Operation of the qubit based on photon-assisted tunneling in a coupled quantum-dot system and the influence of dephasing

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We derive the generalized rate equation for the coupled quantum-dot (QD) system irradiated by a microwave field in the presence of a quantum point contact. It is shown that when a microwave field is tuned in resonance with the energy difference between the ground states of two QD's, the photon-assisted tunneling occurs and, as a result, the coupled QD system may be used as the single qubit. Furthermore, we show that the oscillating current through the detector decays drastically as the dephasing rate increases, indicating clearly the influence of the dephasing effect induced by the quantum point contact used as a detecting device.

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The phenomena of the coherent tunneling in a coupled quantum-dot (QD) system has received much attention, both experimentally[1–3] and theoretically.[4–7] Recently, Blick *et al.*[8] measured the tunnel splitting of a double QD structure and showed the formation of artificial molecular states. The spectrum of a two-coupled QD molecule in the presence of a microwave field was experimentally studied by Oosterkamp *et al.*[9] They demonstrated that when the frequency of the microwave field is equal to the difference between the two uncoupled QD levels, the photon-assisted resonances occur for the weak-coupling case and coherent oscillations of the electron in the coupled QD system are observed as well. For the theoretical aspect, Tsukada *et al.*[10] showed that an electron initially localized in one of the two asymmetric quantum dots begins to oscillate back and forth between them when the two ground energy levels of the two isolated dots are tuned in resonance. The coupled QD molecule system in the presence of an electromagnetic field is investigated by Wu *et al.*[11] It is shown that this coupled system may be used as the qubit for quantum computing and information. However, all of these studies did not take the influence of the measurement into account.

It is well known that the measurement itself will certainly induce dephasing. This effect was studied by Aleiner *et al.*,[12] Levinson,[13] and Buks *et al.*[14] Starting from the Schrödinger equation, Gurvitz *et al.*[15] derived modified rate equations and studied quantum transport in the QD system. Also, Gurvitz[16] demonstrated the dephasing effects induced by measuring the electron state in a coupled QD system via a quantum point contact. Motivated by these studies, we derive, in the present article, the more generalized rate equation for the coupled QD system irradiated by a microwave field in the presence of a quantum point contact, which is used as a detecting device. We investigate the quantum dynamics of the coupled QD system, and find that the photon-assisted tunneling in the coupled QD system occurs when the frequency of the microwave field matches the energy difference between the ground states of the two dots. It is also shown that measurement enforces the coupled QD system to dephase at the rate of $e^{(-\Gamma_d t/2)}$, where Γ_d is the dephasing rate.

The proposed coupled QD system and the detector are schematically shown in Fig. 1. A ballistic one-dimensional point contact is placed near one of the dots (dot 1) as the detector. Its resistance is very sensitive to the electrostatic potential which may be influenced by the electron filled in the measured quantum dots. The detector is represented by a barrier, sandwiched between an emitter (S) and a collector (D). The chemical potentials of the emitter and the collector are denoted as μ_S and μ_D, respectively. $V_d = \mu_S - \mu_D$ represents the applied voltage between the emitter and the collector. Since $\mu_S > \mu_D$, the current flowing through the point contact is given by[17] $I = eTV_d/2\pi$, where e is the electron charge and T is the transmission coefficient of the point contact. In fact, the penetrability of the point contact is affected by the electron tunneling inside the two dots in a very sensitive way.

The entire system that we consider consists of the coupled QD system, the detecting device, and the microwave field. The Hamiltonian of this system can be written as[10,15]

$$H = H_{PC} + H_{DD} + H_I + H_{FD}, \tag{1}$$

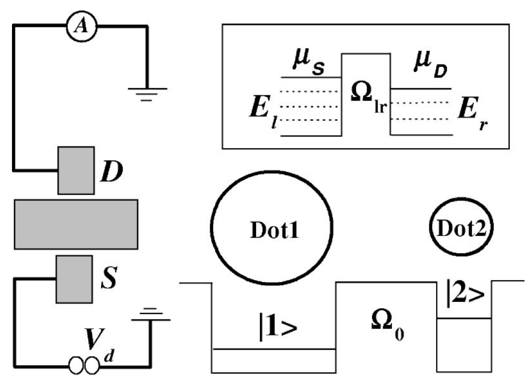

FIG. 1. Schematic illustration of the coupled quantum-dot system and the quantum point contact detector. E_1 and E_2 are the energy levels of the ground states $|1\rangle$ (dot 1) and $|2\rangle$ (dot 2), and Ω_0 denotes the coupling between them. The left part is the detector and its energy levels are shown on right upper. V_d is the applied voltage between the source S and the drain D with chemical potentials μ_S and μ_D, respectively. Ω_{lr} is the coupling between the drain energy level E_l and the source energy E_r.

with

$$H_{DD}=E_1c_1^+c_1+E_2c_2^+c_2+\Omega_0(c_2^+c_1+c_1^+c_2), \tag{2}$$

$$H_{PC}=\sum_l E_l a_l^+a_l+\sum_r E_r a_r^+a_r+\sum_{lr}\Omega_{lr}(a_l^+a_r+a_r^+a_l) \tag{3}$$

and

$$H_I=-\sum_{lr}\Omega'_{lr}c_1^+c_1(a_l^+a_r+a_r^+a_l), \tag{4}$$

$$H_{FD}=-\boldsymbol{P}\cdot\boldsymbol{E}(t)(c_1^+c_2+c_2^+c_1). \tag{5}$$

Here, H_{DD} and H_{PC} are the Hamiltonians describing the isolated QD system and the quantum point contact, respectively. E_1 and E_2 denote the ground-state energies of the two dots and Ω_0 is the coupling between them, while E_l and E_r denote the energy levels in the emitter and collector and Ω_{lr} is the coupling between them. Since the presence of an electron in dot 1 results in an effective increase of the point contact barrier, one can model the interaction between the QD system and the point contact as H_I, where Ω'_{lr} is the variation of the coupling. H_{FD} is the interaction between the coupled QD system and the microwave field, where $\boldsymbol{P}$ is the dipole transition moment and $\boldsymbol{E}(t)=\boldsymbol{E}_\omega\cos(\omega t+\delta)$ denotes the microwave field having frequency ω and initial phase δ.

For convenience we assume that the temperature of the reservoirs is zero and the entire system is initially in a pure state. So the wavefunction describing the entire system can be given by[15]

$$|\Psi\rangle=\Bigg[b_1(t)c_1^+ +\sum_{lr} b_{1lr}c_1^+a_r^+a_l + \sum_{l<l',r<r'} b_{1ll'rr'}c_1^+a_r^+a_{r'}^+a_la_{l'} + b_2(t)c_2^+ +\sum_{lr} b_{2lr}c_2^+a_r^+a_l + \sum_{l<l',r<r'} b_{2ll'rr'}c_2^+a_r^+a_{r'}^+a_2a_{l'}+\cdots\Bigg]|0\rangle. \tag{6}$$

Here, $b(t)$ are the amplitudes of the probability finding the system in the states defined by the corresponding creation and annihilation operators. The vacuum state $|0\rangle$ corresponds to the situation where the states of the emitter and the collector are filled up to their Fermi levels, respectively. The quantum evolution of the whole system is described by the time-dependent Schrödinger equation $i|\dot{\Psi}\rangle=H|\Psi\rangle$ and the corresponding density matrix is given by $\sigma=|\Psi\rangle\langle\Psi|$. In the two-dimensional Fock space composed of the two states $|1\rangle$ and $|2\rangle$, the elements of the density matrix can be expressed as

$$\dot{\sigma}_{11}^{(n)}=-D_1\sigma_{11}^{(n)}+D_1\sigma_{11}^{(n-1)}+i\Omega_0(\sigma_{12}^{(n)}-\sigma_{21}^{(n)}) -i\boldsymbol{P}\cdot\boldsymbol{E}(\sigma_{12}^{(n)}-\sigma_{21}^{(n)}), \tag{7a}$$

$$\dot{\sigma}_{22}^{(n)}=-D_2\sigma_{22}^{(n)}+D_2\sigma_{22}^{(n-1)}-i\Omega_0(\sigma_{12}^{(n)}-\sigma_{21}^{(n)}) +i\boldsymbol{P}\cdot\boldsymbol{E}(\sigma_{12}^{(n)}-\sigma_{21}^{(n)}), \tag{7b}$$

$$\dot{\sigma}_{12}^{(n)}=i\varepsilon\sigma_{12}^{(n)}+i\Omega_0(\sigma_{11}^{(n)}-\sigma_{22}^{(n)})-i\boldsymbol{P}\cdot\boldsymbol{E}(\sigma_{11}^{(n)}-\sigma_{22}^{(n)}) -(D_1+D_2)\sigma_{12}^{(n)}/2+(D_1D_2)^{(1/2)}\sigma_{12}^{(n)}, \tag{7c}$$

where $\varepsilon=E_2-E_1$ and the index n denotes the number of electrons coming to the collector at time t. $D_{1(2)}=T_{1(2)}V_d/2\pi$ is the transition rate of an electron hoping from the emitter to the collector as the electron stays at state $|1\rangle$ ($|2\rangle$). From Eqs. (7) we can obtain the current flowing through the detector

$$I_d(t)=\frac{dQ_d(t)}{dt}=D_2\sigma_{11}(t)+D_1\sigma_{22}(t). \tag{8}$$

Without considering the coupling between the coupled QD system and the point contact, the current through the point contact should be[15] $I_d^{(0)}=D_2$, and the variation of the point contact current is

$$\Delta I_d(t)=(T_2-T_1)V_d\sigma_{22}(t)/2\pi, \tag{9}$$

which measures directly the charge in the dot 2. In order to determine the influence of the point contact on the QD system, we trace out the point contact states in Eqs. (7) and obtain

$$\dot{\sigma}_{11}(t)=i\Omega_0(\sigma_{12}-\sigma_{21})-i\boldsymbol{P}\cdot\boldsymbol{E}(\sigma_{12}-\sigma_{21}), \tag{10a}$$

$$\dot{\sigma}_{22}(t)=i\Omega_0(\sigma_{21}-\sigma_{12})-i\boldsymbol{P}\cdot\boldsymbol{E}(\sigma_{21}-\sigma_{12}), \tag{10b}$$

$$\dot{\sigma}_{12}(t)=i\varepsilon\sigma_{12}+i\Omega_0(\sigma_{11}-\sigma_{22}) -i\boldsymbol{P}\cdot\boldsymbol{E}(\sigma_{11}-\sigma_{22})-\Gamma_d\sigma_{12}/2, \tag{10c}$$

where $\Gamma_d=(\sqrt{D_2}-\sqrt{D_1})^2$ is the dephasing rate. Equations (10) involve the reduced density matrixes, which describe the dynamics of the coupled QD system. The measurement-induced effects on the coupled QD system are included in the last term of Eq. 10(c). The nondiagonal density matrix element damps with the rate $e^{(-\Gamma_d t/2)}$ and it becomes zero as $t \gg 1/\Gamma_d$, implying that the pure state of the system now becomes a statistical mixed one and the coupled QD system loses its coherence. The diagonal density matrixes σ_{11} and $\sigma_{22}(=1-\sigma_{11})$ denote the electron probabilities in dots 1 and 2.

In order to elucidate the quantum dynamical behavior of the coupled QD system and the dephasing effects induced by the measurement, we solved Eqs. (10) numerically. In our calculations the frequency of the microwave field is kept equal to the energy difference between the two ground energy levels of the uncoupled QD system with initial phase $\delta=\pi/2$. The coupling coefficient Ω_0 between two dots for electrons is taken as, for example, -1. First, in order to

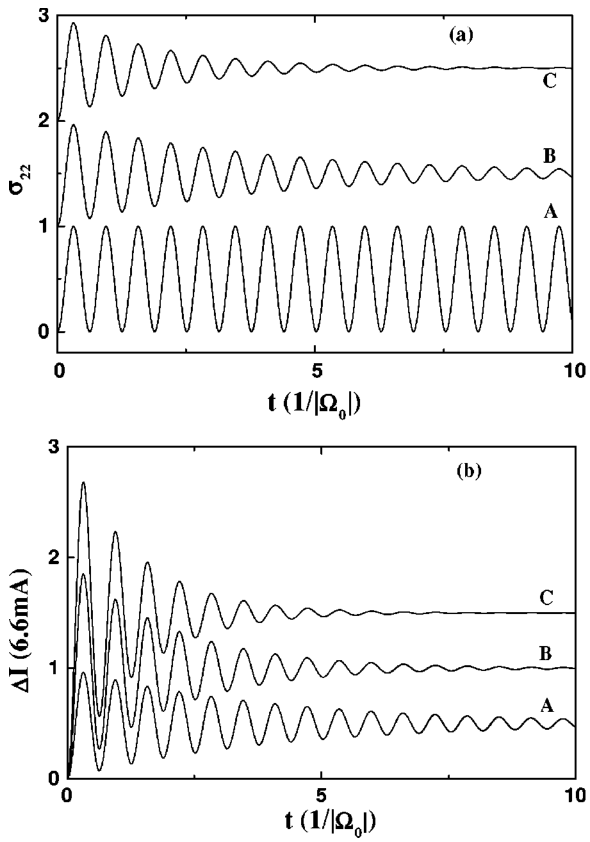

FIG. 2. (a) Evolution of the electron probability in dot 2 for dephasing rates $\Gamma_d = 0$ (curve A), $1\Omega_0$ (curve B), and $2\Omega_0$ (curve C). The curves are offset vertically for clarity. (b) Currents through the detector for dephasing rates $\Gamma_d = 1\Omega_0$ (curve A), $2\Omega_0$ (curve B), and $3\Omega_0$ (curve C).

visualize the dephasing effects induced by the detecting, we plot the time evolution of the probability σ_{22} in Fig. 2(a) for different dephasing rates. In the case of $\Gamma_d = 0$ the probability exhibits sinusoidal oscillations in the curve A of Fig. 2(a). For $\Gamma_d > 0$, the oscillations decay and become nonsinusoidal. Moreover, when the dephasing rate increases, the oscillations decay more quickly [see curves B and C in Fig. 2(a)]. Also, it can be seen from the curve C in Fig. 2(a) that the probabilities σ_{22} approach to 1/2 at sufficiently later time, revealing that the coupled QD system lost its coherence. In Fig. 2(b) we show the current through the quantum point-contact detector. Comparing curves A (B) in Fig. 2(b) with curves B (C) in Fig. 2(a), we can see that when the electron probability approaches the maximum (i.e., the electron localizes mainly in dot 2), the current also reaches the maximum in phase. This observation implies that by means of measuring the current variation, one can truly extract the information of the coupled QD system. Also, it is clear that the current through the detector shows temporal oscillations, and with increasing the dephasing rate, the amplitude of the current increases in the initial stage of the time. When Γ_d is large enough, the current decays quickly to a constant, implying that the detector is no longer able to resolve any information about the coupled QD system as t is much longer than $1/\Gamma_d$.

We then investigate the quantum dynamics of the electron

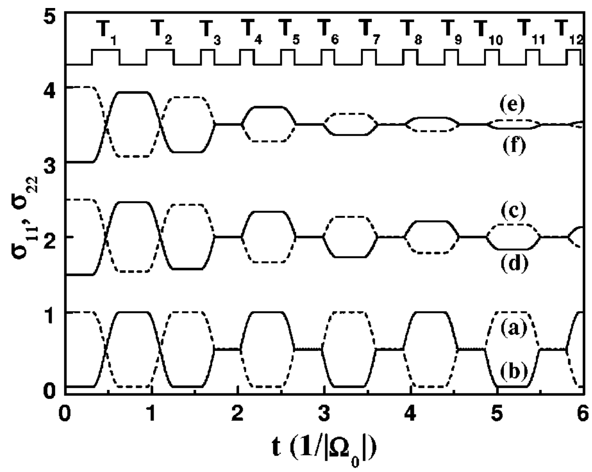

FIG. 3. Probabilities of the electron as a function of time when the magnitude of the microwave field $\boldsymbol{P}\cdot\boldsymbol{E}_\omega = 10\Omega_0$ for different dephasing rates (a) and (b) $\Gamma_d = 2\Omega_0$, (c) and (d) $\Gamma_d = 1\Omega_0$, (e) and (f) $\Gamma_d = 0\Omega_0$. The lines are offset vertically for clarity. The upper part shows the microwave field pulse. The curve denoted by solid (dotted) line represents the probability of the state $|2\rangle(|1\rangle)$.

in the coupled QD system irradiated by a series of microwave pulses. Figure 3 shows the time evolutions of the occupation probabilities σ_{11} and σ_{22} of the electron in states $|1\rangle$ and $|2\rangle$ with $\boldsymbol{P}\cdot\boldsymbol{E}_\omega = 10\Omega_0$ while the dephasing rate Γ_d takes different values. Among the pulses, T_1 and T_2 are $\pi/2$ pulses, while others $(T_3 - T_{12})$ are $\pi/4$ pulses. It can be seen from Figs. 3(a) and 3(b) $(\Gamma_d = 0)$ that the state of the coupled QD system evolves from $|1\rangle(|2\rangle)$ to $|2\rangle(|1\rangle)$ due to the irradiation of the microwave field by the pulse $T_1(T_2)$. When a $\pi/4$ pulse, such as T_3, is applied to the coupled QD system, the state $|1\rangle$ evolves into $(|1\rangle + |2\rangle)/\sqrt{2}$, while $|2\rangle$ is transformed to $(-|1\rangle + |2\rangle)/\sqrt{2}$ as another $\pi/4$ pulse, T_5, comes. These transformations are just the NOT and Hadamard op-

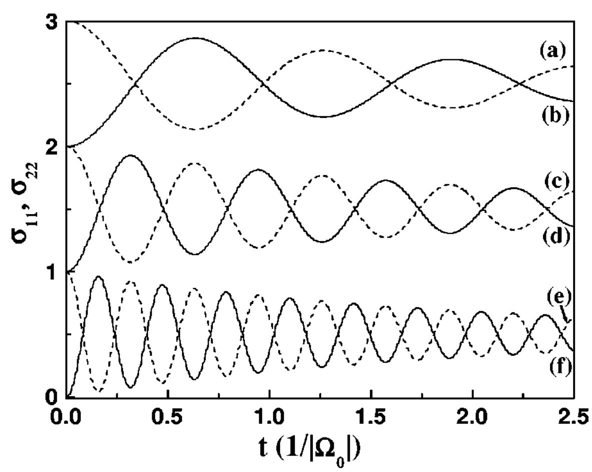

FIG. 4. Evolution of the electron probability as a function of time with the dephasing rate $\Gamma_d = 2\Omega_0$ for different magnitudes of the microwave field: (a) and (b) $\boldsymbol{P}\cdot\boldsymbol{E}_\omega = 5\Omega_0$, (c) and (d) $\boldsymbol{P}\cdot\boldsymbol{E}_\omega = 10\Omega_0$; (e) and (f) $\boldsymbol{P}\cdot\boldsymbol{E}_\omega = 20\Omega_0$. The solid (dotted) line is the probability of state $|2\rangle(|1\rangle)$. The lines are offset vertically forclarity.

erations for the single qubit. Also, we can see that the system begins to evolve whenever it is irradiated by the microwave pulses. Figures 3(e) and 3(f) display the evolutions of the electron probabilities with $\Gamma_d = 2\Omega_0$. It can be seen clearly that the probability difference ($|\sigma_{11} - \sigma_{22}|$) is 0.456 when the pulse T_4 is over, and does not reach the maximum value 1 as shown by Figs. 3(a) and 3(b). When the dephasing rate is set to $1\Omega_0$ the probability difference reaches 0.735 [see Figs. 3(c) and 3(d)]. As a result, we can draw the conclusion that the probability difference decreases for the same time t during any $\pi/4$ pulse when the dephasing rate becomes large.

Finally, we also study the evolutions of the electron occupation probabilities for different magnitude $|\boldsymbol{E}_\omega|$ of the microwave field (see Fig. 4). Here, the dephasing rate is taken to be $\Gamma_d = 2\Omega_0$. As shown by the dotted lines in Figs. 4(a), 4(c), and 4(e), they are nonsinusoidal oscillations with periods 1.255, 0.62, and 0.314 when $\boldsymbol{P}\cdot\boldsymbol{E}_\omega$ are equal to $5\Omega_0$, $10\Omega_0$, and $20\Omega_0$, respectively. These demonstrate that when the quantity $\boldsymbol{P}\cdot\boldsymbol{E}_\omega$ increases, the period of the electron tunneling between the two dots is shortened, in agreement with the formula of the one-qubit logic gates[18]

$$U = \begin{bmatrix} \cos\theta & -ie^{-i\varphi}\sin\theta \\ -ie^{i\varphi}\sin\theta & \cos\theta \end{bmatrix}, \tag{11}$$

where $\theta = (\boldsymbol{P}\cdot\boldsymbol{E}_\omega \cdot t)/2$ and φ is the initial phase of the microwave field. This is just what the photon-assisted tunneling should be. It is easy to see that when suitable pulses are applied to the coupled QD system, all one-qubit operations can be performed. This verifies that the coupled QD system, studied here, may be used as the single qubit for quantum computing.

In conclusion, we use the modified rate equation to investigate the quantum dynamics of the coupled QD system irradiated by a microwave field in the presence of dephasing induced by measurement. It is shown that the photon-assisted tunneling occurs when the microwave field is tuned in resonance with the energy difference between the ground states of the two dots. Our results demonstrate that this coupled QD system may perform all the operation of single qubit. Also, we show that when the dephasing rate increases, the oscillating current through the detector decays drastically. For the application of this coupled QD system in the quantum computing and information, keeping an appropriate dephasing rate is necessary so that the measurements should be as strong as possible to resolve the read-out data, and at the same time the required quantum-computing operations can be implemented.

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