

Effective action approach and Carlson-Goldman mode in d -wave superconductors

Sergei G. Sharapov* and Hans Beck†

Institut de Physique, Université de Neuchâtel, 2000 Neuchâtel, Switzerland

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We theoretically investigate the Carlson-Goldman (CG) mode in two-dimensional clean d -wave superconductors using the effective “phase-only” action formalism. In conventional s -wave superconductors, it is known that the CG mode is observed as a peak in the structure factor of the pair susceptibility $S(\Omega, \mathbf{K})$ only just below the transition temperature T_c and only in dirty systems. On the other hand, our analytical results support the statement by Ohashi and Takada [Phys. Rev. B **62**, 5971 (2000)] that in d -wave superconductors the CG mode can exist in clean systems down to much lower temperatures, $T \approx 0.1T_c$. We also consider the manifestations of the CG mode in the density-density and current-current correlators and discuss the gauge independence of the obtained results.

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I. INTRODUCTION

More than 25 years ago, an unusual propagating soundlike ($\Omega/K = v_{CG}$) Carlson-Goldman (CG) mode in *charged* superconducting systems with the velocity $v_{CG} = 10^3 - 10^4$ m/c was discovered¹ (see also Ref. 2). It was a widespread opinion before the CG mode discovery that since in the charged systems, the soundlike Bogolyubov-Anderson^{3,4} (BA) mode associated with *neutral* superconductors is converted to the plasma mode due to the Anderson-Higgs mechanism, there is no soundlike phase mode in charged systems.

A magnificent effort (see, for example, Refs. 5–8) was made to understand the mechanism responsible for the appearance of the CG mode and its relation to other phenomena of nonequilibrium superconductivity. While the majority theories of the CG mode^{5–8} were essentially based on the kinetic equations that are usually derived using the quasiclassical Green’s functions, the paper by Kulik, Entin-Wohlman, and Orbach⁹ used a more conventional approach based on the Matsubara Green’s functions without kinetic equations. In the subsequent papers of Takada and co-workers (see Refs. 10 and 11 and references therein) the approach of Ref. 9 was further developed and very recently applied for the case of d -wave superconductivity.¹² The collective oscillations in d -wave superconductors were also studied using the kinetic equations for Green’s functions¹³ (see also Ref. 14).

Since the discovery of high- T_c compounds, $d_{x^2-y^2}$ superconductivity has attracted much attention¹⁵ and the claim of Ref. 12 that the CG mode in clean d -wave superconductors may survive in a much wider region of temperatures down to $0.2T_c$ appears to be very different from the established properties of the CG mode in s -wave superconductors, so that it is important to check it by an independent calculation.

On the other hand, the importance of phase fluctuations in high-temperature superconductors (HTSC) (see, e.g., Ref. 16) stimulated interest in the derivation of the “phase-only” effective actions from the microscopic theory. It is important to emphasize that although there is no commonly accepted theory of HTSC, it seems reasonable that one can use a simple BCS-like approach to describe the properties of HTSC below the critical temperature T_c even though such an

approach fails above T_c . Relying on this argument the “phase-only” actions for d -wave superconductors were recently derived in Refs. 17–19. However, the only phase excitations that are described by these actions are the BA mode in the *neutral* superconductor¹⁸ and the plasma mode,^{17,19} which appears when the Coulomb interaction is taken into account. This corresponds to the standard paradigm that does not yield the existence of the CG mode. Thus the purpose of the present paper is to investigate which ingredient is missing in the treatments of Refs. 17–19 so that the CG mode does not appear in these approaches and to establish a link between the results of Ref. 12 and the phase-only action formalism. This missing link is established here and the CG mode is obtained within the effective-action formalism. Our main results can be summarized as follows.

(1) We extend the phase-only effective-action formalism for *charged* systems to incorporate the *density-current* coupling, which was so far considered only using other methods^{9–12} (see also Refs. 20–22, where the effect of this coupling appears to be important for the description of dirty superconductors). In neutral systems the density-current correlator *does contribute* in so-called *Landau terms* of the effective action,^{18,24} so that the correct expression for these terms can only be obtained when this correlator is taken into account.

(2) We show that when the density-current coupling is included it becomes possible to obtain the CG mode using the phase-only action. In particular, we derive an analytical expression for the pair-susceptibility structure factor and solve numerically the equation for the CG-mode velocity.

(3) We show the gauge-independent character of the equation for the collective phase excitations, one of the solutions of which is the CG mode. Establishing a link between the pair susceptibility and this gauge-independent equation for the phase excitations we argue that the peaks in the structure factor associated with the CG mode are independent of the choice of the gauge. The gauge independence of the equation for the CG mode used in the previous papers^{10–12} is also shown, applying the identity derived recently in Ref. 22.

(4) We consider possible manifestations of the CG mode in the gauge-independent density-density and current-current response functions.

(5) We derive analytical expressions for the density-density, current-current, and density-current polarization functions for two-dimensional (2D) clean d -wave superconductors at $T \ll T_c$.

The paper is organized as follows. In Sec. II we introduce our model. In Sec. III, we describe all details about the formalism used in the paper, necessary for further understanding. The various forms of the effective actions (for the phase and electric potential, phase only and electric potential only) are expressed in terms of density-density, current-current, and density-current polarization functions in Sec. IV. The general expressions for these polarization functions are given in Appendix A, their derivation for d -wave superconductors is considered in Appendix C, and the nodal approximation employed for this derivation is briefly discussed in Appendix B. In Sec. IV we also discuss in detail the difference between the present and other^{9–12,20–22} approaches. In Sec. V we briefly recall the properties of the phase excitations in the absence of the density-current coupling and stress some points such as gauge independence of the equation for the collective phase excitations and the properties of the gauge-independent density-density and current-current correlators, which are particularly useful for better understanding of our main results, which are presented in Sec. VI. In particular, in Sec. VI A we derive the equation for the CG mode, give its physical interpretation, and discuss the gauge independence of the present and previous approaches. In Sec. VI B we present the results for the velocity of the CG mode and Sec. VI C is devoted to the structure factor (the calculational details for these sections are given in Appendix D). We conclude in Sec. VII with a discussion and summary of our results.

II. MODEL HAMILTONIAN

Let us consider the following action (in our notations the functional integral is expressed via e^S):

$$S = - \int_0^\beta d\tau \left[\sum_\sigma \int d^2r \psi_\sigma^\dagger(\tau, \mathbf{r}) [\partial_\tau - ieA_0(\tau, \mathbf{r})] \psi_\sigma(\tau, \mathbf{r}) + H(\tau) \right], \quad \mathbf{r} = (x, y), \quad \beta \equiv \frac{1}{T}, \quad (1)$$

where the Hamiltonian $H(\tau)$ is

$$H(\tau) = \sum_\sigma \int d^2r \psi_\sigma^\dagger(\tau, \mathbf{r}) \left[\varepsilon \left(-i\nabla - \frac{e}{c} \mathbf{A}(\tau, \mathbf{r}) \right) - \mu \right] \psi_\sigma(\tau, \mathbf{r}) - \frac{1}{2} \sum_\sigma \int d^2r_1 \int d^2r_2 \psi_\sigma^\dagger(\tau, \mathbf{r}_1) \psi_\sigma(\tau, \mathbf{r}_2) \times \psi_\sigma^\dagger(\tau, \mathbf{r}_1) V(\mathbf{r}_1; \mathbf{r}_2) \psi_\sigma(\tau, \mathbf{r}_1) \psi_\sigma(\tau, \mathbf{r}_2) + \frac{1}{2} \int d^2r_1 \int d^2r_2 \left(\sum_\sigma \psi_\sigma^\dagger(\tau, \mathbf{r}_1) \psi_\sigma(\tau, \mathbf{r}_1) - n \right) \times V_c(\mathbf{r}_1 - \mathbf{r}_2) \left(\sum_{\sigma'} \psi_{\sigma'}^\dagger(\tau, \mathbf{r}_2) \psi_{\sigma'}(\tau, \mathbf{r}_2) - n \right). \quad (2)$$

Here $\psi_\sigma(\tau, \mathbf{r})$ is a fermion field with spin $\sigma = \uparrow, \downarrow$, $\bar{\sigma} \equiv -\sigma$, τ is the imaginary time, $V(\mathbf{r}_1; \mathbf{r}_2)$ is an attractive short-range potential, $V_c(\mathbf{r}_1 - \mathbf{r}_2)$ is the long-range Coulomb interaction, and n is the neutralizing background charge den-

sity. Throughout the paper we call the superconducting system *neutral* if the last term of Eq. (2) is omitted and *charged* if this term is taken into account. Even in the latter case the whole superconductor remains, of course, neutral due to the neutralizing ionic background.

We assume that the momentum representation of $V(\mathbf{r}_1; \mathbf{r}_2)$ contains attraction only in the d -wave channel (see the discussion in Ref. 17). The Fourier transform of the Coulomb interaction $V_c(\mathbf{q})$ depends on the detail of the model. It can, for example, be taken as $V_c(\mathbf{q}) = 4\pi e^2/q^2$ in 3D, $V_c(\mathbf{q}) = 2\pi e^2/q$ in 2D, or a more complicated expression (see, e.g., Refs. 12,17) if the layered structure of HTSC is taken into account. However, the detailed expression is not crucial for the CG mode, because the mode appears when the Coulomb interaction is screened out by the quasiparticles. (The form of the expression would be, of course, essential for the analysis of the plasma mode.^{17,23}) The form of dispersion law, $\varepsilon(\mathbf{k})$, is also not essential because the final results for the d -wave case will be formulated in terms of the noninteracting Fermi velocity $\mathbf{v}_F \equiv \partial\varepsilon(\mathbf{k})/\partial\mathbf{k}|_{\mathbf{k}=\mathbf{k}_F}$ and the gap velocity $\mathbf{v}_\Delta \equiv \partial\Delta(\mathbf{k})/\partial\mathbf{k}|_{\mathbf{k}=\mathbf{k}_F}$, where $\Delta(\mathbf{k})$ is the momentum-dependent superconducting gap. We will also use the parameter $\alpha_D \equiv v_F/v_\Delta$, which is called the anisotropy of the Dirac cone. Throughout the paper $\hbar = k_B = 1$ units are chosen. An external electromagnetic field $A = (A_0, \mathbf{A})$ was introduced in the action (1) to calculate various correlation functions using the functional derivatives with respect to this external source field.

III. DERIVATION OF THE EFFECTIVE ACTION AND THE STRUCTURE FACTOR

The derivation of the effective “phase-only” action for neutral (see, e.g., Refs. 16,18,24) and charged^{17,19,25,26} s - and d -wave superconducting systems is widely discussed in the literature, so we briefly recap the main steps, including the functional integral representation for the structure factor, and making in Sec. IV a point on the appearance of the term that couples density and current.

The first step of the derivation is to use the appropriate Hubbard-Stratonovich transformations to decouple four-fermion interaction terms in the attractive and repulsive channels. The attractive part of the interaction was recently considered in detail in Sec. II of Ref. 18 using the bilocal Hubbard-Stratonovich fields $\Phi(\tau, \mathbf{r}_1; \mathbf{r}_2)$ and $\bar{\Phi}^\dagger(\tau, \mathbf{r}_1; \mathbf{r}_2)$ (see Ref. 27 for a review), so we show explicitly the corresponding transformation only for the Coulomb interaction,

$$\exp \left[-\frac{1}{2} \int d\tau \int d^2r_1 \int d^2r_2 \left(\sum_\sigma \psi_\sigma^\dagger(\tau, \mathbf{r}_1) \psi_\sigma(\tau, \mathbf{r}_1) - n \right) \times V_c(\mathbf{r}_1, \mathbf{r}_2) \left(\sum_{\sigma'} \psi_{\sigma'}^\dagger(\tau, \mathbf{r}_2) \psi_{\sigma'}(\tau, \mathbf{r}_2) - n \right) \right] = \int \mathcal{D}\varphi \exp \left\{ - \int d\tau \int d^2r_1 \int d^2r_2 \times \left[\frac{1}{2} e\varphi(\tau, \mathbf{r}_1) V_c^{-1}(\mathbf{r}_1, \mathbf{r}_2) e\varphi(\tau, \mathbf{r}_2) - ie\varphi(\tau, \mathbf{r}_1) \times \left(\sum_\sigma \psi_\sigma^\dagger(\tau, \mathbf{r}_1) \psi_\sigma(\tau, \mathbf{r}_1) - n \right) \delta(\mathbf{r}_1 - \mathbf{r}_2) \right] \right\}, \quad (3)$$

where the Hubbard-Stratonovich field φ has the meaning of electric potential. Thus, the partition function is

$$Z = \int \mathcal{D}\Psi^\dagger \mathcal{D}\Psi \mathcal{D}\Phi^\dagger \mathcal{D}\Phi \mathcal{D}\varphi \exp[S(\Psi^\dagger, \Psi, \Phi^\dagger, \Phi, \varphi, A)],$$

$$S(\Psi^\dagger, \Psi, \Phi^\dagger, \Phi, \varphi, A) = \int_0^\beta d\tau \int d^2r_1 \int d^2r_2 \left\{ -\frac{1}{V(\mathbf{r}_1; \mathbf{r}_2)} |\Phi(\tau, \mathbf{r}_1; \mathbf{r}_2)|^2 - \frac{1}{2} e\varphi(\tau, \mathbf{r}_1) V_c^{-1}(\mathbf{r}_1, \mathbf{r}_2) e\varphi(\tau, \mathbf{r}_2) + \Psi^\dagger(\tau, \mathbf{r}_1) \right.$$

$$\times \left[-\partial_\tau + ie\tau_3 A_0 + ie\tau_3 \varphi(\tau, \mathbf{r}_1) - \tau_3 \xi \left(-i\nabla - \tau_3 \frac{e}{c} \mathbf{A} \right) \right] \Psi(\tau, \mathbf{r}_2) \delta(\mathbf{r}_1 - \mathbf{r}_2) - ie\varphi(\tau, \mathbf{r}_1) n \delta(\mathbf{r}_1 - \mathbf{r}_2)$$

$$\left. + \Phi^\dagger(\tau, \mathbf{r}_1; \mathbf{r}_2) \Psi^\dagger(\tau, \mathbf{r}_1) \tau_- \Psi(\tau, \mathbf{r}_2) + \Psi^\dagger(\tau, \mathbf{r}_1) \tau_+ \Psi(\tau, \mathbf{r}_2) \Phi(\tau, \mathbf{r}_1; \mathbf{r}_2) \right\}, \quad (4)$$

where Ψ and Ψ^\dagger are the Nambu spinors, $\xi(-i\nabla) \equiv \varepsilon(-i\nabla) - \mu$ and τ_3 , $\tau_\pm = (\tau_1 \pm i\tau_2)/2$ are Pauli matrices.

To consider the Hubbard-Stratonovich field Φ it is convenient to use the relative coordinate $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ and center-of-mass coordinate $\mathbf{R} = (\mathbf{r}_1 + \mathbf{r}_2)/2$, so that $\Phi(\tau, \mathbf{r}_1, \mathbf{r}_2) \equiv \Phi(\tau, \mathbf{R}, \mathbf{r})$. Now, using the functional integral representation, the imaginary-time pair susceptibility is defined as

$$\eta(\tau, \mathbf{R}_1 - \mathbf{R}_2) = -\frac{1}{Z(A=0)} \int \mathcal{D}\Psi^\dagger \mathcal{D}\Psi \mathcal{D}\Phi^\dagger \mathcal{D}\Phi \mathcal{D}\varphi$$

$$\times \Phi(\tau, \mathbf{R}_1, \mathbf{0}) \Phi^\dagger(0, \mathbf{R}_2, \mathbf{0})$$

$$\times \exp[S(\Psi^\dagger, \Psi, \Phi^\dagger, \Phi, \varphi)]. \quad (5)$$

Since the distance $|\mathbf{R}_1 - \mathbf{R}_2|$ is expected to be larger than the internal Cooper-pair scale, it is possible to put $\mathbf{r} = \mathbf{0}$ in Eq. (5). The structure factor $S(\Omega, \mathbf{K})$, which is used to present the experimental data,^{2,6} is related to the real-frequency pair susceptibility $\eta(i\Omega_n \rightarrow \Omega + i0, \mathbf{K})$ by

$$S(\Omega, \mathbf{K}) = -2[1 - \exp(-\beta\Omega)]^{-1} \text{Im} \eta(\Omega + i0, \mathbf{K})$$

$$\approx -\frac{2T}{\Omega} \text{Im} \eta(\Omega + i0, \mathbf{K}). \quad (6)$$

The definition of the pair susceptibility (5) is apparently gauge dependent, since the auxiliary Hubbard-Stratonovich field is gauge dependent. Nevertheless, as we discuss later, the poles of $\eta(\Omega, \mathbf{K})$ are *gauge independent* and this justifies the use of Eqs. (5) and (6) to extract the observable values.

The simplest way to study the low-energy phase dynamics²⁸ is to employ the canonical gauge transformation²⁹

$$\Psi(x) \rightarrow \begin{pmatrix} e^{i\theta(x)/2} & 0 \\ 0 & e^{-i\theta(x)/2} \end{pmatrix} \Upsilon(x), \quad x \equiv (\tau, \mathbf{r}) \quad (7)$$

separating the phase of the ordering field,

$$\Phi(\tau, \mathbf{R}, \mathbf{r}) \approx \Delta(\tau, \mathbf{R}, \mathbf{r}) \exp[i\theta(\tau, \mathbf{R})]. \quad (8)$$

Then after the integration over the Fermi fields the partition function becomes

$$Z = \int \Delta \mathcal{D}\Delta \mathcal{D}\theta \mathcal{D}\varphi \exp[-\beta\Omega\{\Delta, \partial\theta, \varphi, A\}], \quad (9)$$

where the effective potential

$$\beta\Omega\{\Delta, \partial\theta, \varphi, A\} = \int_0^\beta d\tau \int d^2r_1 \int d^2r_2 \left[\frac{\Delta^2(\tau, \mathbf{R}, \mathbf{r})}{V(\mathbf{r}_1 - \mathbf{r}_2)} \right.$$

$$+ \frac{1}{2} e\varphi(\tau, \mathbf{r}_1) V_c^{-1}(\mathbf{r}_1, \mathbf{r}_2) e\varphi(\tau, \mathbf{r}_2)$$

$$\left. + ie\varphi(\tau, \mathbf{r}_1) n \delta(\mathbf{r}_1 - \mathbf{r}_2) \right] - \text{Tr} \text{Ln} G^{-1} \quad (10)$$

with the inverse Green's function

$$G^{-1} = \mathcal{G}^{-1} - \Sigma, \quad (11)$$

$$\langle \tau_1, \mathbf{r}_1 | \Sigma(\partial\theta, \varphi, A) | \tau_2, \mathbf{r}_2 \rangle$$

$$= \left[\tau_3 \left(i \frac{\partial_{\tau_1} \theta}{2} - ie\varphi(\tau_1, \mathbf{r}_1) - ieA_0(\tau_1, \mathbf{r}_1) \right) \right.$$

$$\left. + \tau_3 \mathcal{O}_1(\mathbf{r}_1) + \hat{I} \mathcal{O}_2(\mathbf{r}_1) \right] \delta(\tau_1 - \tau_2) \delta(\mathbf{r}_1 - \mathbf{r}_2). \quad (12)$$

For $T \ll T_c$ it is reasonable to neglect the amplitude fluctuations and assume that the amplitude of the order parameter $\Delta(\tau, \mathbf{R}, \mathbf{r})$ does not depend on \mathbf{R} . Then the frequency-momentum representation of \mathcal{G} in Eq. (11) is the usual Nambu-Gor'kov Green's function

$$\mathcal{G}(i\omega_n, \mathbf{k}) = -\frac{i\omega_n \hat{I} + \tau_3 \xi(\mathbf{k}) - \tau_1 \Delta(\mathbf{k})}{\omega_n^2 + \xi^2(\mathbf{k}) + \Delta^2(\mathbf{k})}, \quad (13)$$

where, because *d*-wave pairing is considered, $\Delta(\mathbf{k}) = \Delta/2(\cos k_x a - \cos k_y a)$ (a is the lattice constant) and $\omega_n = \pi(2n+1)T$ is fermionic (odd) Matsubara frequency. Since, in what follows, only the low temperatures $T \ll \Delta(T)$ are considered, we can replace the temperature-dependent amplitude $\Delta(T)$ by its zero-temperature value $\Delta_0 \equiv \Delta(T=0)$. Thus all linear low-temperature dependences of the

polarization functions considered below are due to the nodes of $\Delta(\mathbf{k})$, but not to the temperature dependence of $\Delta(T)$ itself.

The precise form of the operators O_1 and O_2 in Eq. (12) which depends on the particular form of the tight-binding spectrum $\varepsilon(\mathbf{k})$ is given in Ref. 18 (see also Ref. 19 for the formulation of the general rules for representation of Σ). It is essential, however, that the coordinate representation of Σ does not depend on the phase θ itself and contains only its derivatives. Thus the coordinate representation of Ω_{kin} is also expressed via the derivatives of θ . This property is particularly convenient for studying 2D models when a constant space-independent phase is prohibited by the Coleman-Mermin-Wagner-Hohenberg (CMWH) theorem.

Since we are interested only in the phase dynamics in the presence of Coulomb interaction, in what follows we consider only the phase- (θ) and the electric-potential- (φ) dependent parts of the thermodynamical potential (10). This part of Ω , which we denote as Ω_{kin} , can be presented as a series

$$\begin{aligned} \Omega_{kin}\{\partial\theta, \varphi\} = & \int_0^\beta d\tau \int d^2r_1 \int d^2r_2 \left[\frac{1}{2} e\varphi(\tau, \mathbf{r}_1) V_c^{-1}(\mathbf{r}_1, \mathbf{r}_2) \right. \\ & \left. \times e\varphi(\tau, \mathbf{r}_2) + ie\varphi(\tau, \mathbf{r}_1) n \delta(\mathbf{r}_1 - \mathbf{r}_2) \right] \\ & + T \text{Tr} \sum_{n=1}^{\infty} \frac{1}{n} (\mathcal{G}\Sigma)^n. \end{aligned} \quad (14)$$

This way of deriving the effective action has many advantages. The main among them is that the gauge-invariant combinations $\hbar\partial_\tau\theta/2 - e\varphi - eA_0$ and $\hbar\nabla\theta/2 - e/c\mathbf{A}$ are explicitly present during all stages of the derivation.³⁰ This property is obviously related to the introduction of the phase via the gauge transformation (7). There is no need because of this to keep the external electromagnetic field (A_0, \mathbf{A}) during the intermediate stages of the derivation since it can be easily restored following the above-mentioned prescription, which in the frequency-momentum space is

$$\begin{aligned} i\Omega_n\theta(K) & \rightarrow i\Omega_n\theta(K) + 2eA_0(K), \\ iK_\alpha\theta(K) & \rightarrow iK_\alpha\theta(K) - \frac{2e}{c}A_\alpha(K). \end{aligned} \quad (15)$$

Differentiating with respect to this source field we will derive physical correlation functions (see the discussion in Ref. 17) in what follows. It has to be stressed that the *minimal-coupling* prescription (15) does not guarantee itself the gauge independence of the final result. The gauge independent treatment of the transformations (7) and (8) for *d*-wave pairing is, in particular, one of the complications.²⁹

In the previous studies of the CG mode⁹⁻¹² the phase field was introduced using the expansion of the ordering field $\Phi(x)$ around the equilibrium value Δ via $\Phi(x) = \Delta + \Phi_1(x) + i\Phi_2(x)$ and associating the fields $\Phi_1(x)$ with the amplitude and $\Phi_2(x)$ (or to be more precise $\Phi_2(x)/\Delta_0$) with the phase fluctuations. Although, as will be discussed below, the final result obtained in the both the methods agrees, the present method of the investigation of the CG mode is more transparent because it explicitly uses the gauge-independent combinations of the fields over the whole derivation. For

example, one can easily recognize that Σ contains a gauge-independent Cooper-pair chemical potential $\theta/2 - e\varphi$,⁸ which in other approaches has to be collected from the different parts of the equations. Other advantages of the present approach will be discussed in the subsequent sections where the effective action is presented.

To consider the phase and charge fluctuations at the Gaussian level it is sufficient to include only the terms with $n=1,2$ in the infinite series in Eq. (14). Finally, we rewrite the pair susceptibility (5) in terms of the new variables,

$$\begin{aligned} \eta(\tau, \mathbf{R}_1 - \mathbf{R}_2) = & - \int \Delta \mathcal{D}\Delta \mathcal{D}\theta \mathcal{D}\varphi \Delta(\tau, \mathbf{R}_1, \mathbf{0}) \exp[i\theta(\tau, \mathbf{R}_1)] \\ & \times \Delta(0, \mathbf{R}_2, \mathbf{0}) \exp[-i\theta(\tau, \mathbf{R}_2)] \\ & \times \exp[-\beta\Omega\{\Delta, \partial\theta, \varphi\}]. \end{aligned} \quad (16)$$

As was mentioned above, we are interested only in the phase-fluctuation structure factor neglecting the presence of the amplitude fluctuations. This implies that one can use a saddle-point approximation for Δ , so that omitting unimportant constants in the expansion of the exponent in Eq. (16) one arrives at

$$\begin{aligned} \eta(\tau, \mathbf{R}_1 - \mathbf{R}_2) = & - \frac{\Delta_0^2}{4} \int \mathcal{D}\theta \mathcal{D}\varphi \theta(\tau, \mathbf{R}_1) \theta(\tau, \mathbf{R}_2) \\ & \times \exp[-\beta\Omega_{kin}\{\partial\theta, \varphi\}]. \end{aligned} \quad (17)$$

In writing Eq. (17) we have also expanded the exponents that were present in Eq. (16), because there are no free vortices in the system for $T < T_c$ and the multivalued character of the phase is irrelevant. This approximated form of the pair susceptibility is equivalent to the expressions for susceptibility used in Refs. 9–12. Expanding the exponents we neglect a widening of the structure-factor peaks, which is related to the absence of the long-range order in 2D (CMWH theorem). It is known, for example, from the analysis of the dynamic structure factor of lattices^{31,32} that the δ -function phonon resonance obtained in 3D harmonic crystals in the 2D case is converted to the power-law singularity,

$$S(\omega, \mathbf{q}) \sim \frac{f(\alpha)}{|\omega^2 - v^2 q^2|^{1-\alpha(T)}}, \quad (18)$$

where $f(\alpha)$ is a function of α that goes to zero as $\alpha \rightarrow 0$, so that in this limit the structure factor transforms to a δ function. Since for low temperatures $\alpha(T) \ll 1$ we may safely neglect this effect of widening because it does not move the position of the peak and we are primarily interested in the temperatures $T \ll T_c$.

IV. GENERAL FORM OF THE EFFECTIVE ACTION

In this section we present the effective potential $\Omega_{kin}\{\theta, \varphi\}$ and discuss the term that leads to the appearance of the CG mode. We also derive the effective phase-only and electric-potential-only actions integrating out the electric potential φ and the phase θ , respectively.

A. The effective action and polarization functions

Calculating the terms with $n=1,2$ in Eq. (14) (see, e.g., Refs. 18,24) one arrives at

$$\begin{aligned}
\beta\Omega_{kin}\{\theta, \varphi\} = & \frac{T}{8} \sum_{n=-\infty}^{\infty} \int \frac{d\mathbf{K}}{(2\pi)^2} [\varphi(-K)4e^2V_c^{-1}(\mathbf{K})\varphi(K) + \{i\Omega_n\theta(-K) - 2e\varphi(-K)\}\Pi_{33}(K) \\
& \times \{i\Omega_n\theta(K) + 2e\varphi(K)\} + \theta(-K)\{\Lambda_0^{\alpha\beta} + \Pi_{00}^{\alpha\beta}(K)\}K_\alpha K_\beta \theta(K) + \theta(-K)(-K_\alpha)\Pi_{30}^\alpha(K) \\
& \times \{i\Omega_n\theta(K) + 2e\varphi(K)\} + \{i\Omega_n\theta(-K) - 2e\varphi(-K)\}(-K_\alpha)\Pi_{03}^\alpha(K)\theta(K)], \quad (19)
\end{aligned}$$

where we introduced short-hand notations $K = (i\Omega_n, \mathbf{K})$ with \mathbf{K} being a 2D vector (summation over dummy indices $\alpha, \beta = 1, 2$ is implied). In Eq. (19) the current-current polarization function Π_{00} is

$$\begin{aligned}
\Pi_{00}^{\alpha\beta}(i\Omega_n, \mathbf{K}) \equiv & T \sum_{l=-\infty}^{\infty} \int \frac{d^2k}{(2\pi)^2} \pi_{00}(i\Omega_n, \mathbf{K}; i\omega_l, \mathbf{k}) \\
& \times v_{F\alpha}(\mathbf{k})v_{F\beta}(\mathbf{k}) \quad (20)
\end{aligned}$$

with the Fermi velocity $v_{F\alpha}(\mathbf{k}) = \partial\xi(\mathbf{k})/\partial k_\alpha$; the density-density polarization function Π_{33} is

$$\Pi_{33}(i\Omega_n, \mathbf{K}) \equiv T \sum_{l=-\infty}^{\infty} \int \frac{d^2k}{(2\pi)^2} \pi_{33}(i\Omega_n, \mathbf{K}; i\omega_l, \mathbf{k}), \quad (21)$$

and the density-current polarization function Π_{03}^α is

$$\Pi_{03}^\alpha(i\Omega_n, \mathbf{K}) \equiv T \sum_{l=-\infty}^{\infty} \int \frac{d^2k}{(2\pi)^2} \pi_{03}(i\Omega_n, \mathbf{K}; i\omega_l, \mathbf{k})v_{F\alpha}(\mathbf{k}). \quad (22)$$

π_{ij} in Eqs. (20)–(22) is given by

$$\begin{aligned}
\pi_{ij}(i\Omega_n, \mathbf{K}; i\omega_l, \mathbf{k}) \equiv & \text{Tr}[\mathcal{G}(i\omega_l + i\Omega_n, \mathbf{k} + \mathbf{K}/2)\tau_i\mathcal{G} \\
& \times (i\omega_l, \mathbf{k} - \mathbf{K}/2)\tau_j], \quad (\tau_0 \equiv \hat{I}) \quad (23)
\end{aligned}$$

and $\Lambda_0^{\alpha\beta}$ in Eq. (19) is the first-order contribution in the superfluid stiffness,

$$\Lambda_0^{\alpha\beta} = \int \frac{d^2k}{(2\pi)^2} n(\mathbf{k})m_{\alpha\beta}^{-1}(\mathbf{k}), \quad m_{\alpha\beta}^{-1}(\mathbf{k}) \equiv \partial^2\xi(\mathbf{k})/\partial k_\alpha\partial k_\beta \quad (24)$$

with

$$n(\mathbf{k}) = 1 - \frac{\xi(\mathbf{k})}{E(\mathbf{k})} \tanh \frac{E(\mathbf{k})}{2T}, \quad E(\mathbf{k}) = \sqrt{\xi^2(\mathbf{k}) + \Delta^2(\mathbf{k})}. \quad (25)$$

In writing Eq. (19) we omitted the linear time-derivative term (see, e.g., Refs. 17,18), which is irrelevant for the present analysis.

The general expressions for the polarizations (20)–(22) are given in Appendix A (see also Ref. 18) and calculated analytically for 2D clean d -wave superconductors in Appendix C (a brief discussion of the nodal approximation used to

calculate these polarizations and the transformation to the global coordinate system are given in Appendixes B and D, respectively).

As an example we show in Fig. 1 the real part of the density-density polarization function $\Pi_{33}(\Omega, \mathbf{K})$ (this function is in fact just the Lindhard's function for the superconducting state), which is given by Eq. (C3) [and its dimensionless form is given by Eqs. (C4) and (C5)] as a function of $\Omega/v_F K$ for the different directions of \mathbf{K} . The angle ϕ determines the direction of \mathbf{K} with respect to the (10) direction in such a way that $\phi = \pi/4$ corresponds to the nodal direction [see also Eq. (D5) and the explanation in Appendix D]. Comparing this figure with Fig. 7 from Ref. 12 (our definition of ϕ is equivalent to the definition of the angle θ_q used in Ref. 12), which was obtained by numerical integration of Eq. (A1), one can see that our analytical expression (C3) gives essentially the same result. In particular, Fig. 1 shows that Π_{33} has a peak at $\Omega/v_F K = \cos(\phi - \pi/4)$. Furthermore, as shown, for instance, for $\phi = \pi/16$, there is a lower peak in $\text{Re}\Pi_{33}(\Omega, \mathbf{K})$ at $\Omega/v_F K = \cos(\phi - \pi/4 + \pi/2)$. Note that the cases $\phi = 0$ and $\phi = \pi/4$ are “degenerate” because for $\phi = 0$ the lower peak coincides with the upper one and for $\phi = \pi/4$ the lower peak is at $\Omega/v_F K = 0$ (see also Fig. 5,¹² where the case $\phi = 0.2, 0.23\pi$ is shown).

In Ref. 12 the origin of these peaks was related to the gap nodes, which can be regarded as two “one-dimensional normal-state electronic bands” toward $\phi = \pm \pi/4$. These “normal bands” are able to screen out the Coulomb interaction in certain regions of the Fermi surface even for $T \ll T_c$ and this screening along with the presence of Π_{03}^α will make

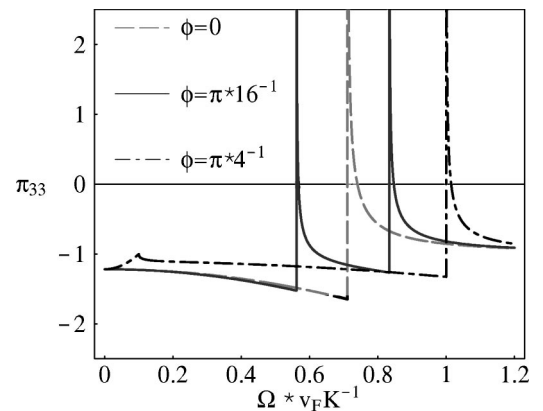


FIG. 1. Real part of $\Pi_{33}(\Omega, \mathbf{K})$ [or more precisely $\tilde{\Pi}_{33}(\Omega, \mathbf{K}) = (\pi/m)\Pi_{33}(\Omega, \mathbf{K})$, see Eq. (C5)]. We put $T = 0.5T_c$ (T_c and the angle ϕ are defined in Appendixes C and D, respectively) and $\alpha_D = 10$.

the appearance of the CG mode possible. Substituting Eq. (D5) in the analytical expression (C3) it is indeed easy to see that these peaks are due to the square-root singularities of Π_{33} . These are the same singularities that are present in 2D normal-state Lindhard's function due to the lowered dimensionality of the momentum integration,³³ but since the d -wave superconducting state is considered, the position of these singularities does depend on the direction of \mathbf{K} with respect to the Fermi surface. Finally we note that these singularities in $\text{Re}\Pi_{33}$ at $\Omega = v_F K \cos(\phi - \pi/4) + 0$ are accompanied by the singularity in $\text{Im}\Pi_{33}$ at $\Omega = v_F K \cos(\phi - \pi/4) - 0$, which was considered in Ref. 18.

The effective potential (19) becomes more tractable in the matrix form,

$$\beta\Omega_{kin}\{\theta, \varphi\} = \frac{T}{8} \sum_{n=-\infty}^{\infty} \int \frac{d\mathbf{K}}{(2\pi)^2} [\theta(-K)e\varphi(-K)] \times \mathcal{M}^{-1} \begin{bmatrix} \theta(K) \\ e\varphi(K) \end{bmatrix}, \quad (26)$$

where

$$\mathcal{M}^{-1} = \begin{bmatrix} -\Omega_n^2 \Pi_{33}(K) + \Lambda^{\alpha\beta}(K) K_\alpha K_\beta - i\Omega_n K_\alpha \Pi_{03}^\alpha(K) - i\Omega_n K_\alpha \Pi_{30}^\alpha(K) & 2i\Omega_n \Pi_{33}(K) - 2K_\alpha \Pi_{30}^\alpha(K) \\ -2i\Omega_n \Pi_{33}(K) + 2K_\alpha \Pi_{30}^\alpha(K) & 4[-\Pi_{33}(K) + V_c^{-1}(\mathbf{K})] \end{bmatrix} \quad (27)$$

with the bare (unrenormalized by the phase fluctuations) superfluid stiffness $\Lambda^{\alpha\beta} = \Lambda_0^{\alpha\beta} + \Pi_{00}^{\alpha\beta}(K)$.

B. Comparison with other approaches

Let us compare our effective action, Eqs. (26) and (27), with the action obtained in Ref. 17 and see the differences between the present and previous^{9–12} approaches. As one can notice, the only difference between Eqs. (26), (27) and Eqs. (25), (26) in Ref. 17 is due to the presence of the density-current polarization function Π_{03}^α . It is a general belief that this term has to be zero due to the ‘‘symmetry arguments.’’²⁵ However, as shown in Ref. 24 (see also Ref. 18, where the d -wave case is considered) this correlator has to be taken into account to obtain the correct expressions for the *Landau terms* of the effective action. This is the term that nontrivially couples phase and density fluctuations and makes the CG mode possible in the present approach.

From this point of view, the role of Π_{03}^α is the same as the role of the phase-charge coupling

$$\Pi_{23}(i\Omega_n, \mathbf{K}) \equiv T \sum_{l=-\infty}^{\infty} \int \frac{d^2k}{(2\pi)^2} \pi_{23}(i\Omega_n, \mathbf{K}; i\omega_l, \mathbf{k}) \gamma(\mathbf{k}),$$

$$\gamma(\mathbf{k}) \equiv \frac{2\Delta(\mathbf{k})}{\Delta_0}, \quad (28)$$

with π_{23} given by Eq. (23) in the approach of Kulik *et al.*⁹ and Takada with co-workers.^{10–12} Note that $\Pi_{23} = -\Pi_{32}$, while $\Pi_{03}^\alpha = \Pi_{30}^\alpha$.

It is interesting that techniques essentially similar to those of Refs. 9–12 have been used in Refs. 20–22 to consider suppression of the critical temperature in disordered superconductors. In Refs. 21 and 22 both amplitude and phase fluctuations were taken into account, and to consider the influence of nonmagnetic impurities, the electronic Green's functions had a 4×4 matrix structure. The main difference between the present approach and Refs. 9–12, 20–22,

nevertheless, remained the same, viz., in these papers the order-parameter phase itself was expressed via the operator $O_2 = \Psi^\dagger \tau_2 \Psi$, as summarized in Table II of Ref. 21. Thus τ_2 also enters the phase-density correlator, Π_{23} in the notations of Refs. 9–12 (or $\Pi_{\phi\rho}$ in the notations of Refs. 21 and 22).

In our opinion, the physical meaning of Π_{23} is, however, more obscure than that of Π_{03}^α . Indeed, since Π_{23} is expressed via the Pauli matrix τ_2 , so that it seems like the phase itself is a dynamical variable on its own, while physically meaningful are only the space and time derivatives of the phase.

These derivatives can only enter into the formal expressions as a current via \hat{I} and as a density via τ_3 matrices, respectively. This property is obviously present in the definition (22) of Π_{03}^α , which thus has the more clear meaning of a density-current polarization function.

Another important difference between the present and previous^{9–12,20–22} approaches is that the present derivation does not need an explicit use of the gap equation for Δ . For example, in Ref. 9 the charge conservation follows from the explicit use of the gap equation, while in the present approach as we will discuss later, the charge conservation is already built in the phase dynamics itself.

The fact that the present approach does not rely on the particular form of the gap equation is more convenient for modeling HTSC, where the gap does not close at the critical temperature T_c , so that the equation $\Delta(T_c^0) = 0$ gives only the mean-field transition temperature T_c^0 . Thus another definition of the true critical temperature T_c is necessary. As recently discussed in Ref. 34 (see also Ref. 16), it is reasonable for HTSC to estimate T_c as the temperature of the Berezinskii-Kosterlitz-Thouless transition [see Eq. (C10) in Appendix C].

Finally, it is worth mentioning here the recent papers³⁵ where an approach very similar to that of the present paper was employed to study the CG mode in the model of color superconducting quark matter. One of the advantages of Ref.

35 is that it treats the problem in an explicitly gauge-invariant way, while here, the Coulomb gauge is already imposed in writing the Hamiltonian Eq. (2) and then, when necessary, the gauge independence of the results obtained is discussed in a more intuitive way. In general, however, to prove the gauge independence of the results, an arbitrary gauge has to be considered to show that the physical observables do not depend on the gauge-fixing parameter as has been done in Ref. 35 (see also Ref. 36, where the gauge invariance of the physical quantities calculated using the $T=0$ effective potential is discussed). Following this route one can obtain the “relativistic” [see Ref. 35(b) for the details of the proof] generalization of Eq. (19), which contains a gauge-fixing parameter λ ,

$$\begin{aligned} \beta\Omega_{kin}\{\theta, A_\mu\} = T \sum_K \frac{1}{8} \left\{ \frac{1}{\pi} \left[A_\mu(-K) (K^2 \delta_{\mu\nu} - K_\mu K_\nu) A_\nu(K) \right. \right. \\ \left. \left. + \frac{1}{\lambda} A_\mu(-K) K_\mu K_\nu A_\nu(K) \right] \right. \\ \left. + [-iK_\mu \theta(-K) - 2eA_\mu(-K)] \Pi_{\mu\nu}(K) \right. \\ \left. \times [iK_\nu \theta(K) - 2eA_\nu(K)] \right\}, \quad (29) \end{aligned}$$

with $K_\mu \equiv (\Omega_n, \mathbf{K})$, $K^2 = K_\mu K_\mu = \Omega_n^2 + \mathbf{K}^2$, $\mu, \nu = 0, 1, 2$, $A_0(K) = -\varphi(K)$, and $c = 1$. Note that in Eq. (29) we have the whole electromagnetic potential A_μ instead of the Coulomb component present in Eq. (19). The polarization tensor $\Pi_{\mu\nu}(K)$ is obviously related to the polarizations used in Eq. (19). The question of gauge independence (or dependence) can be addressed considering how the calculated quantities depend on λ .

C. Effective actions for the phase and electric potential

Integrating out φ and θ from Eq. (26) one can obtain, respectively,

$$\begin{aligned} \beta\Omega_{kin}\{\theta\} = \frac{T}{8} \sum_{n=-\infty}^{\infty} \int \frac{d\mathbf{K}}{(2\pi)^2} \theta(-K) \mathcal{M}_\theta^{-1}(i\Omega_n, \mathbf{K}) \theta(K), \\ \mathcal{M}_\theta^{-1} = M_{11}^{-1} - M_{22} M_{12}^{-1} M_{21}^{-1} \quad (30) \end{aligned}$$

and

$$\begin{aligned} \beta\Omega_{kin}\{\varphi\} = \frac{T}{8} \sum_{n=-\infty}^{\infty} \int \frac{d\mathbf{K}}{(2\pi)^2} \varphi(-K) \mathcal{M}_\varphi^{-1}(i\Omega_n, \mathbf{K}) \varphi(K), \\ \mathcal{M}_\varphi^{-1} = e^2 (M_{22}^{-1} - M_{11} M_{12}^{-1} M_{21}^{-1}). \quad (31) \end{aligned}$$

It is evident that $\mathcal{M}_\varphi^{-1} = e^2 M_{22}^{-1} M_{11} \mathcal{M}_\theta^{-1} = e^2 M_{22}^{-1} \det \mathcal{M}^{-1}$, so that if $M_{11}^{-1}, M_{22}^{-1} \neq 0$ the equations $\mathcal{M}_\theta^{-1} = \mathcal{M}_\varphi^{-1} = \det \mathcal{M}^{-1} = 0$ are equivalent.

Using Eqs. (17) and (31) it is straightforward to obtain that the structure factor (6) is given by

$$S(\Omega, \mathbf{K}) = \frac{T}{\Omega} \frac{\Delta_0^2}{8} \text{Im} \mathcal{M}_\theta(\Omega + i0, \mathbf{K}). \quad (32)$$

V. PHASE EXCITATIONS IN THE CHARGED SYSTEM IN THE ABSENCE OF THE DENSITY-CURRENT COUPLING

A. Equation for the collective phase excitations

Let us assume for a moment that there is no density-current coupling ($\Pi_{03}^\alpha = \Pi_{30}^\alpha = 0$) and discuss briefly the collective excitations that follow from Eqs. (27), (30), and (31). As mentioned above, the matrix \mathcal{M}^{-1} reduces in this case to the known expression.¹⁷ Therefore, it is not surprising that the phase-only action (30) takes the form

$$\begin{aligned} \mathcal{M}_\theta^{-1}(i\Omega_n, \mathbf{K}) = -\Omega_n^2 \bar{\Pi}_{33} + \Lambda^{\alpha\beta} K_\alpha K_\beta, \\ \bar{\Pi}_{33} = \frac{\Pi_{33}(K)}{1 - \Pi_{33}(K) V_c(\mathbf{K})}, \quad (33) \end{aligned}$$

which coincides with the corresponding expression in Ref. 17 (see also Ref. 25). The dispersion law of the collective phase modes is defined by the equation

$$\mathcal{M}_\theta^{-1}(i\Omega_n \rightarrow \Omega + i0, \mathbf{K}) = 0. \quad (34)$$

This equation can also be regarded as a direct consequence of charge conservation,

$$\frac{\partial \rho(t, \mathbf{r})}{\partial t} + \nabla \cdot \mathbf{j}(t, \mathbf{r}) = 0, \quad (35)$$

where the current and charge density are defined via

$$\mathbf{j} = -c \frac{\delta \Omega_{kin}\{\theta, A\}}{\delta \mathbf{A}}, \quad \rho = \frac{\delta \Omega_{kin}\{\theta, A\}}{\delta A_0}, \quad (36)$$

where the electromagnetic field A in Ω_{kin} was restored using the rule (15). Evaluating Eq. (36) one arrives at Eq. (45) with $\Pi_{03}^\alpha = 0$, so that Eq. (34) is indeed recovered. There is, in fact, no surprise in the link between Eqs. (35) and (34), which is just a consequence of the way we introduced the phase θ in Eq. (7).

As discussed in Refs. 17 and 25 the only solution of this equation for $\mathbf{K} \rightarrow 0$ is the plasma mode. Using as an example the 3D form of the Coulomb potential and assuming that $\Lambda^{\alpha\beta} = \Lambda \delta_{\alpha\beta}$, which is valid for isotropic system with $m_{\alpha\beta} = m \delta_{\alpha\beta}$, one obtains from this equation that the plasma frequency $\omega_p = \sqrt{4\pi\Lambda e^2}$ for the limit $\mathbf{K} \rightarrow 0$. The expression for ω_p can be reduced to the standard one, $\sqrt{4\pi n e^2 / m}$, if one uses the superfluid stiffness $\Lambda = n/m$ obtained for the continuum model with s -wave pairing.

It is clear that for the plasma mode $\Omega_p(\mathbf{K})/K \rightarrow \infty$ as $\mathbf{K} \rightarrow 0$. This property remains valid even if 2D Coulomb potential is used and it makes plasmons different from any sound mode with $\Omega/K \rightarrow \text{const}$ as $\mathbf{K} \rightarrow 0$. For example, if a neutral superconductor were considered the polarization $\bar{\Pi}_{33}$

in Eq. (33) would be replaced by Π_{33} and the solution of Eq. (34) for $\mathbf{K} \rightarrow 0$ is the soundlike BA mode and its velocity is given by Eq. (C9).

B. Phase excitations via electric-potential propagator and gauge-independent density-density and current-current correlators

It is instructive also to look at the form \mathcal{M}_φ for the electric potential φ which is,

$$\mathcal{M}_\varphi^{-1}(i\Omega_n, \mathbf{K}) = 4e^2 \left(\frac{-\Pi_{33}\Lambda^{\alpha\beta}K_\alpha K_\beta}{-\Omega_n^2\Pi_{33} + \Lambda^{\alpha\beta}K_\alpha K_\beta} + V_c^{-1}(\mathbf{K}) \right). \quad (37)$$

Considering the same example of an isotropic system with a 3D Coulomb potential, Eq. (37) in the limit $\mathbf{K} \rightarrow 0$ and $\Omega/K \rightarrow \infty$ can be reduced to the known expression (see, e.g., Ref. 25)

$$\mathcal{M}_\varphi^{-1}(i\Omega_n, \mathbf{K}) = \frac{\mathbf{K}^2}{\pi} \left(1 + \frac{\omega_p^2}{\Omega_n^2} \right). \quad (38)$$

It is obvious that the above-discussed plasma mode can be also seen in \mathcal{M}_φ . Indeed after the analytical continuation $i\Omega_n \rightarrow \Omega + i0$, $\mathcal{M}_\varphi(\Omega, \mathbf{K})$ acquires a pole at $\Omega = \omega_p$.

It is also useful to evaluate gauge-independent density-density and current-current correlators, which are defined as

$$\begin{aligned} \chi(i\Omega_n, \mathbf{K}) &= -\frac{\delta^2 T \ln Z\{A\}}{\delta A_0(-K) \delta A_0(K)}, \\ \chi^{\alpha\beta}(i\Omega_n, \mathbf{K}) &= -\frac{\delta^2 T \ln Z\{A\}}{\delta A_\alpha(-K) \delta A_\beta(K)}, \end{aligned} \quad (39)$$

where

$$Z\{A\} = \int \mathcal{D}\theta \mathcal{D}\varphi \exp[-\beta \Omega_{kin}\{\theta, \varphi, A\}] \quad (40)$$

with the external field (A_0, \mathbf{A}) restored using the rule (15). Then we arrive at the standard expressions¹⁷

$$\chi(i\Omega_n, \mathbf{K}) = -\frac{e^2 \bar{\Pi}_{33} \Lambda^{\alpha\beta} K_\alpha K_\beta}{\Lambda^{\alpha\beta} K_\alpha K_\beta - \bar{\Pi}_{33} \Omega_n^2} \quad (41)$$

and

$$\chi^{\alpha\beta}(i\Omega_n, \mathbf{K}) = e^2 \left(\Lambda^{\alpha\beta} - \frac{\Lambda^{\alpha\mu} \Lambda^{\nu\beta} K_\mu K_\nu}{-\Omega_n^2 \bar{\Pi}_{33} + \Lambda^{\alpha\beta} K_\alpha K_\beta} \right). \quad (42)$$

Again assuming that $\Lambda^{\alpha\beta} = \Lambda \delta_{\alpha\beta}$ one can reduce Eq. (42) to the known form of the current-current correlator

$$\chi^{\alpha\beta}(i\Omega_n, \mathbf{K}) = e^2 \Lambda \frac{-\Omega_n^2 \bar{\Pi}_{33} \delta_{\alpha\beta} + \Lambda K^2 \left(\delta_{\alpha\beta} - \frac{K_\alpha K_\beta}{K^2} \right)}{-\Omega_n^2 \bar{\Pi}_{33} + \Lambda K^2}. \quad (43)$$

The difference between the gauge-independent equations (41) and (42) and the gauge-dependent $\bar{\Pi}_{33}$, $\Lambda^{\alpha\beta}$ in Eq. (33) was recently discussed in Ref. 17. The link considered above between charge-conservation expressed by Eq. (35) and Eq. (34) shows that the solutions of Eq. (34) are gauge independent, even though $\bar{\Pi}_{33}$, $\Lambda^{\alpha\beta}$ are gauge dependent. At a more formal level it can be argued that Eqs. (41) and (42) are gauge independent because even if one derives them starting from Eq. (29) the gauge-fixing parameter λ *does not enter* the final result and one obtains the same expressions.

Putting this in another way, one can say that the position of zeros of $\mathcal{M}_\theta^{-1}(\Omega, \mathbf{K})$ is gauge independent, because these zeros coincide with the poles of the gauge-independent $\chi(\Omega, \mathbf{K})$. As we will see in Sec. VI D, density-current coupling modifies both Eqs. (41) and (42); nevertheless the general argument about zeros of $\mathcal{M}_\theta^{-1}(\Omega, \mathbf{K})$ [poles of $\mathcal{M}_\theta(\Omega, \mathbf{K})$] remains valid.

VI. PHASE EXCITATIONS IN THE PRESENCE OF DENSITY-CURRENT COUPLING

Here we generalize all expressions from the preceding section for nonzero Π_{30}^α . In particular, Eq. (33) becomes

$$\begin{aligned} \mathcal{M}_\theta^{-1}(i\Omega_n, \mathbf{K}) &= -\Omega_n^2 \bar{\Pi}_{33} - i\Omega_n K_\alpha (\bar{\Pi}_{03}^\alpha(K) + \bar{\Pi}_{30}^\alpha(K)) \\ &\quad + \bar{\Lambda}^{\alpha\beta} K_\alpha K_\beta, \end{aligned} \quad (44a)$$

$$\begin{aligned} \bar{\Pi}_{03}^\alpha(K) &\equiv \frac{\Pi_{03}^\alpha(K)}{1 - \Pi_{33}(K) V_c(\mathbf{K})}, \\ \bar{\Lambda}^{\alpha\beta} &\equiv \Lambda^{\alpha\beta} + \frac{\Pi_{03}^\alpha(K) \Pi_{30}^\beta(K)}{-\Pi_{33}(K) + V_c^{-1}(\mathbf{K})}. \end{aligned} \quad (44b)$$

The dispersion law for all collective phase excitations is still defined by Eq. (34), but with \mathcal{M}_θ given by Eq. (44a). It has to be pointed out that the interpretation of Eqs. (34) and (44) as a charge-conservation law, Eq. (35), remains valid even for $\Pi_{30}^\alpha \neq 0$. The only difference is that the expressions (36) for current and density

$$\begin{aligned} j_\alpha(i\Omega_n, \mathbf{K}) &= \frac{e}{2} [\bar{\Lambda}^{\alpha\beta}(K) iK_\beta - i\bar{\Pi}_{03}^\alpha(K) i\Omega_n] \theta(K), \\ \rho(i\Omega_n, \mathbf{K}) &= -\frac{e}{2} [\bar{\Pi}_{33}(K) i\Omega_n + i\bar{\Pi}_{03}^\alpha(K) iK_\alpha] \theta(K) \end{aligned} \quad (45)$$

are now more complicated and contain $\bar{\Pi}_{03}^\alpha$. Nevertheless substitution of Eq. (45) in Eq. (35) indeed results in Eq. (34) with \mathcal{M}_θ given by Eqs. (44a). Hence, even in the most general case $\Pi_{03}^\alpha \neq 0$, the position of zeros of \mathcal{M}_θ^{-1} (poles of \mathcal{M}_θ) is gauge independent.

A. Equation for the CG mode, its physical interpretation, and gauge independence

It can be checked that in the limit $\mathbf{K} \rightarrow 0$ and $\Omega/K \rightarrow \infty$ one of the collective modes is the plasma mode considered

above. We are, however, more interested in whether a sound-like mode ($\Omega/K = \text{const}$ for $\mathbf{K} \rightarrow 0$) that would be similar to the BA mode in neutral superconductors can exist in the charged system. It is seen from Eqs. (44) that if the ratio Ω/K is fixed and one is interested in the low-energy excitations with $\Omega \rightarrow 0$, only the last term of Eq. (44a) is relevant because $\bar{\Pi}_{33}(K)$ and $\bar{\Pi}_{03}^\alpha(K)$ are $\sim V_c^{-1}(\mathbf{K})$ in this limit. In this case the equation $\mathcal{M}_\theta^{-1}(\Omega, \mathbf{K}) = 0$ after the analytical continuation $i\Omega_n \rightarrow \Omega + i0$ reduces to

$$\bar{\Lambda}^{\alpha\beta}(\Omega, \mathbf{K}) K_\alpha K_\beta = \left(\Lambda^{\alpha\beta}(\Omega, \mathbf{K}) - \frac{\Pi_{03}^\alpha(\Omega, \mathbf{K}) \Pi_{30}^\beta(\Omega, \mathbf{K})}{\Pi_{33}(\Omega, \mathbf{K})} \right) \times K_\alpha K_\beta = 0, \quad (46)$$

so that the detailed form of the Coulomb interaction becomes irrelevant, as was mentioned above. The solutions of Eq. (46) are gauge independent because for $\mathbf{K} \rightarrow \mathbf{0}$ this equation is equivalent to the gauge-independent equation $\mathcal{M}_\theta^{-1}(\Omega, \mathbf{K}) = 0$.

It is interesting that practically the same arguments about the gauge independence of the CG mode studied using the formalism of Refs. 9–12 can be made. The corresponding equation for the CG mode is

$$\bar{\Pi}_{22}(\Omega, \mathbf{K}) - \Pi_{22}(0, \mathbf{0}) = 0, \quad (47)$$

where

$$\bar{\Pi}_{22}(\Omega, \mathbf{K}) = \Pi_{22}(\Omega, \mathbf{K}) + \Pi_{23}(\Omega, \mathbf{K}) \frac{V_c(\mathbf{K})}{1 - V_c(\mathbf{K}) \Pi_{33}(\Omega, \mathbf{K})} \times \Pi_{32}(\Omega, \mathbf{K}). \quad (48)$$

Π_{23} is given by Eq. (28) and the definition of Π_{22} is given in Ref. 12 [in addition to two obvious τ_2 matrices, it also contains the factor $\gamma^2(\mathbf{k})$ because d -wave pairing is considered]. One can show that in the limit $\mathbf{K} \rightarrow 0$, Eq. (47) reduces to the Ward identity

$$[-2/V + \Pi_{22}(\Omega, \mathbf{K} \rightarrow 0)] \Pi_{33}(\Omega, \mathbf{K} \rightarrow 0) + \Pi_{23}^2(\Omega, \mathbf{K} \rightarrow 0) = 0. \quad (49)$$

Exactly this identity was recently proven in Ref. 22 (to be precise, s -wave pairing was considered) using the gauge-independence (charge-conservation) arguments (see, e.g., Ref. 37) and the mean-field gap equation

$$\frac{2}{V} + \Pi_{22}(0, \mathbf{0}) = 0. \quad (50)$$

As has already been mentioned this equation has to be explicitly used in the formalism of Refs. 9–12.

It is possible to establish a link between Eq. (46) and a simple and transparent interpretation of the CG mode suggested by Schmid and Schön³⁸ (see also Chap. 13 in Ref. 8). Comparing the expression for current in Eqs. (45) and (46) one can notice that since $\bar{\Pi}_{03}^\alpha(K)$ disappears in the limit $\mathbf{K} \rightarrow 0$, Eq. (46) is just a condition

$$\nabla \cdot \mathbf{j} = 0, \quad j_\alpha(i\Omega_n, \mathbf{K}) = \frac{e}{2} \bar{\Lambda}^{\alpha\beta}(K) i K_\beta \theta(K), \quad (51)$$

which implies that if there is a supercurrent in the system, it should be compensated by some normal current minimizing the total current. Exactly the same condition, which relates the CG mode to a counterflow of supercurrent and normal current, is discussed in Ref. 8. Thus using the two-fluid picture one may say that $\bar{\Lambda}^{\alpha\beta}(K)$ in Eq. (46) consists of the supercurrent $\Lambda^{\alpha\beta}$ and normal-current $\bar{\Lambda}^{\alpha\beta}(K) - \Lambda^{\alpha\beta}(K)$ parts.

Although this counterflow resembles second sound in He,⁴ it was pointed out in the earliest studies³⁸ (see also Ref. 6 and 8) that the CG mode is not the second sound, which is a hydrodynamic mode since in the CG mode the normal fluid and superfluid are not in the local thermodynamic equilibrium and this mode is not a hydrodynamic mode.

It has to be stressed that even though the solutions of Eqs. (46) and (47) are gauge independent for $\mathbf{K} \rightarrow 0$, the existence of such solutions *is not required* by the gauge invariance because no general statement can be made about the polarizations Π_{33} , $\Lambda^{\alpha\beta}$, and Π_{03}^α for arbitrary values of Ω and \mathbf{K} . Thus, in contrast to the BA and plasma modes, the existence of which is guaranteed by the Goldstone theorem³⁹ and the Anderson-Higgs mechanism, the CG mode does not obey any theorem and its existence is a fortunate result of many subtle features of the system dynamics.

B. Velocity of the CG mode

It is difficult to solve Eq. (46) analytically for d -wave pairing and even numerically, a more simple equation for real part $\text{Re} \bar{\Lambda}^{\alpha\beta}(\Omega, \mathbf{K}) = 0$ is usually considered.^{11,12} This significantly simplifies its solution but, in general, this approximation can be justified only *a posteriori*, when the imaginary part is estimated. It is possible to study this equation in two ways. The first way is to extract the dispersion law $\Omega(\mathbf{K})$ for, in general, arbitrary \mathbf{K} . The second way is to find the velocity of the CG mode, $v_{CG}(\phi) \equiv \Omega/|\mathbf{K}|$ in the limit $\mathbf{K} \rightarrow 0$. The extraction of the dispersion law is more sensitive to the approximations that were made in the calculation of the polarization operators. In particular, usage of the approximated expressions (A3), (A4), and (A5), which neglect the pair breaking for $\Omega \geq 2\Delta_0$, introduces the restriction $v_F K < \Delta_0$. Moreover, for the polarization equations (C4), (C7), and (C11) calculated using the nodal approximation (see Appendix B) the condition of smallness of \mathbf{K} becomes even more strict, so that here we study only the equation for v_{CG} . As also discussed in Appendix B the nodal approximation is valid for $T \ll \Delta_0$. This is the reason why in the present paper only the temperatures $T < 0.6T_c$ are considered, where T_c is defined by Eq. (C10). Nevertheless, the presented formalism allows, in principle, to study the phase-fluctuation structure factor up to T_c if no additional approximations are made in the calculation of the polarizations.

Thus for illustrative purposes and comparison with Ref. 12 we also solve numerically the equation

$$\text{Re}\bar{\Lambda}^{\alpha\beta}\left(\frac{\Omega}{K}=v_{CG}(\phi), K\rightarrow 0\right)\frac{K_\alpha K_\beta}{K^2}=0 \quad (52)$$

[it is rewritten in dimensionless form (D1) for the numerical work] instead of Eq. (46) and the results are presented in Fig. 2. We stress that for $\phi=\pi/16$ we obtain two solutions: $v_{CG}\approx 0.83v_F$ and $v_{CG}\approx 0.56v_F$. These results are in fact in

$$v_{CG}=v_F\times\begin{cases} 0.707 & (0.711 \text{ at } T=0.5T_c): & \phi=0 & (\text{antinode direction}) \\ 0.831 & (0.833 \text{ at } T=0.5T_c): & \phi=\pi/16 \\ 0.555 & (0.562 \text{ at } T=0.5T_c): & \phi=\pi/16 & (\text{other pair of nodes}) \\ 1 & (1.00 \text{ at } T=0.5T_c): & \phi=\pi/4 & (\text{nodal direction}). \end{cases} \quad (53)$$

Interestingly we obtain that the CG mode disappears even at somewhat lower temperatures than in Ref. 12, $T=0.1T_c$. This lowest value of T when the CG mode still exists is, however, more the result of a numerical solution of Eq. (52) than a real threshold temperature, because the peaks of the structure factor considered below disappear rather gradually.

C. Structure factor

The knowledge of the structure factor $S(\Omega, \mathbf{K})$ is even more important than the value of the velocity v_{CG} because this factor contains information about the damping of the CG mode.⁴⁰ Furthermore, as we have already mentioned, this is the quantity that is measured in the Carlson-Goldman experiment.^{1,2}

One of the main results of the present paper is that we obtain a closed *analytical* expression for the structure factor (32) for a clean d -wave superconductor, which is just a substitution of Π_{33} , Π_{03}^α and Λ given, respectively, by Eqs.

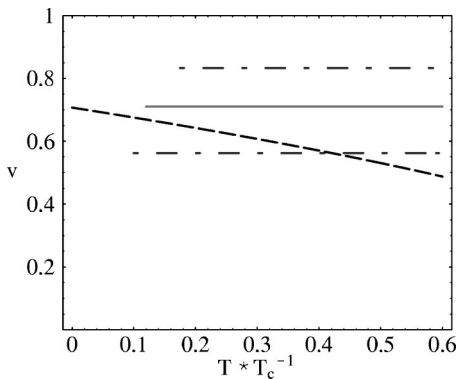


FIG. 2. Temperature dependence of the velocities of the BA mode (dashed line) and, the CG mode for $\phi=0$ (full line) and $\phi=\pi/16$ (dot-dashed lines). We put $\alpha_D=10$ and the velocities are expressed in units of v_F .

very good agreement with the results shown in Figs. 2(a) and Fig. 2(b) of Ref. 12. In particular, we also obtain that the velocity of the CG mode is practically temperature independent and its value is well described by $v_{CG}=v_F\cos(\phi-\pi/4)$ [or $v'_{CG}=v_F\cos(\phi-\pi/4+\pi/2)$ for the other pair of nodes, which is rotated with respect to the first pair by $\pi/2$]. Indeed, this equation gives

(C4), (C11), and (C7) in Eq. (44b) and the inverse imaginary part of Eq. (44a) in Eq. (32). The dimensionless form of M_θ^{-1} which is convenient for numerical evaluation, is given in Appendix D [Eqs. (D2) and (D3)]. In our formalism $M_\theta(\Omega, \mathbf{K})$ for a fixed value of $v_F K$ depends on the following dimensionless ratios: (i) $\alpha_D=v_F/v_\Delta$, (ii) $T/\epsilon_F \equiv T/T_c(T_c/\epsilon_F)$ [see Eq. (C10) where T_c is expressed via ϵ_F], (iii) $v_F^2 K^2/\omega_p^2 \equiv v_F^2 K^2/\epsilon_F^2(\epsilon_F^2/\omega_p^2)$, and the angle ϕ that characterizes the direction of \mathbf{K} with respect to the Fermi surface (see Appendix D). The ratio $v_F^2 K^2/\omega_p^2$, which was absent in Eq. (52) is now present because the full expression for M_θ^{-1} contains the Coulomb potential $V_c(\mathbf{K})$ [see its representation in terms of $v_F K$ and ω_p in Eq. (D4)], which we chose in the simplest 3D form. It is indeed easy to see that since $v_F K \ll \omega_p$, the detailed form of V_c is not important and here we compute $S(\Omega, \mathbf{K})$ including the Coulomb potential only to demonstrate this explicitly.

Despite the gauge-dependent definition of the pair susceptibility (5), the peaks of the structure factor (32) considered below, which are associated with the singularities of $\mathcal{M}_\theta(\Omega, \mathbf{K})$ [zeros of $\mathcal{M}_\theta^{-1}(\Omega, \mathbf{K})$], can be regarded as *gauge independent* in the sense that the position of these singularities is gauge independent as was argued above. Furthermore, as shown in Ref. 35, a more general expression for \mathcal{M}_θ derived from Eq. (29) *does depend* on the gauge-fixing parameter λ , but in such a way that the position of the *pole* of \mathcal{M}_θ and *its residue* are *gauge invariant*. This justifies the use of the structure factor $S(\Omega, \mathbf{K})$ which is expressed via $\text{Im}\mathcal{M}_\theta(\Omega, \mathbf{K})$.

In Figs. 3(a)–3(c) we show the structure factor calculated using the analytical expressions mentioned above for different temperatures. The position of the peaks defined by the ratio $\Omega/v_F K$ is well fitted by Eq. (53). In particular, for $\phi=\pi/16$ two peaks are seen. This allows one to associate the origin of these peaks with the gap nodes. As expected, the peaks are getting less sharp and higher as the temperature increases. The width of the peaks also depends on the direction of \mathbf{K} : for a larger value of $\phi(0<\phi<\pi/4)$ the corre-

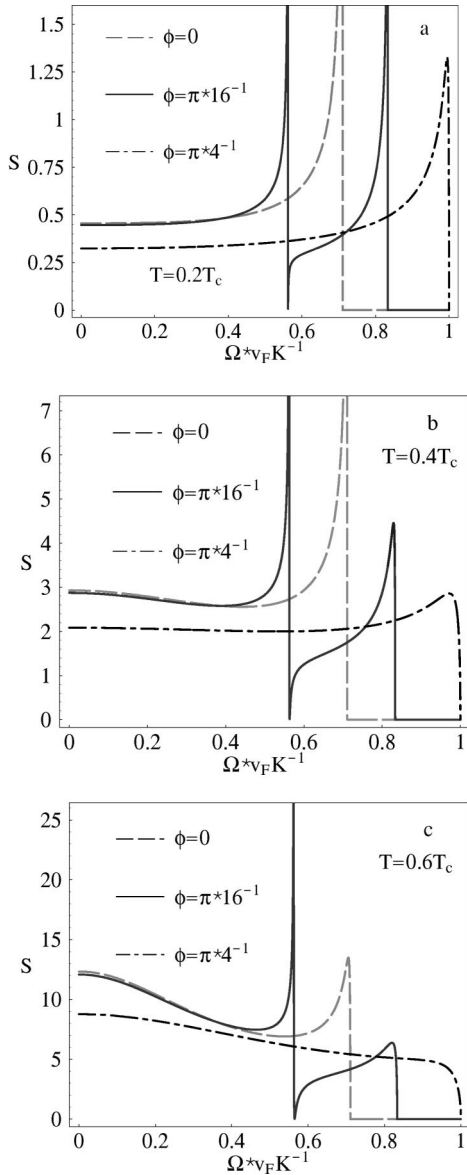


FIG. 3. Structure factor $S(\Omega, \mathbf{K})$ in arbitrary units: (a) $T = 0.2T_c$, (b) $T = 0.4T_c$, and (c) $T = 0.6T_c$. We put $\omega_p = 0.5\epsilon_F$, $v_F K = 0.01\epsilon_F$ (i.e., $v_F K = 0.02\omega_p$), and $\alpha_D = 10$, so that $T_c \approx 0.063\epsilon_F$.

sponding peak is wider. All these results are in agreement with Fig. 3 from Ref. 12, but due to the analytical character of the calculation the subtle peak features have a better resolution.

To argue that these peaks in Fig. 3 are indeed due to the density-current coupling in Fig. 4 we show for comparison the structure factor, which was calculated setting $\Pi_{30}^\alpha = 0$. The disappearance of the peaks confirms our claim that the CG mode demands nonzero density-current coupling.

This procedure of setting Π_{03}^α equal to zero is in fact very convenient in clarifying the origin of the peaks, because even for $\Pi_{03}^\alpha = 0$ some peaks can be seen, because both Π_{33} and $\Pi_{00}^{\alpha\beta}$ have the square-root singularities discussed in Sec. IV. For example, for $\pi/16 < \phi < \pi/4$ (see Fig. 5) the lower peak becomes even sharper, but in contrast to Fig. 4 it *does not disappear* when we set $\Pi_{03}^\alpha = 0$.

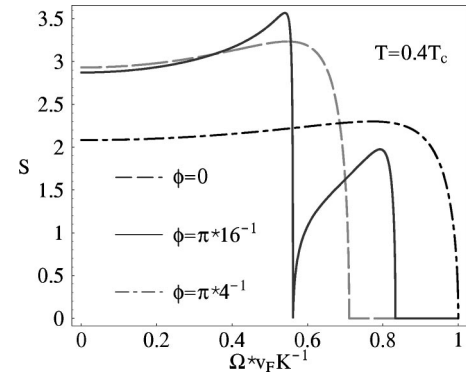


FIG. 4. Structure factor $S(\Omega, \mathbf{K})$ in arbitrary units for $T = 0.4T_c$. The rest of the parameters is the same as in Fig. 3, except for the fact that the density-current coupling is “switched off” ($\Pi_{03}^\alpha = 0$).

D. Manifestations of the CG mode in the classical response functions

The structure factor considered above is very important for the investigation of the CG mode because for superconductors there is no classical laboratory field that couples to, and in the static limit is the thermodynamic conjugate of, the order parameter. The reason why there is no laboratory conjugate field in the superconductor and superfluid cases is that these order parameters are off-diagonal in number space.² Nevertheless it is interesting to investigate whether or not the

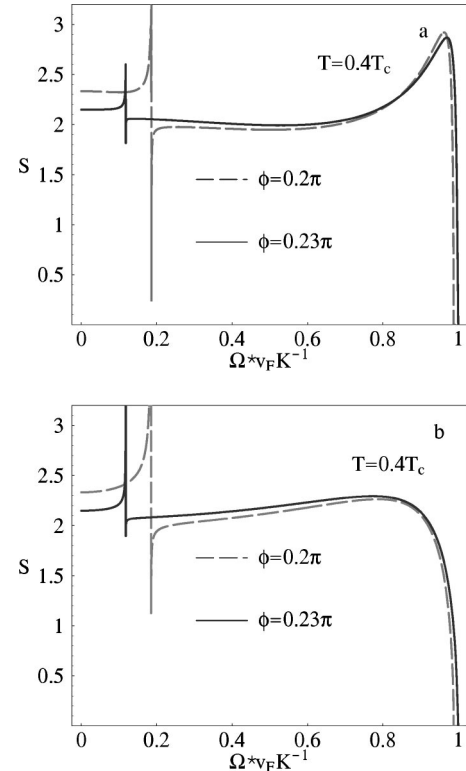


FIG. 5. Structure factor $S(\Omega, \mathbf{K})$ in arbitrary units for $T = 0.4T_c$. (a) $\Pi_{03}^\alpha \neq 0$, (b) $\Pi_{03}^\alpha = 0$. The rest of the parameters is the same as in Fig. 3.

CG mode can manifest itself in the ‘‘classical’’ correlators when one goes beyond the static limit.

Let us first consider the form M_φ for the electric potential. Substituting the elements of the matrix (27) in Eq. (31) we arrive at the following expression [compare with Eq. (37)]:

$$\begin{aligned} \mathcal{M}_\varphi^{-1}(i\Omega_n, \mathbf{K}) &= 4e^2 \left(\frac{-\Pi_{33}\Lambda^{\alpha\beta}K_\alpha K_\beta + \Pi_{03}^\alpha \Pi_{30}^\beta K_\alpha K_\beta}{-\Omega_n^2 \Pi_{33} + \Lambda^{\alpha\beta} K_\alpha K_\beta - i\Omega_n K_\alpha \Pi_{03}^\alpha - i\Omega_n K_\alpha \Pi_{30}^\alpha} \right. \\ &\quad \left. + V_c^{-1}(\mathbf{K}) \right). \end{aligned} \quad (54)$$

One can verify that both plasma and CG modes are present in the equation $\mathcal{M}_\varphi^{-1}=0$.

Although the equation $\mathcal{M}_\varphi^{-1}=0$ is gauge independent, \mathcal{M}_φ^{-1} itself as well as \mathcal{M}_θ^{-1} is gauge dependent. This time, however, we can take instead of \mathcal{M}_φ , truly gauge-independent correlators (39). Note that the definition (39) with the source fields (A_0, \mathbf{A}) clearly shows that the ‘‘classical laboratory field’’ is used to probe the corresponding response. Then repeating the calculation of Eqs. (41) and (42) with nonzero Π_{03}^α we arrive at the gauge-independent density-density correlator

$$\chi(i\Omega_n, \mathbf{K}) = \frac{e^2(\bar{\Pi}_{03}^\alpha \bar{\Pi}_{30}^\beta - \bar{\Pi}_{33} \bar{\Lambda}^{\alpha\beta}) K_\alpha K_\beta}{\bar{\Lambda}^{\alpha\beta} K_\alpha K_\beta - \bar{\Pi}_{33} \Omega_n^2 - i\Omega_n K_\alpha \bar{\Pi}_{03}^\alpha - i\Omega_n K_\alpha \bar{\Pi}_{30}^\alpha} \quad (55)$$

and current-current correlator

$$\chi^{\alpha\beta}(i\Omega_n, \mathbf{K}) = e^2 \left(\bar{\Lambda}^{\alpha\beta} - \frac{(i\Omega_n \bar{\Pi}_{03}^\alpha - \bar{\Lambda}^{\alpha\mu} K_\mu)(i\Omega_n \bar{\Pi}_{30}^\beta - \bar{\Lambda}^{\nu\beta} K_\nu)}{\bar{\Lambda}^{\alpha\beta} K_\alpha K_\beta - \bar{\Pi}_{33} \Omega_n^2 - i\Omega_n K_\alpha \bar{\Pi}_{03}^\alpha - i\Omega_n K_\alpha \bar{\Pi}_{30}^\alpha} \right). \quad (56)$$

In Fig. 6 we show the spectral density

$$B(\Omega, \mathbf{K}) = \frac{1}{\pi} \text{Im} \chi(\Omega, \mathbf{K}) \quad (57)$$

calculated for the density-density correlator (55). The plasma mode [see Fig. 6(b)] and the CG mode [see Fig. 6(a)], as proven by Fig. 6(c), are clearly seen in the density-density correlator. We note that for $\phi = \pi/16$ the lower peak does not change strongly, showing that it is due to the density-density correlations and not the CG mode.

As the value of K decreases, the relative weight of the CG mode with respect to the plasma mode becomes smaller and smaller as shown in Fig. 7. Finally in the limit

$$\chi(i\Omega_n, \mathbf{K}) \sim - \frac{e^2 \bar{\Pi}_{33} \bar{\Lambda}^{\alpha\beta} K_\alpha K_\beta}{\bar{\Lambda}^{\alpha\beta} K_\alpha K_\beta}, \quad \frac{\Omega}{K} = \text{const}, \quad \mathbf{K} \rightarrow 0, \quad (58)$$

the CG mode disappears from the density-density correlator. This agrees with the statement of Ref. 23 that the density-density correlator in the limit $\mathbf{K} \rightarrow 0$ is dominated by the plasma oscillation only.

Treating the current-current correlator (56) in the same way one obtains

$$\begin{aligned} \chi^{\alpha\beta}(i\Omega_n, \mathbf{K}) &\sim e^2 \left(\bar{\Lambda}^{\alpha\beta} - \frac{\bar{\Lambda}^{\alpha\mu} K_\mu \bar{\Lambda}^{\nu\beta} K_\nu}{\bar{\Lambda}^{\alpha\beta} K_\alpha K_\beta} \right), \\ \frac{\Omega}{K} &= \text{const}, \quad \Omega \rightarrow 0. \end{aligned} \quad (59)$$

Taking into account that the structure of $\bar{\Lambda}^{\alpha\beta}$ is $\bar{\Lambda}^{\alpha\beta} = \Lambda \delta_{\alpha\beta} - F v_{F\alpha} v_{F\beta}$ one can check that the term with F can-

cel out from the transverse correlator. This shows that the longitudinal CG mode cannot be seen in the transverse current-current correlator and its presence does not have any influence on the Meissner effect in the limit $\mathbf{K} \rightarrow 0$.

VII. DISCUSSION

In the present paper we have shown that the 2D model of clean d -wave superconductors predicts the existence of the CG mode in a wide temperature region down to $T=0.1T_c$. This is done using the analytical expression for the structure factor, which has peaks associated with the CG mode, and solving numerically the equation for the CG-mode velocity v_{CG} . All our results are in a good agreement with Ref. 12 where a similar model has been studied numerically using the formalism of Ref. 9. It was also shown in Ref. 12 that in contrast to s -wave superconductors, where the impurities suppressing the Landau damping result in more favorable conditions for the observation of the CG mode, in d -wave superconductors the CG mode disappears in the dirty system.

Thus the main physical question is whether our prediction of the CG mode in a clean d -wave superconductor is relevant for HTSC cuprates, which are very complex compounds. Recent measurements⁴¹ done in high-purity $\text{YBa}_2\text{Cu}_3\text{O}_7$ crystals show that the in-plane mean free path l increases to $\approx 1 \mu\text{m}$ below $T=20 \text{ K}$ ($\approx 0.22T_c$). This suggests that these systems are deeply in the clean limit $l \gg \xi \sim 10 \text{ \AA}$ (ξ is the in-plane superconducting coherence length) and the model we considered can be applied to describe the CG mode in these compounds. Using the value of the Fermi velocity $v_F \approx 1.8 \times 10^7 \text{ cm/s}$ from Ref. 41 we predict that the velocity of the CG mode is expected to be within the range $v_{CG} = (1/\sqrt{2}-1)v_F \approx (1.3-1.8) \times 10^5 \text{ m/c}$ depending on the direction of \mathbf{K} , which is one or two orders of magni-

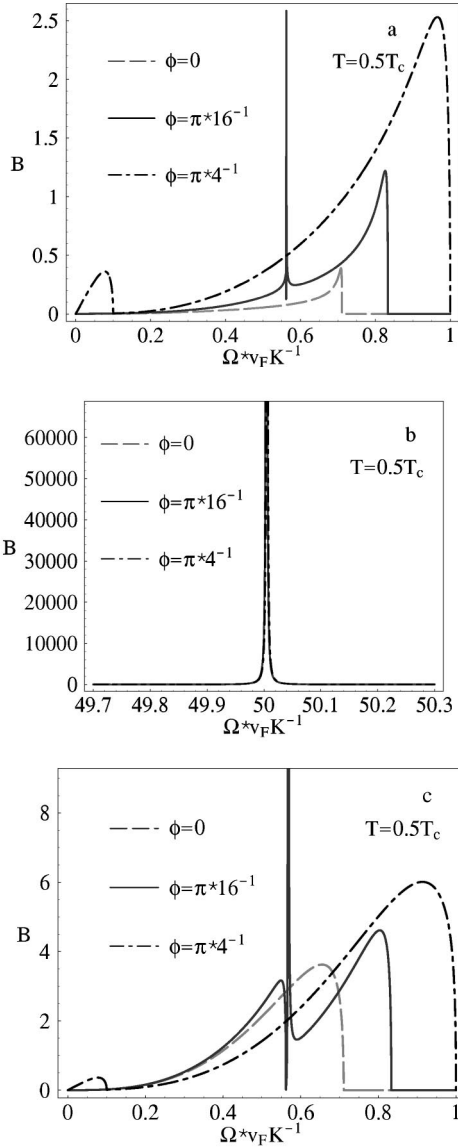


FIG. 6. Spectral density $B(\Omega, \mathbf{K})$ in arbitrary units for $T = 0.5T_c$. (a) $0 \leq \Omega/v_F K \leq 1$ and $\Pi_{03}^\alpha \neq 0$, (b) $49 \leq \Omega/v_F K \leq 51$ and $\Pi_{03}^\alpha \neq 0$, (c) $0 \leq \Omega/v_F K \leq 1$ and $\Pi_{03}^\alpha = 0$. We put $\omega_p = 0.5\epsilon_F$, $v_F K = 0.01\epsilon_F$ (i.e., $\omega_p = 50v_F K$), and $\alpha_D = 10$. To display the δ -like plasma peak seen in (b) a small imaginary part is added to ω_p .

tude faster than the velocity of the CG mode observed in conventional superconductors.

From the theoretical point of view there are many questions that deserve further theoretical investigation. First of all, it would still be important to consider the influence of impurities and inelastic scattering by antiferromagnetic spin fluctuations within the proposed formalism. It is also interesting to estimate the widening of the structure-factor peaks, which is discussed above Eq. (18). This widening may become important if temperatures $T \lesssim T_c$ are considered.

The discovery of the CG mode in conventional superconductors has led to much deeper understanding of superconductivity, so we hope that the investigation of the same problem in HTSC would also increase the understanding of these complex systems.

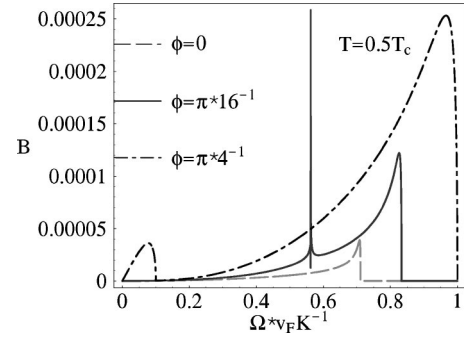


FIG. 7. Spectral density $B(\Omega, \mathbf{K})$ in arbitrary but the same units in Fig. 6 for $T = 0.5T_c$. We put $\omega_p = 0.5\epsilon_F$, $v_F K = 0.001\epsilon_F$ (i.e., $\omega_p = 500v_F K$), and $\alpha_D = 10$.

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APPENDIX A: GENERAL EXPRESSIONS FOR POLARIZATIONS Π_{ij}

The general expressions for the polarization functions Π_{ij} are (see, e.g., Refs. 11, 18, and 24)

$$\begin{aligned} \left[\begin{array}{c} \Pi_{00}^{\alpha\beta}(i\Omega_n, \mathbf{K}) \\ \Pi_{33}(i\Omega_n, \mathbf{K}) \end{array} \right] = & - \int \frac{d^2k}{(2\pi)^2} \left\{ \frac{1}{2} \left(1 - \frac{\xi_- \xi_+ \pm \Delta_- \Delta_+}{E_- E_+} \right) \right. \\ & \times \left[\frac{1}{E_+ + E_- + i\Omega_n} + \frac{1}{E_+ + E_- - i\Omega_n} \right] \\ & \times [1 - n_F(E_-) - n_F(E_+)] \\ & + \frac{1}{2} \left(1 + \frac{\xi_- \xi_+ \pm \Delta_- \Delta_+}{E_- E_+} \right) \\ & \times \left[\frac{1}{E_+ - E_- + i\Omega_n} + \frac{1}{E_+ - E_- - i\Omega_n} \right] \\ & \left. \times [n_F(E_-) - n_F(E_+)] \right\} V_{\pm}(\mathbf{k}). \quad (\text{A1}) \end{aligned}$$

$$V_{\pm}(\mathbf{k}) \equiv \begin{cases} v_{F\alpha}(\mathbf{k})v_{F\beta}(\mathbf{k}), & \text{“+”}, \\ 1, & \text{“−”}, \end{cases}$$

and

$$\begin{aligned}
 \Pi_{03}^\alpha(i\Omega_n, \mathbf{K}) = & \int \frac{d^2k}{(2\pi)^2} \left\{ \left(\frac{\xi_+}{2E_+} - \frac{\xi_-}{2E_-} \right) \left[\frac{1}{E_+ + E_- + i\Omega_n} \right. \right. \\
 & \left. \left. - \frac{1}{E_+ + E_- - i\Omega_n} \right] [1 - n_F(E_-) - n_F(E_+)] \right. \\
 & + \left(\frac{\xi_+}{2E_+} + \frac{\xi_-}{2E_-} \right) \left[\frac{1}{E_+ - E_- + i\Omega_n} \right. \\
 & \left. - \frac{1}{E_+ - E_- - i\Omega_n} \right] [n_F(E_-) \\
 & \left. - n_F(E_+)] \right\} v_{F\alpha}(\mathbf{k}), \quad (\text{A2})
 \end{aligned}$$

where $\xi_\pm \equiv \xi(\mathbf{k} \pm \mathbf{K}/2)$, $E_\pm \equiv E(\mathbf{k} \pm \mathbf{K}/2)$, and $\Delta_\pm \equiv \Delta(\mathbf{k} \pm \mathbf{K}/2)$. One can also check that $\Pi_{30}^\alpha(i\Omega_n, \mathbf{K}) = \Pi_{03}^\alpha(i\Omega_n, \mathbf{K})$.

The first and second terms in Eqs. (A1) and (A2) have a clear physical interpretation.³⁷ The first term proportional to $1 - n_F(E_-) - n_F(E_+)$ gives the contribution from ‘‘superfluid’’ electrons. The second term gives the contribution of the thermally excited quasiparticles (i.e., ‘‘normal’’ fluid component). This is the term responsible for the appearance of the *Landau terms* in the effective action (see, e.g., Refs. 18 and 24). The imaginary part of these terms is the only source of damping of the phase excitations in the clean system considered here. The physical origin of this damping is due to the scattering of the thermally excited quasiparticles from the phase excitations.

Since in what follows we are interested in the limit $\Omega, v_F K \ll \Delta_0$ we may safely neglect Ω and K in the first terms of Eqs. (A1) and (A2) and write

$$\begin{aligned}
 \Pi_{33}(\Omega, \mathbf{K}) \approx & - \int \frac{d^2k}{(2\pi)^2} \left\{ \frac{\Delta^2(\mathbf{k})}{E^3(\mathbf{k})} \tanh \frac{E(\mathbf{k})}{2T} \right. \\
 & + \frac{\xi^2(\mathbf{k})}{E^2(\mathbf{k})} \frac{2(E_+ - E_-)}{(E_+ - E_-)^2 - \Omega^2} \\
 & \left. \times [n_F(E_-) - n_F(E_+)] \right\}, \quad (\text{A3})
 \end{aligned}$$

$$\begin{aligned}
 \Pi_{00}^{\alpha\beta}(\Omega, \mathbf{K}) \approx & - \int \frac{d^2k}{(2\pi)^2} \frac{2(E_+ - E_-)}{(E_+ - E_-)^2 - \Omega^2} \\
 & \times [n_F(E_-) - n_F(E_+)] v_{F\alpha}(\mathbf{k}) v_{F\beta}(\mathbf{k}), \quad (\text{A4})
 \end{aligned}$$

$$\begin{aligned}
 \Pi_{03}^\alpha(\Omega, \mathbf{K}) \approx & - \int \frac{d^2k}{(2\pi)^2} \frac{\xi(\mathbf{k})}{E(\mathbf{k})} \frac{2\Omega}{(E_+ - E_-)^2 - \Omega^2} \\
 & \times [n_F(E_-) - n_F(E_+)] v_{F\alpha}(\mathbf{k}). \quad (\text{A5})
 \end{aligned}$$

This approximation is in agreement with that used in Appendix C of Ref. 11.

APPENDIX B: NODAL APPROXIMATION

The key values that are necessary for evaluation of the polarizations, Eqs. (A3), (A4), and (A5), are the differences $E_+ - E_-$ and $n_F(E_+) - n_F(E_-)$. Expanding in \mathbf{K} ,

$$E(\mathbf{k} + \mathbf{K}/2) - E(\mathbf{k} - \mathbf{K}/2) = \mathbf{v}_g(\mathbf{k}) \mathbf{K}, \quad (\text{B1})$$

where the quasiparticle group velocity is given by

$$\mathbf{v}_g(\mathbf{k}) = \nabla_{\mathbf{k}} E(\mathbf{k}) = \frac{1}{E(\mathbf{k})} [\xi(\mathbf{k}) \mathbf{v}_F + \Delta(\mathbf{k}) \mathbf{v}_\Delta]. \quad (\text{B2})$$

It is obvious that due to the gap \mathbf{k} dependence, Eq. (B1) differs from the *s*-wave case²⁴ by the second term.⁴² To perform the calculation analytically it is useful to rewrite Eqs. (B1) and (B2) in terms of the nodal approximation described in detail in Ref. 42 (see also Sec. VII of Ref. 18 where the imaginary parts of these polarizations were calculated). In particular, using this approximation one has $\varepsilon(\mathbf{k}) \approx p_1 \equiv v_F k_1 = p \cos \delta$, $\Delta(\mathbf{k}) \approx p_2 \equiv v_\Delta k_2 = p \sin \delta$ with $E(\mathbf{k}) \approx p = \sqrt{p_1^2 + p_2^2} = \sqrt{v_F^2 k_1^2 + v_\Delta^2 k_2^2}$, where the quasiparticle momentum $\mathbf{k} = (k_1, k_2)$ is written in the nodal coordinate system $\hat{\mathbf{k}}_1, \hat{\mathbf{k}}_2$ associated with the *j*th node ($j = 1, \dots, 4$). (Note that the angle δ was denoted in Refs. 18 and 42 as φ .) Then

$$\mathbf{v}_g \mathbf{K} = v_F K_1 \cos \delta + v_\Delta K_2 \sin \delta \equiv P \cos(\delta - \psi),$$

$$E(\mathbf{k} \pm \mathbf{K}/2) \ll \Delta_0, \quad (\text{B3})$$

where the momentum $\mathbf{K} = (K_1, K_2)$ of the θ particle is also expressed in the nodal coordinate system $\hat{\mathbf{k}}_1, \hat{\mathbf{k}}_2$, so that $P^1 \equiv v_F K_1 = P \cos \psi$, $P^2 \equiv v_\Delta K_2 = P \sin \psi$, and $P = \sqrt{(P^1)^2 + (P^2)^2} = \sqrt{v_F^2 K_1^2 + v_\Delta^2 K_2^2}$. (We denoted the components of \mathbf{P} as P^1, P^2 to make them different from the node label P_j used in what follows.) Finally, we can approximate the difference $n_F(E_+) - n_F(E_-)$ as

$$n_F(E_+) - n_F(E_-) \approx \frac{dn_F(E)}{dE} \mathbf{v}_g \mathbf{K} = \frac{dn_F(E)}{dE} P \cos(\delta - \psi). \quad (\text{B4})$$

Recall also that in the nodal approximation the integral over the original Brillouin zone is replaced by the integration over four nodal subzones as

$$\begin{aligned}
 \int \frac{d^2k}{(2\pi)^2} & \rightarrow \sum_{j=1}^4 \int \frac{dk_1 dk_2}{(2\pi)^2} \\
 & \rightarrow \sum_{j=1}^4 \int \frac{d^2p}{(2\pi)^2 v_F v_\Delta} \\
 & = \sum_{j=1}^4 \int_0^{p_{max}} \frac{p dp}{2\pi v_F v_\Delta} \int_0^{2\pi} \frac{d\varphi}{2\pi}, \\
 p_{max} & = \sqrt{\pi v_F v_\Delta} / a. \quad (\text{B5})
 \end{aligned}$$

It is necessary to underline that the nodal approximation is designed for the low-temperature regime $T \ll \Delta_0$. Thus all polarizations derived in Appendix C are applicable for $T \ll \Delta_0$.

APPENDIX C:

CALCULATION OF Π_{ij} AND EQUATION FOR T_c

After the substitution of Eqs. (B3), and (B4) in Eq. (A3) and integration over p (see Ref. 18) Π_{33} is expressed via the integral

$$I = \int_0^{2\pi} d\delta \frac{\cos^2 \delta \cos^2(\delta - \psi)}{2\pi \cos^2(\delta - \psi) - b^2} \quad (C1)$$

with $b = |\Omega|/P_j$. (Note that we have omitted the subzone index j in ψ and b .) This integral can be calculated using the table integral (3.682) from Ref. 43 for $b^2 > 1$ giving

$$I = \frac{1}{2} + b^2 \cos 2\psi - \frac{|b|^3}{\sqrt{b^2 - 1}} \cos^2 \psi + |b| \sqrt{b^2 - 1} \sin^2 \psi. \quad (C2)$$

Then using the analytical continuation in the region $b^2 < 1$, we finally arrive at the result

$$\begin{aligned} \Pi_{33}(\Omega, \mathbf{K}) = & -\kappa(T) - \sum_{j=1}^4 \frac{\ln 2}{\pi} \frac{T}{v_F v_\Delta} \\ & \times \left\{ \frac{1}{2} + \frac{\Omega^2}{P_j^2} \cos 2\psi_j - \frac{|\Omega|}{P_j} \frac{1}{\sqrt{\frac{\Omega^2}{P_j^2} - 1}} \right. \\ & \times \left[\frac{\Omega^2}{P_j^2} \cos^2 \psi_j - \left(\frac{\Omega^2}{P_j^2} - 1 \right) \sin^2 \psi_j \right] \Theta \left(\frac{|\Omega|}{P_j} - 1 \right) \\ & + i \frac{\Omega}{P_j} \frac{1}{\sqrt{1 - \frac{\Omega^2}{P_j^2}}} \left[\frac{\Omega^2}{P_j^2} \cos^2 \psi_j \right. \\ & \left. \left. + \left(1 - \frac{\Omega^2}{P_j^2} \right) \sin^2 \psi_j \right] \Theta \left(1 - \frac{|\Omega|}{P_j} \right) \right\}, \quad (C3) \end{aligned}$$

where $\kappa(T)$ denotes the term that originates from the first term in the braces of Eq. (A3), which cannot be accurately calculated using the nodal approximation.¹⁸ It is easy to show, however, that $\kappa = m/\pi$ at $T=0$ for the continuum s -wave pairing model. This can be used to write Π_{33} in the dimensionless form,

$$\Pi_{33}(\Omega, \mathbf{K}) = \frac{m}{\pi} \tilde{\Pi}_{33}(\Omega, \mathbf{K}), \quad (C4)$$

where

$$\tilde{\Pi}_{33}(\Omega, \mathbf{K}) = -\tilde{\kappa} - \sum_{j=1}^4 \frac{\ln 2}{2} \frac{T}{\epsilon_F} \alpha_D \{ \dots \}, \quad (C5)$$

where \dots are the terms in the braces in Eq. (C3), $\epsilon_F = mv_F^2/2$ is the Fermi energy, $\alpha_D = v_F/v_\Delta$ (see Sec. II), and we assume that $\tilde{\kappa} \sim 1$. One can also check that the imaginary part of Π_{33} coincides with the expression calculated in Ref. 18 by integrating $\delta(P \cos[\delta - \psi] - \Omega)$.

In the same way we arrive at the expression for the current-current polarization function, Eq. (A4),

$$\begin{aligned} \Pi_{00}^{\alpha\beta}(\Omega, \mathbf{K}) K_\alpha K_\beta / K^2 \\ = - \sum_{j=1}^4 \frac{\ln 2}{\pi} \frac{T}{v_F v_\Delta} \frac{P_j^2 \cos^2 \psi_j}{K^2} \left[1 - \frac{|\Omega|}{P_j} \frac{1}{\sqrt{\frac{\Omega^2}{P_j^2} - 1}} \right. \\ \left. \times \Theta \left(\frac{|\Omega|}{P_j} - 1 \right) + i \frac{\Omega}{P_j} \frac{1}{\sqrt{1 - \frac{\Omega^2}{P_j^2}}} \Theta \left(1 - \frac{|\Omega|}{P_j} \right) \right]. \quad (C6) \end{aligned}$$

The zero-order superfluid stiffness $\Lambda_0^{\alpha\beta} = \delta_{\alpha\beta} n/m$ for the continuum $[m_{\alpha\beta}(\mathbf{k}) = m \delta_{\alpha\beta}]$ system at $T=0$, where n is the total carrier density which, of course, coincides with the density of the neutralizing background. Using the expression $\epsilon_F = \pi n/m$, which is, strictly speaking, valid only for the 2D systems with the quadratic dispersion law, we can also rewrite the superfluid stiffness $\Lambda_{\alpha\beta}$ in the dimensionless form

$$\begin{aligned} \Lambda^{\alpha\beta}(\Omega, \mathbf{K}) \frac{K_\alpha K_\beta}{K^2} \equiv \frac{\epsilon_F}{\pi} \tilde{\Lambda} = \frac{\epsilon_F}{\pi} \left[\tilde{\Lambda}_0 - \sum_{j=1}^4 \ln 2 \frac{T}{\epsilon_F} \alpha_D \right. \\ \left. \times \frac{P_j^2}{(v_F K)^2} \cos^2 \psi_j \dots \right]. \quad (C7) \end{aligned}$$

As was already mentioned after Eq. (C3), the terms that contain the averaging over the Fermi surface [see, e.g., Eqs. (24) and (25)] cannot be accurately calculated using the nodal approximation.¹⁸ Thus, in general, $\tilde{\Lambda}_0$ as well as $\tilde{\kappa}$ should be considered as a free parameter of the model. In particular, decreasing the value of $\tilde{\Lambda}_0$ it is possible to describe a lowering of the zero-temperature superfluid stiffness in HTSC. Nevertheless, for the numerical computations we will assume that $\tilde{\Lambda}_0 \sim 1$. It is easy to obtain (see, e.g., Refs. 18 and 42) the static, zero-momentum bare superfluid stiffness

$$\Lambda = \frac{\epsilon_F}{\pi} \left(\tilde{\Lambda}_0 - 2 \ln 2 \frac{T}{\epsilon_F} \alpha_D \right) \quad (C8)$$

and the velocity of the BA mode

$$v = \sqrt{\frac{\Lambda}{\Pi_{33}(0, \mathbf{0})}} = \frac{v_F}{\sqrt{2}} \sqrt{\frac{\tilde{\Lambda}_0 - 2 \ln 2 \alpha_D T / \epsilon_F}{\tilde{\kappa}}} \quad (C9)$$

so that for $\tilde{\Lambda}_0 = \tilde{\kappa} = 1$ the BA-mode velocity $v(T=0) = v_F/\sqrt{2}$. Using Eq. (C8) one can estimate the temperature of the Berezinskii-Kosterlitz-Thouless transition from the equation $T_c = \pi/2\Lambda(T_c)$, which gives

$$T_c = \frac{\epsilon_F \tilde{\Lambda}_0}{2(1 + \ln 2 \alpha_D)}. \quad (\text{C10})$$

We will use this definition of T_c to express the temperature T in units of T_c and ϵ_F .

Finally, we obtain that expression for Eq. (A5),

$$\begin{aligned} & \Pi_{03}^\alpha(\Omega, \mathbf{K}) K_\alpha / K \\ &= - \sum_{j=1}^4 \frac{T}{\pi v_F} \left\{ \alpha_D \ln 2 \frac{\Omega}{v_F K} \cos \psi_j \right. \\ & \quad \times \left[\cos \psi_j - \frac{|\Omega|}{P_j} \frac{\cos \psi_j}{\sqrt{\frac{\Omega^2}{P_j^2} - 1}} \Theta \left(\frac{|\Omega|}{P_j} - 1 \right) \right. \\ & \quad \left. \left. + i \frac{\Omega}{P_j} \frac{\cos \psi_j}{\sqrt{1 - \frac{\Omega^2}{P_j^2}}} \Theta \left(1 - \frac{|\Omega|}{P_j} \right) \right] \right\}. \quad (\text{C11}) \end{aligned}$$

where we put inside the braces the dimensionless part $\tilde{\Pi}_{03}^\alpha K_\alpha / K$.

APPENDIX D: EQUATIONS FOR V_{CG} , STRUCTURE FACTOR AND TRANSFORMATION TO THE GLOBAL COORDINATE SYSTEM

Substituting Eqs. (C4), (C7), and (C11) in Eq. (46), we obtain the equation for CG mode in the dimensionless form, which is convenient for numerical investigation,

$$\tilde{\Lambda} = \frac{T^2 (\tilde{\Pi}_{03}^\alpha K_\alpha / K)^2}{2\epsilon_F^2 \tilde{\Pi}_{33}}. \quad (\text{D1})$$

The whole expression (44a) for M_θ^{-1} can also be written as

$$M_\theta^{-1}(\Omega, \mathbf{K}) = \frac{m}{\pi} v_F^2 K^2 \tilde{M}_\theta^{-1}(\Omega, \mathbf{K}), \quad (\text{D2})$$

where

$$\begin{aligned} \tilde{M}_\theta^{-1}(\Omega, \mathbf{K}) &= \frac{\Omega^2}{v_F^2 K^2} \hat{\Pi}_{33}(\Omega, \mathbf{K}) - \frac{\Omega}{v_F K} \hat{\Pi}_{03}^\alpha(\Omega, \mathbf{K}) \frac{K_\alpha}{K} \\ & \quad + \frac{1}{2} \hat{\Lambda}^{\alpha\beta} \frac{K_\alpha K_\beta}{K^2}, \quad (\text{D3}) \end{aligned}$$

where the dimensionless polarization functions $\hat{\Pi}_{33}$, $\hat{\Pi}_{03}^\alpha$, and $\hat{\Lambda}^{\alpha\beta}$ were made from the full polarization functions $\tilde{\Pi}_{33}$, $\tilde{\Pi}_{03}^\alpha$, $\tilde{\Lambda}^{\alpha\beta}$ [see Eqs. (33) and (44b)] in the same way as the polarizations Eqs. (C4), (C7), and (C11). The only difference is that these full polarizations include the Coulomb potential $V_c(\mathbf{K})$ (for simplicity we take the 3D potential), which for our purposes is convenient to rewrite as

$$V_c^{-1}(\mathbf{K}) = \frac{m}{2\pi} \frac{v_F^2 K^2}{\omega_p^2}, \quad (\text{D4})$$

where ω_p is the plasma frequency defined after Eq. (36).

Although the local nodal coordinate systems (P_j, ψ_j) are very convenient for calculating the polarization functions (C5), (C7), and (C11), the final expressions for them and, for example, Eq. (D2) have to be calculated in the global or laboratory coordinate system (K, ϕ) . It is convenient to measure the angle ϕ from the vector \hat{k}_x , so that $\phi=0$ corresponds to the corner of the Fermi surface (see, e.g., Fig. 1 in Ref. 18) and the first node is at $\phi=\pi/4$. Thus the transformations from the global coordinate system into the local system related to the j th node are

$$\begin{aligned} P_j &= K \sqrt{v_F^2 \cos^2 \left(\phi - \frac{\pi}{4} + \frac{\pi}{2}(j-1) \right) + v_\Delta^2 \sin^2 \left(\phi - \frac{\pi}{4} + \frac{\pi}{2}(j-1) \right)}, \\ \cos \psi_j &= \frac{v_F K}{P_j} \cos \left(\phi - \frac{\pi}{4} + \frac{\pi}{2}(j-1) \right), \quad \sin \psi_j = \frac{v_\Delta K}{P_j} \sin \left(\phi - \frac{\pi}{4} + \frac{\pi}{2}(j-1) \right), \quad j=1, \dots, 4. \quad (\text{D5}) \end{aligned}$$

*On leave of absence from Bogolyubov Institute for Theoretical Physics, Kiev, Ukraine.

Electronic address: Sergei.Sharapov@unine.ch

†URL: <http://www.unine.ch/phys/theocond/>. Electronic address: Hans.Beck@unine.ch

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