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Vortex lattice stability in the SO(5) model

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We study the energetics of superconducting vortices in the SO(5) model for high-T_c materials proposed by Zhang. We show that for a wide range of parameters normally corresponding to type-II superconductivity, the free energy per unit flux $\mathcal{F}(m)$ of a vortex with m flux quanta is a decreasing function of m, provided the doping is close to its critical value. This implies that the Abrikosov lattice is unstable, a behavior typical of type-I superconductors. For dopings far from the critical value, $\mathcal{F}(m)$ can become very flat, indicating a less rigid vortex lattice, which would melt at a lower temperature than expected for a BCS superconductor.

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I. INTRODUCTION

The phase diagrams of all high-temperature superconductors have a rich structure, with two prominent features at low temperatures: antiferromagnetism and superconductivity. Antiferromagnetism (AF) is seen at low doping, while superconductivity (SC) is observed if the doping exceeds a critical value.

A description of these phenomena was proposed by Zhang,[1] who observed that both superconductivity and antiferromagnetism involve spontaneous symmetry breaking. Borrowing heavily on ideas from particle physics, he suggested that the symmetries involved are unified into a larger approximate symmetry group. He presented a strong case for the group SO(5), with the SC and AF order parameters combined, forming a fundamental representation of this group.

The parameters of the potential of the Ginzburg-Landau theory determine the ground state of the model; at high temperatures, the symmetry is unbroken, while at low temperatures, either the AF or SC order parameter attains an expectation value, depending on the doping. Because of the coupling between the AF and SC order parameters, exotic possibilities for solitons in the model can arise, as was observed by Zhang in his original paper. These ideas were developed in Refs. 2–5; for related work see Refs. 6–8.

In this paper, we wish to further analyze the properties of exotic solitons in the SO(5) model. We will consider in detail SC vortices,[2,3] although other possibilities[4,5] can be analyzed similarly. We will first introduce the SO(5) model and review the reasons for suspecting that SC vortices might have AF cores.

We will then study the free energy of vortices as a function of their winding number.[9] Normally, in a type-II superconductor (one for which the Ginzburg-Landau parameter κ satisfies $\kappa > \kappa_c = 1/\sqrt{2}$) the energy divided by winding number m (or energy per flux quantum) of a vortex is an increasing function of m. This implies that vortices of winding number greater than 1 are unstable, which is one way of seeing that the vortex lattice is the preferred (lowest-energy) configuration of a superconductor placed in an external magnetic field. For a type-I superconductor ($\kappa < \kappa_c$), the situation is reversed: the energy divided by m is a decreasing function of m and the vortex lattice is unstable.

However, as will be seen below, this is not necessarily true in the SO(5) model. Under certain circumstances, the vortex energy per flux of a type-II superconductor can be a *decreasing* function of flux, indicating an instability of the vortex lattice: type-I behavior.

The underlying reason is the possibility of AF vortex cores, which have been experimentally demonstrated in underdoped La_2CuO_4 samples.[8] When AF cores occur in SC vortices, the AF order parameter makes a contribution to the vortex free energy, which is increasingly negative with increasing m.

Two factors are involved. The first is the degree to which a superconductor is type-II; the second is the proximity to SO(5) symmetry, which is explicitly broken away from ''critical doping'' (that which corresponds to the SC-AF phase boundary). These factors reinforce one another, so that a mildly type-II superconductor can easily exhibit type-I behavior, while a strongly type-II superconductor requires a doping exceedingly close to critical.

This feature of the SO(5) model gives, in principle, a dramatic prediction of that model. If one varies the doping in a given superconductor, the vortex lattice should become less and less rigid, melting more and more easily as critical doping is approached. Eventually, type-I behavior should appear.

It must be noted that the region of parameter space corresponding to critical doping appears to be experimentally delicate; in particular, the appearance of inhomogeneities (stripe formation, phase separation), Refs. 11–13, could mask the appearance of type-I behavior. Nonetheless, reduced melting temperatures should appear away from this delicate region, so that an experimental signature is still possible. Indeed, Sonier *et al.* have studied the melting of the vortex lattice in high-temperature superconductors and have observed melting at temperatures lower than expected in underdoped cuprates.[14]

II. VORTICES IN THE SO(5) MODEL

According to the SO(5) model, the low-energy dynamics of high-temperature superconductors is written in terms of a five-component real field transforming as a fundamental representation of SO(5). The upper two components, say, of this real field are the real and imaginary components of the complex order parameter of superconductivity, while the lower three components are the AF order parameter. We will call these fields $\phi=\phi_1+i\phi_2$ and $\boldsymbol{\eta}=(\eta_1,\eta_2,\eta_3)$, respectively.

The low-energy effective theory can be described in terms of the following free energy:

$$\hat{F}=\int d^2x\left[\frac{\hat{h}^2}{8\pi}+\frac{\hbar^2}{2m^*}\left|\left(-i\boldsymbol{\nabla}-\frac{e^*}{\hbar c}\hat{\mathbf{A}}\right)\phi\right|^2+\frac{\hbar^2}{2m^*}(\boldsymbol{\nabla}\boldsymbol{\eta})^2+V(\phi,\boldsymbol{\eta})\right], \tag{1}$$

where $\hat{h}=\boldsymbol{\nabla}\times\hat{\mathbf{A}}$ is the microscopic magnetic field (hats will simplify notation shortly, when we go to a description in terms of dimensionless quantities).

Much information (including the ground state) can be found by examining the potential. Including even powers of the fields up to fourth order, the most general potential is

$$V(\phi,\eta)=-\frac{a_1^2}{2}\phi^2-\frac{a_2^2}{2}\eta^2+\frac{b_1\phi^4+2b_3\phi^2\eta^2+b_2\eta^4}{4}, \tag{2}$$

where we have written $\phi=|\phi|$ and $\eta=|\boldsymbol{\eta}|$. We have given the quadratic terms negative coefficients since this is what is phenomenologically interesting. In order for the potential to be bounded from below, the quartic terms must obey the following inequalities: $b_{1,2}>0$, $b_3>-\sqrt{b_1b_2}$.

Strictly speaking, the model should be called an SO(3) $\times$SO(2) model, since this is the actual symmetry of the model. Nonetheless, the potential is invariant under the larger group SO(5) if the two mass parameters are equal and if the three quartic couplings are equal. It will be an approximate symmetry if these couplings are approximately equal. In what follows, for simplicity we will set the three quartic couplings to the same value, $b_1=b_2=b_3=b$.

In order to study SC vortices, we must restrict ourselves to the region in the parameter space that gives a SC ground state. This will be the case if the global minimum of the potential has a nonzero value of ϕ and a zero value of η. Examination of the potential shows this to be true if $\beta\equiv a_2^2/a_1^2<1$. Then the ground state is $(\phi,\eta)=(v,0)$, where $v=a_1/\sqrt{b}$. It is convenient to add a constant $a_1^4/4b$ to the potential, so that the free energy of the superconducting state in the absence of a magnetic field is zero. Note that $\beta=1$ corresponds to the SO(5) symmetric limit of the potential, and also to critical doping, since neither the SC or AF state is preferred at that value.

It is easy to see qualitatively why the core of a vortex *might* have an AF core (i.e., a core where $\eta\neq0$). In a vortex [in the SO(5) model as well as in the familiar case of conventional superconductors], the field ϕ changes in phase by 2π at spatial infinity. By continuity, ϕ must have a zero at some point, chosen to be the origin. Now let us look at how the field η fits into the situation. At infinity, $|\phi|=v$ and the energy is minimized for $\eta=0$. Inside the vortex core, however, $|\phi|\to0$. This means that the potential, viewed as a function of η with $\phi=0$, is minimized at $\eta\neq0$. Were the potential energy the only factor, η would certainly develop a nonzero expectation value inside the core of the vortex. However, the potential and gradient energies are in competition (the gradient energy being minimized if η is zero everywhere), and the minimum-energy configuration may or may not have $\eta\neq0$ in the core of the vortex, depending on which of these two competing factors dominates. The form of the potential suggests that as β is increased, there is greater likelihood of an AF core; this is indeed what is found numerically (see below, as well as Refs. 2 and 5).

As ansatz for the vortex, we use that of a conventional vortex (generalized to winding number m) with in addition an ansatz for η (whose orientation is taken to be constant), which allows for the possibility of a nonzero core,

$$\phi(x)=vf(s)e^{im\theta}, \tag{3}$$

$$\hat{A}_i(x)=a_1c\sqrt{\frac{m^*}{e^*}}\epsilon_{ij}\frac{s_j}{s}A(s), \tag{4}$$

$$\eta(x)=vn(s), \tag{5}$$

where $s=r/\lambda$, being the penetration depth, $\lambda=(m^*c^2/4\pi e^{*2}v^2)^{1/2}$.

The equations of motion of the dimensionless fields f, n, and A are (prime denotes derivative with respect to s) as follows:

$$\frac{1}{\kappa^2}\left[f''+\frac{1}{s}f'-\left(\frac{m}{s}+\kappa A\right)^2f\right]+f(1-f^2-n^2)=0, \tag{6}$$

$$\frac{1}{\kappa^2}\left(n''+\frac{1}{s}n'\right)+n(\beta-f^2-n^2)=0, \tag{7}$$

$$h'+\left(\frac{m}{\kappa s}+A\right)f^2=0, \tag{8}$$

where in the last equation h is the dimensionless magnetic field, defined by $h=-A'-A/s$. The dimensionless free energy $F=(2e^{*2}/a_1^2m^*c^2)\hat{F}$ of a vortex of winding number m is given by

$$F(m)=\int_0^\infty ds\frac{s}{2}\left\{\left(A'+\frac{A}{s}\right)^2+\kappa^{-2}\left[f'^2+\left(\frac{m}{s}+\kappa A\right)^2f^2+n'^2\right]-f^2-\beta n^2+\frac{1}{2}(f^2+n^2)^2+\frac{1}{2}\right\}. \tag{9}$$

These expressions contain three parameters: the Ginzburg-Landau parameter $\kappa=\lambda/\xi$ [where the coherence length is $\xi=(\hbar^2/m^*a_1^2)^{1/2}$], the parameter β defined above, and the winding number of the vortex m. For high-temperature superconductors, κ is usually quite large, while β is determined in sample preparation by varying the doping. [Specifically, β can be written in terms of more physical quantities as $\beta=1-8m^*\xi(T)^2\chi(\mu^2-\mu_c^2)/\hbar^2$.] $\beta>1$ corresponds to the AF phase, while $\beta<1$ describes the SC phase. We will

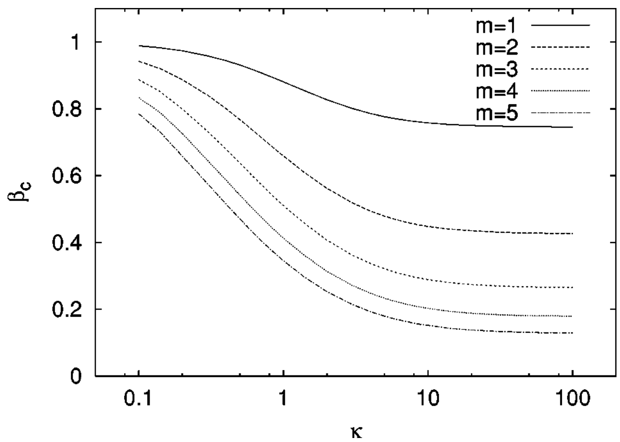

FIG. 1. β_c as a function of κ for various winding numbers.

be particularly interested in $\beta \lesssim 1$. The equations of motion are solved numerically, using the relaxation algorithm described in Ref. 15.

III. VORTEX ENERGETICS

For a given m, the vortex may or may not have an AF core, depending on the parameters of the model. We define $\beta_c(\kappa,m)$, the critical value of β, such that for $\beta > \beta_c$ the vortex core is AF, while for $\beta < \beta_c$ it is normal. Figure 1 shows β_c as a function of κ, for various values of m. One sees that as m increases, β_c decreases. This can be understood intuitively: higher m corresponds to a wider vortex core, and thus greater impetus for n to attain a nonzero value in the core.

A very useful quantity for given values of κ and β is the free energy per winding number of a vortex as a function of m, $\mathcal{F}(m) = F(m)/m$. This quantity clearly influences the behavior of a superconductor when placed in a magnetic field: if $\mathcal{F}$ increases with m, the field will penetrate in vortices of winding number 1, while if $\mathcal{F}$ decreases with m, vortices will coalesce to form large normal regions.

For a conventional superconductor, $\mathcal{F}$ increases or decreases with m for type-II or type-I superconductors, respectively.[16] The SO(5) model gives $\mathcal{F}(m)$ for a conventional superconductor by setting $\beta = 0$; then, $n(s) = 0$ and the vortex free energy (9) is identical to that of a conventional superconductor.

Figure 2 shows $\mathcal{F}(m)$ for various values of β and κ. In the first three plots, the upper curve ($\beta = 0$) represents a conventional superconductor: $\mathcal{F}(m)$ is decreasing, constant and increasing for type-I, borderline I-II, and type-II superconductors, respectively. The remaining curves reflect the effect of an AF core in the SO(5) model. The fourth plot corresponds to a large value of κ; $\beta = 0$ is not displayed in order to resolve different values of β very close to 1.

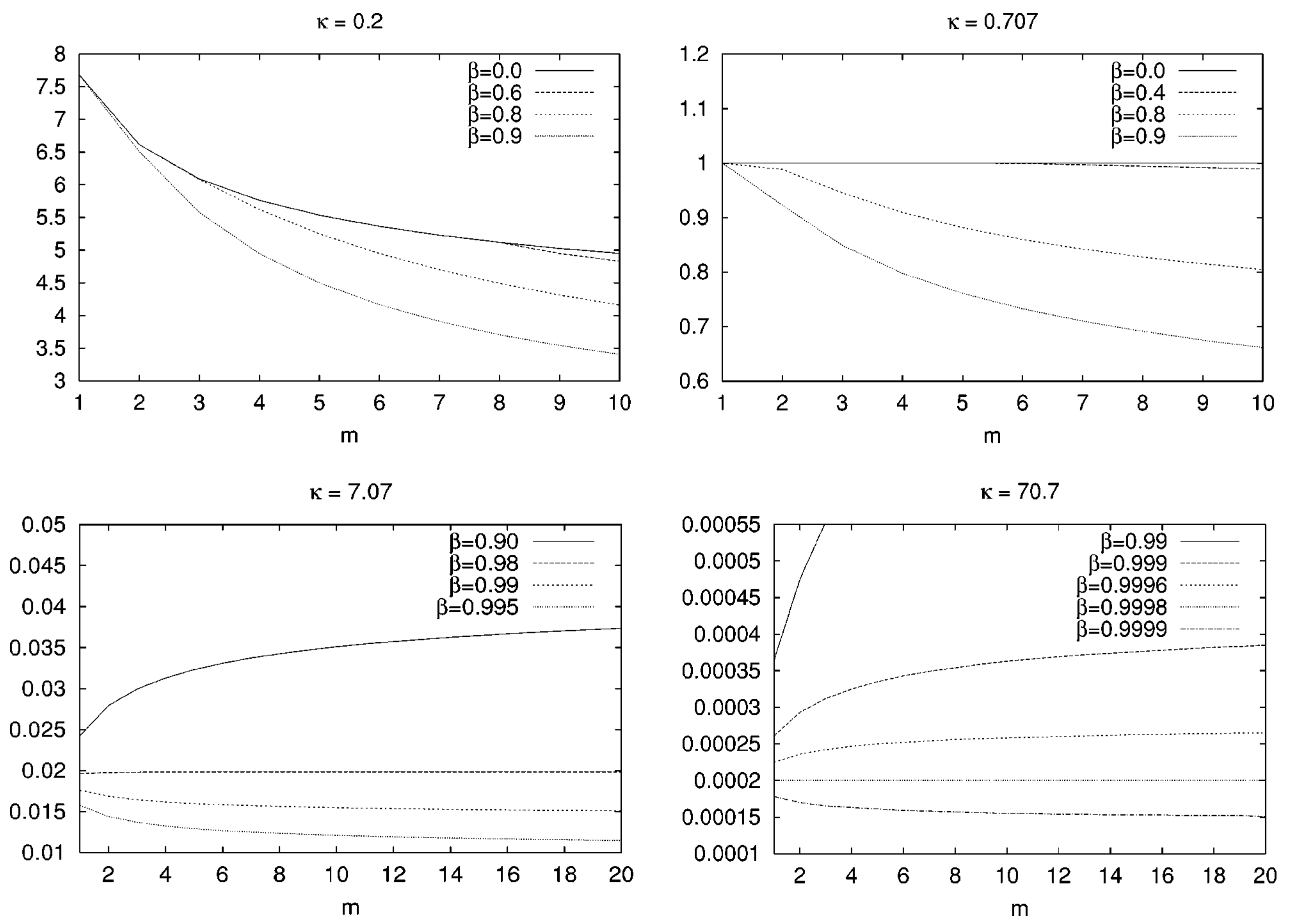

FIG. 2. $\mathcal{F}(m)$ for various values of parameters κ and β.

It is clear that the development of an AF core has a profound effect on $\mathcal{F}(m)$. This can be understood qualitatively in the following way. As mentioned above, as m increases, the vortex core width increases. This is already true for conventional superconductors, but the effect is more pronounced for SO(5) superconductors when the core becomes AF, since in that case the free-energy difference between the AF and SC states is reduced, and the potential energy (which tends to reduce the core size) is less important. A larger core size permits a more spread out magnetic field, and an overall reduced energy. [Note that anomalously large core sizes in $YBa_2Cu_3O_{7-\delta}$ at low magnetic fields have been observed,[17] though whether the SO(5) model can explain this has not yet been addressed.]

In a type-I superconductor [Fig. 2(a)] $\mathcal{F}(m)$ decreases more quickly once an AF core develops. This changes in a quantitative way, but not a qualitative way, the behavior of the material.

Things are more interesting in the case of a type-II superconductor [Figs. 2(c) and 2(d)], where a qualitative transition from type-II to type-I behavior can be achieved. This occurs at approximately $\beta=0.98$ and $\beta=0.9998$ for $\kappa=7.07$ and 70.7, respectively.

Clearly for strongly type-II superconductors (as is the case with high-temperature superconductors), β must be extremely close to 1 (doping extremely close to critical) for this transition to occur. Even before this point, there is a substantial decrease in $\mathcal{F}(m)$, meaning that the energetic savings in forming a vortex lattice (as compared to a large, normal region where the magnetic field penetrates) are substantially reduced. This would be reflected in a less rigid, more easily melted lattice. Such behavior has in fact been seen in underdoped cuprates.[14]

We have also calculated the surface energy at a normal-superconducting boundary as a function of β and κ, and find results consistent with the above analysis: a positive or negative surface energy when $\mathcal{F}(m)$ is of negative or positive slope, respectively. This will be reported elsewhere.

In summary, by analyzing the energy per unit flux of vortices as a function of winding number in the SO(5) model, we find that the development of an antiferromagnetic core has a profound effect on the behavior of a superconductor in a magnetic field. This effect depends on the doping of the material, becoming more and more strong as the doping is reduced to the critical value (that corresponding to the AF/SC transition). More specifically, we find that the degree to which a given superconductor behaves as a type-II superconductor decreases as the doping is reduced. This can result in a less rigid (more easily melted) vortex lattice, and as the doping approaches its critical value type-I behavior results.

Speight[18] has recently analyzed the static intervortex force in conventional superconductivity, by treating the vortices as point sources. It would be interesting to repeat this analysis in the SO(5) model to see the effect of the n field on these forces, and to see if the above behavior can be understood in terms of long-range forces between vortices.

It would also be interesting to extend the work of Bogomol'nyi[10] to the SO(5) model. This would circumvent much of the numerical work done in the present paper. We have not yet succeeded in doing so, however.

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