

## Effect of charge fluctuations on the persistent current through a quantum dot

Kicheon Kang,<sup>1,2</sup> Sam Young Cho,<sup>3</sup> and Ju-Jin Kim<sup>2</sup>

<sup>1</sup>Basic Research Laboratory, Electronics and Telecommunications Research Institute, Taejon 305-350, Korea

<sup>2</sup>Department of Physics, Chonbuk National University, Chonju 561-756, Chonbuk, Korea

<sup>3</sup>Institute of Physics and Applied Physics, Yonsei University, Seoul 120-749, Korea

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We study coherent charge transfer between an Aharonov-Bohm ring and a side-attached quantum dot. The charge fluctuation between the two substructures is shown to give rise to an algebraic suppression of the persistent current circulating the ring as the size of the ring becomes relatively large. The charge fluctuation at resonance provides a transition between diamagnetic and paramagnetic states. Universal scaling, the crossover behavior of a persistent current from a continuous to a discrete energy limit in the ring, is also discussed.

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Coherent electron transfer from one region to another region in a composite mesoscopic structure may drastically affect the quantum-mechanical properties of the coupled system. Coupling between the subsystems due to electron transfer accompanies charge and/or spin fluctuation, and significantly modifies the characteristics of the system. A prototype to investigate such kinds of effects is a mesoscopic Aharonov-Bohm (AB) ring tunnel coupled to a quantum dot (QD) (Refs. 1–8) or connected to a finite-size wire.<sup>9–14</sup> A purely one-dimensional (1D) ring of spinless electrons exhibits persistent current (PC), being either diamagnetic or paramagnetic depending on the number of electrons being odd or even,<sup>15</sup> respectively. This so-called “Leggett’s conjecture” breaks down when the ring is coupled to an additive structure such as a QD, because the system is not purely 1D any longer. In a side-branched QD case of Ref. 1, it was shown that coherent charge transfer from a QD to a ring induces a sharp transition between plateaus of diamagnetic and paramagnetic states. This phenomenon is quite understandable in terms of a single charge transfer that alters the number of electrons in the ring and in the QD one by one. However, discussion of Ref. 1 was limited to a relatively small ring where the energy level spacing ( $\delta$ ) of the ring is much larger than the tunnel coupling strength ( $\Gamma'$ ) between the QD and the ring.

The role of spin fluctuation associated with the Kondo effect on the PC recently became an issue of serious debate.<sup>3–7</sup> The central point of the debate is whether the following argument of the correspondence between an open system and its closed counterpart can be applied to a system with a Kondo correlated quantum dot: *The PC in a 1D non-interacting scattering model with a fixed electron number  $N_e$  is completely determined by the transmission probability at the Fermi level, to the leading order of  $1/L$ , with  $L$  being the length of the ring.*<sup>16</sup> Transport current (TC) through a 1D quantum wire with a side branch QD is completely suppressed in the Kondo limit.<sup>17</sup> Conversely, it was shown by an exact Bethe ansatz<sup>4</sup> and by a diagrammatic expansion<sup>5</sup> that the PC of a mesoscopic ring with a side-branch QD is not affected by the presence of the Kondo effect in the continuum limit (supporting the spin-charge separation). On the other hand, Affleck and Simon<sup>6</sup> obtained a result opposite to that of Refs. 4 and 5 by using a renormalization-group analy-

sis combined with the above described argument of Ref. 16, and concluded that the normalized PC is completely suppressed in the continuum limit.

At this stage, there arise fundamental questions associated with strong charge fluctuations that have not been clarified up to now: (i) whether the PC and TC are straightforwardly interrelated each other or not, and (ii) the nature of the universal scaling behavior of the PC in the limit of  $\delta/\Gamma' \rightarrow 0$ . To clarify these points let us consider a QD side coupled to a mesoscopic ring. To understand the effect of charge fluctuation on the PC and TC, it will be very instructive to study the case of a noninteracting QD, because this model is exactly solvable. We show that charge fluctuations between the ring and the QD suppress the PC algebraically. We argue that the suppression can be understood without the assumption that the PC is a Fermi-surface effect. Although this result cannot be directly applied to a system with a Kondo correlation, the comparison between the PC’s due to charge and spin fluctuations may provide deeper insights to resolve the ongoing issue of a Kondo-correlated quantum dot side coupled to a mesoscopic ring.

We begin with the model Hamiltonian of the system,

$$H = H_0 + H_D + H_T, \quad (1a)$$

where  $H_0$ ,  $H_D$ , and  $H_T$  represent the AB ring, the QD, and the tunnel coupling, respectively:

$$H_0 = -t \sum_{j=0}^{N-1} (e^{i\phi/N} c_j^\dagger c_{j+1} + \text{H.c.}), \quad (1b)$$

$$H_D = \varepsilon_d d^\dagger d, \quad (1c)$$

$$H_T = -t'(d^\dagger c_0 + c_0^\dagger d). \quad (1d)$$

For the ring we employ a 1D periodic tight-binding model of  $N$  lattice sites ( $c_N = c_0$ ) with the hopping integral  $t$  taken to be real without loss of generality. The phase factor  $\phi$  is defined by  $\phi = 2\pi\Phi/\Phi_0$ , with  $\Phi$  and  $\Phi_0 = hc/e$  being the AB flux and the flux quantum, respectively. The QD, which is much smaller than the ring, is modeled as a single resonant level of the energy  $\varepsilon_d$ . For our concern, this model is sufficient to study the effect of a charge fluctuation on the PC. The QD is coupled to the ring at site “0” by the tunneling

matrix element  $t'$ . Throughout our discussion,  $t'$  is considered to be much smaller than  $t$ . Also, for simplicity, we consider only the half-filled case,  $N_e = (N+1)/2$  with odd  $N$ , where the total number of lattice sites including the dot is  $N+1$ . This assumption does not affect the conclusion we will draw in the present study.

The essential parameter representing the coupling strength between the ring and the QD, namely,  $\Gamma'$ , is defined by

$$\Gamma' = \pi \rho(\varepsilon_F) |t'(\varepsilon_F)|^2, \quad (2)$$

where  $\rho(\varepsilon_F)$  and  $t'(\varepsilon_F)$  represent the density of states and the hopping amplitude at the Fermi energy  $\varepsilon_F$ , respectively. For the half-filled case ( $\varepsilon_F=0$ ) at the continuum limit,  $\rho(0) = \delta^{-1} = N/(2\pi t)$  and  $t'(0) = -t'/\sqrt{N}$ . Thus  $\Gamma'$  is given by

$$\Gamma' = \frac{t'^2}{2t}. \quad (3)$$

Hamiltonian (1) can be exactly diagonalized, which gives the  $N+1$  eigenvalues  $\{E_\alpha\}$ . After some algebra, one finds that the eigenvalues can be obtained from the equation

$$E_\alpha - \varepsilon_d = \frac{\Gamma'}{\pi} \delta \sum_m \frac{1}{E_\alpha - \varepsilon_m}, \quad (4)$$

where  $\varepsilon_m = -2t \cos[(2m\pi - \phi)/N]$  are the eigenvalues of the ring Hamiltonian  $H_0$  with  $m$  being integers corresponding to the angular momenta.

At zero temperature, the PC  $I(\phi)$  is given by the coherent charge response of the ground state to the AB flux as

$$I(\phi) = -\frac{e}{\hbar} \sum_{\alpha}^{\text{occ}} \frac{\partial E_\alpha}{\partial \phi}, \quad (5)$$

with the summation to be taken only for occupied levels of  $\{E_\alpha\}$ . Combining Eqs. (4) and (5), one obtains the expression for the current,

$$I(\phi) = \sum_m F(\varepsilon_m) I_m(\phi), \quad (6a)$$

where

$$I_m(\phi) = -\frac{e}{\hbar} \frac{\partial \varepsilon_m}{\partial \phi} \quad (6b)$$

is the current contribution by the bare energy level  $\varepsilon_m$  for the ring, and

$$F(\varepsilon_m) = \frac{\Gamma'}{\pi} \delta \sum_{\alpha}^{\text{occ}} \frac{A_\alpha}{(E_\alpha - \varepsilon_m)^2} \quad (6c)$$

represents the effective distribution function of electrons at the level  $\varepsilon_m$  with

$$A_\alpha = \left[ 1 + \frac{\Gamma'}{\pi} \delta \sum_m \frac{1}{(E_\alpha - \varepsilon_m)^2} \right]^{-1}. \quad (6d)$$

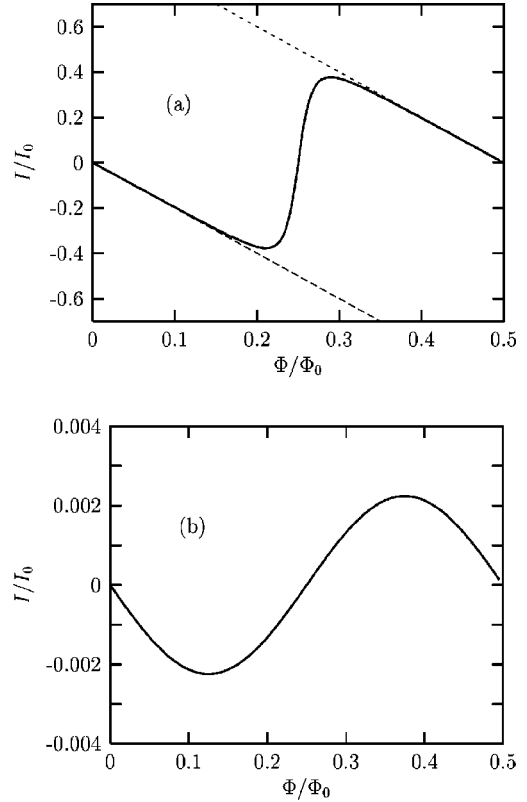


FIG. 1. Charge resonant ( $\varepsilon_d=0$ ) persistent current phase relations (a) for weak coupling ( $\delta/\Gamma' = 1000$ ) and (b) for strong coupling ( $\delta/\Gamma' = 0.5$ ). The dashed and dotted lines in (a) correspond to the current of an ideal ring with the number of electrons being odd and even, respectively.

$F(\varepsilon_m)$  corresponds to the probability of an electron occupied in the bare energy level  $\varepsilon_m$ , with its summation on  $m$  being just the average number of electrons in the ring part,

$$\sum_m F(\varepsilon_m) = N_e - n_d, \quad (7)$$

where  $n_d$  is the occupation number of electrons in the QD.

First we discuss the current-phase relation at resonance ( $\varepsilon_d=0$ ), where the charge fluctuation is strongest. Figure 1 shows the PC in two different cases: (a) weak and (b) strong couplings, respectively.  $I_0$  denotes the PC amplitude of an ideal ring with the same radius,  $I_0 = ev_F/L$ , with  $v_F$  being the Fermi velocity. Some interesting results are found both in the weak- and strong-coupling cases. First, the current displays no parity effect, that is, it does not depend on the number of electrons being even or odd in contrast to pure 1D systems. Second, its period is half ( $\Phi_0/2$ ) of that in an isolated ring. It should be noted that  $I/I_0$  is a universal function of  $\delta/\Gamma'$  for  $\varepsilon_d=0$ .

For a level spacing  $\delta$  much larger than the coupling strength  $\Gamma'$  [Fig. 1(a)], the behavior of the PC shows a crossover from the diamagnetic state to the paramagnetic state of an ideal ring with a sawtooth-shaped functional form. The crossover of the ground state occurs at an AB flux  $\Phi = \Phi_0/4$ , which leads to a period halving of the PC. This

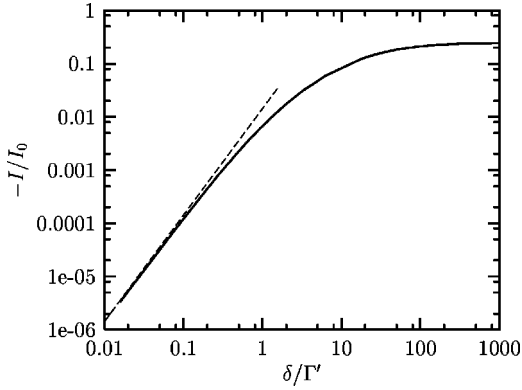


FIG. 2. Universal curve of  $I/I_0$  on resonance ( $\varepsilon_d=0$ ) as a function of  $\delta/\Gamma'$ , for  $\Phi=\Phi_0/8$ . The dashed line corresponds to  $I/I_0 \propto -(\delta/\Gamma')^2$ .

crossover can be understood in the following way. Only the topmost level of the ring with  $N_e$  electrons mainly contributes to the PC in the  $\delta \gg \Gamma'$  limit (to be precise, in the  $\delta \gg t'$  limit). The PC follows that of an ideal ring with  $N_e$  electrons or with  $N_e - 1$  electrons, depending on whether the topmost level is occupied or not. The criterion of which curve the PC will follow depends on the relative magnitude of  $E_{N_e}^0$  and  $E_{N_e-1}^0$  where  $E_{N_e}^0$  represents the ground-state energy of an ideal ring with  $N_e$  electrons. With this criterion, one can find that the ground state of the system should be diamagnetic at  $\Phi < \Phi_0/4$ , and paramagnetic at  $\Phi > \Phi_0/4$ . This fact is not affected by the total number of electrons  $N_e$  being even or odd, and thus the parity effect disappears in this special case.

Figure 1(b) shows that, for a relatively strong coupling  $I/I_0$  is much suppressed due to the strong charge fluctuation and level hybridization. The current-phase relation resembles a sinusoidal function. This behavior is quite universal in the  $\delta/\Gamma' \rightarrow 0$  limit, with a much suppressed value of  $I/I_0$ . Here we emphasize that *this feature of the transition from a diamagnetic state to a paramagnetic state is a unique phenomenon of charge fluctuation which is not present in a Kondo system with  $\delta \rightarrow 0$  limit*. That is, the effect of a charge fluctuation does not provide a one-to-one correspondence to that of the spin fluctuation studied in Refs. 4–6.

The universal feature of the PC as a function of  $\delta/\Gamma'$  can be understood from Fig. 2.  $I/I_0$  as a function of  $\delta/\Gamma'$  is plotted on a log-log scale.  $\Phi=\Phi_0/8$  is chosen for this figure, but the overall feature including the universal scaling is independent of a given value of the flux. In the very weak tunneling limit ( $\delta/\Gamma' \rightarrow \infty$ ), the current saturates to a value of an ideal ring,  $I/I_0 = -0.25$  for  $\Phi=\Phi_0/8$ . This is exactly what is expected from Fig. 1(a).  $I/I_0$  is diminished as  $\delta/\Gamma'$  decreases. One can find that at small  $\delta/\Gamma' (\ll 1)$  the curve of the PC becomes linear, with its slope being exactly 2. This implies that  $I/I_0$  is proportional to  $(\delta/\Gamma')^2$  in this region, and eventually reduces to zero in the continuum limit of  $\delta/\Gamma' \rightarrow 0$  ( $L \rightarrow \infty$ ).

The vanishing of  $I/I_0$  at  $\delta/\Gamma' \rightarrow 0$  limit could be interpreted in two different ways, as follows. The first interpretation is based on an analogy between the open system and the

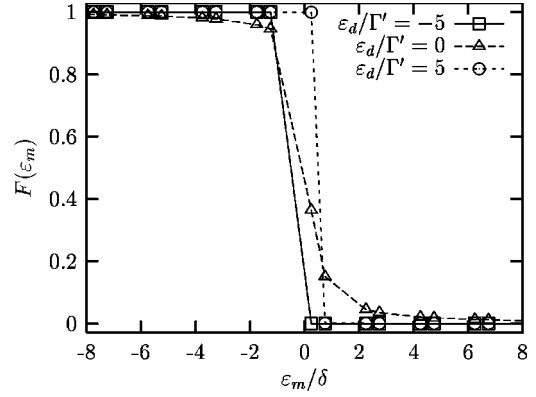


FIG. 3. Effective distribution function of the ring energy levels in the strong-coupling limit ( $\delta \leq \Gamma'$ ). The parameters are given as  $N_e=388$ ,  $\Phi=\Phi_0/8$ , and  $\delta/\Gamma'=5.06 \times 10^{-2}$ .

ring with  $L \rightarrow \infty$ .<sup>16</sup> It was shown in Ref. 17 that the linear response conductance through a 1D perfect quantum wire with a side-coupled QD is given by

$$G = \frac{2e^2}{h} T(\varepsilon_F), \quad (8)$$

where  $T(\varepsilon_F) = \cos^2 \pi n_d$  is the transmission probability at the Fermi level for spinless electrons. At resonance,  $n_d = 1/2$ ; thus the conductance reduces to zero. This perfect reflection is understood as a result of the destructive interference between the direct transmission through the wire and the resonant transmission via the QD. The vanishing of the normalized PC in our study might be regarded as the same characteristics with the perfect reflection in the corresponding open system.

However, the following alternative viewpoint can elucidate the suppression of the PC in a closed system without introducing the analogy with the open system. For  $\varepsilon_d=0$  with  $\delta/\Gamma' \ll 1$ , various configurations of the distribution in the ring states contribute to the current, with its total number of electrons being  $N_e$  or  $N_e - 1$ . These include many excited states with excitation energies lower than  $\Gamma'$ , with fluctuating numbers, of electrons in the ring part. Each configuration contributes to the current, with its magnitude and size depending very much on the configuration of excited states. These configurations modify the occupation weight of each states  $\varepsilon_m$  in the ring. The effective distribution is plotted in Fig. 3 as a function of the ring energy level. One can find that, for  $\varepsilon_d \ll -\Gamma'$ ,  $F(\varepsilon_m)$  coincides with the distribution function of an ideal ring with  $N_e - 1$  electrons. That is,  $F(\varepsilon_m) = 1$  for  $N_e - 1$  lowest energy states, and  $F(\varepsilon_m) = 0$  otherwise. Similarly, for  $\varepsilon_d \gg \Gamma'$ ,  $F(\varepsilon_m)$  corresponds to the distribution function of an ideal ring with  $N_e$  electrons. On the other hand, at the charge resonance point ( $\varepsilon_d=0$ ),  $F(\varepsilon_m)$  shows a partial occupation weight for the levels adjacent to  $\varepsilon_d$ . Actually, this induces an algebraic suppression of the PC. It is important to note that this alternative interpretation does not require an assumption that the PC can be regarded as a Fermi-surface effect. Furthermore, this argument cannot be applied to the Kondo system where the charge fluctuation is completely suppressed.

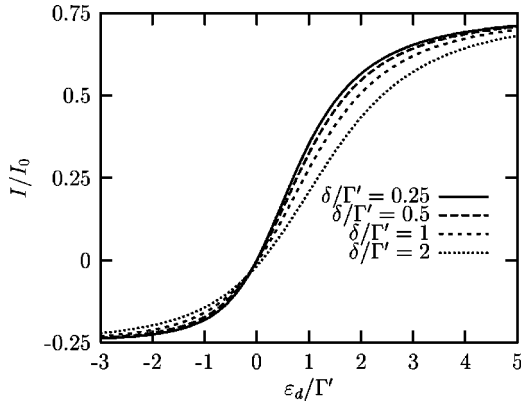


FIG. 4. Crossover of the persistent current from  $I_{ideal}^{N_e-1}(\phi)$  to  $I_{ideal}^{N_e}(\phi)$  as a function of the dot level position. Here the parameters are given as  $N_e = 188$ , and  $\Phi = \Phi_0/8$ .

In Fig. 4, the PC as a function of  $\varepsilon_d$  is displayed for different values of  $\delta/\Gamma'$ . The current shows a crossover from  $I = I_{ideal}^{N_e-1}$  at  $\varepsilon_d \ll -\Gamma'$  to  $I = I_{ideal}^{N_e}$  at  $\varepsilon_d \gg \Gamma'$ . The crossover occurs around  $-\varepsilon_d^* \leq \varepsilon_d \leq \varepsilon_d^*$ , where  $\varepsilon_d^* \approx \Gamma'$ , for small enough  $\delta/\Gamma'$ . In fact,  $I/I_0$  is expected to be a universal function of the renormalized parameter  $\varepsilon_d/\Gamma'$  in the limit of  $\delta/\Gamma' \rightarrow 0$ . One can find that the crossover value of the dot level  $\varepsilon_d^*$  increases as the level discreteness of the ring becomes important. In the  $\delta/\Gamma' \gg 1$  limit, the crossover value  $\varepsilon_d^*$  no longer corresponds to  $\Gamma'$ . This can be easily seen by considering the extremely discrete limit of the ring where  $\delta \gg 2t'$ . In this limit, only the topmost energy level, namely,  $\varepsilon_L$ , will participate in the charge transfer to the dot.<sup>1</sup> Then one can write the current in the simplified form

$$I(\phi) = I_{ideal}^{N_e-1}(\phi) - \frac{e}{2\hbar} \frac{\partial \varepsilon_L}{\partial \phi} \left( 1 + \frac{\varepsilon_d - \varepsilon_L}{\sqrt{(\varepsilon_d - \varepsilon_L)^2 + (2t')^2}} \right), \quad (9)$$

from which one can find that  $I \approx I_{ideal}^{N_e-1}$  at  $\varepsilon_d - \varepsilon_L \ll -2t'$  and  $I \approx I_{ideal}^{N_e}$  at  $\varepsilon_d - \varepsilon_L \gg 2t'$ . The crossover energy value is  $\varepsilon_d^* \approx 2t'$  instead of  $\Gamma'$ . Because  $\Gamma' = t'^2/2t \ll 2t'$ , in our study  $\varepsilon_d^*$  increases as  $\delta$  increases, which indicates that the discreteness of the ring eigenstates weakens the effect of charge transfer resonance. A similar kind of evolution from the continuum to the discrete limit was studied in Ref. 18 for a double-barrier resonant tunneling model, which shows a transition from true resonance to a semiresonance behavior of the PC. Our case can be considered as another manifestation of the crossover from true resonance to semiresonance for the case of a side-attached QD.

In conclusion, the effects of resonant charge transfer between a QD and an AB ring have been investigated by considering the PC circulating the ring. We have discovered a nontrivial algebraic suppression of the PC due to charge fluctuations in the continuum limit of ring energy levels. We have also found that the transition of diamagnetic to paramagnetic states is closely interrelated to the PC suppression, and that this property does not provide an exact one-to-one correspondence to the coherent charge response to the AB flux at Kondo resonance.

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