and t'' are restricted to $t',t''\lesssim 1/\Delta$. This becomes clear if we set $\delta\mathbf{r}=0$ and express the contribution in Eq. (14) containing the dynamics of the SC as

$$\sum_{\mathbf{kk}'} u_{\mathbf{k}}v_{\mathbf{k}}u_{\mathbf{k}'}v_{\mathbf{k}'}\, e^{-i(E_{\mathbf{k}}t'-E_{\mathbf{k}'}t'')} = (\pi\nu_S\Delta/2)^2 H_0^{(1)}(t''\Delta)H_0^{(2)}(t'\Delta), \qquad (15)$$

where $H_0^{(1)}$ and $H_0^{(2)}$ are Hankel functions of the first and second kinds, and ν_S is the energy density of states (DOS) per spin in the SC at μ_S. For times $t',t''>1/\Delta$, the Hankel functions are rapidly oscillating, since for large x we have $\mathrm{H}_0{}^{(1/2)}(x)\sim\sqrt{2/\pi x}\exp(\pm[ix-(\pi/4)])$. In contrast, the time-dependent phase in Eq. (14) containing the bias μ suppresses the integrand in Eq. (14) only for times $|2t-t'-t''|>1/\mu>1/\Delta$. Being interested only in the leading order in μ/Δ, we can assume that $|t|>t',t''$ in the current formula [Eq. (14)], since the LL correlation functions are slowly decaying in time with the main contribution (in the integral) coming from large times $|t|$. In addition, since $1/\mu>\Lambda/v_F$, we can neglect the term containing the Fermi velocity v_F in Eq. (13). To test the validity of our approximations, we first consider the noninteracting limit with $K_\nu=1$ and $u_\nu=v_F$, for which an analytical expression is also available for higher-order terms in μ/Δ.

VI. NONINTERACTING LIMIT FOR CURRENT I_1

Let us first consider a one-dimensional (1D) Fermi gas (i.e., $K_\nu=1$ and $u_\nu=v_F$), and evaluate the integral over t in Eq. (14) in the noninteracting limit. The LL correlation functions simplify to $G^1(t)=(1/2\pi)\lim_{\alpha\to 0}1/(\alpha+iv_Ft)$, and we are left with the integral

$$\int_{-\infty}^{\infty} dt e^{i2t\mu}\{G^1(t-t'')G^1(t-t')+G^1(t-t'-t'')G^1(t)\}$$
$$=\left(\frac{1}{2\pi}\right)^2\int_{-\infty}^{\infty} dt e^{i2t\mu}\left\{\frac{1}{[\alpha+iv_F(t-t'')][\alpha+iv_F(t-t')]} + \frac{1}{[\alpha+iv_F(t-t'-t'')][\alpha+iv_Ft]}\right\}\Bigg|_{\alpha\to 0}, \qquad (16)$$

which can be evaluated by closing the integration contour in the upper complex plane. Inserting the result into Eq. (14), we obtain

$$I_1=(32e|t_0|^4/V^2)\lim_{\eta\to 0}\int_0^{\infty}dt'\int_0^{\infty}dt''e^{-\eta(t'+t'')}$$
$$\times\sum_{\mathbf{kk}'} u_{\mathbf{k}}v_{\mathbf{k}}u_{\mathbf{k}'}v_{\mathbf{k}'}e^{-i[E_{\mathbf{k}}t'-E_{\mathbf{k}'}t''-(\mathbf{k}+\mathbf{k}')\delta\mathbf{r}]}$$
$$\times\frac{1}{\pi v_F^2}\left\{\frac{\sin[(t''-t')\mu]}{t''-t'}+\frac{\sin[(t'+t'')\mu]}{t'+t''}\right\}. \qquad (17)$$

The sine functions in Eq. (17) can be expanded in powers of μ, and for $\mu<\Delta$ it is sufficient to keep just the leading order term in μ since the integrals over t' and t'' have the form

$$\int_0^{\infty} dt e^{-(\eta\pm iE_{\mathbf{k}})t}t^n=\frac{n!}{(\eta\pm iE_{\mathbf{k}})^{n+1}}, \qquad (18)$$

where $n=1,2,3,\ldots$. Since $E_{\mathbf{k}}\geqslant\Delta$, higher powers in t' and t'' produce higher powers in μ/Δ, and, as expected, we can therefore ignore the dependence on t' and t'' in the LL correlation functions. In contrast to this, when we consider the current for tunneling of two electrons into the same (interacting) LL lead, we will see that the two-particle correlation function will not allow for such a simplification. In leading order in μ/Δ, the integrals over t' and t'' are evaluated according to Eq. (18) with $n=0$, and we obtain an (effective) momentum sum for the SC correlations $[\Sigma_{\mathbf{k}}(u_{\mathbf{k}}v_{\mathbf{k}}/E_{\mathbf{k}})\cos(\mathbf{k}\cdot\delta\mathbf{r})]^2$. To evaluate this sum we use $u_{\mathbf{k}}v_{\mathbf{k}}=\Delta/(2E_{\mathbf{k}})$ and linearize the spectrum around the Fermi level μ_S, since the Fermi energy in the SC $\varepsilon_F>\Delta$, with the Fermi wave vector k_F. We then obtain (δr denotes $|\delta\mathbf{r}|$)

$$\sum_{\mathbf{k}}\frac{u_{\mathbf{k}}v_{\mathbf{k}}}{E_{\mathbf{k}}}\cos(\mathbf{k}\cdot\delta\mathbf{r})=\frac{\pi}{2}\nu_S\frac{\sin(k_F\delta r)}{k_F\delta r}e^{-(\delta r/\pi\xi)}. \qquad (19)$$

In Eq. (19) we introduced the coherence length of a Cooper pair in the SC, $\xi=v_F/\pi\Delta$. We finally obtain I_1^0, the current I_1 in the noninteracting limit:

$$I_1^0=e\pi\gamma^2\mu\left[\frac{\sin(k_F\delta r)}{k_F\delta r}\right]^2\exp\left(-\frac{2\delta r}{\pi\xi}\right). \qquad (20)$$

Here we have defined $\gamma=4\pi\nu_S\nu_l|t_0|^2/LV$, which is the dimensionless conductance per spin to tunnel from the SC to the LL leads. The non-interacting DOS of the LL per spin ν_l is given by $\nu_l=L/\pi v_F$. [We remark in passing that this result agrees with a T-matrix calculation in the energy domain.[40] In this case we sum explicitly over the final states, given by a singlet $|f\rangle=(1/\sqrt{2})[a^\dagger_{1p\uparrow}a^\dagger_{2q\downarrow}-a^\dagger_{1p\downarrow}a^\dagger_{2q\uparrow}]|i\rangle$, where the a operators describe electrons in a noninteracting 1D Fermi gas. Note that triplet states are excluded as final states since our Hamiltonian H does not change the total spin.] We see that the current I_1^0 is exponentially suppressed on the scale of ξ, if the tunneling of the two (coherent) electrons takes place from different points $\mathbf{r}_1$ and $\mathbf{r}_2$ of the SC. For conventional s-wave SC the coherence length ξ is typically on the order of micrometers, and therefore this poses not severe experimental restrictions. Thus, in the regime of interest $\delta r<\xi$, the suppression of the current I_1^0 is only polynomial, i.e., $\propto(1/k_F\delta r)^2$. It was shown[41] that a superconductor on top of a two-dimensional electron gas (2DEG) can induce superconductivity (via the proximity effect) in the 2DEG with a finite order parameter. The 2DEG then becomes an effective 2D SC. More recently, it was suggested that superconductivity should also be present in ropes of single-walled carbon nanotubes,[42] which are 1D systems. It is therefore interesting also to calculate Eq. (19) in two dimensions and one dimension. In the case of a 2D SC we evaluate $\Sigma_{\mathbf{k}}(u_{\mathbf{k}}v_{\mathbf{k}}/E_{\mathbf{k}})\cos(\mathbf{k}\cdot\delta\mathbf{r})$ in leading order in $\delta r/\pi\xi$, and we find

$$\sum_{\mathbf{k}(2D)}\frac{u_{\mathbf{k}}v_{\mathbf{k}}}{E_{\mathbf{k}}}\cos(\mathbf{k}\cdot\delta\mathbf{r})=\frac{\pi}{2}\nu_S\left(J_0(k_F\delta r)+2\sum_{\nu=1}^{\infty}\frac{J_{2\nu}(k_F\delta r)}{\pi\nu}\right), \tag{21}$$

where $J_\nu(x)$ denotes the Bessel function of order ν. For large $k_F\delta r$, we obtain $J_\nu(k_F\delta r)\sim\sqrt{2/\pi k_F\delta r}\cos[k_F\delta r-(\nu\pi/2)-(\pi/4)]$, which allows an approximation of the right-hand side of Eq. (21) for large $k_F\delta r$ by $(\pi/2)\nu_S J_0(k_F\delta r)[1-(2/\pi)\ln 2]$. This result is exact to leading order in an expansion in $1/k_F\delta r$. So asymptotically, the current decays only $\propto 1/k_F\delta r$. For $\delta r=0$, the bracket on the right-hand side of Eq. (21) becomes 1 as in the 3D case.

In the case of a 1D SC we obtain

$$\sum_{\mathbf{k}(1D)}\frac{u_{\mathbf{k}}v_{\mathbf{k}}}{E_{\mathbf{k}}}\cos(\mathbf{k}\cdot\delta\mathbf{r})=\frac{\pi}{2}\nu_S\cos(k_F\delta r)e^{-(\delta r/\pi\xi)}, \tag{22}$$

where there are only oscillations and no decay of the Andreev amplitude (for $\delta r/\pi\xi<1$). We see that the suppression of the current due to a finite separation of the tunneling points on the SC can be reduced considerably (or even excluded completely) by going over to lower-dimensional SC's.

VII. CURRENT I_1 INCLUDING INTERACTION

We now are ready to treat the interacting case. Having obtained confidence in our approximation schemes from the noninteracting case above, we can now neglect the t' and t'' dependences of the LL correlation function appearing in Eq. (14), valid in leading order in μ/Δ. In this limit the t integral considerably simplifies to

$$\int_{-\infty}^{\infty}dt e^{i(2t-t'-t'')\mu}\{G^1(t-t'')G^1(t-t')+G^1(t-t'-t'')G^1(t)\}\sim\frac{1}{(2\pi)^2}\frac{2\Lambda^{2(\gamma_\rho+\gamma_\sigma)}}{\prod_{\nu=\rho,\sigma}u_\nu^{2\gamma_\nu+1}}\int_{-\infty}^{\infty}dt\frac{e^{i2\mu t}}{\prod_{\nu=\rho,\sigma}[(\Lambda/u_\nu)+it]^{2\gamma_\nu+1}}. \tag{23}$$

An analytical expression for this integral is available,[43] and is given in Appendix B. The treatment of the remaining integrals over t' and t'' and the calculation of the Andreev contribution is the same as in the noninteracting case and for the current I_1, in leading order in μ/Δ and in the small parameters $2\Lambda\mu/u_\nu$, we obtain

$$I_1=\frac{I_1^0}{\Gamma(2\gamma_\rho+2)}\frac{v_F}{u_\rho}\left[\frac{2\mu\Lambda}{u_\rho}\right]^{2\gamma_\rho}. \tag{24}$$

In Eq. (24) we used $K_\sigma=1$ and $u_\sigma=v_F$. The interaction suppresses the current considerably, and the bias dependence has a characteristic nonlinear form $I_1\propto(\mu)^{2\gamma_\rho+1}$, with an interaction dependent exponent $2\gamma_\rho+1$. The parameter γ_ρ is the exponent for tunneling into the bulk of a single LL,[29] i.e., $\rho(\varepsilon)\sim|\varepsilon|^{\gamma_\rho}$, where $\rho(\varepsilon)$ is the single-particle DOS. Note that the current I_1 does not show a dependence on the correlation time $1/\Delta$, which is a measure of the time separation between the two electron-tunneling events. This is so since the two partners of a Cooper pair tunnel to *different* LL leads with no interaction-induced correlations between the leads.

VIII. CURRENT I_2 FOR TUNNELING OF TWO ELECTRONS INTO THE SAME LEAD

The main feature of the case where two electrons, originating from an Andreev process, tunnel into the same lead, is now that the four-point correlation function of the LL no longer factorizes, as is the case when the two electrons tunnel into different leads [see Eq. (8)]. In addition the two electrons will tunnel into the lead preferably from the same spatial point on the SC, i.e., $\delta r=0$. We denote by I_2 the current for coherent transport of two electrons into the *same* lead, either lead 1 *or* lead 2. It can be written in a similar way as I_1 [see Eq. (7)], with the difference that now we consider final states with two additional electrons (of opposite spin) in the same lead (either 1 or 2) compared to the initial state. For I_2, we then obtain

$$I_2=4e\lim_{\eta\to0}\sum_{s,s'}\int_{-\infty}^{\infty}dt\int_0^{\infty}dt'\int_0^{\infty}dt''e^{-\eta(t'+t'')+i(2t-t'-t'')\mu}\times\langle H^\dagger_{Tn-s'}(t-t'')H^\dagger_{Tns'}(t)H_{Tn-s}(t')H_{Tns}\rangle, \tag{25}$$

where we have assumed that leads 1 and 2 are identical, which results in an additional factor of 2. Again, the thermal average is to be taken at $T=0$, and this ground-state expectation value factorizes into a SC part times a LL part. However, the LL part no longer factorizes due to strong correlations between the two tunneling electrons. In this case we obtain

$$\sum_{s,s'}\langle H^\dagger_{Tn-s'}(t-t'')H^\dagger_{Tns'}(t)H_{Tn-s}(t')H_{Tns}\rangle$$
$$=|t_0|^4\sum_s\langle\psi_{ns}(t-t'')\psi_{n-s}(t)\psi^\dagger_{n-s}(t')\psi^\dagger_{ns}\rangle\times\langle\Psi^\dagger_s(\mathbf{r}_n,t-t'')\Psi^\dagger_{-s}(\mathbf{r}_n,t)\Psi_{-s}(\mathbf{r}_n,t')\Psi_s(\mathbf{r}_n)\rangle$$
$$+|t_0|^4\sum_s\langle\psi_{n-s}(t-t'')\psi_{ns}(t)\psi^\dagger_{n-s}(t')\psi^\dagger_{ns}\rangle\times\langle\Psi^\dagger_{-s}(\mathbf{r}_n,t-t'')\Psi^\dagger_s(\mathbf{r}_n,t)\Psi_{-s}(\mathbf{r}_n,t')\Psi_s(\mathbf{r}_n)\rangle. \tag{26}$$

The four-point correlation functions for the SC in Eq. (26) are the same as in Eq. (8) for the case when the two electrons tunnel into different leads, except that now $\delta\mathbf{r}=0$. The (normalized) four-point correlation functions in Eq. (26) for the LL are

$$G^2_{rr'1}(t,t',t'')\equiv\langle\psi_{nrs}(t-t'')\psi_{nr'-s}(t)\psi^\dagger_{nr'-s}(t')\psi^\dagger_{nrs}\rangle/\langle\psi_{nrs}(t-t'')\psi^\dagger_{nrs}\rangle\langle\psi_{nr'-s}(t-t')\psi^\dagger_{nr'-s}\rangle,$$

which can be calculated using similar methods as described above for the single-particle correlation function. After some calculation we obtain

$$G^2_{rr'1}(t,t',t'')=\prod_{\nu=\rho,\sigma}\left(\frac{\Lambda-iu_\nu t''}{\Lambda+iu_\nu(t-t'-t'')}\right)^{\lambda_{\nu rr'}}\left(\frac{\Lambda+iu_\nu t'}{\Lambda+iu_\nu t}\right)^{\lambda_{\nu rr'}}. \tag{27}$$

where $\lambda_{\nu rr'}=\xi_\nu(rr'K_\nu+(1/K_\nu))/4$ with $\xi_{\rho/\sigma}=\pm1$. For the other sequence

$$G^2_{rr'2}=\langle\psi_{nr-s}(t-t'')\psi_{nr's}(t)\psi^\dagger_{nr-s}(t')\psi^\dagger_{nr's}\rangle/\langle\psi_{nr-s}(t-t'-t'')\psi^\dagger_{nr-s}\rangle\langle\psi_{nr's}(t)\psi^\dagger_{nr's}\rangle,$$

we obtain

$$G^2_{rr'2}(t,t',t'')=-\prod_{\nu=\rho,\sigma}\left(\frac{\Lambda-iu_\nu t''}{\Lambda+iu_\nu(t-t'')}\right)^{\lambda_{\nu rr'}}\left(\frac{\Lambda+iu_\nu t'}{\Lambda+iu_\nu(t-t')}\right)^{\lambda_{\nu rr'}}. \tag{28}$$

We remark that contributions from other combinations of left and right movers, as indicated in Eqs. (27) and (28), are negligible. A contribution like $\langle\psi_{nr's}(t-t'')\psi_{nr-s}(t)\psi^\dagger_{nr'-s}(t')\psi^\dagger_{nrs}\rangle_{r\neq r'}$ is only nonzero if spin exchange between right and left movers is possible, but this is a backscattering process which we explicitly exclude. Using Eqs. (25)–(28) together with Eq. (15), we obtain a formal expression for I_2 (with $\delta r=0$):

$$\begin{aligned}I_2=4e\left(\frac{\pi\nu_S\Delta|t_0|^2}{V}\right)^2\lim_{\eta\to0}\int_{-\infty}^{\infty}dt\int_0^\infty dt'\\ \times\int_0^\infty dt''e^{i(2t-t'-t'')\mu-\eta(t'+t'')}H_0^{(1)}(t''\Delta)H_0^{(2)}(t'\Delta)\\ \times\sum_{b=\pm1}[G^2_{b1}(t,t',t'')G^1(t-t'')G^1(t-t')\\ -G^2_{b2}(t,t',t'')G^1(t-t'-t'')G^1(t)].\end{aligned} \tag{29}$$

In Eq. (29) the meaning of the summation index is $b\equiv+1$ for $rr'=+1$, and $b\equiv-1$ for $rr'=-1$. We proceed to evaluate the current I_2 with $K_\sigma=1$ and $u_\sigma=v_F$. To calculate the current I_2 we assume that the time scales Λ/u_ρ and Λ/v_F are the smallest ones in the problem. The times Λ/u_ρ and Λ/v_F are both on the order of the inverse Fermi energy in the LL, which is larger than the energy gap Δ and the bias μ. By applying the same arguments as in Sec. V for the current I_1, we approximate the current I_2, assuming $|t|>t',t''>\Lambda/v_F,\Lambda/u_\rho$, which is accurate in leading order in the small parameters μ/Δ, $\Lambda\Delta/u_\nu$ and $\Lambda\mu/u_\nu$. In this limit we obtain $G^2_{rr'1}=-G^2_{rr'2}=(t't'')^{\gamma_{\rho rr'}}/[(\Lambda/u_\rho)+it]^{2\lambda_{\rho rr'}}[(\Lambda/v_F)+it]^{-(1+rr')/2}$, where $\gamma_{\rho rr'}=[(1/K_\rho)+rr'K_\rho-(1+rr')]/4>0$. The exponent $\gamma_{\rho rr'}$ is related to γ_ρ, introduced in the single-particle correlation function [Eq. (13)] via $\gamma_{\rho rr'}=\gamma_\rho$ for $r=r'$, and $\gamma_{\rho rr'}=\gamma_\rho+(1-K_\rho)/2$ for $r\neq r'$. The current I_2 for tunneling of two electrons into the *same* lead 1 or 2 then becomes (for $\delta r=0$)

$$\begin{aligned}I_2=2e\left(\frac{2\pi\nu_S\Delta|t_0|^2}{V}\right)^2\frac{\Lambda^{2\gamma_\rho}}{u_\rho^{2\gamma_\rho+1}v_F}\\ \times\lim_{\eta\to0}\sum_{b=\pm1}\int_0^\infty dt'\int_0^\infty dt''e^{-\eta(t'+t'')}(t't'')^{\gamma_{\rho b}}\\ \times H_0^{(1)}(t''\Delta)H_0^{(2)}(t'\Delta)\frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}dt\\ \times\frac{e^{i2\mu t}}{\left(\frac{\Lambda}{u_\rho}+it\right)^{2\lambda_{\rho b}+2\gamma_\rho+1}\left(\frac{\Lambda}{v_F}+it\right)^{(1-b)/2}}.\end{aligned} \tag{30}$$

Again, the integrals appearing in Eq. (30) can be evaluated analytically[43] with the results given in Appendix B. Note that according to the two-particle LL correlation functions [Eqs. (27) and (28)], we find that the dynamics coming from the delay times t' and t'' cannot be neglected anymore, as was done in Ref. 33. We evaluate Eq. (30) in leading order in $2\mu\Lambda/u_\nu$ and finally obtain, for the current I_2,

$$I_2=I_1\sum_{b=\pm1}A_b\left(\frac{2\mu}{\Delta}\right)^{2\gamma_{\rho b}}. \tag{31}$$

The interaction dependent constant A_b in Eq. (31) is given by

$$A_b=\frac{2^{2\gamma_{\rho b}-1}}{\pi^2}\frac{\Gamma(2\gamma_\rho+2)}{\Gamma(2\gamma_{\rho b}+2\gamma_\rho+2)}\Gamma^4\left(\frac{\gamma_{\rho b}+1}{2}\right), \tag{32}$$

which is decreasing when interactions in the leads are increased. The function $\Gamma(x)$ is the gamma function. We remark that in Eq. (31) the current I_1 is to be taken at $\delta r=0$. The noninteracting limit $I_2=I_1=I_1^0$ is recovered by putting $\gamma_\rho=\gamma_{\rho b}=0$ and $u_\rho=v_F$. The result for I_2 shows that the unwanted injection of two electrons into the same lead is suppressed compared to I_1 by a factor of $A_+(2\mu/\Delta)^{2\gamma_{\rho+}}$ if both electrons are injected into the same branch (left or right movers), or by $A_-(2\mu/\Delta)^{2\gamma_{\rho-}}$ if the two electrons travel in different directions. Since it holds that $\gamma_{\rho-}=\gamma_{\rho+}+(1-K_\rho)/2>\gamma_{\rho+}$, it is more favorable that two electrons travel in the same direction than in opposite directions. The suppression of the current I_2 by $1/\Delta$ very nicely shows the two-particle correlation effect for the coherent tunneling of two electrons into the same lead. The larger Δ is, the shorter the delay time between the arrivals of the two partner electrons of a given Cooper pair, and, in turn, the more the second electron will be influenced by the presence of the first one already in the LL. By increasing the bias μ the electrons can tunnel faster through the barrier due to more channels becoming available into which the electron can tunnel; therefore, the effect of Δ is less pronounced. Also note that this correlation effect disappears when interactions are absent in the LL ($\gamma_\rho=\gamma_{\rho b}=0$).

IX. EFFICIENCY AND DISCUSSION

We have established that there indeed exists a suppression for the tunneling of two spin-entangled electrons into the same LL lead compared to the desired process where the two electrons tunnel into different leads. However, we have to take into account that the process into different leads also suffers a suppression due to a finite tunneling separation δr of the two electrons forming a Cooper pair in the SC. In Sec. VI we showed that this suppression can be considerably reduced if one uses effectively low-dimensional SC's. To estimate the efficiency of the entangler, we form the ratio I_1/I_2 and demand that it is larger than 1. This requirement is fulfilled if approximately

$$A_+\left(\frac{2\mu}{\Delta}\right)^{2\gamma_{\rho+}}<1/(k_F\delta r)^{d-1}, \tag{33}$$

where d is the dimension of the SC, and it is assumed that the coherence length ξ of the SC is large compared to δr. The leading term of I_2 is proportional to $(2\mu/\Delta)^{2\gamma_{\rho+}}$ describing the power-law suppression, with an exponent $2\gamma_{\rho+}=2\gamma_\rho$, of the process where two electrons, entering the same lead, will propagate in the same direction. The exponent $\gamma_{\rho+}$ is the exponent for the single-particle tunneling DOS from a metal (SC) into the center (bulk) of a LL. Experimentally accessible systems which exhibit LL behavior are metallic carbon nanotubes. It was pointed out[44,45] that the long range part of the Coulomb interaction, which is dominated by forward-scattering events with small momentum transfer, can lead to a LL behavior in carbon nanotubes with very small values of $K_\rho\sim0.2$–0.3, as measured experimentally[24,46] and predicted theoretically.[44] This would correspond to an exponent $2\gamma_\rho\sim0.8$–1.6, which seems very promising. Note that in this scenario of small K_ρ the interaction dependent constants A_b also become much smaller than unity. For $K_\rho=0.3$ we obtain from Eq. (32) that $A_+\sim0.11$, and for $K_\rho=0.2$ we obtain $A_+\sim0.02$ (note that $A_-<A_+$). In addition, single-wall nanotubes show similar tunneling exponents, as derived here. The tunneling DOS for a single-wall nanotube is predicted[44,45] to be $\rho(\varepsilon)\sim|\varepsilon|^\eta$ with $\eta=(K_\rho^{-1}+K_\rho-2)/8$, which is half of γ_ρ, and was measured[24,46] to be ~0.3–0.4. Similar values were found also in multiwall nanotubes.[47] It is known that the power-law suppression of the single-particle DOS is even larger if one considers tunneling into the *end* of a LL. For single-wall nanotubes one finds[44,45] $\eta_{end}=(K_\rho^{-1}-1)/4>\eta$, or for conventional LL theory again an enhancement by a factor of 2.[48] We therefore expect to obtain an even stronger suppression if the Cooper pairs tunnel into the end of the LL's. We remark that the nonlocality of the two electrons could be probed via the Aharonov-Bohm oscillations in the current, when leads 1 and 2 are formed into a loop enclosing a magnetic flux. Due to the different paths by which the electrons can choose to go around the loop, we expect to see h/e and $h/2e$ oscillation periods, as a function of magnetic flux, in the current like in the case of noninteracting leads.[16] The interference of contributions where the two electrons travel through different leads, with contributions where they travel through the same lead, then leads to h/e oscillations, whereas the interference of contributions where both electrons travel through the same arm 1 or 2 of the loop leads to the $h/2e$ oscillations. The amplitudes of these oscillations must be related to the currents describing the interfering processes. We expect that the h/e oscillation contribution should be $\propto(I_1I_2)^\alpha$ and the $h/2e$ oscillation contribution should be $\propto I_2^{2\alpha}$, with an exponent α that has to be determined by explicit calculations. In the noninteracting limit α should be 1/2.[16] The different periods then allow for an experimental test of how successful the separation of the two electrons is. For instance, if the two electrons can only tunnel into the same lead, e.g., if $k_F\delta r$ is too large or the interaction in the leads too weak, then $I_1\sim0$ and we would only see the $h/2e$ oscillations in the current. The determination of the precise value of the exponents will be deferred to another publication since it requires a separate calculation including finite size properties of the LL along the lines discussed in Ref. 49.

X. DECAY OF THE ELECTRON-SINGLET DUE TO LL-INTERACTIONS

We have shown in the preceding sections that the interaction in a LL lead can help to separate two spin-entangled electrons so that the two electrons enter *different* leads. A natural question then arises: what is the lifetime of a (nonlocal) spin-singlet state formed of two electrons which are injected into different LL leads, one electron per lead? To address this issue we introduce the following correlation function:

$$P(\mathbf{r},t)=|\langle S(\mathbf{r},t)|S(0,0)\rangle|^2. \tag{34}$$

This function is the probability density that a singlet state, injected at point $\mathbf{r}\equiv(x_1,x_2)=0$ and at time $t=0$, is found at some later time t and at point $\mathbf{r}$. Therefore, $P(\mathbf{r},t)$ is a measure of how much of the initial singlet state remains after the two injected electrons have interacted with all the other electrons in the LL during the time interval t. Here

$$|S(\mathbf{r},t)\rangle=\sqrt{\pi\alpha}[\psi^\dagger_{1\uparrow}(x_1,t)\psi^\dagger_{2\downarrow}(x_2,t) - \psi^\dagger_{1\downarrow}(x_1,t)\psi^\dagger_{2\uparrow}(x_2,t)]|0\rangle \tag{35}$$

is the electron singlet state created on top of the LL ground states. The extra normalization factor $\sqrt{2\pi\alpha}$ is introduced to guarantee $\int d\mathbf{r}\,P(\mathbf{r},t)=1$ in the noninteracting limit, and corresponds to the replacement of ψ_{ns} by $(2\pi\alpha)^{1/4}\psi_{ns}$. The singlet-singlet correlation function factorizes into two single-particle Green's functions due to the negligible interaction between the leads 1 and 2. Therefore we have $P(\mathbf{r},t)=(2\pi\alpha)^2\Pi_n|\langle\psi_{ns}(x_n,t)\psi^\dagger_{ns}(0,0)\rangle|^2$, with

$\langle \psi_{ns}(x_n,t)\psi^\dagger_{ns}(0,0)\rangle = \Sigma_{r=\pm} e^{ik_F r x_n} G^1_{nrs}(x_n,t)$. For simplicity we just study the slow spatial variations of $|\langle \psi_{ns}(x_n,t)\psi^\dagger_{ns}(0,0)\rangle|^2$. Using Eq. (13) together with $\pi\delta(x) = \lim_{\alpha\to 0}\alpha/(\alpha^2+x^2)$, we can then write the remaining probability of the singlet as

$$P(\mathbf{r},t) = \prod_n \frac{1}{2}\sum_{r=\pm} F(t)\,\delta(x_n - r v_F t), \tag{36}$$

with a time decaying weight factor of the δ function

$$F(t) = \prod_{\nu=\rho,\sigma} \sqrt{\frac{\Lambda^2}{\Lambda^2+(v_F-u_\nu)^2 t^2}} \times \left(\frac{\Lambda^4}{[\Lambda^2+(v_F^2-u_\nu^2)t^2]^2+(2\Lambda u_\nu t)^2}\right)^{\gamma_\nu/2}. \tag{37}$$

From Eqs. (36) and (37) we see that charge and spin of an electron propagate with velocity v_F, whereas charge (spin) excitations of the LL propagate with u_ρ (u_σ). Without interaction, i.e., $u_\nu = v_F$ and $K_\nu = 1$, we have $F(t) = 1$, which means that there is no decay of the singlet state. Using again $u_\sigma = v_F$, $K_\sigma = 1$ and $u_\rho = v_F/K_\rho$ we see from Eq. (37) that as interactions are turned on, the singlet state starts to decay on a time scale given by Λ/u_ρ. For long times t and for $u_\sigma = v_F$, $K_\sigma = 1$, and $u_\rho = v_F/K_\rho$, the asymptotic behavior of the decay is

$$F(t) \sim \frac{\Lambda}{u_\rho(1-K_\rho)}\left[\frac{\Lambda^2}{u_\rho^2(1-K_\rho^2)}\right]^{\gamma_\rho}\left[\frac{1}{t}\right]^{2\gamma_\rho+1},$$

which for very strong interactions in the LL leads, i.e., K_ρ much smaller than 1, becomes $F(t) \sim [\Lambda/u_\rho t]^{2\gamma_\rho+1}$. We will show in Sec. XI that although the singlet gets destroyed due to interactions, we still can observe charge and spin of the initial singlet via the spin and charge density fluctuations of the LL.

XI. PROPAGATION OF CHARGE AND SPIN

The charge and spin propagations as functions of time in a state $|\Psi\rangle$ can be described by the correlation function $\langle\Psi|\rho(x,t)|\Psi\rangle$ for the charge, and $\langle\Psi|\sigma_z(x,t)|\Psi\rangle$ for the spin. The normal-ordered charge density operator for LL n is $\rho_n(x_n) = \Sigma_s :\psi^\dagger_{ns}(x_n)\psi_{ns}(x_n): = \Sigma_{sr} :\psi^\dagger_{nsr}(x_n)\psi_{nsr}(x_n):$, if we only consider the slow spatial variations of the density operator. Similarly, the normal-ordered spin-density operator in the z direction is $\sigma^z_n(x_n) = \Sigma_{sr} s :\psi^\dagger_{nsr}(x_n)\psi_{nsr}(x_n):$. These density fluctuations can be expressed in a bosonic form (see Appendix A) as

$$\rho_n(x_n) = \frac{\sqrt{2}}{\pi}\,\partial_x\phi_{n\rho}(x_n), \tag{38}$$

and for the spin

$$\sigma^z_n(x_n) = \frac{\sqrt{2}}{\pi}\,\partial_x\phi_{n\sigma}(x_n). \tag{39}$$

We now consider a state $|\Psi\rangle = \psi^\dagger_{nsr}(x_n)|0\rangle$ where we inject an electron at time $t=0$ into branch r on top of the LL ground state in lead n, and calculate the time-dependent charge and spin-density fluctuations according to $\langle 0|\psi_{nsr}(x_n)\rho_n(x'_n,t)\psi^\dagger_{nsr}(x_n)|0\rangle$ for the charge and similarly for the spin, $\langle 0|\psi_{nsr}(x_n)\sigma^z_n(x'_n,t)\psi^\dagger_{nsr}(x_n)|0\rangle$. If we express the bosonic field operators $\phi_{n\nu}$ and $\theta_{n\nu}$ in terms of the boson modes shown in Eq. (10) and (11) and the Fermi operators according to the bosonization dictionary [Eq. (4)] we obtain for the charge fluctuations

$$\begin{aligned}(2\pi\alpha)\langle 0|\psi_{nsr}(x_n)\rho_n(x'_n,t)\psi^\dagger_{nsr}(x_n)|0\rangle &= \tfrac{1}{2}(1+rK_\rho)\,\delta(x'_n - x_n - u_\rho t)\\ &\quad + \tfrac{1}{2}(1-rK_\rho)\,\delta(x'_n - x_n + u_\rho t),\end{aligned} \tag{40}$$

and for the spin fluctuations we obtain

$$\begin{aligned}(2\pi\alpha)\langle 0|\psi_{nsr}(x_n)\sigma^z_n(x'_n,t)\psi^\dagger_{nsr}(x_n)|0\rangle &= \frac{s}{2}(1+rK_\sigma)\,\delta(x'_n - x_n - u_\sigma t)\\ &\quad + \frac{s}{2}(1-rK_\sigma)\,\delta(x'_n - x_n + u_\sigma t).\end{aligned} \tag{41}$$

Results (40) and (41) are obtained by sending $\Lambda \to 0$. We see that in contrast to the singlet, the charge- and spin-density fluctuations in the LL created by the injected electron do not decay and show a pulse shape with no dispersion in time. This is due to the linear energy dispersion relation of the LL model. In carbon nanotubes such a highly linear dispersion relation is indeed realized, and, therefore, nanotubes should be well suited for spin transport. Another interesting effect that shows up in Eqs. (40) and (41) is the different velocities of spin and charge, which is known as spin-charge separation. It would be interesting to test Bell inequalities[4] via spin-spin correlation measurements between the two LL leads, and see if the initial entanglement of the spin singlet is still observable in the spin-density-fluctuations. Although the detection of single spins with magnitudes on the order of electron spins still has not been achieved, magnetic resonance force microscopy seems to be very promising for doing so.[50] Another scenario is to use the LL just as an intermediate medium which is needed to first separate the two electrons of a Cooper pair and then to take them (in general other electrons) out again into two (spatially separated) Fermi-liquid leads where the (possibly reduced) spin entanglement could be measured via the current noise in a beamsplitter experiment.[15] Similarly, to test Bell inequalities one can measure the spin via the charge of the electron.[9,19,20] In this context we finally note that the decay of the singlet state given by Eq. (36) sets in almost immediately after the injection into the LL's (the time scale is approximately the inverse of the Fermi energy); however at least at zero temperature, the suppression is only polynomial in time, which suggests that some fraction of the singlet state can still be recovered.

XII. CONCLUSIONS

We proposed an *s*-wave superconductor (SC), coupled to two spatially separated Luttinger-liquid (LL) leads, as an entangler for electron spins. We showed that the strong correlations present in the LL can be used to separate two electrons, forming a spin-singlet state, which originate from an Andreev tunneling process of a Cooper pair from the SC to the leads. We have shown that the coherent tunneling of two electrons into the same lead is suppressed by a characteristic power law in the small parameter μ/Δ, where μ is the applied bias between the SC and the LL leads, and Δ is the gap in the SC. On the other hand, when the two electrons tunnel into different leads, the current is suppressed by the initial separation of the two electrons. This suppression, however, can be considerably reduced by going over to effective lower-dimensional SC. We also addressed the question of how much of the initial singlet can be taken out of the LL at some later time, and we found that the probability is decreasing in time, again with a power law (at zero temperature). Nevertheless, the spin information can still be transported through the wires by means of the (proper) spin excitations of the LL.

While preparing this manuscript we have learned of related and independent efforts by Bena *et al.*[51] who considered a similar setup as proposed here thereby arriving at similar conclusions.

ACKNOWLEDGMENTS

We thank W. Belzig, C. Bruder, F. Meier, and E. V. Sukhorukov for useful discussions. We also gratefully acknowledge discussions with S. Vishveshwara, C. Bena, L. Balents, and M. P. A. Fisher. This work was supported by the Swiss NSF, NCCR Basel, DARPA, and ARO.

APPENDIX A: FINITE-SIZE DIAGONALIZATION OF THE LL HAMILTONIAN

In this appendix we derive the diagonalized form of the LL Hamiltonian [Eq. (3)] including terms of order $1/L$, which describe integer charge and spin excitations. For simplicity, we consider only one LL and will therefore suppress the subscript n for the leads. We start with the exact bosonization dictionary for the Fermi operator for electrons on branch $r=\pm$,[30,31]

$$\psi_{rs}(x)=\lim_{\alpha\to 0}\frac{U_{r,s}}{\sqrt{2\pi\alpha}}\exp\left\{ir(p_F-\pi/L)x+\frac{ir}{\sqrt{2}}\{\phi_\rho(x)+s\phi_\sigma(x)-r[\theta_\rho(x)+s\theta_\sigma(x)]\}\right\}. \tag{A1}$$

The $U_{r,s}$ operator (often denoted as the Klein factor) is unitary, and decreases the number of electrons with spin s on branch r by one. This operator also ensures the correct anticommutation relations for $\psi_{rs}(x)$. The normal ordered charge density operator is $\rho(x)=\Sigma_{sr}:\psi^\dagger_{rs}(x)\psi_{rs}(x):$, where $::$ measures the corresponding quantity relative to the ground state. The normal-ordered spin-density operator is defined by $\sigma^z(x)=\Sigma_{sr}s:\psi^\dagger_{rs}(x)\psi_{rs}(x):$. In addition, one can define (bare) current densitiy operators for charge $j_\rho(x)=\Sigma_{sr}r\psi^\dagger_{rs}(x)\psi_{rs}(x)$, and for the spin $j_\sigma(x)=\Sigma_{sr}rs\,\psi^\dagger_{rs}(x)\psi_{rs}(x)$, respectively. Note that the current density has not to be normal ordered, since its ground-state expectation value vanishes. The normal-ordered product $:\psi^\dagger_{rs}(x)\psi_{rs}(x):$ is calculated according to

$$:\psi^\dagger_{rs}(x)\psi_{rs}(x):=\lim_{\Delta x\to 0}:\psi^\dagger_{rs}(x+\Delta x)\psi_{rs}(x):. \tag{A2}$$

By expanding the operator product in Eq. (A2) within the normal-order sign, the right-hand side of Eq. (A2) is equal to $(1/2\pi)\partial_x(\phi_\rho(x)+s\phi_\sigma(x)-r[\theta_\rho(x)+s\theta_\sigma(x)])/\sqrt{2}$, from which one easily finds

$$\rho(x)=\frac{\sqrt{2}}{\pi}\partial_x\phi_\rho(x),\qquad \sigma^z(x)=\frac{\sqrt{2}}{\pi}\partial_x\phi_\sigma(x) \tag{A3}$$

and, for the current densities,

$$j_\rho(x)=-\sqrt{2}\Pi_\rho(x),\qquad j_\sigma(x)=-\sqrt{2}\,\Pi_\sigma(x). \tag{A4}$$

The field $\Pi_\nu(x)$ is related to $\theta_\nu(x)$ by $\partial_x\theta_\nu(x)=\pi\Pi_\nu(x)$. We decompose the phase fields into $\phi_\nu(x)=\phi^P_\nu(x)+\phi^0_\nu(x)$ and $\Pi_\nu(x)=\Pi^P_\nu(x)+\Pi^0_\nu(x)$, where the part with nonzero momentum, ϕ^P_ν and Π^P_ν, can be expanded in a series of normal modes

$$\phi^P_\nu(x)=\frac{1}{\sqrt{L}}\sum_{p\neq 0}\frac{1}{\sqrt{2\omega_{p\nu}}}e^{ipx}e^{-\alpha|p|/2}(b_{\nu p}+b^\dagger_{\nu -p}), \tag{A5}$$

and, for the canonical momentum,

$$\Pi^P_\nu(x)=\frac{-i}{\sqrt{L}}\sum_{p\neq 0}\sqrt{\frac{\omega_{p\nu}}{2}}e^{ipx}e^{-\alpha|p|/2}(b_{\nu p}-b^\dagger_{\nu -p}). \tag{A6}$$

These fields have to satisfy bosonic commutation relations $[\phi^P_\nu(x),\Pi^P_\mu(x')]=i\delta_{\nu\mu}[\delta(x-x')-1/L]$, which in turn demands $[b_{\nu p},b^\dagger_{\mu p'}]=\delta_{\nu\mu}\delta_{pp'}$ and $[b_{\nu p},b_{\mu p'}]=[b^\dagger_{\nu p},b^\dagger_{\mu p'}]=0$. The zero mode parts ϕ^0_ν and Π^0_ν can be found by considering the integrated charge (spin) and charge (spin) currents, respectively. For instance the integrated charge density $\Sigma_{rs}N_{rs}=\int dx\rho(x)=(\sqrt{2}/\pi)[\phi_\rho(L/2)-\phi_\rho(-L/2)]=(\sqrt{2}/\pi)[\phi^0_\rho(L/2)-\phi^0_\rho(-L/2)]$. Similar results are obtained for the other density operators, which then implies the zero modes to be $\phi^0_\nu(x)=(\pi/L)(N_{+\nu}+N_{-,\nu})x$ and $\Pi^0_\nu=-(1/L)(N_{+\nu}-N_{-,\nu})$, where $N_{r\rho/\sigma}=(N_{r\uparrow}\pm N_{r\downarrow})/\sqrt{2}$. The LL Hamiltonian [Eq. (3)] is then diagonalized by the following expansion of the bosonic fields,

$$\phi_\nu(x)=\sum_{p\neq 0}\sqrt{\frac{\pi K_\nu}{2|p|L}}e^{ipx}\,e^{-\alpha|p|/2}(b_{\nu p}+b^\dagger_{\nu -p})+\frac{\pi}{L}(N_{+\nu}+N_{-,\nu})x, \tag{A7}$$

and for the canonical conjugate momentum operator,

$$\Pi_\nu(x) = -i\sum_{p\neq 0}\sqrt{\frac{|p|}{2\pi L K_\nu}}e^{ipx}e^{-\alpha|p|/2}(b_{\nu p}-b^\dagger_{\nu -p}) - \frac{1}{L}(N_{+\nu}-N_{-,\nu}), \quad \text{(A8)}$$

where we have used $\omega_{p\nu}=|p|/K_\nu\pi$. For the operator $K_L = H_L - \mu_l N$, we then obtain

$$K_L = \sum_{p\neq 0,\nu} u_\nu |p| b^\dagger_{\nu p} b_{\nu p} + \frac{2\pi}{L}\left[\frac{u_\nu}{K_\nu}(N_{+\nu}+N_{-,\nu})^2 + u_\nu K_\nu (N_{+\nu}-N_{-,\nu})^2\right]. \quad \text{(A9)}$$

In Eq. (A9) we have subtracted the zero-point energy $(1/2)\Sigma_{p\neq 0,\nu}u_\nu|p|$, which originates from an infinite filled Dirac sea of negative energy particle states in the LL model. The zero modes in Eqs. (A7) and (A8) give rise to contributions of order $1/L$ in Hamiltonian (A9), and they are also responsible for a shift of the Fermi wave vector p_F, appearing in the Fermionic field operator $\psi_{ns}(x)$, by a contribution of the same order. Since we are only interested in the thermodynamic limit, we have neglected the zero-mode contributions in explicit calculations in the main text.

APPENDIX B: EXACT RESULTS FOR THE TIME INTEGRALS

In this appendix we give the exact results for the time integrals in Eqs. (23) and (30). The integrals over the time variable t appearing in Eqs. (23) and (30) have the form

$$\int_{-\infty}^{\infty} dt \frac{e^{i2\mu t}}{\left(\frac{\Lambda}{u_\rho}+it\right)^Q\left(\frac{\Lambda}{u_\sigma}+it\right)^R} = \frac{2\pi e^{-2\mu\frac{\Lambda}{u_\rho}}(2\mu)^{Q+R-1}}{\Gamma(Q+R)} \times {}_1F_1[R;Q+R;2\Lambda(u_\rho^{-1}-u_\sigma^{-1})\mu], \quad \text{(B1)}$$

This integral formula is valid for Q and R satisfying $\mathrm{Re}(Q+R)>1$ (see Ref. 43, p. 345) which is in the range of our interest since we have $Q+R>2$. The function $\Gamma(x)$ in Eq. (B1) is the gamma function, and ${}_1F_1(\alpha;\gamma;z)$ is the confluent hypergeometric function given by

$${}_1F_1(\alpha;\gamma;z) = 1 + \frac{\alpha}{\gamma}\frac{z}{1!} + \frac{\alpha(\alpha+1)}{\gamma(\gamma+1)}\frac{z^2}{2!} + \cdots. \quad \text{(B2)}$$

In the main text we considered only the leading order term of ${}_1F_1$, since higher-order terms are smaller by the parameters $2\mu\Lambda/u_\rho$ and $2\mu\Lambda/u_\sigma$. The integrals over the delay times t' and t'' in Eq. (30) contain Hankel functions of the first and second kinds, which are linear combinations of Bessel functions of the first and second kinds, i.e., $H_0^{(1/2)}(t\Delta) = J_0(t\Delta) \pm iY_0(t\Delta)$. The integrals over t' and t'' in Eq. (30) are therefore linear combinations of integrals of the form

$$\lim_{\eta\to 0}\int_0^\infty dt e^{-\eta t} Y_0(t\Delta) t^\delta = \lim_{\eta\to 0}\left(-\frac{2}{\pi}\Gamma(\delta+1)(\Delta^2+\eta^2)^{-(1/2)(\delta+1)} \times Q_\delta(\eta/\sqrt{\eta^2+\Delta^2})\right), \quad \text{(B3)}$$

and

$$\lim_{\eta\to 0}\int_0^\infty dt e^{-\eta t} J_0(t\Delta) t^\delta = \lim_{\eta\to 0}[\Gamma(\delta+1)(\Delta^2+\eta^2)^{-(1/2)(\delta+1)} \times P_\delta(\eta/\sqrt{\eta^2+\Delta^2})]. \quad \text{(B4)}$$

This result is valid for $\delta>-1$ (see Ref. 43 p. 691). The functions Q and P are Legendre functions. The limit $\eta\to 0$ for $Q_\delta(\eta/\sqrt{\eta^2+\Delta^2})$ is (see Ref. 43, p. 959)

$$Q_\delta(0) = -\frac{1}{2}\sqrt{\pi}\sin(\delta\pi/2)\frac{\Gamma\left(\frac{\delta+1}{2}\right)}{\Gamma\left(\frac{\delta}{2}+1\right)}, \quad \text{(B5)}$$

and the limit $\eta\to 0$ for $P_\delta(\eta/\sqrt{\eta^2+\Delta^2})$ is

$$P_\delta(0) = \frac{\sqrt{\pi}}{\Gamma\left(\frac{\delta}{2}+1\right)\Gamma\left(\frac{1-\delta}{2}\right)} = \frac{1}{\sqrt{\pi}}\cos(\delta\pi/2)\frac{\Gamma\left(\frac{\delta+1}{2}\right)}{\Gamma\left(\frac{\delta}{2}+1\right)}. \quad \text{(B6)}$$

[1] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935).
[2] C. H. Bennett and G. Brassard, in *Proceedings of the IEEE International Conference on Computers, Systems and Signal Processing, Bangalore, India* (IEEE, New York, 1984), p. 175.
[3] C.H. Bennett and D.P. DiVincenzo, Nature (London) **404**, 247 (2000).
[4] J.S. Bell, Rev. Mod. Phys. **38**, 447 (1966).
[5] A. Aspect, J. Dalibard, and G. Roger, Phys. Rev. Lett. **49**, 1804 (1982).
[6] K. Mattle, H. Weinfurter, P.G. Kwiat, and A. Zeilinger, Phys. Rev. Lett. **76**, 4656 (1996).
[7] D. Bouwmeester, J.-W. Pan, K. Mattle, M. Eibl, H. Weinfurter, and A. Zeilinger, Nature (London) **390**, 575 (1997).

[8] D. Boschi, S. Branca, F. De Martini, L. Hardy, and S. Popescu, Phys. Rev. Lett. **80**, 1121 (1998).
[9] D. Loss and D. P. DiVincenzo, Phys. Rev. A **57**, 120 (1998); cond-mat/9701055 (unpublished).
[10] G. Burkard, H.-A. Engel, and D. Loss, Fortschr. Phys. **48**, 965 (2000); cond-mat/0004182 (unpublished).
[11] J.M. Kikkawa, I.P. Smorchkova, N. Samarth, and D.D. Awschalom, Science **277**, 1284 (1997).
[12] J.M. Kikkawa and D.D. Awschalom, Phys. Rev. Lett. **80**, 4313 (1998).
[13] D.D. Awschalom and J.M. Kikkawa, Phys. Today **52**(6), 33 (1999).
[14] D.P. DiVincenzo and D. Loss, J. Magn. Magn. Mater. **200**, 202 (1999); cond-mat/9901137 (unpublished).
[15] G. Burkard, D. Loss, and E.V. Sukhorukov, Phys. Rev. B **61**, R16 303 (2000).
[16] P. Recher, E.V. Sukhorukov, and D. Loss, Phys. Rev. B **63**, 165314 (2001); cond-mat/0009452 (unpublished).
[17] That superconducting correlations can induce nonlocal spin-entanglement was pointed out already in Ref. 25. There two superconductors, held at the same chemical potential, are tunnel coupled to two spatially separated quantum dots containing one electron spin each and which are not tunnel coupled among themselves. The tunnel coupling to the superconductors then induces an effective Heisenberg spin interaction between the two spins on the dots.
[18] L.P. Kouwenhoven, G. Schön, and L.L. Sohn, in *Mesoscopic Electron Transport*, Vol. 345 of *NATO Advanced Study Institute Series E: Applied Sciences*, edited by L.P. Kouwenhover, G. Schön, and L.L. Sohn (Kluwer, Dordrecht, 1997).
[19] P. Recher, E.V. Sukhorukov, and D. Loss, Phys. Rev. Lett. **85**, 1962 (2000).
[20] H.-A. Engel and D. Loss, Phys. Rev. Lett. **86**, 4648 (2001).
[21] G.B. Lesovik, T. Martin, and G. Blatter, Eur. Phys. J. B **24**, 287 (2002).
[22] G. Falci, D. Feinberg, and F.W.J. Hekking, Europhys. Lett. **54**, 255 (2002).
[23] R. Mélin, cond-mat/0105073 (unpublished).
[24] M. Bockrath, D.H. Cobden, J. Lu, A.G. Rinzler, R.E. Smalley, L. Balents, and P.L. McEuen, Nature (London) **397**, 598 (1999).
[25] M.-S. Choi, C. Bruder, and D. Loss, Phys. Rev. B **62**, 13 569 (2000); cond-mat/0001011 (unpublished); and in *Lecture Notes in Physics*, edited by R. Haug and H. Schoeller (Springer, Berlin, 2001), Vol. 579, pp. 46–66.
[26] M.R. Geller and D. Loss, Phys. Rev. B **62**, R16 298 (2000).
[27] J. R. Schrieffer, *Theory of Superconductivity* (Benjamin/Cummings, New York, 1964).
[28] In context with nanotubes we remark, that it was pointed out in Refs. 44 and 45 that for the (metallic) armchair nanotubes (N,N) with $N \gtrsim 10$ it is appropriate to use a Luttinger-liquid model where backscattering and Umklapp scattering can be neglected. This is so because for large N, the probability for two electrons to be near each other is small $(\sim 1/N)$, and therefore one can neglect the short-range part $r \sim a$, with a the lattice spacing, of the Coulomb potential which corresponds to large momentum transfers $\sim 2p_F$ at which backscattering and Umklapp scattering takes place.
[29] H.J. Schulz, Phys. Rev. Lett. **64**, 2831 (1990).
[30] F.D.M. Haldane, J. Phys. C **14**, 2585 (1981).
[31] R. Heidenreich, R. Seiler, and A. Uhlenbrock, J. Stat. Phys. **22**, 27 (1980).
[32] F.W.J. Hekking, L.I. Glazman, K.A. Matveev, and R.I. Shekhter, Phys. Rev. Lett. **70**, 4138 (1993).
[33] R. Fazio, K.H. Wagenblast, C. Winkelholz, and G. Schön, Physica B **222**, 364 (1996).
[34] E. Merzbacher, *Quantum Mechanics*, 3rd ed. (Wiley, New York, 1998) Chap. 20.
[35] G. D. Mahan, *Many-Particle-Physics*, 3rd ed. (Kluwer, Dordrecht, 2000).
[36] J. Sólyom, Adv. Phys. **28**, 201 (1979).
[37] Y. Suzumura, Prog. Theor. Phys. **63**, 5 (1980).
[38] H.J. Schulz, J. Phys. C **16**, 6769 (1983).
[39] J. Voit, J. Phys. C **5**, 8305 (1993).
[40] P. Recher and D. Loss (unpublished).
[41] A.F. Volkov, P.H.C. Magne, B.J. van Wees, and T.M. Klapwijk, Physica C **242**, 261 (1995).
[42] M. Kociak, A.Yu. Kasumov, S. Guron, B. Reulet, I.I. Khodos, Yu.B. Gorbatov, V.T. Volkov, L. Vaccarini, and H. Bouchiat, Phys. Rev. Lett. **86**, 2416 (2001).
[43] I.S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 6th ed. (Academic, San Diego, 2000).
[44] R. Egger and A. Gogolin, Phys. Rev. Lett. **79**, 5082 (1997); R. Egger, *ibid.* **83**, 5547 (1999).
[45] C. Kane, L. Balents, and M.P.A. Fisher, Phys. Rev. Lett. **79**, 5086 (1997).
[46] J. Nygård, Ph.D. thesis, Faculty of Science University of Copenhagen, 2000.
[47] A. Bachthold, M. de Jonge, K. Grove-Rasmussen, P.L. McEuen, M. Buitelaar, and C. Schönenberger, Phys. Rev. Lett. **87**, 166801 (2001).
[48] L. Balents and R. Egger, Phys. Rev. B **64**, 035310 (2001).
[49] M.R. Geller and D. Loss. Phys. Rev. B **56**, 9692 (1997).
[50] D. Rugar, B.C. Stipe, H.J. Mamin, C.S. Yannoni, T.D. Stowe, K.Y. Yasumura, and T.W. Kenny, Appl. Phys. A: Mater. Sci. Process. **72**, S3-S10 (2001).
[51] C. Bena, S. Vishveshwara, L. Balents, and M.P.A. Fisher, cond-mat/0202102 (unpublished).