p-wave Cooper pairing of fermions in mixtures of dilute Fermi and Bose gases

D. V. Efremov[1,2] and L. Viverit[3]
[1]*Max-Planck-Institut für Physik Komplexer Systeme, Nöthnitzer Str. 38, 01187 Dresden, Germany*
[2]*Kapitza Institute for Physical Problems, 117334 Moscow, Russia*
[3]*Dipartimento di Fisica, Università di Milano, via Celoria 16, 20133 Milano, Italy*
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We predict a p-wave Cooper pairing of the spin-polarized fermions in a binary fermion-boson mixture due to the exchange of density fluctuations of the bosonic medium. We then examine the dependence of the Cooper pairing temperature on the parameters of the system. We finally estimate the effect of combining the boson-induced interaction with other pairing mechanisms, e.g., the Kohn-Luttinger one, and find that the critical temperature of p-wave Cooper pairing can be realistic for experiment.

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The investigation of quantum degenerate gases has a long history. The latest step ahead in this direction has been the realization of Bose-Einstein condensation in gases of the alkali elements ^{87}Rb, ^{7}Li, and ^{23}Na (Refs. 1–3) in the confined geometry of optomagnetic traps. Currently experimentalists are concerned with obtaining fermionic superfluidity. The regime of quantum degeneracy was already achieved experimentally in samples of potassium[4] and lithium atoms.[5] Degeneracy was reached by the first group by evaporatively cooling together two hyperfine states of the same fermionic element (^{40}K), and by the second group by sympathetically cooling the fermionic isotope ^{6}Li with the bosonic one ^{7}Li. Other groups are progressing along similar lines.

Various papers appeared lately discussing what type of Cooper pairing (s wave or p wave) is more realistic to achieve.[6–11] From one side s-wave pairing (with orbital angular momentum of the pair $L=0$) has a higher critical temperature than p-wave (pairing $L=1$) for typical parameters. However, it takes place only between atoms of different spins (hyperfine components). This imposes a very stringent constraint on the densities n_1 and n_2 of the components which form the pairs. One should have $|(n_1-n_2)/(n_1+n_2)|\leqslant T_c/\varepsilon_F \ll 1$. (Here and below, we let $k_B=\hbar=1$). From the experimental point of view, with alkalis this condition may be hard to achieve as the two hyperfine components are manipulated independently. Since, on the other hand, a p-wave Cooper pair is formed by atoms in the same spin component, the restriction on the densities is taken away, and it is therefore relevant to ask how realistic it would be to consider observing that instead.

As a contribution to this field, in this paper we want to determine the effect of the presence of a Bose gas (Bose condensate) on the p-wave Cooper pairing of fermions. Low densities of atomic gases give one the possibility to introduce small parameters in the theory—namely, the gas parameters $(an^{1/3})\ll 1$, where a stands for the appropriate two-particle scattering length in vacuo and n for the density of the Bose or the Fermi gas. Using these small parameters we calculate the effective interaction. We show that the exchange of boson density fluctuations gives an attractive contribution to the effective interaction of two fermions with bosonic superfluid background in channels with nonzero angular momenta. The largest one corresponds to the p-wave channel. Hence fermions in binary mixtures of Bose and Fermi gases are unstable toward p-wave Cooper pairing. We calculate the associated T_c, and determine the Fermi and Bose densities which provide the highest value of T_c compatible with the constraints imposed by the instability to phase separation. At the end of the paper we show that the boson-induced interaction can be combined with some other p-pairing mechanism, in which case it acts to increase the critical temperature significantly.

We start with the Hamiltonian of almost ideal Fermi and Bose gases,

$$H=H_F+H_B+H_{BF}, \tag{1}$$

with

$$H_F=\sum_{\alpha,p}\xi_{\alpha,p}a^{\dagger}_{\alpha,p}a_{\alpha,p}+\frac{U_{FF}}{2}\sum_{\alpha,\beta;pp'q}a^{\dagger}_{\alpha,p-q}a^{\dagger}_{\beta,p'+q}a_{\beta,p'}a_{\alpha,p},$$

$$H_B=\sum_{p}\varepsilon_p b^{\dagger}_p b_p+\frac{U_{BB}}{2}\sum_{pp'q}b^{\dagger}_{p-q}b^{\dagger}_{p'+q}b_{p'}b_p,$$

$$H_{BF}=U_{BF}\sum_{pp'q}a^{\dagger}_{\alpha,p-q}b^{\dagger}_{p'+q}b_{p'}a_{\alpha,p},$$

where $\xi_{\alpha,p}=(p^2/2m_F-\varepsilon_{F,\alpha})$ and $\varepsilon_p=p^2/2m_B$ are the kinetic energies of the Fermi and Bose gases respectively, $\varepsilon_{F,\alpha}$ is the Fermi energy of the Fermi gas with spin (hyperfine component) α, and U_{FF}, U_{FB}, and U_{BB} are two-particle interaction constants for Fermi-Fermi, Fermi-Bose, and Bose-Bose interactions. The constants are related to the two-body scattering lengths a_{BB}, a_{BF}, and a_{FF} *in vacuo* as $U_{FF}=4\pi a_{FF}/m_F$, $U_{BB}=4\pi a_{BB}/m_B$, and $U_{BF}=4\pi a_{BF}/m_{BF}$; and $m_{BF}=2m_B m_F/(m_B+m_F)$.

In general (apart from the case of resonance scattering) the harmonics of the scattering amplitude for slow particles are proportional to $f_l\sim a(ap_F)^{2l}$,[12] where a is a length of the order of a s-scattering length. For example, the $l=1$ bare contribution goes like $U^l_{FF}\nu_f\sim(ap_F)^3$. This contribution is very small and can in general be neglected if some other triplet pairing mechanism is present. In the case of fermions in two-spin states, for instance, the indirect interaction by polarization of the fermions in the other spin state (Kohn-Luttinger mechanism),[13] provides a contribution of order $(ap_F)^2$, and is therefore more important than the bare one.

A standard procedure[14] yields the critical temperature for pairing with given angular momentum l,

$$T_{cl}=\tilde{\varepsilon}_F\ \exp\left\{-\frac{1}{\nu_F|\tilde{\Gamma}_l|}\right\}, \tag{2}$$

where $\tilde{\Gamma}_l<0$ is the lth spherical harmonic of the irreducible vertex, $\tilde{\varepsilon}_F$ is some energy parameter of the order of the Fermi energy, and ν_F is the density of states on the Fermi level. The real transition corresponds to the angular momentum with the maximum allowed temperature. The effective interaction between two Fermi particles $\tilde{\Gamma}$ is the sum of the bare one U_{FF}, the interaction via polarization of the bosonic medium (exchange of density fluctuations) U_{FBF}, and possibly the interaction via polarization of the other fermionic species U_{FFF} if fermions in more than one spin orientation are present.

We assume temperatures much smaller than those of degeneracy. The correctness of this hypothesis will be confirmed by the results found. The effective interaction of Fermi quasiparticles on a mass surface with zero transfer energy is given by $U_{FBF}(q)=U_{BF}^2\chi(q,\omega=0)$, where $\chi(q,\omega)$ is the density-density response function for an almost ideal Bose gas at zero temperature.[15] Since we are interested in the low-density limit of Bose and Fermi gases and $U_{BB}\gg\nu_F U_{BF}^2\sim(a_{BF}p_F)U_{BF}$, we can neglect the renormalization of boson density-density correlation function due to Bose-Fermi interaction, and we can write, to first order in the gas parameter,[16]

$$\chi(q,\omega)=\frac{1}{\omega^2-\varepsilon_q^0(\varepsilon_q^0+2n_BU_{BB})}\frac{n_Bq^2}{m_B}. \tag{3}$$

So the effective interaction of Fermi atoms with zero transfer energy reads

$$U_{FBF}(q,0)=-\frac{U_{BF}^2}{U_{BB}}\left[1+\left(\frac{q}{2m_Bs}\right)^2\right]^{-1}, \tag{4}$$

where $s=(n_BU_{BB}/m_B)^{1/2}$ is the sound velocity in the boson gas. We recall that stability requires $U_{BB}>0$.

A direct calculation of the first three partial components of U_{FBF} gives the result

$$\nu_F U_{FBF}^l=-\nu_F\frac{U_{BF}^2}{U_{BB}}R_l(p_F/m_Bs), \tag{5}$$

with

$$R_0(x)=\frac{\ln(1+x^2)}{x^2},$$

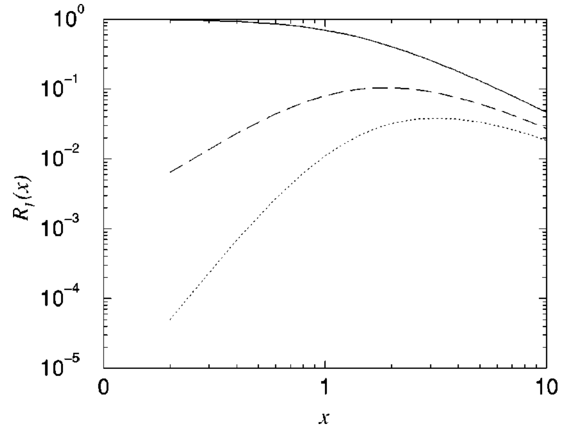

FIG. 1. Functions $R_0(x)$ (solid line), $R_1(x)$ (dashed line), and $R_2(x)$ (dotted line).

$$R_1(x)=\frac{2}{x^2}\left[\left(\frac{1}{x^2}+\frac{1}{2}\right)\ln(1+x^2)-1\right],$$

$$R_2(x)=\frac{6}{x^4}\left[\left(\frac{1}{x^2}+1+\frac{x^2}{6}\right)\ln(1+x^2)-\left(1+\frac{x^2}{2}\right)\right].$$

The functions $R_0(x)$, $R_1(x)$, and $R_2(x)$ are shown in Fig. 1. The strongest interaction is in the channel with $l=0$. For large l one can show that R_l drops off exponentially in l. Therefore this contribution to the effective interaction for $l>2$ can be neglected. The functions for $l\neq 0$ are strongly nonmonotonic, in contrast with the zero angular momentum case. For instance, the maximum of $R_1(x)$ is obtained for $x_{opt}=1.86$ $[R_1(1.86)=0.1]$. The maximum gives the optimal ratio of Bose and Fermi components for given scattering lengths. We would like to note that our approach differs from the one that was proposed by Bardeen, Baym, and Pines[17] for the description of the effective interaction between fermions due to exchange of density fluctuation of the Bose condensate in solutions ^{3}He-^{4}He. Using $v_F\ll s$ they approximated the spectrum of the Bose condensate of ^{4}He by a phonon spectrum. This is equivalent to neglecting the momentum dependent term in Eq. (4). In this approximation only s-wave pairing is possible, as all harmonics with $l\geq 1$ vanish. The effective attraction we point out is thus due to the specific form of the Bogoliubov spectrum of the bosons.

Let us consider a binary gas consisting of one fermionic and one bosonic species (structureless fermions and bosons). The Cooper pairing in the s-wave channel is prohibited by the Pauli principle. It can exist in the p-wave channel owing to density fluctuations of bosonic medium with the effective attractive interaction given by formula (5). Note that the value of the ratio U_{BF}^2/U_{BB} and the densities of the gases cannot be arbitrary. The restriction is associated with the phase separation at high densities of the binary mixture into two regions: a Fermi-Bose mixture (with densities n_B^{sep} and n_{F1}) and a pure fermionic region (with density n_{F2}). This phase separation into two large regions is a full analog of that observed in the mixtures of ^{3}He-^{4}He. To check the sta-

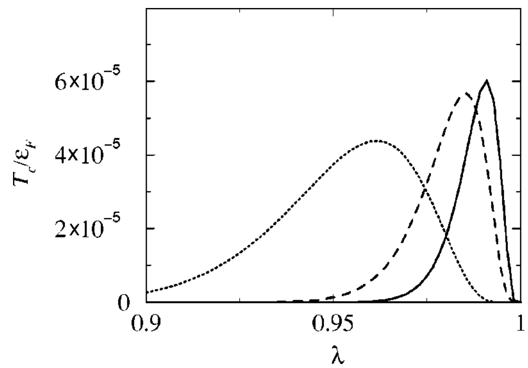

FIG. 2. T_c/ε_F vs λ for a binary boson-fermion mixture for different values of the coefficient β. The solid curve is for $\beta=5$, the dashed one for $\beta=3$, and the dotted one for $\beta=1$.

bility of the mixture against phase separation we rewrite the expression for U_{FBF} in dimensionless parameters λ, α, and β in the spirit of Ref. 18:

$$\lambda=\frac{\nu_F U_{BF}^2}{U_{BB}}=\frac{2}{\pi}\frac{m_B m_F}{m_{BF}^2}\frac{a_{BF}}{a_{BB}}a_{BF}p_F. \tag{6}$$

The expression for λ is exactly the factor in front of R_l in Eq. (5),

$$n_B^{sep}=\frac{\varepsilon_F}{U_{BF}}(y^2-1)=\frac{(6\pi^2 n_F)^{2/3}}{8\pi a_{BF}}\frac{m_{BF}}{m_F}(y^2-1), \tag{7}$$

where $y\geqslant 1$ is solution to the equation

$$-15\lambda^{-1}(y+1)^2+8y^3+16y^2+24y+12=0. \tag{8}$$

We then find

$$x^{-2}=\lambda^{-1}\beta[y(\lambda)^2-1], \tag{9}$$

where

$$\beta=\frac{\alpha}{\pi}\frac{m_B}{m_{BF}}p_F a_{BF}, \quad \alpha=\frac{n_B}{n_B^{sep}(\lambda)}.$$

In the case of phase separation y is the ratio of fermionic densities in the two regions $y=(n_{F2}/n_{F1})^{1/3}\geqslant 1$ and n_B^{sep} is the density of bosonic component in bosonic-fermionic mixture region. The authors of Ref. 18 showed that there are three possibilities: (a) a single uniform phase, (b) a purely fermionic phase coexisting with a mixed phase, and (c) a purely fermionic phase coexisting with a purely bosonic. Let us examine all three possibilities.

The single uniform phase is stable provided the conditions $\lambda\leqslant 1$ and $\alpha\leqslant 1$ are fulfilled. This immediately gives $\nu_F U_{BF}^2/U_{BB}<1$, and the value of the effective interaction is restricted by $\nu_F U_{FBF}<0.1$, which corresponds to temperatures of Cooper pairing in the binary mixtures about five orders of magnitude less than the Fermi energy. In Fig. 2 we plot the critical temperature as a function of λ for given β. We see that the maximum of the critical temperature increases with an increase of β. Note that for given scattering lengths the maximum is in region of the parameters close to the phase separation ($\alpha=n_B/n_B^{sep}\rightarrow 1$). If the total density of the boson gas is larger than n_B^{sep} ($\alpha>1$), the binary mixture undergoes a phase separation into two phases: a purely fermionic phase and a mixed Fermi-Bose phase. In this case in the mixed phase the density of the bosonic gas is $n_{B1}=n_B^{sep}$, and the density of the Fermi gas: n_{F1}. These are determined by the system of equations (6)–(9). Our result, obtained for the single uniform phase, is still valid within the mixed phase if the appropriate densities of Fermi and Bose gases are used. The third possibility is the coexistence of a purely fermionic phase with a purely bosonic one, which exists for much higher total densities of bosons and fermions. In this case of course there is no effective interaction between fermions due to the exchange of boson density fluctuations.

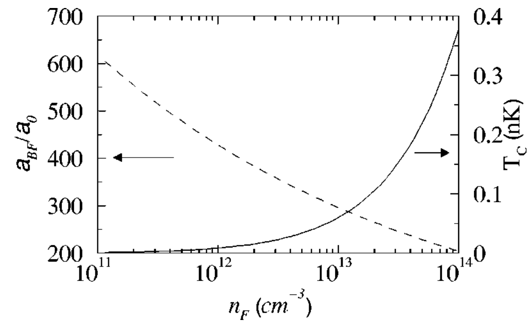

FIG. 3. Optimal scattering length a_{BF} and corresponding T_c for Cooper pairing, vs the fermionic density in a binary boson-fermion mixture of ^{6}Li and ^{87}Rb.

We can conclude that the contribution of the exchange of density fluctuations of the bosonic medium has its maximum for the set of parameters close to those of phase separation of a single uniform phase into two coexisting phases: a mixed phase and a pure fermionic one.

Let us make some estimates for real systems. Take a binary mixture of fermionic ^{6}Li and bosonic ^{87}Rb, and choose for ^{6}Li the density $n_F\sim 10^{12}$ cm^{-3}. This corresponds to $\varepsilon_F/k_B\sim 600$ nK. The boson-boson scattering length is $a_{BB}=110a_0$, a_0 being the Bohr radius. The boson-fermion scattering length is unknown. In order to obtain T_c close to its maximum value for binary boson-fermion mixture, we should obtain λ close to 0.95. For the given mixture this corresponds to $a_{BF}\sim 450a_0$. Substituting these values into (5)–(9), and taking for the Bose component the density $n_B\sim 10^{13}$ cm^{-3} we get $T_c\sim 10^{-2}$ nK. Similar calculations for $n_F\sim 10^{14}$ cm^{-3} imply $a_{BF}\sim 200a_0$, and with $n_B\sim 5\times 10^{14}$ cm^{-3}, $T_c\sim 0.5$ nK. In Fig. 3 we summarize the optimal parameters of the system and the corresponding critical temperatures for *p*-wave pairing in a binary mixture.

There is a possibility of increasing T_c by combining the above-mentioned mechanism with either *p*-wave quasibound resonance for the scattering of Fermi atoms, or by considering a mixture of two spin states of fermions with one of bosons. In the former case the irreducible vertex in Eq. (2) is determined by the sum of the interactions due to polarization of bosons and the (now large) bare *p*-wave attractive scattering of Fermi atoms.[19]

In the case of two species of fermions with one of bosons, again the effective interaction has two contributions: from boson density fluctuations and from the Kohn-Luttinger mechanism. The effective interaction in the latter is a non-monotonic function of the ratio of the densities of the different hyperfine components (see Refs. 10 and 11). Its maximum is $\nu_F U_{eff} \sim 0.058(ap_F)^2$ which corresponds to a ratio $n_1 \sim 2.8 n_2$.[20]

For optimal parameters[21] and a density $n_1 = 10^{14}$ cm^{-3} the critical temperature reads

$$T_c \sim \tilde{\varepsilon}_F \exp\left\{ -\frac{1}{\nu_F U_{FBF} + 0.058(a_{FF}^s p_F)^2} \right\}.$$

$T_c \sim 1$ nK and 20 nK, respectively, for bare s-wave Fermi-Fermi scattering lengths $|a_{FF}^s| = 500 a_0$ and $1000 a_0$. Note that the value of the critical temperature obtained is valid for $a_{FF}^s > 0$ as well as for $a_{FF}^s < 0$. A pure Kohn-Luttinger mechanism would give $T_c = 10^{-30}$ and 10^{-5} nK, respectively, for the given scattering lengths, so that the main contribution comes from the boson-induced term. For ^{6}Li, however, the s-wave scattering length between two different hyperfine states is $a_{FF}^s = -2160 a_0$. The critical temperature with this scattering length is ~ 0.5 μK. For a pure Kohn-Luttinger mechanism it would be ~ 0.1 μK, which shows the strong effect that bosons also have in this case.[22] The full analogy with mixtures ^{3}He-^{4}He shows that here both single uniform phase and phase-separated states are possible, and explicit calculations for the case of two Fermi species and a Bose one need to be carried out.

In conclusion we showed that the fermions in a (typical) dilute binary mixtures of Fermi and Bose gases are unstable toward p-wave Cooper pairing. This is due to their effective attraction arising from boson polarization. We then calculated how the associated T_c can be maximized. Although the highest T_c's found do not seem to be presently experimentally observable, we showed that the mechanism may be used to enhance pairing when combined with others.

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