Thermoelectric power of nondegenerate Kane semiconductors under the conditions of mutual electron-phonon drag in a high electric field

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The thermoelectric power of nondegenerate Kane semiconductors with due regard for the electron and phonon heating and their thermal and mutual drags is investigated. The electron spectrum is taken in the Kane two-band form. It is shown that the nonparabolicity of electron spectrum significantly influences the magnitude of the thermoelectric power and leads to a change of its sign and dependence on the heating electric field. The field dependence of the thermoelectric power is determined analytically under various drag conditions.

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I. INTRODUCTION

Recently, the interest in thermoelectric power both theoretically and experimentally in various systems, mesoscopic quantum dots,[1,2] quantum wires,[3] heterojunctions and quantum well structures,[4–11] as well as the bulk materials,[11,12] has been intensified. Almost all of the earlier theoretical investigations for analyzing the diffusion[3,15–17] and phonon drag[7–9,18] components of the thermoelectric power in macroscopic systems are based on the Boltzmann equation. In these works, the weakly nonuniform systems under the linear transport conditions are considered in the absence of external electric field and in the presence of lattice temperature gradient.

There are some theoretical investigations of thermoelectric and thermomagnetic effects in semiconductors at high external electric and nonquantizing magnetic fields.[19–23] In these studies, heating of electrons and phonons, and their thermal and mutual drags for the parabolic spectrum of nondegenerate electrons and for the nonparabolic spectrum of degenerate electrons are considered. These investigations are based on the solution of the coupled system of kinetic equations of hot electrons and phonons in nonlinear transport conditions. There are also theoretical investigations of this problem in the hydrodynamic approximation.

Lei theoretically discussed the thermoelectric power of both bulk materials and quantum wells in the presence of charge carrier heating with a high applied electric field by using the so-called ''balance equation approximation'' for weakly nonuniform systems.[11,13,24,25] These calculations indicate that the hot electron effect on the thermoelectric power may not only change its magnitude but also change its sign at high electric fields. This result has been confirmed by Xing *et al.*[12] using the nonequilibrium statistical operator method of Zubarev[14] jointly with the Lei-Ting balance equation approach.[24] In Refs. 11 and 12 the phonon drag contribution to thermoelectric power is neglected at electron temperatures of interest for hot electron transport. Thus, in both treatments this contribution which is known to be important in linear transport at low temperatures in bulk semiconductors[10] and two-dimensional systems[4–6,10] is missed. By using the hydrodynamic balance equation transport theory extended to weakly nonuniform systems, Wu *et al.* carried out a calculation of the phonon drag contribution to thermoelectric power of bulk semiconductors and quantum well structures.[26] According to the authors, the balance equation approach has the advantage of easy inclusion of hot electron effect and claims the importance of the phonon drag contribution to thermoelectric power in hot electron transport condition. They note that their consideration is applicable in the regime where the electron drift velocity is lower than the sound velocities for materials having high impurity concentrations and intermediate electric field strength. Contrary to the assumptions of Xing *et al.*,[12] their results demonstrate that the phonon drag contribution is remarkably enhanced at low lattice temperature under the conditions considered. It is shown in Ref. 11 that the diffusion component of the thermoelectric power may be negative within a low enough lattice temperature range at high electric field while the phonon drag component is still positive. In connection with these conclusions, it is necessary to note that such a result was obtained in 1977 by Babaev and Gassymov in Ref. 20. In that paper, the thermoelectric power and transverse Nernst-Ettingshausen (NE) effect in semiconductors at high electric and nonquantizing magnetic fields are studied by solving the coupled system of kinetic equations for electrons and phonons. In the investigation, both the heating of electrons and phonons, and the phonon drag are taken into account. It is shown that when the temperature gradient of hot electrons (∇T_e) is produced by the lattice temperature gradient (∇T), $\nabla E=0$, and $\nabla T_e=(\partial T_e/\partial T)\nabla T$, the electronic parts of the thermoelectric and the NE fields reverse their sign. In the case of heated phonons and $T_p=T_e \gg T$, both electronic and phonon parts of the thermoelectric and thermomagnetic fields reverse their sign for all cases considered. Here T_e, T_p, and T are the temperature of electrons, phonons, and lattice, respectively. In Ref. 12 the thermoelectric power of charge carriers heated under a strong applied electric field in semiconductors is obtained by making use of the nonequilibrium statistical operator method. The final Eqs. (18) and (19) for thermopower and the conclusion that the hot electron effect may change both the magnitude and sign of the thermopower repeat the results obtained in Ref. 20 for a special case (when ∇T_e is realized by ∇T). Moreover, we

note that for the high field case considered in Ref. 12, hot electrons (or semiconductor) are in the regime of phonon generation. Therefore, both the distribution function and the state of phonons are nonstationary as a result of the mutual drag of charge carriers and phonons at high electric field, which is considered in Refs. 27–29. For the role of the mutual electron-phonon drag and phonon generation at high external electric and magnetic fields, see Refs. 28–30.

Recently, the interest in the study of thermoelectric and NE effect in II-VI semiconductors has been intensified.[31–34] Earlier investigations of the magnetic field dependence of the longitudinal NE effect in HgSe (Refs. 35,36) and lead chalcogenides[37,38] in the region of comparatively high temperatures ($T \geqslant 77$ K) demonstrated that the thermoelectromotive force (emf) exhibits saturation in the classical region of strong magnetic fields H irrespective of the dominant scattering mechanism of charge carriers in the conduction band. However, measurements of the longitudinal NE effect in iron-doped HgSe samples at low temperatures ($20 \leqslant T \leqslant 60$ K), revealed presence of a maxima in the change of thermoelectric power $\Delta\alpha(H)=|\alpha(H)-\alpha(0)|$. $\Delta\alpha(H)$ first increases quadratically with increasing H for $\Omega\tau<1$, then passes through a maximum for some $H=H_m$, and finally decreases as the field increases further. Here, $\Omega=eH/mc$ is the cyclotron frequency, and τ is the electron relaxation time. Another unusual fact is the sign reversal of the transverse NE coefficient $Q_{\perp}(H)$ with magnetic field increasing in the range $\Omega\tau>1$.[33,34] The experiments in Ga-doped HgSe demonstrated that at low temperatures, NE coefficients change sign with increasing Ga concentration or the applied magnetic field strength. The unusual features of the NE effect observed in HgSe crystals may be attributed to the effect of mutual drag, which can experimentally be detected in semiconductors with high concentration of conduction electrons.[39] As it is shown in the present paper, these conditions can be realized more easily under high external electric field at arbitrary temperatures.

A consistent microscopic theory of transport phenomena in semiconductors and semimetals in high external electric and magnetic fields with due regard for the heating of charge carriers and phonons, their thermal and mutual drags, and the possible phonon generation by the drift charge carriers must be based on the solution of coupled system of kinetic equations for charge carriers and phonons. Such a problem was formulated and solved for the first time by Gassymov,[28] see also Ref. 27. In the statement of the problem, it should be noted that the traditional approximation of small anisotropy of phonon distribution function (so-called ''diffusion approximation'') is applicable to phonons whose drift velocities (u) is much smaller than the sound velocity (s_0) in crystal. In the presence of external electric and magnetic fields, this condition obviously is not fulfilled. This violation shows up particularly in several ways under the acoustical instability conditions ($u \geqslant s_0$). Actually, both spherically symmetric $N_s(q)$ and antisymmetric $N_a(q)$ parts of the phonon distribution function as well as $N_a(q)/N_s(q)$ grow as u increases. Indeed, $N_a(q)/N_s(q) \to 1$ as $u \to s_0$, and $N_a(q)/N_s(q) \gg 1$ when $u \gg s_0$. The general solution of the Boltzmann equation for phonons shows that $N(q)$ is stationary for $u<s_0$, and nonstationary for $u \geqslant s_0$. These results are obtained by solving the nonstationary kinetic equation for phonons interacting with charge carriers at high electric and arbitrary magnetic fields in the nondiffusion approximation.[27–29]

In the light of the foregoing discussion, we must note that the method of calculation used in Refs. 11, 12, and 26 has intrinsically questionable assumptions. Actually in the process of obtaining the force and energy balance equations, it is assumed that the distribution function of electrons has the form of drifted Fermi distribution function, and that of phonons has the form of drifted Planck's distribution function with effective electron temperature T_e and electron drift velocity v_d as a result of the electron-phonon collisions. These assumptions mean that this method is applicable only in the strong mutual drag conditions when $\nu_p \gg \nu_i$ and $\beta_e \gg \beta_p$, i.e., electrons and phonons transfer their energy and momentum to each other, and as a result they have the same effective temperature and drift velocity. Note that here ν_p and ν_i are the collision frequencies of electrons with phonons and impurities β_e and β_p are the collision frequencies of phonons with electrons and phonons, respectively. Under the strong mutual drag conditions, drift velocities of electrons and phonons are the same, $u=s_0$, only at the acoustical instability threshold (AIT). At AIT, the distribution function of phonons is nonstationary and grows linearly in time. In other words, drift velocities of electrons and phonons may be equal to each other only at the nonstationary conditions of phonon generation or amplification in external electric and magnetic fields.[28,29] Thus, the assumptions made in Refs. 11, 12, and 26 make it possible to use this method only under the strong mutual drag conditions and in the region of drift velocities $v_d \ll s_0$. On the other hand, under the mutual drag conditions and $v_d \ll s_0$, electrons and phonons interacting with electrons may have the same temperature $T_e=T_p$, but their drift velocities may not be equal to each other, i.e., $v_d \neq u$.

What about the terminology of thermal drag (or the drag of electrons by phonons), and mutual drag of electrons and phonons? There is a misunderstanding. Actually, the terminology of mutual drag covers the drag of electrons by phonons if $\nu_i \gg \nu_p$ and $\beta_e \gg \beta_{pb}$ as well as the drag of phonons by electrons if $\nu_p \gg \nu_i$ and $\beta_e \gg \beta_{pb}$. Here β_{pb} is the collision frequency of phonons with phonons (p), and boundaries of the crystal (b); and it is defined as $\beta_{pb}=\beta_p+\beta_b$. Therefore, the mutual drag covers both the drag of electrons by phonons (it is called ''thermal drag'') and the drag of phonons by electrons. The latter is named in the literature incorrectly as ''mutual drag.'' However, the mutual drag is the sum of both drags and, for this reason, it is sometimes called as ''veritable drag.'' In the mutual drag, electrons and phonons are scattered preferably by each other, and the strong mutual drag may form a coupled system with joint temperature $T_e=T_p$ and drift velocity $v_d=u$.

In the literature, usually the phonon drag effect (thermal drag) is studied in the absence of heating external electric field and in the presence of small ∇T in impure semiconductors when the collision frequency of electrons with impurity ions is much greater than that of electrons with phonons (low mobility, low temperature, and high impurity concentration).

In this situation the drag of phonons by electrons is less than the drag of electrons by phonons (thermal drag). In high external electric field, electrons are heated and the frequency of their scattering by impurity ions decreases; meanwhile their scattering frequency by phonons increases.

For the nondegenerate hot electrons with parabolic spectrum and effective temperature T_e, the ratio $\nu_i/\nu_p \sim (T_e/T)^{-3}$ decreases sharply, and becomes unity at some critical value of the electric field $E=E_{\rm cr}$. For $E>E_{\rm cr}$, electrons and phonons scatter from each other, and the effect of their mutual drag becomes important. The experiments for investigation of the effect of phonon drag in specimens of InSb or Ge are usually carried out at external fields $E>10$ V cm^{-1} and lattice temperatures $T<20$ K. At these conditions $T_e \approx 10^2, 10^3$ T.

The effect of high electric field is not limited by the heating of electrons; it also leads to the following effects.

(a) The drift velocity of electrons increases. Indeed, when $\nabla T_e \| \nabla T, v_d \gg v_{\nabla T}$. Here $v_{\nabla T}$ is the drift velocity of phonons in the presence of ∇T.

(b) The ratio β_e/β_p increases as T_e/T increases.

(c) The momentum range of phonons interacting with electrons increases by T_e as $0<q<2\overline{p}=\sqrt{8mT_e} \equiv 2p_T(T_e/T)^{1/2}$.

(d) The number of phonons interacting with electrons increases by T_e linearly. Namely, $N(q)=T_e/\hbar\omega_q^{\star}$. This is the most important finding.

(e) Under the mutual drag conditions, the inelasticity of scattering of electrons by phonons is obtained from $\hbar\omega_q^{\star}=\hbar\omega_q-\mathbf{u}\cdot\mathbf{q}$. It decreases with increasing u, and $N(q)=N(q,T_e)/(1-\mathbf{u}\cdot\mathbf{q}/\hbar\omega_q)$ increases as u increases. Because, the denominator goes to zero as $u\to s_0$. At these drift velocities, the phonon generation or amplification by the external electric field starts, and the state of phonons becomes nonstationary. Under these conditions the thermal drag, which is proportional to the degree of the inelasticity of the electron-phonon scattering, tends to zero, and the mutual drag of electrons and phonons is strong. Therefore, electrons and phonons form a system coupled by the mutual drag with common temperature T_e and drift velocity $\mathbf{u}$.[27–29]

The organization of the paper is as follows. The theoretical analysis of the problem is given in Sec. II. In Sec. III we discuss the results of the present work in detail. Finally, the conclusion is given in Sec. IV.

II. THEORY

The two-band Kane spectrum of electrons is

$$p(\varepsilon)=(2m_n\varepsilon)^{1/2}\left(1+\frac{\varepsilon}{\varepsilon_g}\right)^{1/2}, \tag{1}$$

where m_n is the effective mass of electrons at the bottom of the conduction band, ε_g is the band gap, p and ε are the electron momentum and energy, respectively.[17]

The physical process considered is the thermoelectric Seebeck effect in the presence of a heating electric field $\mathbf{E}$ and ∇T_e, which can be produced by ∇E or ∇T. The basic equations of the problem are the coupled Boltzmann transport equations for electrons and phonons. The quasielastic scattering of electrons by acoustic phonons is considered. For the case considered, the distribution functions of electrons $f(\mathbf{p},\mathbf{r})$ and phonons $N(\mathbf{q},\mathbf{r})$ may be presented in the form

$$f(\mathbf{p},\mathbf{r})=f_0(\varepsilon,\mathbf{r})+\mathbf{f}_1(\varepsilon,\mathbf{r})\frac{\mathbf{p}}{p}, \quad |\mathbf{f}_1|\ll f_0, \tag{2}$$

$$N(\mathbf{q},\mathbf{r})=N_0(q,\mathbf{r})+\mathbf{N}_1(q,\mathbf{r})\frac{\mathbf{q}}{q}, \quad |\mathbf{N}_1|\ll N_0. \tag{3}$$

Here f_0 and $\mathbf{f}_1$ and N_0 and $\mathbf{N}_1$ are the isotropic and the anisotropic parts of the electron and phonon distribution functions, respectively.

If the interelectronic collision frequency ν_{ee} is much greater than the collision frequency of electrons for the energy transfer to lattice ν_ε, then $f_0(\varepsilon,\mathbf{r})$ is the Fermi distribution function with an effective electron temperature T_e. We consider the case that there is a ''thermal reservoir'' of short-wavelength (SW) phonons for the long-wavelength (LW) phonons, with maximum quasimomentum $q_{\max}\approx 2\overline{p} \ll T/s_0$, interacting with electrons. In this case $N_0(q,\mathbf{r})$ has the form

$$N_0(q,\mathbf{r})\approx\frac{T_p(\mathbf{r})}{s_0 q}, \tag{4}$$

where T_p is the effective temperature of LW phonons.[40]

Starting from the Boltzmann transport equations, we obtain the following relations for $\mathbf{f}_1$ and $\mathbf{N}_1$ in the steady state:

$$\frac{p}{m(\varepsilon)}\nabla f_0-e\mathbf{E}_{\mathbf{c}}\frac{p}{m(\varepsilon)}\frac{\partial f_0}{\partial\varepsilon}+\nu(\varepsilon)\mathbf{f}_1 + \frac{2\pi m(\varepsilon)}{(2\pi\hbar)^3 p^2}\frac{\partial f_0}{\partial\varepsilon}\int_0^{2p}\mathbf{N}_1(q)W(q)\hbar\omega_q q^2 dq=0, \tag{5}$$

$$S_0\nabla N_0+\beta(q)\mathbf{N}_1-\frac{4\pi m(\varepsilon)}{(2\pi\hbar)^3}W(q)N_0(q)\int_{q/2}^{\infty}\mathbf{f}_1 dp=0, \tag{6}$$

where e is the absolute value of the electronic charge, $\mathbf{E}_c=\mathbf{E}+\mathbf{E}_T$, where $\mathbf{E}_T$ is the thermoelectric field, $m(\varepsilon)$ is the effective mass of electron, $\hbar\omega_q=s_0 q$ is the phonon energy, $W(q)=W_0 q^t$ is the square of the matrix element of the electron-phonon interaction ($t=1$ for deformation and $t=-1$ for piezoelectric interaction), and $\beta(q)$ and $\nu(\varepsilon)$ are the total phonon and electron momentum scattering rates, respectively.

For the Kane semiconductors with electron spectrum given by Eq. (1), $m(\varepsilon)$ and $\nu(\varepsilon)$ have the form[17]

$$m(\varepsilon)=m_n\left(1+\frac{2\varepsilon}{\varepsilon_g}\right), \tag{7}$$

$$\nu(\varepsilon)=\nu_0(T)\left(\frac{T_p}{T}\right)^l\left(1+\frac{2\varepsilon}{\varepsilon_g}\right)\left(1+\frac{\varepsilon}{\varepsilon_g}\right)^{-r}\left(\frac{\varepsilon}{T}\right)^{-r}, \tag{8}$$

where $r=3/2, l=0$ for the scattering of electrons by impurity ions, and $r=-t/2, l=1$ for the scattering of electrons by acoustic phonons. When LW phonons are scattered by SW phonons or by crystal boundaries, $\beta(q)$ does not depend on the spectrum of electrons and has the form[40]

$$\beta_p(q)=\frac{T^4}{4\pi\rho\hbar^4 s_0^4}q, \quad \beta_b(q)=\frac{s_0}{L}, \tag{9}$$

where the indices p and b denote the scattering of LW phonons by SW phonons and crystal boundaries and ρ and L are the density and the minimum size of the specimen, respectively. On the other hand, when LW phonons are scattered by electrons, $\beta_e(q)$ depends on the spectrum of electrons, and for the spectrum given by Eq. (1) we obtain

$$\beta_e(q)=\left(\frac{m_n s_0^2}{8\pi T_e}\right)^{1/2}\frac{NW_0}{T_e}\left(1+\frac{2T_e}{\varepsilon_g}\right)^2\left(1+\frac{3T_e}{2\varepsilon_g}\right)^{-3/2}q^t, \tag{10}$$

where N is the concentration of electrons.

Solving the coupled Eqs. (5) and (6) by the same way as in Ref. 23, it is easy to calculate the electric current density of electrons,[17]

$$\mathbf{J}=-\frac{e}{3\pi^2\hbar^3}\int_0^\infty \mathbf{f}_1(\varepsilon)p^2(\varepsilon)d\varepsilon. \tag{11}$$

Let the external electric field be directed along the x axis, and ∇T (or the external electric field gradient ∇E) along the z axis. Under these conditions the electron part (α_e) and phonon part (α_p) of the thermoelectric power (α) are obtained from equation $J_z=0$ as

$$\alpha=\alpha_e+\alpha_p, \quad \alpha_e=-\frac{\beta_{11}^{(e)}}{\sigma_{11}}, \quad \alpha_p=-\frac{\beta_{11}^{(p)}}{\sigma_{11}}, \tag{12}$$

where

$$\sigma_{11}=\int_0^\infty a(x)[1+b(x)]dx, \tag{13}$$

$$\beta_{11}^{(e)}=\frac{1}{e}\int_0^\infty a(x)\left\{x-\frac{\zeta(T_e)}{T_e}+\left[1-\frac{\zeta(T_e)}{T_e}\right]b(x)\right\}dx, \tag{14}$$

$$\beta_{11}^{(p)}=\frac{1}{e}\int_0^\infty a(x)\{\lambda(x)+\lambda(\vartheta_e)b(x)\}dx, \quad x=\frac{\varepsilon}{T_e},$$

$$\vartheta_e=\frac{T_e}{T}, \quad \vartheta_p=\frac{T_p}{T}. \tag{15}$$

Here $\zeta(T_e)$ is the chemical potential of hot electrons

$$a(x)=\frac{e^2}{3\pi^2\hbar^3}\frac{p^3(x)}{m(x)\nu(x)}\exp\left[\frac{\zeta(T_e)}{T_e}-x\right], \tag{16}$$

$$b(x)=\frac{\gamma(x)}{1-\gamma(\vartheta_e)}\frac{m(x)}{m(\vartheta_e)}\frac{\nu(x)}{\nu(\vartheta_e)}, \tag{17}$$

$$\gamma(x)=\frac{3+t}{(2p)^{3+t}}\frac{\nu_p(x)}{\nu(x)}\int_0^{2p}\frac{\beta_e(q)}{\beta(q)}q^{2+t}\,dq, \tag{18}$$

$$\lambda(x)=\frac{3+t}{(2p)^{3+t}}\frac{m(x)s_0^2}{T_p}\nu_p(x)\int_0^{2p}\frac{1}{\beta(q)}q^{2+t}dq, \tag{19}$$

where $\nu_p(x)$ is the scattering frequency of electrons by phonons. The coefficient $\lambda(x)$ characterizes the efficiency of the thermal drag, and $\gamma(x)$ describes the same for the mutual drag.

As it follows from Eq. (12), by taking into account Eqs. (13)–(15), α_p consists of ''thermal drag'' and ''mutual drag'' terms. Actually, the first term in Eq. (15) considers ''the drag of electrons by phonons'' (thermal drag) and the second term considers ''the drag of phonons by electrons'' (mutual drag).

In Eq. (15), the first term is dominant if $\nu_i \gg \nu_p$ and $\beta_e \gg \beta_{pb}$, i.e., phonons are scattered preferably by electrons, but electrons are scattered by impurity ions (thermal drag). The second term is dominant, on the other hand, if $\nu_i \ll \nu_p$ and $\beta_e \gg \beta_{pb}$. Since at high electric fields $\nu_i(\varepsilon)/\nu_p(\varepsilon)=\nu_i(T)/\nu_p(T)(T_e/T)^{-3}=E_{cr}/E$, the mutual drag dominates for $E>E_{cr}$. Using the total collision frequency $\nu(\varepsilon)=\nu_i(\varepsilon)+\nu_p(\varepsilon)$, we study E dependence of the thermal and mutual drags by using Eq. (15).

The ratio of the second and first terms in Eq. (15) is $[\lambda(\vartheta)/\lambda(x)]b(x)$. When $x=\bar{x}=T_e/T$, $[\lambda(\vartheta)/\lambda(\bar{x})]=1$. Therefore, we have $[\lambda(\vartheta)/\lambda(\bar{x})]b(\bar{x})\approx b(\vartheta)=\gamma(\vartheta)/[1-\gamma(\vartheta)]$. As it follows from this result, $\gamma(\vartheta)/[1-\gamma(\vartheta)]$ is smaller than 1 for $\gamma(\vartheta)<\frac{1}{2}$, equal to 1 for $\gamma(\vartheta)=\frac{1}{2}$, and larger than 1 for $\frac{1}{2}<\gamma(\vartheta)<1$. Moreover, it tends to infinity as $\gamma(\vartheta)\to 1$. Therefore, at high electric field the mutual drag is more important.

Because of the complexity of general analysis of Eqs. (12)–(15), hereafter we examine the dependence of electron momentum on its energy in the form

$$p(\varepsilon)=(2m_n\varepsilon_g)^{1/2}\left(\frac{\varepsilon}{\varepsilon_g}\right)^s. \tag{20}$$

This form, for the spectrum given by Eq. (1), corresponds to parabolic case for $T_e \ll \varepsilon_g, s=\frac{1}{2}$, and strongly nonparabolic case for $T_e \gg \varepsilon_g, s=1$. In these cases $m(\varepsilon)$, $\nu(\varepsilon)$, and $\beta(q)$ may be presented as

$$m(\varepsilon)=2sm_n\left(\frac{\varepsilon}{\varepsilon_g}\right)^{2s-1}, \tag{21}$$

$$\nu(\varepsilon)=2s\nu_0(T)\vartheta_p^l\left(\frac{\varepsilon}{\varepsilon_g}\right)^{(2s-1)(1-r)}\left(\frac{\varepsilon}{T}\right)^{-r}, \tag{22}$$

$$\beta(q)=\beta(T)\vartheta_e^{n(s-2)}\left(\frac{T}{\varepsilon_g}\right)^{n(s-1/2)}\left(\frac{s_0 q}{T}\right)^k, \tag{23}$$

where $n=1, k=t$ for scattering of LW phonons by electrons, $n=0, k=0$ for scattering by the crystal boundaries, and $n=0, k=1$ for scattering by SW phonons.

For the spectrum expressed by Eq. (20), from Eqs. (12)–(19) we obtain

$$\alpha_e=-\frac{1}{e}\left(1+C_1\frac{\gamma_0}{1-\gamma_0}\right)^{-1}\left\{3-s+2sr-\frac{\zeta(T_e)}{T_e}+\left[1-\frac{\zeta(T_e)}{T_e}\right]C_1\frac{\gamma_0}{1-\gamma_0}\right\}, \quad (24)$$

$$\alpha_p=-\frac{1}{e}\frac{C_2+(C_1-C_2)\gamma_0}{1+(C_1-1)\gamma_0}\times\frac{(3+t)2^{(2-3k/2)}s^2}{3+t-k}\left(\frac{m_n s_0^2}{T}\right)^{(1-k/2)}\times\left(\frac{T\vartheta_e}{\varepsilon_g}\right)^{(s-1/2)(4+t-k-n)}\vartheta_e^{(3n+t-k)/2}\frac{\nu_{p0}(T)}{\beta(T)}, \quad (25)$$

where

$$C_1=\frac{\Gamma(1+3s+2sr+2st-sk)}{\Gamma(3-s+2sr)},\qquad C_2=\frac{\Gamma(1+3s+2sr+st-sk)}{\Gamma(3-s+2sr)}, \quad (26)$$

$$\gamma_0=\frac{(3+t)2^{3(t-k)/2}}{3+2t-k}\left(\frac{m_n s_0^2}{T}\right)^{(t-k)/2}\times\left(\frac{T\vartheta_e}{\varepsilon_g}\right)^{(s-1/2)(2r+2t-k-n+1)}\times\vartheta_e^{[r+t+(3n-3-k)/2]}\vartheta_p^{1-l}\ \frac{\beta_e(T)}{\beta(T)}\frac{\nu_{p0}(T)}{\nu_0(T)}. \quad (27)$$

The chemical potential of nondegenerate electrons for the spectrum in Eq. (20) becomes

$$\zeta(T_e)=T_e\ln\left\{\frac{3\pi^2\hbar^3N}{\Gamma(1+3s)(2m_nT)^{3/2}}\left(\frac{T}{\varepsilon_g}\right)^{-3(s-1/2)}\vartheta_e^{-3s}\right\}. \quad (28)$$

Consider the limits $\gamma_0\ll 1$ and $\gamma_0\to 1$. The first limit corresponds to the weak mutual drag case. In this case, by using Eqs. (24) and (25), the components of the thermoelectric power is found to be

$$\alpha_e=-\frac{1}{e}\left\{3-s+2sr-\frac{\zeta(T_e)}{T_e}-C_1(2-s+2sr)\gamma_0\right\} \quad (29)$$

and

$$\alpha_p=-\frac{1}{e}\{C_2+C_1(1-C_2)\gamma_0\}\frac{(3+t)2^{(2-3k/2)}s^2}{3+t-k}\left(\frac{m_n s_0^2}{T}\right)^{(1-k/2)}\left(\frac{T\vartheta_e}{\varepsilon_g}\right)^{(s-1/2)(4+t-k-n)}\times\vartheta_e^{(3n+t-k)/2}\frac{\nu_{p0}(T)}{\beta(T)}. \quad (30)$$

TABLE I. Dependences of ϑ_e on E in the condition $\gamma_0\to 1$.

	$s=\frac{1}{2}$	$s=1$
Case i	$\vartheta_e\sim E^{4/3}$	$\vartheta_e\sim E^{1/2}$
Case ii	$\vartheta_e\sim E^{1/3}$	$\vartheta_e\sim E^{1/5}$
Case iii	$\vartheta_e\sim E^{4/11}$	$\vartheta_e\sim E^{2/9}$

Since $C_1>0$, and $2-s+2sr\geqslant 0$ for all real scattering mechanisms and the spectrum of electrons with $s\geqslant\frac{1}{2}$, from Eq. (29) we find that the mutual drag leads to a decrease of α_e both in the parabolic and nonparabolic cases.

The $\gamma_0\to 1$ limit, on the other hand, corresponds to the strong mutual electron-phonon drag. In this case $k=t$, $n=1$, $r=-t/2$, $l=1$, and $\vartheta_p=\vartheta_e$. From Eq. (27) we obtain $\gamma_0=[\beta_e(T)/\beta(T)][\nu_{p0}(T)/\nu_0(T)]\to 1$. Hence, α_e and α_p take the form

$$\alpha_e=-\frac{1}{e}\left\{1-\frac{\zeta(T_e)}{T_e}\right\}, \quad (31)$$

$$\alpha_p=-\frac{1}{e}\frac{4\sqrt{2}\ (2s)^2}{3\pi^{3/2}}\left(\frac{T}{\varepsilon_g}\right)^{3(s-1/2)}\frac{(m_nT)^{3/2}}{\hbar^3N}\vartheta_e^{3s}. \quad (32)$$

One can also see the decrease of α_e by the influence of mutual drag, from a comparison of Eqs. (31) and (29). As it follows from Eq. (28), for nondegenerate electrons we have

$$\frac{(m_nT)^{3/2}}{\hbar^3N}\left(\frac{T}{\varepsilon_g}\right)^{3(s-1/2)}\approx\exp\left[-\frac{\zeta(T)}{T}\right]\gg 1. \quad (33)$$

The E dependence of ϑ_e in the weak mutual drag case was considered elsewhere.[21] Here we investigate the same dependence in the strong mutual drag conditions. In this case the electron temperature is determined by the energy balance equation

$$\sigma_{11}(\vartheta_e)E^2=W_{pp}(\vartheta_e), \quad (34)$$

where $W_{pp}(\vartheta_e)$ is the power transferred by LW phonons to the "thermal reservoir" of SW phonons. Now we consider the following limiting cases

$$\text{(i)}\ \ \frac{\beta_p+\beta_b}{\beta_e}\ll\frac{\nu_i}{\nu_p},\quad \text{(ii)}\ \ \beta_p\gg\beta_b,\ \ \frac{\beta_p}{\beta_e}\gg\frac{\nu_i}{\nu_p},$$

$$\text{(iii)}\ \ \beta_p\ll\beta_b,\ \ \frac{\beta_b}{\beta_e}\gg\frac{\nu_i}{\nu_p}. \quad (35)$$

The results obtained for $\vartheta_p=\vartheta_e\gg 1$ are given in Table I.

As it is seen in Table I, the nonparabolicity of the electron spectrum strongly changes E dependence of the electron temperature. Using Table I, one can easily obtain E dependence of α for the cases considered in Eq. (35). For instance, if the first inequality is satisfied, then $\alpha_p\sim E^2$ for the parabolic, and $\alpha_p\sim E^{3/2}$ for the strong nonparabolic spectrum of electrons.

Let us consider the dependences of V_e, α_p, and V_p on E for different scattering mechanisms of electrons and phonons. As it follows from the results obtained above, the dependence of α_e on ϑ_e or E is weak (logarithmic) for the

TABLE II. Dependences of V_e, α_p, and V_p on E in the condition $\gamma_0 \to 1$.

		$s=\frac{1}{2}$	$s=1$
Case i	V_e	$\sim E^{4/3}$	$\sim E^{1/2}$
	α_p	$\sim E^{2}$	$\sim E^{3/2}$
	V_p	$\sim E^{10/3}$	$\sim E^{2}$
Case ii	V_e	$\sim E^{1/3}$	$\sim E^{1/5}$
	α_p	$\sim E^{1/2}$	$\sim E^{3/5}$
	V_p	$\sim E^{5/6}$	$\sim E^{4/5}$
Case iii	V_e	$\sim E^{4/11}$	$\sim E^{2/9}$
	α_p	$\sim E^{6/11}$	$\sim E^{2/3}$
	V_p	$\sim E^{10/11}$	$\sim E^{8/9}$

limiting cases $\gamma_0 \to 0$ and $\gamma_0 \to 1$. If $\vartheta_e \gg 1$ at one end of the specimen, and $\vartheta_e = 1$ at the other end, $V_e \sim \vartheta_e$ by the accuracy of logarithmic dependence. When $\gamma_0 \to 1$, $\alpha_p \sim \vartheta_e^{3s}$, and $V_p \sim \vartheta_e^{3s+1}$.

Taking into account the foregoing discussion and Table I, one can find the dependences of V_e, α_p, and V_p on E as $\gamma_0 \to 1$. The results are given in Table II.

In the weak mutual drag case, for $T_p = T_e \gg 1$, α_p, and ϑ_e are given by

$$\alpha_p \sim \vartheta_e^{(4+t-k-n)+2n-2}, \quad \vartheta_e = \left(\frac{E}{E_i}\right)^{2/(8s-1-2rs+l)}, \tag{36}$$

where E_i is

$$E_i = \left(\frac{T}{\varepsilon_g}\right)^{(s-1/2)(4-r)} \left(\frac{m_n T}{\hbar^2 N^{2/3}}\right)^{3/4} \left(\frac{m_n T}{e^2}\right)^{1/2} [\nu_e(T)\beta_p(T)]^{1/2}. \tag{37}$$

We find dependence of V_e on E for several interaction mechanisms as shown in Table III. In the weak mutual drag case, we obtain the E dependence of α_p and V_p for several scattering mechanisms as follows.

(1) Electrons are scattered by deformation acoustical (DA) phonons; phonons transfer their energy to electrons, but momentum to the crystal boundaries. $t=1, r=-1/2, l=1, k=1, n=1$ (drag of phonons by electrons case):

$$\alpha_p \sim E^{2/9} \quad (s=1/2), \quad \sim E^{2/3} \quad (s=1),$$

$$V_p \sim E^{2/3} \quad (s=1/2), \quad \sim E^{8/9} \quad (s=1). \tag{38}$$

TABLE III. Dependences of V_e on E in the condition $\gamma_0 \ll 1$.

Interaction		$s=\frac{1}{2}$	$s=1$
DA interaction of electrons with acoustical phonons ($t=1$, $r=-1/2$)	V_e	$\sim E^{4/9}$	$\sim E^{2/9}$
PA interaction ($t=-1$, $r=1/2$)	V_e	$\sim E^{4/7}$	$\sim E^{2/7}$
The momentum scattering of electrons by impurity ions ($r=3/2$)	V_e	$\sim E^{4/3}$	$\sim E^{1/2}$

(2) Electrons are scattered by DA phonons, and phonons by electrons. $t=1, r=-1/2, l=1, k=1, n=1$ (the mutual drag case):

$$\alpha_p \sim E^{2/3} \quad (s=1/2), \quad \sim E^{2/3} \quad (s=1),$$

$$V_p \sim E^{10/9} \quad (s=1/2), \quad \sim E^{8/9} \quad (s=1). \tag{39}$$

(3) Electrons are scattered by piezo acoustical (PA) phonons; phonons transfer their energy to electrons and momentum to the crystal boundaries. $t=-1, r=1/2, l=1, k=0, n=0$ (drag of phonons by electrons case):

$$\alpha_p \sim E^{-2/7} \quad (s=1/2), \quad \sim E^{2/7} \quad (s=1),$$

$$V_p \sim E^{2/7} \quad (s=1/2), \quad \sim E^{4/7} \quad (s=1). \tag{40}$$

(4) Electrons are scattered by PA phonons, and phonons by electrons. $t=-1, r=1/2, l=1, k=-1, n=1$ (the mutual drag case):

$$\alpha_p \sim E^{6/7} \quad (s=1/2), \quad \sim E^{6/7} \quad (s=1),$$

$$V_p \sim E^{10/7} \quad (s=1/2), \quad \sim E^{8/7} \quad (s=1). \tag{41}$$

(5) Electrons transfer their momentum to impurity ions, energy to DA phonons; and phonons transfer their energy to electrons, momentum to the boundaries. $t=1, r=3/2, l=0, k=0, n=0$ (''thermal drag'' or drag of electrons by phonons):

$$\alpha_p \sim E^{2/3} \quad (s=1/2), \quad \sim E^{3/2} \quad (s=1),$$

$$V_p \sim E^{2} \quad (s=1/2), \quad \sim E^{2} \quad (s=1). \tag{42}$$

(6) The momentum of electrons is transferred to impurity ions, energy to DA phonons; and phonons transfer their energy and momentum to electrons. $t=1, r=3/2, l=0, k=1, n=1$ (drag of electrons by phonons or ''thermal drag'' case):

$$\alpha_p \sim E^{2} \quad (s=1/2), \quad \sim E^{3/2} \quad (s=1),$$

$$qV_p \sim E^{10/3} \quad (s=1/2), \quad \sim E^{2} \quad (s=1). \tag{43}$$

(7) The momentum of electrons is transferred to impurity ions, energy to PA phonons, and phonons transfer their energy to electrons and momentum to the boundaries. $t=-1, r=3/2, l=0, k=0, n=0$ (drag of electrons by phonons ''thermal drag''):

$$\alpha_p \sim E^{-2/3} \quad (s=1/2), \quad \sim E^{1/2} \quad (s=1),$$

$$qV_p \sim E^{2/3} \quad (s=1/2), \quad \sim E \quad (s=1). \tag{44}$$

(8) The momentum of electrons is transferred to impurity ions, energy to PA phonons; and phonons transfer their energy and momentum to electrons. $t=-1, r=3/2, l=0, k=-1, n=1$ (''thermal drag'' case):

$$\alpha_p \sim E^2 \quad (s=1/2), \quad \sim E^{3/2} \quad (s=1),$$

$$V_p \sim E^{10/3} \quad (s=1/2), \quad \sim E^2 \quad (s=1). \tag{45}$$

It should be noted that the cases 6 and 8 lead to the same results, because in both cases $r=3/2$, $l=1$, $k=t$, and $n=1$.

III. DISCUSSION

The nonparabolicity of electron spectrum significantly influences the thermoelectric power of hot charge carriers and leads to a change of its electron temperature dependence, as it is seen from Eqs. (24) and (25). For all scattering mechanisms $4+t-k-n>0$. Therefore, the nonparabolicity of the spectrum leads to a more rapid increase of α_p with increasing T_e. Moreover, α_p consists of the factor $\nu_{p0}(T)/\beta(T) \gg 1$.

As it follows from Eqs. (29) and (30), in the weak mutual drag case α_e does not depend on T_e or E by the accuracy of logarithmic dependence, and the thermoelectric field (or voltage) depends on T_e linearly. Indeed, $\alpha_e \ll \alpha_p$, and α_p depends on T_e and E more strongly.

For nondegenerate electrons, the factor in Eq. (31) is

$$\frac{(m_n T)^{3/2}}{\hbar^3 N}\left(\frac{T}{\varepsilon_g}\right)^{3(s-1/2)} \approx \exp\left(-\frac{\zeta(T)}{T}\right) \gg 1. \tag{46}$$

By comparing Eqs. (31) and (32) we may easily see that under the strong mutual drag condition $\alpha_e \ll \alpha_p$. In other words, the thermoelectric power mainly consists of the phonon part. Indeed, we again see that the nonparabolicity of the electron spectrum strongly changes the dependence of α_p on T_e. In the weak mutual drag case, $\alpha_p \sim T_e^{(3n+t-k)/2}$ for the parabolic, and $\alpha_p \sim T_e^{(2+n-k-t)}$ for the strong nonparabolic spectrum of electrons. In the strong mutual drag, $\alpha_p \sim T_e^{3/2}$ for the parabolic, and $\alpha_p \sim T_e^3$ for the strong nonparabolic spectrum cases.

According to Eq. (31) in the strong mutual drag case, the dependences of α_e on ϑ_e and E are logarithmic and $V_e \sim \vartheta_e$. In Table I we see that under the strong mutual drag conditions, V_e, α_p, and V_p grow as E increases in the limiting cases given in Eq. (35). According to Table II in the strong mutual drag case, the nonparabolicity of the spectrum leads to a weaker dependence of V_e on E than in the parabolic one. In other words, as E increases, V_e grows faster in the parabolic case. The influence of the nonparabolicity of the spectrum on α_p and V_p is more complicated. In the case (i), α_p and V_p grow more rapidly with E for the parabolic spectrum. However, in the cases (ii) and (iii), α_p grows more rapidly with E for the nonparabolic spectrum. On the other hand, the dependence of V_p on E approximately is the same for both parabolic and nonparabolic spectrum of electrons.

In the weak mutual drag case, according to Table III, for the scattering of electrons by phonons, if V_e is proportional to E^n for the nonparabolic spectrum, then, it is proportional to E^{2n} for the parabolic spectrum of electrons. What about the dependences of α_p and V_p on E for the weak mutual drag case? One can see from Eqs. (38)–(45) that for all the cases considered, the thermoelectric voltage V_p grows as E increases.

The cases 2 and 4 consider the mutual drag condition for the region of common drift velocities $u \ll s_0$. In this case the dependence of α_p on E is exactly the same for both parabolic and nonparabolic spectrums. But, the dependences of V_p are different. Actually, V_p increases faster for the parabolic spectrum with increasing E.

The cases 1 and 3 consider the drag of phonons by electrons under the conditions of scattering of electrons by DA and PA phonons. As it is seen from Eqs. (38) and (40), in these cases α_p and V_p grow more rapidly as E increases for the nonparabolic spectrum.

The cases 6 and 8 consider the drag of electrons by phonons or the ''thermal drag.'' As it follows from Eqs. (43) and (45), the dependences of α_p and V_p on E are the same independent of the type of the scattering of electrons by DA or PA phonons. Moreover, α_p and V_p grow faster as E increases for the parabolic spectrum.

In cases 5 and 7 we have the condition of drag of electrons by phonons with common drift velocities equal to that of phonons u. In case 5, the dependence of V_p on E is the same for both the parabolic and nonparabolic spectrums, whereas α_p grows more rapidly for nonparabolic case. On the other hand, both α_p and V_p grow faster for the nonparabolic spectrum as E increases in case 7.

In the weak mutual drag case, ϑ_e is proportional to $E^{s[4+(t-k)-n]+2n-2}$. Therefore, when $t=k$ and $n=1$ we have $\vartheta_e \sim E^{3s}$.

In the absence of mutual drag, electronic part of the thermoelectric field (or the integral thermoelectric power) is

$$E_{cz} = -\frac{1}{e}(2rs-4s+3)\nabla_z T_e. \tag{47}$$

For the strong nonparabolic spectrum, when electrons are scattered by PA phonons ($r=1/2$), E_{cz} vanishes. However, when electrons are scattered by DA phonons ($r=-1/2$), E_{cz} reverses its sign compared to the parabolic spectrum case. Thus, the nonparabolicity of the electron spectrum leads to a change of the sign of the thermoelectric field.

In the case of the parabolic spectrum and heated electrons, if the electron temperature gradient is produced by the lattice temperature gradient, then the electronic part of the thermoelectric field reverses its sign in comparison to the case of nonheated electrons ($T_e=T$). For the case $T_p=T_e \gg T$, $\partial T_e/\partial T<0$ is negative. Therefore, both electronic and phonon parts of the thermoelectric field reverse their signs compared to the nonheating case for all considered situations.

IV. CONCLUSION

In the present work, we show that the nonparabolicity of electron spectrum significantly influences the magnitude of the thermoelectric power and leads to a change of its sign compared to the parabolic spectrum case. The nonparabolicity also remarkably changes the heating electric field dependence of the thermoelectric power.

It is shown that in the strong mutual drag conditions, the phonon part of the thermoelectric power dominates over the electron part. Indeed, the thermoelectric power increases with the electronic temperature as $\sim T_e^{3/2}$ for the parabolic, and as $\sim T_e^3$ for the strong nonparabolic spectrum of electrons. For all the cases considered α_p, and the thermoelectric fields V_e and V_p grow as E increases. Indeed, we show that this growth is more rapid for the parabolic spectrum of electrons.

In the weak mutual drag case for the scattering of electrons by phonons, it is found out that V_e grows faster with increasing E for the parabolic spectrum case. Moreover, for all the cases studied V_p grows as E increases.

It is shown that in both weak and strong mutual drag cases, electronic part of the thermoelectric power does not depend on T_e or E by the accuracy of logarithmic dependence. Hence, V_e depends on T_e linearly.

It is found out that under the mutual drag conditions, for the drift velocities much smaller than the sound velocity in the crystal, the E dependences of α_p are exactly the same for both parabolic and nonparabolic spectrum of electrons. However, the dependences of V_p are different.

Under the drag of phonons by electrons conditions, for the scattering of electrons by DA and PA phonons, it is shown that α_p and V_p grow more rapidly as E increases for the nonparabolic spectrum of electrons. In the thermal drag case, the dependences of α_p and V_p on E are the same independent of the type of interaction of electrons by DA or PA phonons. In the case of drag of electrons by phonons with common drift velocities of phonons, the dependence of V_p on E is the same for both parabolic and nonparabolic spectrum of electrons, whereas α_p grows faster for the nonparabolic spectrum case.

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