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Screening behavior of a charged Bose-Einstein condensate including many-body effects

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We study static screening effects of a charged Bose condensate in two and three dimensions. Using the Kukkonen and Overhauser approach, we show that the screening function depends on the nature of the screened particles. We derive the effective interaction between equally charged bosons screened by a charged Bose condensate. We find that screening including many-body effects gives rise to attraction between equally charged bosons.

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I. INTRODUCTION

Screening effects are essential in interacting quantum liquids and have been discussed extensively in many textbooks. For electron screening, most work was done using the random-phase approximation (RPA).[1] Many-body effects described by the local-field correction (LFC) are known to modify the screening function, compared to the RPA, when the density of charged carriers become small.[2] For electron screening, due to the spin effects, one has to use two LFCs in order to describe the effective electron-electron interaction, for instance, within the Kukkonen-Overhauser approach.[3] For electrons, the Kukkonen-Overhauser approach is a theoretical frame to describe many-body effects for interacting systems.

Our study is motivated by the discovery of the Bose condensation of neutral atoms at very low temperature. Collective modes have been studied.[4] In systems with neutral atoms, the interaction potential is modeled by a short-range interaction potential. In this paper we generalize the Kukkonen-Overhauser approach to boson systems and we apply it here to a charged Bose condensate (i) in order to see what is modified compared to electrons and (ii) to point out the implications of LFC screening on physical properties of charged boson systems. In fact, we present some interesting results concerning the screened potential in the normal space and in the Fourier space. The present study of a long-range interaction potential can also be applied to a short-range interaction potential.

For an electron gas, due to the spin, two LFCs are necessary to describe the effective electron-electron interaction.[3] If the screening is provided by a Bose condensate, a single LFC is sufficient to describe the effective boson-boson interaction. In the following, we generalize the Kukkonen-Overhauser approach to boson systems and we study the effective boson-boson interaction in the real space and the inverse space.

In Sec. II we shortly describe the model. Our theoretical results for the screened potential and the effective dielectric function in the q space are given in Sec. III. The results for the screened potential in the normal space are presented and discussed in Sec. IV. The conclusion is found in Sec. V. The detailed calculations of the screening functions for the Bose condensate using the Kukkonen and Overhauser approach is given in the Appendix.

II. MODEL

As the model, we use a d-dimensional Bose gas ($d=2,3$) with parabolic dispersion and boson density N_d. The relevant length is the effective Bohr radius $a^*=\varepsilon_L/m^*e^2$ with the Planck constant $h/2\pi=1$. m^* is the effective mass and ε_L is the dielectric constant of the background. The relevant energy scale is the effective Rydberg Ry$^*=m^*e^4/2\varepsilon_L{}^2$. The density parameter r_s is given by $r_s=[3/4\pi N_3 a^{*3}]^{1/3}$ for $d=3$ and by $r_s=[1/\pi N_2 a^{*2}]^{1/2}$ for $d=2$.

We consider a jellium model. For a negatively charged Bose condensate, a positive background charge ensure charge neutrality. In the Fourier space, the interaction potential between the bosons is denoted by $V_d(q)$. The unscreened Coulomb interaction potential between two equally charged particles is repulsive and given by $V(q)=V_d(q)$ with $V_3(q)=4\pi e^2/\varepsilon_L q^2$ and $V_2(q)=2\pi e^2/\varepsilon_L q$. A Bose particle is assumed to hold a single elementary charge. By a simple rescaling, our results could by applied to particles with different mass and charge.

III. THEORY

The screened potential $V_{ij,\mathrm{sc}}(q)$ between particle i and j is written in terms of the screening function $\varepsilon_{ij}(q)$ by

$$V_{ij,\mathrm{sc}}(q)=\frac{V(q)}{\varepsilon_{ij}(q)}. \tag{1}$$

i and j stand for charged test particles (t) or bosons (b). Our main interest in this paper is the effective boson-boson interaction $V_{b\text{-}b,\mathrm{sc}}(q)$. A test charge is, by definition, distinct from the boson medium providing the screening. In the following we use the Kukkonen-Overhauser[3] approach to calculate $V_{ij,\mathrm{sc}}(q)$. Details can be found in the Appendix.

The dielectric function $\varepsilon_{t\text{-}t}(q)$ is given by $1/\varepsilon_{t\text{-}t}(q)=[1-V_d(q)G_d(q)X_0(q)]/\{1+V_d(q)[1-G_d(q)]X_0(q)\}$ where $G_d(q)$ is the LFC function. $X_0(q)=4N_d m^*/q^2$ is the static density-density response function of the free Bose condensate.[5,6] The calculation for Bose condensate screening is given in the Appendix. In the following $G_2(q)$ and $G_3(q)$ denote the LFC functions in $d=2$ and $d=3$, respectively. For explicit forms concerning the LFC for bosons, see Ref. 7. The LFC we use in this paper fulfills the compressibility sum rule.

The dielectric function for the test-charge–test-charge interaction is given by

$$\frac{1}{\varepsilon_{t\text{-}t}(q)}=1-\frac{1}{1-G_d(q)+q^{d+1}/q_d{}^{d+1}}, \tag{2}$$

with $q_3a^*=12^{1/4}/r_s{}^{3/4}$ and $q_2a^*=2/r_s{}^{2/3}$. $1/q_d$ is the relevant length scale for screening in the Bose condensate and goes to infinity for $r_s\to\infty$, which corresponds to the unscreened limit. The second term on the right-hand side (rhs) of Eq. (2) describes screening effects. The LFC $G_d(q)$ in Eq. (2) describes many-body effects.

The screened test-charge–boson (t-b) interaction $V_{t\text{-}b,\text{sc}}(q)$ is written as $V_{t\text{-}b,\text{sc}}(q)=V(q)/\varepsilon_{t\text{-}b}(q)$ where the inverse dielectric function for test-charge–boson (t-b) interaction is $1/\varepsilon_{t\text{-}b}(q)=1/\{1+V_d(q)[1-G_d(q)]X_0(q)\}$; see Appendix. Of course, the symmetry relation $\varepsilon_{t\text{-}b}(q)=\varepsilon_{b\text{-}t}(q)$ is fulfilled. This leads to

$$\frac{1}{\varepsilon_{t\text{-}b}(q)}=1-\frac{1-G_d(q)}{1-G_d(q)+q^{d+1}/q_d{}^{d+1}}. \tag{3}$$

The screened boson-boson (b-b) interaction $V_{b\text{-}b,\text{sc}}(q)$ is given by $V_{b\text{-}b,\text{sc}}(q)=V(q)/\varepsilon_{b\text{-}b}(q)$. The inverse dielectric function for the boson-boson interaction is given by $1/\varepsilon_{b\text{-}b}(q)=\{1+V_d(q)[1-G_d(q)]G_d(q)X_0(q)\}/\{1+V_d(q)\times[1-G_d(q)]X_0(q)\}$; for details see the Appendix. Explicitly we find for the b-b interaction

$$\frac{1}{\varepsilon_{b\text{-}b}(q)}=1-\frac{[1-G_d(q)]^2}{1-G_d(q)+q^{d+1}/q_d{}^{d+1}}. \tag{4}$$

We mention that the form given above for $1/\varepsilon_{b\text{-}b}(q)$ is similar to those of $1/\varepsilon_{t\text{-}t}(q)$ and $1/\varepsilon_{t\text{-}b}(q)$: replacing one test charge by one boson introduces a factor $1-G_d(q)$ in the second term on the rhs of Eq. (2); see Eq. (3). For the b-b interaction both test charges must be replaced by bosons and a factor $[1-G_d(q)]^2$ enters the second term of the rhs of Eq. (4).

For $G_d(q)=0$ one gets the familiar RPA expression $\varepsilon_{\text{RPA}}(q)=1+V_d(q)X_0(q)$, which can be written as

$$\varepsilon_{\text{RPA}}(q)=1+q_d{}^{d+1}/q^{d+1}, \tag{5}$$

with $V_{\text{RPA}}(q)=V(q)/\varepsilon_{\text{RPA}}(q)$. For $r_s\ll 1$, many-body effects can be neglected and one obtains $\varepsilon_{t\text{-}t}(q)\approx\varepsilon_{t\text{-}b}(q)\approx\varepsilon_{\text{RPA}}(q)$. For $r_s>1$, however, one must specify which kind of interaction one wants to study. $\varepsilon_{t\text{-}b}(q)$ takes a different form from $\varepsilon_{t\text{-}t}(q)$ to account for the indistinguishability of bosons.

We obtain the long wavelength limits of the inverse dielectric functions as

$$1/\varepsilon_{\text{RPA}}(q\to 0)=q^{d+1}/q_d{}^{d+1}\propto q^{d+1}, \tag{6a}$$

$$1/\varepsilon_{t\text{-}t}(q\to 0)=-G_d(q\to 0)\propto -q^{d-1}, \tag{6b}$$

$$1/\varepsilon_{t\text{-}b}(q\to 0)=q^{d+1}/q_d{}^{d+1}\propto q^{d+1}, \tag{6c}$$

and

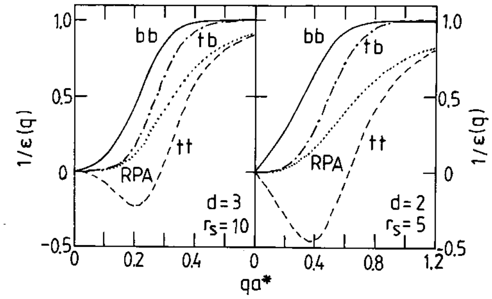

FIG. 1. Inverse dielectric function $1/\varepsilon_{ij}(q)$ versus wave number for $d=3$ ($r_s=10$) and $d=2$ ($r_s=5$). The solid (dotted) line represents the dielectric function of the boson-boson (b-b) (RPA) interaction. The dashed (dashed-dotted) line represents the dielectric function of the test-charge–test-charge interaction (t-t) [test-charge–boson interaction (t-b).].

$$1/\varepsilon_{b\text{-}b}(q\to 0)=G_d(q\to 0)\propto q^{d-1}. \tag{6d}$$

We mention that $G_d(q\to 0)$ is related to the compressibility of the Bose condensate. In the limit of short wavelengths or large wave numbers we find $1/\varepsilon_{\text{RPA}}(q\to\infty)=1/\varepsilon_{t\text{-}t}(q\to\infty)=1/\varepsilon_{t\text{-}b}(q\to\infty)=1/\varepsilon_{b\text{-}b}(q\to\infty)=1$.

In Fig. 1 we show $1/\varepsilon_{ij}(q)$ $(i,j=t,b)$ versus q for $d=3$ and $d=2$, with density parameters $r_s=10$ and $r_s=5$, respectively. Many-body effects are more important in $d=2$ than in $d=3$. Figure 1 illustrates that one must be careful in choosing the appropriate screening function, corresponding to the physical problem one considers. Even the sign of the inverse screening function can change. It strongly points to the fact that many-body effects are an essential ingredient in order to get a "correct" screening function.

IV. RESULTS AND DISCUSSION

Now we present results for the screened potential in the real space. This is of importance because our physical intuition is more adapted to this space. The screened Coulomb interaction is given by

$$V_{ij,\text{sc}}(r)=\frac{1}{2\pi^2 r}\int_0^\infty dq\, q\,\sin(qr)V_{ij,\text{sc}}(q) \tag{7}$$

in $d=3$ and by

$$V_{ij,\text{sc}}(r)=\frac{1}{2\pi}\int_0^\infty dq\, qJ_0(qr)V_{ij,\text{sc}}(q) \tag{8}$$

in $d=2$. $J_0(x)$ is the zero-order Bessel function of the first kind. The above formulas allow to calculate $V_{ij,\text{sc}}(r)$ including many-body effects. Bound state energies for t-t, t-b, and b-b interactions are given elsewhere.[8] In the following, we concentrate on the b-b-interaction, the interaction potential between equally charged screened bosons.

Our results for the screened boson-boson interaction $V_{b\text{-}b,\text{sc}}(r)$ versus r are given in Fig. 2 for $d=3$ and $d=2$ with $r_s=0.7$. A small r_s value was chosen in order to keep the difference with $V_{\text{RPA}}(r)$ small. Again we notice that many-body effects are larger in $d=2$ than in $d=3$. For small

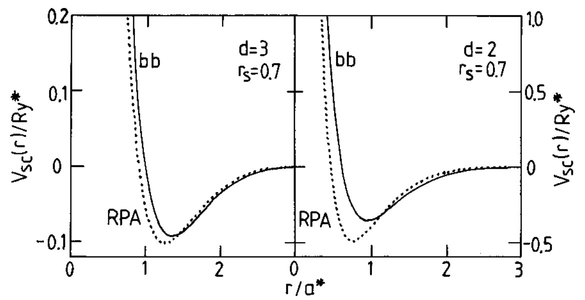

FIG. 2. Screened potential $V_{\mathrm{BCb\text{-}b}}(r)$ versus distance r for $r_s = 0.7$ in $d=3$ and $d=2$ for equally charged particles. The solid (dotted) lines represent the boson-boson (b-b) (RPA) interaction.

distances, the screened potential is strongly repulsive and becomes attractive at intermediate distances with a negative minimum $V_{b\text{-}b,\mathrm{sc}}(r_{\mathrm{min}})$ at $r=r_{\mathrm{min}}$. In the following we study in detail the behavior of $V_{b\text{-}b,\mathrm{sc}}(r_{\mathrm{min}})$ and $r=r_{\mathrm{min}}$ versus r_s.

The minimum of the screened potential represents an overscreening effect. Within the RPA one gets $V_{\mathrm{RPA}}(r_{\mathrm{min}})/\mathrm{Ry}^* = -0.0781/r_s^{3/4}$ in $d=3$ and $V_{\mathrm{RPA}}(r_{\mathrm{min}})/\mathrm{Ry}^* = -0.384/r_s^{2/3}$ in $d=2$.[8] It is clear from these results that overscreening effects are more important for *small* r_s. $V_{b\text{-}b,\mathrm{sc}}(r_{\mathrm{min}})$ and r_{min} versus r_s are shown in Fig. 3 for $d=3$ and $d=2$. In the high density limit, as expected, one finds $V_{b\text{-}b,\mathrm{sc}}(r_{\mathrm{min}}) = V_{\mathrm{RPA}}(r_{\mathrm{min}})$. On the other hand, for large r_s one finds $|V_{b\text{-}b,\mathrm{sc}}(r_{\mathrm{min}})| \ll |V_{\mathrm{RPA}}(r_{\mathrm{min}})| < |V_{t\text{-}b,\mathrm{sc}}(r_{\mathrm{min}})| \ll |V_{t\text{-}t,\mathrm{sc}}(r_{\mathrm{min}})|$. This means that overscreening effects are less pronounced for the b-b-interaction than for the RPA.

From Fig. 1 we conclude that no evident criterion for the existence of an attractive part can be found from the screened potential in the Fourier space. For instance, the screened potential $V_{\mathrm{RPA}}(q)$ and $V_{b\text{-}b,\mathrm{sc}}(q)$ are positive for any q. On the other hand, in the real space, it is quite clear that the screened potential $V_{\mathrm{RPA}}(r)$ and $V_{b\text{-}b,\mathrm{sc}}(r)$ always has an attractive part, see Fig. 2, and bound states exist.[8]

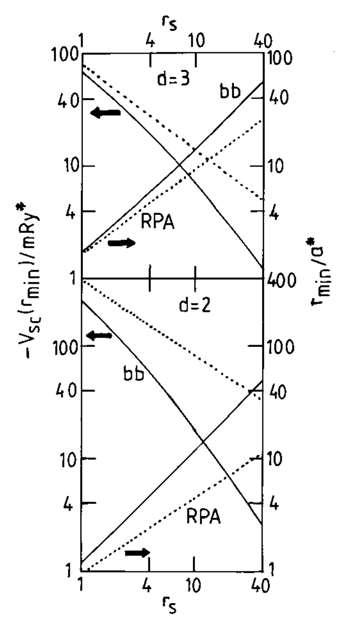

FIG. 3. r_{min} and $V_{\mathrm{SC}}(r_{\mathrm{min}})$ versus r_s for $d=3$ and $d=2$. The solid (dotted) lines represent the boson-boson (b-b) (RPA) interaction.

In order to better understand the origin of the oscillations of the screened potential, as shown in Fig. 2, we consider the case of an electron gas in $d=2$ with a finite valley degeneracy g_v. One finds[9] $X_0(q \leqslant 2k_F) = \rho_F$ and $X_0(q>2k_F) = \rho_F[1-(1-4k_F{}^2/q^2)^{1/2}]$ with the density of states $\rho_F = g_v m^*/\pi$ and the electron density $N_2 = g_v k_F{}^2/2\pi$. For large wave numbers, one gets $X_0(q \gg 2k_F) = 2\rho_F k_F{}^2/q^2 = 4m^* N_2/q^2$, which is just the response function of the Bose condensate. A similar result, $X_0(q \gg 2k_F) = 4m^* N_3/q^2$, is obtained for the three-dimensional electron gas. For a given density and for $g_v \to \infty$ the Fermi wave number k_F goes to zero. Therefore, one can argue that the oscillations in the Bose-condensate screened potential represent Friedel oscillations[10] in the limit of $g_v \to \infty$.

The expression given for the static dielectric function $1/\varepsilon_{ij}(q)$ can be generalized to include the frequency dependence $1/\varepsilon_{ij}(q,\omega)$ when replacing $X_0(q)$ by $X_0(q,\omega)$. $X_0(q,\omega)$ is the dynamical density-density response function of the free charged Bose condensate, see Appendix. In textbooks the collective modes are defined by the equation $1/\varepsilon_{t\text{-}t}(q,\omega)=0$. In fact, the same condition holds for $1/\varepsilon_{t\text{-}t}(q,\omega)=0$, $1/\varepsilon_{t\text{-}b}(q,\omega)=0$, and $1/\varepsilon_{b\text{-}b}(q,\omega)=0$ as well, namely,

$$1 + V_d(q)[1 - G_d(q)]X_0(q,\omega) = 0. \tag{9}$$

This shows that the different dielectric functions described in this paper lead to the same dispersion for the collective modes. In fact, the genuine collective modes should be defined by $1/\varepsilon_{b\text{-}b}(q,\omega)=0$. We note that in general the LFC $G_d(q,\omega)$ depends on the frequency. Here we considered the static LFC only.

We believed that the study of $1/\varepsilon_{ij}(q)$ and of the effective boson-boson interaction $V_{b\text{-}b,\mathrm{sc}}(r)$ needed a detailed discussion, as presented in this paper. We would like to stress that effective interaction potentials and attraction effects between equally charged particles are hot topics, not only for indistinguishable particles[11] as fermions or bosons, but also when particles are distinguishable as for classical particles.[12,13]

V. CONCLUSION

We have studied some screening functions $\varepsilon_{ij}(q)$ of a charged Bose gas in three and two dimensions including many-body effects by generalizing the Kukkonen-Overhauser approach to Bose particles. The inverse dielectric function $1/\varepsilon_{ij}(q)$ was shown to depend on the nature of the particles (test particles or bosons), which are screened by the condensate. Detailed results have been presented for the effective boson-boson interaction. It was shown that overscreening effects are a quite natural phenomena if screening is due to bosons. In two dimensions many-body effects are more important than in three dimensions.

APPENDIX

We apply the Kukkonen-Overhauser approach[3] to a Bose condensate. In the following we assume that the Bose condensate is perturbed by an external test charge $\rho_{ext}(q,\omega)$. The external potential is given by $V_{ext}(q,\omega) = V_d(q)\rho_{ext}(q,\omega)$, with $V_3(q)=4\pi e^2/\varepsilon_L q^2$ and $V_2(q) = 2\pi e^2/\varepsilon_L q$. We write this equation as $V_{ext}=V_d\rho_{ext}$.

The external perturbation induces a density variation Δn in the Bose condensate, which is given in linear response theory as $\Delta n = -X_0 V_{b\text{-}t}$. X_0 as the density-density response function. The test-charge–test-charge interaction potential, which is the total potential seen by another test charge, is written as $V_{t\text{-}t}=V_d(\rho_{ext}+\Delta n)$. The dielectric function $\varepsilon_{t\text{-}t}$ is defined as $V_{t\text{-}t}=V_{ext}/\varepsilon_{t\text{-}t}$. The boson–test-charge interaction potential $V_{b\text{-}t}$ is given by $V_{b\text{-}t}=V_{t\text{-}t}+V_{xc,b\text{-}t}$ with an exchange-correlation potential $V_{xc,b\text{-}t}$ caused by Δn. We use the approximation that $V_{xc,b\text{-}t}$ is a local potential: $V_{xc,b\text{-}t}= -V_d G_d \Delta n$. This equation defines the LFC $G_d(q,\omega)$. Notice that within the jellium model the total direct Coulomb interaction vanishes. The dielectric function ε_r is defined as $V_{t\text{-}b}=V_{ext}/\varepsilon_{t\text{-}b}$.

From these equations it follows that $1/\varepsilon_{b\text{-}t}$ is expressed as $1/\varepsilon_{b\text{-}t}=1/[1+V_d(1-G_d)X_0]$. With $V_{t\text{-}t}=V_{b\text{-}t}-V_{xc,b\text{-}t}$ one can show that the dielectric function $1/\varepsilon_{t\text{-}t}$ is given by $1/\varepsilon_{t\text{-}t}=[1-V_dG_dX_0]/\varepsilon_{t\text{-}b}=[1-V_dG_dX_0]/[1+V_d(1-G_d)X_0]$. From these equations we finally derive

$$1/\varepsilon_{t\text{-}t}(q,\omega)=1-1/[1-G_d(q,\omega)+1/V_d(q)X_0(q,\omega)] \tag{A1}$$

and

$$1/\varepsilon_{t\text{-}b}(q,\omega)=1-[1-G_d(q,\omega)]/[1-G_d(q,\omega) + 1/V_d(q)X_0(q,\omega)]. \tag{A2}$$

In the static case the LFC $G_d(q,\omega)$ and the response function $X_0(q,\omega)$ are replaced by $G_d(q)$ and $X_0(q)$. We obtain expressions for $1/\varepsilon_{ij}(q)$ shown in Fig. 1.

For the boson-boson interaction we find $V_{b\text{-}b0}=V_{b\text{-}t} + V_{xc,b\text{-}b}$ with $V_{b\text{-}t}=V_{t\text{-}t}+V_{xc,b\text{-}t}$, $V_{xc,b\text{-}t}=-V_dG_d\Delta n$, $V_{xc,b\text{-}b}=-V_dG_d\rho$, and $\Delta n=-X_0V_{b\text{-}b0}$. ρ is the charge density of the Bose condensate. Consequently, one obtains $V_{xc,b\text{-}t}=-V_dG_dX_0V_{b\text{-}b0}$. From these equations one derives $V_{b\text{-}b0}=[1-G_d]V_d\rho/[1+V_d(1-G_d)X_0]$.

The real boson-boson interaction potential is given by $V_{b\text{-}b}=V_{b\text{-}b0}+V_dG_d\rho$. We define the inverse dielectric constant $1/\varepsilon_{b\text{-}b}$ by $V_{b\text{-}b}=V_d\rho/\varepsilon_{b\text{-}b}$ and get $1/\varepsilon_{b\text{-}b}=[1+V_d(1-G_d)G_dX_0]/[1+V_d(1-G_d)X_0]$. This can be written as

$$1/\varepsilon_{b\text{-}b}(q,\omega)=1-[1-G_d(q,\omega)]^2/[1-G_d(q,\omega) + 1/V_d(q)X_0(q,\omega)]. \tag{A3}$$

The expression of Kukkonen and Overhauser[3] for the electron-electron interaction $V_{e\text{-}e}(q,\omega)$ contains two terms, one spin-independent term $V_0(q,\omega)$ [depending on $G_+(q,\omega)$] and one spin-dependent term $J(q,\omega)$ [depending on $G_-(q,\omega)$]. The expression (A3) for bosons can be obtained from the expression for electrons in Ref. 3 [Eqs. (34) and (35)] by using $J(q,\omega)=0$ and $G_+(q,\omega)=G_d(q,\omega)$.

Finally, we note that $X_0(q,z)$ with $X_0(q,\omega)=X_0(q,z=\omega+i0)$ is expressed as

$$X_0(q,z)=-2N_d\varepsilon(q)/[z^2-\varepsilon(q)^2], \tag{A4}$$

with $\varepsilon(q)=q^2/2m^*$. For $z=0$ one gets $X_0(q)=X_0(q,z=0)=4N_dm^*/q^2$.

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