Local density of states in superconductor−strong ferromagnet structures

F. S. Bergeret,[1] A. F. Volkov,[1,2] and K. B. Efetov[1,3]

[1]*Theoretische Physik III, Ruhr-Universität Bochum, D-44780 Bochum, Germany*
[2]*Institute of Radioengineering and Electronics of the Russian Academy of Sciences, 103907 Moscow, Russia*
[3]*L. D. Landau Institute for Theoretical Physics, 117940 Moscow, Russia*
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We study the dependence of the local density of states (LDOS) on coordinates for a superconductor-ferromagnet (S/F) bilayer and a $S/F/S$ structure assuming that the exchange energy h in the ferromagnet is sufficiently large: $h\tau \gg 1$, where τ is the elastic relaxation time. This limit cannot be described by the Usadel equation and we solve the more general Eilenberger equation. We demonstrate that, in the main approximation in the parameter $(h\tau)^{-1}$, the proximity effect does not lead to a modification of the LDOS in the S/F system and a nontrivial dependence on coordinates shows up in next orders in $(h\tau)^{-1}$. In the $S/F/S$ sandwich the correction to the LDOS is nonzero in the main approximation and depends on the phase difference between the superconductors. We also calculate the superconducting critical temperature T_c for the bilayered system and show that it does not depend on the exchange energy of the ferromagnet in the limit of large h and a thick F layer.

I. INTRODUCTION

In recent years the interest in the proximity effect between superconducting (S) and ferromagnetic (F) layers has increased considerably. The actual progress in the preparation of high quality metallic multilayer systems allows a careful study of the mutual interaction of superconductivity and ferromagnetism in hybrid S/F structures. The proximity effect manifests itself in, e.g., changes of the density of states[1] (DOS) or in the dependence of the superconducting critical temperature T_c on the thickness of the F layer (see, e.g., Refs. 2–4).

Many theoretical works have been devoted to the study of such structures, which is not a simple task. In most of them the DOS and the critical temperature of the superconducting transition T_c were analyzed in the "dirty limit." This means that the retarded and advanced Green's functions, which determine the thermodynamical properties, were obtained from the Usadel equation (e.g., Refs. 5–10) which is simpler than the more general Eilenberger equation. In other words, it was assumed that the mean free path l is much shorter than any characteristic length (with exception of the Fermi wave length), and that all energies involved in the problem are smaller than τ^{-1}, where τ is the elastic relaxation time. In particular, the condition $h\tau \ll 1$ has to be satisfied (h is the exchange energy). It was shown that in this limit the DOS in the F layer oscillates and decays with increasing exchange energy h. In the limit $h\tau \ll 1$, the period of the oscillations and the decay length are comparable to each other and are of the order $\sqrt{D/h}$.

Calculations in the opposite (purely ballistic) case were performed in Refs. 11,12. Haltermanns and Valls[12] solved the Bogolyubov and self-consistency equations numerically and presented the results for the LDOS in a F/S structure. Zareyan and co-workers[11] calculated the LDOS of a F film in contact with a superconductor. They assumed that the electrons in the F film are scattered only at the rough F/vac surface ("vac" stands for vacuum). However, not so much attention has been payed to the more realistic case $h\tau \gg 1$.

Except for Ref. 13, where this limit was considered for very thin ferromagnetic films, a study of the proximity effect in disordered ferromagnetic conductors with strong exchange energies h is still lacking. This limit is very important when dealing with ferromagnetic layers made of transition metals, as Fe or Ni, for which $h \lesssim 1$ eV, and whose sizes are larger than the mean free path l.

In the present paper, we analyze the DOS and the critical temperature T_c of the superconducting transition in S/F structures for which the condition

$$h\tau \gg 1 \tag{1}$$

is satisfied. Here τ is the elastic relaxation time due to impurity scattering in the bulk. According to condition (1) the exchange interaction, which is proportional to h is much stronger than the interaction with impurities, which is of the order of τ^{-1}. We will show that in this limit the decay length of the superconducting condensate function induced in the ferromagnet is of the order of the mean free path l. Therefore, despite its smallness, it is very important to retain the impurity scattering term if the thickness of the F layer is larger than l. On the other hand, the product τT_c may be smaller or larger than unity. We will see that in the limit (1) thermodynamical quantities, such as DOS and T_c, differ considerably from those in the dirty[5–10] and purely ballistic limits.[11,12]

As it has been pointed out in Refs. 14,15, the Usadel equation is not applicable in the limit (1), and the more general Eilenberger equation should be solved. One can solve the latter equation with the help of an expansion in the parameter $(h\tau)^{-1}$ for both the strong and weak proximity effect. In the case of a weak proximity effect the solution can be obtained even for an arbitrary impurity concentration,[15] i.e., for an arbitrary value of the parameter $h\tau$. It will be shown that in the limit of large $h\tau$ the condensate function f oscillates in space with a period v_F/h and decays on a length of the order of the mean free path l. If one neglects the impurity scattering in the bulk, as was done in Refs. 11,12,

the results for DOS depend essentially on the boundary conditions at the F/vac interface. This is because in that case the condensate function f is formed by interfering waves reflected from the F/vac boundary. Therefore, the thermodynamical properties depend sensitively on whether these reflections are specular[12] or diffusive.[11]

In the present paper, we assume that the thickness of the ferromagnetic layer d_F is much greater than the mean free path l and therefore reflected waves are unimportant provided the bulk impurity scattering dominates. We show that in the main approximation in the parameter $(h\tau)^{-1}$, the LDOS in the ferromagnetic region in a F/S bilayer is not affected by the proximity effect. On the other hand the amplitude of the condensate function in the ferromagnet near the S/F interface is equal to that in the superconductor S if the interface transmittance is high, and decays away from the interface at distances of the order l. In the limit (1), the mean free path l is much larger than the period of the oscillations ($\sim v_F/h$) and may be comparable with d_F. Therefore, one can speak about long-range penetration of the superconducting condensate into the ferromagnet. In the presence of two superconductors (e.g., in a $S/F/S$ sandwich) the situation changes: a correction to the DOS in F arises in the main approximation and its magnitude depends on the phase difference between the superconductors and on the thickness d_F of the F layer vanishing in the limit $d_F \to \infty$. We also analyze a variation of the critical temperature of the superconducting transition T_c for a S/F bilayer for which the condition (1) is satisfied. We find the dependence of T_c on the thickness d_S of the superconductor in the limit $d_F \gg l$ and compare it with the results obtained in the dirty limit.

II. BASIC EQUATIONS

Physical quantities of interest can be computed using quasiclassical retarded $\hat{g}^R$ and advanced $\hat{g}^A$ Green's functions. In order to find the retarded Green's function $\hat{g}^R$ (for brevity we omit the index R) one should solve the Eilenberger equation (the advanced Green's function can be found in the same way)

$$\mu v_F \partial_x \hat{g} - i(\epsilon + h)[\hat{\tau}_3, \hat{g}] - i[\hat{\Delta}, \hat{g}] + (1/2\tau)[\langle \hat{g} \rangle, \hat{g}] = 0. \tag{2}$$

Here $\hat{g}$ is the retarded Green function in the Nambu space. It obeys the normalization condition

$$\hat{g}^2 = 1. \tag{3}$$

The angle brackets denote the averaging over angles: $\langle (\cdots) \rangle = \int_0^1 d\mu (\cdots)$, $\mu = \cos\theta$, θ is the angle between the momentum and the x axis (in all the S/F systems considered below the x axis is perpendicular to the S/F interfaces), and v_F is the Fermi velocity. The exchange field h vanishes in the S layers. $\hat{\Delta}$ is the matrix

$$\hat{\Delta} = \begin{pmatrix} 0 & \Delta \\ -\Delta^* & 0 \end{pmatrix},$$

and Δ is the pair potential in the superconductor. We assume that in the F layers the electron-electron interaction vanishes and therefore $\Delta = 0$. The last term of Eq. (2) describes the effect of nonmagnetic impurities. Equation (2) is supplemented by the Zaitsev boundary condition at the S/F interface.[16]

$$\hat{a}\left(R - R\hat{a}^2 + \frac{T}{4}(\hat{s}_1 - \hat{s}_2)^2 \right) = \frac{T}{4}[\hat{s}_2, \hat{s}_1]. \tag{4}$$

Here $\hat{a}$ and $\hat{s}$ are the antisymmetric and the symmetric (in μ) parts of the Green's function $\hat{g}$; R and T are the reflection and transmission coefficients. Equations (2)–(4) describe the system completely and are the basic equations in our study. In spite of a rather simple symbolic representation they are quite complicated and a general solution can hardly be obtained for an arbitrary impurity concentration. Accordingly, further calculations are performed under certain assumptions that allow us to simplify Eqs. (2)–(4) for values of parameters which still correspond to real systems.

III. LOCAL DENSITY OF STATES IN A S/F BILAYER

In this section, it is assumed that the mean free path l_s in the superconductor is larger than the coherence length ξ_s. In this case the last term in Eq. (2) can be disregarded in the S region. We consider first a S/F bilayer. The S/F interface is located at $x = 0$, the superconductor and ferromagnet occupy the regions $x < 0$ and $x > 0$, respectively. We also assume that the thickness of both layers are much larger than the characteristic lengths over which $\hat{g}$ varies, i.e., ξ_s in the S layer and l in the F layer. We focus first on the LDOS in the superconductor for subgap energies $\epsilon < \Delta$. It is clear that in this energy range and for $|x| \gg \xi_s$ the LDOS vanishes as it should be in a bulk superconductor.

We assume a perfect S/F interface transparency. In this case and according to the boundary condition Eq. (4) both the symmetric and the antisymmetric parts must be continuous at the interface. For simplicity we approximate the dependence $\Delta(x)$ by a steplike function $\Delta(x) = \Delta\Theta(-x)$. Then, the solution of Eq. (2) in the superconducting region has the form

$$\hat{g}_S = (\epsilon/\xi - \text{sgn}\,\mu\Delta/\xi C e^{\kappa_s x})\hat{\tau}_3$$
$$+ [\Delta/\xi - \text{sgn}\,\mu(\epsilon/\xi)C e^{\kappa_s x}]i\hat{\tau}_2 + C\,\text{sgn}\,\mu e^{\kappa_s x}\hat{\tau}_1. \tag{5}$$

Here $\xi = \sqrt{\epsilon^2 - \Delta^2}$, $\kappa_s = -2i\xi/|\mu|v_F$, and C is a constant which has to be determined from the boundary conditions. Note that the solution (5) can also be represented in terms of the eigenfunctions $\hat{U} = (u, v)$ of the Bogolyubov equation, i.e., it can be written as

$$\hat{g}_S = \hat{g}_{Sb} + \hat{\tau}_3 \hat{U}^+(x) \otimes \hat{U}(x), \tag{6}$$

where $\hat{g}_{sb}$ is the Green function in the bulk. For energies $|\epsilon| < \Delta$ the functions $\hat{U}(x)$ decay exponentially away from the S/F interface.

Now we have to solve Eq. (2) in the ferromagnet. In the limit determined by Eq. (1) one can check that the solution of Eq. (2) in the main approximation with respect to $(h\tau)^{-1}$ is

$$\hat{g}_F = \hat{\tau}_3 + D\exp(-\kappa x/|\mu|l)i\hat{\tau}_2 + \text{sgn}\,\mu D\exp(-\kappa x/|\mu|l)\hat{\tau}_1,$$
(7)

where $\kappa = 1 - 2i(\epsilon + h)\tau$. The second and third terms in Eq. (7) describe the condensate function induced in the ferromagnet due to the proximity effect.

As opposed to many authors claiming that the decay length of the condensate function is proportional to v_F/h (see, for example, Ref. 7), Eq. (7) clearly shows that the penetration depth of the condensate is of the order of the mean free path l and does not depend on the exchange field h. Thus, for clean and strong ferromagnets, the superconducting condensate penetrates over long distances compared to the magnetic length v_F/h. As mentioned in the Introduction, in the dirty case, $h\tau \ll 1$, the penetration length of the condensate into the ferromagnet decreases with increasing h, which is true for the singlet component of the condensate function (the triplet component of the condensate may penetrate into the ferromagnet over a long length of the order $\sqrt{D/T}$.[17,18] This triplet component may be induced by the presence of a magnetic inhomogeneity close to the S/F interface.[17]) However, we see from Eq. (7) that a long-range proximity effect exists even for singlet pairing and homogeneous magnetization. Using the fact that $\hat{g}$ is continuous at $x = 0$, one can determine the constants D and C from Eqs. (5) and (7)

$$C = D = \frac{-\xi + \epsilon}{\Delta}.$$
(8)

The normalized LDOS is given by the well known expression

$$\tilde{\nu} = \nu/\nu_0 = (1/2)\text{ReTr}\hat{\tau}_3\hat{g},$$
(9)

where ν_0 is the LDOS in the normal state. From Eqs. (5)–(9) one obtains in the S region ($\epsilon < \Delta$)

$$\tilde{\nu}(x,\epsilon) = \langle\exp(2\sqrt{\Delta^2 - \epsilon^2}\,x/|\mu|v_F)\rangle.$$
(10)

In Fig. 1, we plot the spatial dependence of $\tilde{\nu}$ and the absolute value of the imaginary part of the symmetric condensate function [second term in Eq. (7)]. Due to the proximity effect of the ferromagnet the superconductor becomes gapless in the region close to the S/F interface. According to Eqs. (7) and (9) the LDOS in the ferromagnet remains unchanged despite the presence of the condensate function induced in the F region [second and third term in Eq. (7)]. The reason for that is the cancellation of the symmetric [second term in Eq. (7)] and antisymmetric parts [third term in Eq. (7)] in the normalization condition (3). Thus, the coefficient in front of τ_3 in Eq. (7) is 1 and therefore $\tilde{\nu} = 1$.

The correction to the LDOS in the ferromagnet is not zero if one takes into account higher order terms in the expansion in the parameter $(h\tau)^{-1}$. We calculate this correction assum-
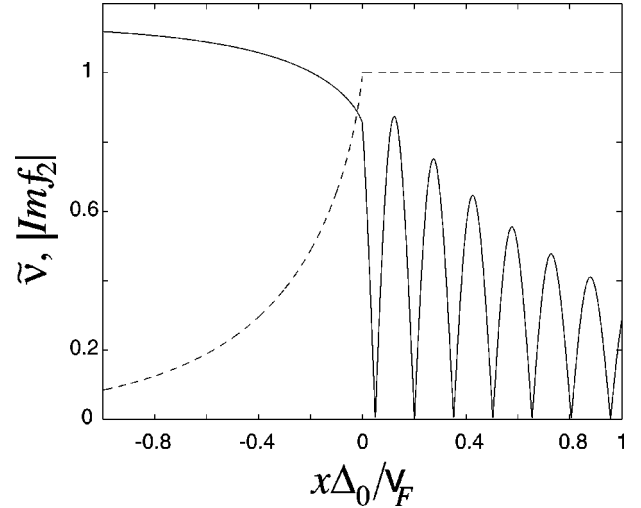

FIG. 1. The spatial dependence of the normalized LDOS $\tilde{\nu}$ (dashed line), and the imaginary part of the symmetric condensate function $|\text{Im}f_2|$ (solid line). The S (F) layer is located in the region $x < 0$ ($x > 0$). Δ_0 is the value of Δ for $T = 0$. $\Delta/\Delta_0 = 0.8$, $\epsilon/\Delta_0 = 0.4$, $\tau\Delta_0 = 1$, and $h/\Delta_0 = 10$. Although the function $|\text{Im}f_2|$ exhibits a strongly oscillating and weakly decaying behavior in the ferromagnet, the LDOS remains unchanged in this region.

ing for simplicity that the transparency of the S/F interface is low. In this case, the Green function in the superconductor is not affected by the proximity effect and is given by

$$\hat{g}_S = G_S\hat{\tau}_3 + F_S i\hat{\tau}_2,$$
(11)

where $G_S = \epsilon/\sqrt{\epsilon^2 - \Delta^2}$ and $F_S = \Delta/\sqrt{\epsilon^2 - \Delta^2}$. For a low interface transparency, the boundary condition (4) reduces to

$$\hat{a} = \gamma[\hat{s}_F, \hat{s}_S] \cong \gamma F_S\hat{\tau}_1.$$
(12)

Here $\gamma = T(\mu)/4R(\mu) \ll 1$ is the parameter describing the transmittance of the interface. The way how to solve Eq. (2) was presented in Ref. 15. The exact Green function is given by

$$\hat{g}_F(x) = \hat{\tau}_3 + f_2(x)i\hat{\tau}_2 + f_1(x)\hat{\tau}_1$$

$$= \hat{\tau}_3 + \int (dk/2\pi)[f_{2k}i\hat{\tau}_2 - (\mu l/\kappa)f_{2k}\hat{\tau}_1\partial_x]e^{ikx},$$
(13)

where

$$f_{2k} = \frac{2\kappa F_S l}{M[1 - \kappa\langle 1/M\rangle]}[-\kappa(\gamma\mu\langle 1/M\rangle - \langle\gamma\mu/M\rangle) + \gamma\mu],$$
(14)

$M = (kl\mu)^2 + \kappa^2$, and $\kappa = 1 - 2i(\epsilon + h)\tau$. In the case under consideration, i.e., $h\tau \gg 1$, one can easily show that $|\kappa\langle 1/M\rangle| < 1$ and hence we can expand Eq. (14):

$$f_{2k} \cong \frac{2\kappa F_S l}{M}(\gamma\mu + \kappa\gamma_0/M_{\mu=1}).$$
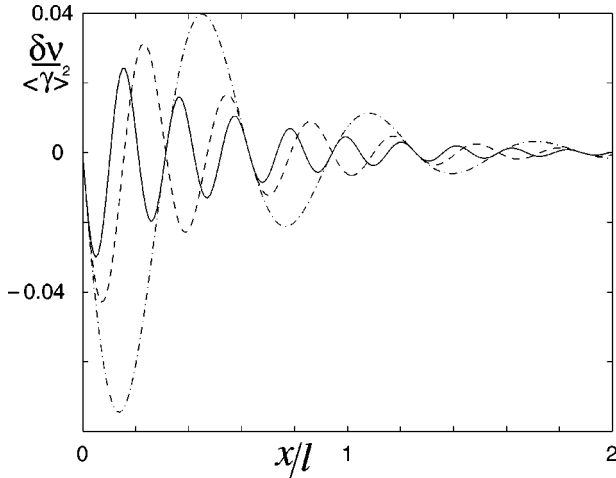(15)

FIG. 2. Spatial dependence of the LDOS in the ferromagnet for different values of $h\tau$. The solid, dashed, and dash-dotted lines correspond to $h\tau=15$, $h\tau=10$, and $h\tau=5$, respectively.

For simplicity we have assumed that the transmission coefficient $T(\mu)$ has a sharp maximum at $\theta=0$, i.e., $\gamma(\mu)=\gamma_0\delta(\mu-1)$. The first correction to the normalized LDOS proportional to $(h\tau)^{-1}$ can be now obtained. After cumbersome but straightforward calculations one obtains at low energies

$$\delta\nu=\frac{1}{2}\mathrm{Re}\langle(f_2)^2-(f_1)^2\rangle=-\gamma_0^2(e^{-2x/l}/4h\tau)\sin(4h\tau x/l).$$
$$(16)$$

Equation (16) shows that for sufficiently large $h\tau$, the correction to the LDOS of the normal metal is small. This correction to the LDOS proportional to $(h\tau)^{-1}$ reveals a damped-oscillatory behavior similar as the one reported in Ref. 10, where the case $h\tau\ll1$ (dirty limit) was considered. The spatial dependence of the local DOS is shown in Fig. 2.

IV. LOCAL DENSITY OF STATES IN A $S/F/S$ SANDWICH

Now we consider a Josephson-like $S/F/S$ structure (see Fig. 3). In this case, the correction to the DOS is not zero even in the main approximation in the parameter $(h\tau)^{-1}$. The thickness of the F layer is d_F, and we assume again that the transparency of the S/F interfaces is low. The solution of Eq. (2) can be sought in the following form:
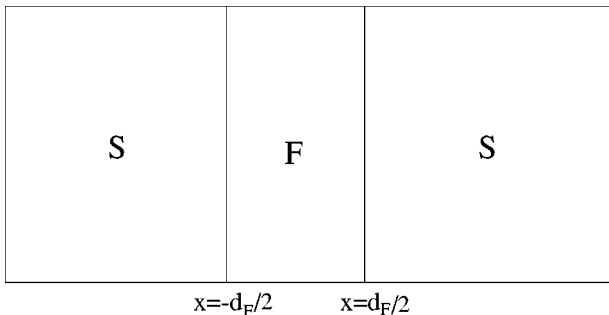

FIG. 3. The $S/F/S$ structure.

$$\hat{g}=\hat{\tau}_3+\hat{s}+\hat{a}, \qquad (17)$$

where $\hat{s}$ and $\hat{a}$ are the symmetric and antisymmetric part of the condensate function induced in the ferromagnet. The equations which determine $\hat{s}$ and $\hat{a}$ can be obtained from Eq. (2) and written in the form

$$(\mu l)^2\partial_{xx}^2\hat{s}-\kappa^2\hat{s}-\kappa\langle\hat{s}\rangle=0,$$

$$\hat{a}=-(\mu l/\kappa)\hat{\tau}_3\partial_x\hat{s}, \qquad (18)$$

where κ is defined in Eq. (14).

In the limit of a weak proximity effect, the boundary conditions at $x=\pm d_F/2$ are [the upper (lower) sign corresponds to $x=d_F/2$ $(x=-d_F/2)$]

$$\hat{a}=\mp\gamma\hat{\tau}_3\hat{F}_S. \qquad (19)$$

$\hat{F}_S$ is the condensate function of the superconductors which now is given by $\hat{F}_S(\varphi)=i\hat{\tau}_2 F_S\exp(\pm i\varphi\hat{\tau}_3)$, where F_S is defined in Eq. (11) and φ is the phase difference between the superconductors. The general solution of Eq. (18) with the boundary conditions Eq. (19) was presented in Ref. 15. In the limit $h\tau\gg1$ one obtains

$$\hat{s}(x)=-2\kappa_F l\mu\gamma F_S\sum_n[i\hat{\tau}_2\cos\varphi/2+i\hat{\tau}_1(-1)^n\sin\varphi/2]$$

$$\times\int dk/2\pi e^{-ikx}e^{ik(2n+1)d_F}/M$$

$$=-\gamma F_S\left[i\hat{\tau}_2\cos(\varphi/2)\frac{\cosh\theta(x)}{\sinh\theta(d_F/2)}\right.$$

$$\left.+i\hat{\tau}_1\sin(\varphi/2)\frac{\sinh\theta(x)}{\cosh\theta(d_F/2)}\right], \qquad (20)$$

where $\theta(x)=\kappa x/l|\mu|$ and n takes integer values from $-\infty$ to $+\infty$. From Eq. (18) one obtains the expression for $\hat{a}$

$$\hat{a}(x)=\gamma F_S\left[\hat{\tau}_2\sin(\varphi/2)\frac{\cosh\theta(x)}{\cosh\theta(d_F/2)}\right.$$

$$\left.-\hat{\tau}_1\cos(\varphi/2)\frac{\sinh\theta(x)}{\sinh\theta(d_F/2)}\right]. \qquad (21)$$

The correction to the LDOS due to the proximity effect is then given by

$$\delta\nu=-\frac{1}{2}\mathrm{Re}(\hat{s}^2+\hat{a}^2)$$

$$=2\frac{\Delta^2}{\epsilon^2-\Delta^2}\mathrm{Re}\left\langle\frac{\gamma^2}{\sinh^2 2\theta(d_F/2)}\right.$$

$$\left.\times[1+\cos\varphi\cosh 2\theta(d_F/2)]\right\rangle, \qquad (22)$$

where $2\theta(d_F/2)=\kappa d_F/l|\mu|$ As one could expect, $\delta\nu$ vanishes in the limit of a very large thickness d_F of the F layer.
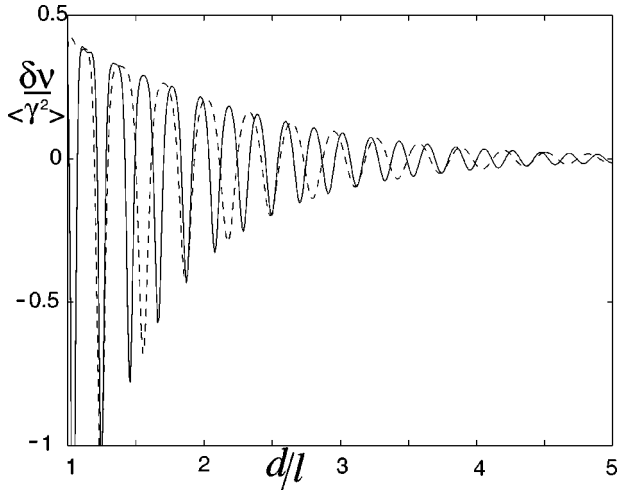
FIG. 4. LDOS versus the thickness d_F of the F layer. Here $\epsilon\tau = 0.1$, $\Delta\tau = 1$, and $\varphi = 0$. The solid and dashed lines correspond to $h\tau = 15$ and $h\tau = 10$.

At the same time, one can see that the correction to the LDOS is not x dependent but it depends on the phase difference φ. This behavior is quite interesting and rather unexpected. The correction to the LDOS, Eq. (22), may be both positive and negative depending on the phase φ. In Figs. 4 and 5 we plot the dependence of $\delta\nu$ on the thickness d_F and on the phase difference φ. As before we assume that $\gamma^2(\mu) = (\gamma_0^2/\delta\mu)\delta(\mu-1)$, where $\delta\mu$ is the width of the peak in the dependence $\gamma(\mu)$. Thus, for a $S/F/S$ sandwich the correction to the LDOS $\delta\nu$ is much larger than in the case of a S/F bilayer. In the latter case the correction is proportional to the small factor $(h\tau)^{-1}$, while in the $S/F/S$ structure $\delta\nu$ is finite in zeroth order in $(h\tau)^{-1}$ and proportional to the large factor $1/\delta\mu$.

V. CRITICAL TEMPERATURE FOR A S/F BILAYER

Another thermodynamic quantity we are interested in, is the superconducting critical temperature of a S/F bilayer in
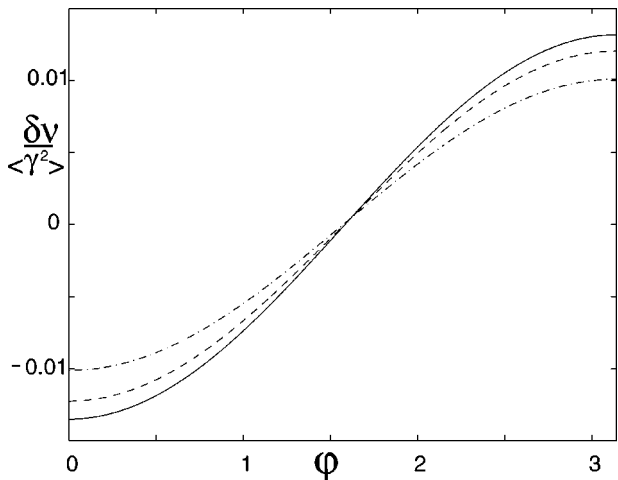

FIG. 5. LDOS versus the phase difference φ for the parameters $\epsilon\tau = 0.1$, $\Delta\tau = 1$, and $d_F/l = 5$. The solid, dashed, and dash-dotted lines correspond to $h\tau = 15$, $h\tau = 10$, and $h\tau = 5$, respectively.

the limit $h\tau \gg 1$. The opposite case ($h\tau \ll 1$) was studied in Refs. 5–7, 9. Here we consider a bilayer system consisting of one S layer of thickness d_S and one F layer of thickness d_F. Of course, the superconducting transition temperature can considerably change only provided the thickness of the superconducting layer is not very large $d_s \lesssim \xi_S$. We assume that the S/F interface has a very high transparency, otherwise the proximity effect does not affect substantially the critical temperature T_c. The computation of the condensate function $\hat{f} = \hat{s} + \hat{a}$ from Eq. (2) follows as before. In the limit $h\tau \gg 1$ the solution in the F layer is given simply by Eq. (7). The solution in the S region requires a little more care. For temperatures close to T_c one can seek for solutions in the form

$$\hat{g}_S = \hat{\tau}_3 + F_2 i\hat{\tau}_2 + F_1\hat{\tau}_1. \tag{23}$$

If we assume that $\Delta(x)$ is a slow varying function of x, then the function F_2 can be written as[19]

$$F_2 = \Delta/\epsilon + \delta F. \tag{24}$$

Bearing in mind that the functions F_1 and F_2 are continuous at the S/F interface, one obtains the following equation for δF:[20]

$$(\mu l)^2 \partial_{xx}^2 \delta F - \kappa_s^2 \delta F - \kappa_s\langle \delta F\rangle = -2|\mu|l\kappa_s D\delta(x), \tag{25}$$

where $\kappa_s = -1 + 2i\epsilon\tau$, and D is the constant from Eq. (7). Equation (25) can be easily solved in the Fourier space. The solution is given by

$$\delta F_k = \frac{2l\kappa_s D}{M_S(1 + \kappa_s\langle 1/M_S\rangle)}(|\mu| + |\mu|\kappa_s\langle 1/M_S\rangle - \kappa_s\langle|\mu|/M_S\rangle). \tag{26}$$

Here $M_S = (kl\mu)^2 + \kappa_s^2$. In the dirty limit, i.e., when $\epsilon\tau \ll 1$, the expression Eq. (26) can be simplified:

$$\delta F_k \cong \frac{3l\kappa_s D}{(lk)^2 - 6i\epsilon\tau} \tag{27}$$

and the value of the function $\delta F(x)$ at $x = 0$ is given by

$$\delta F(0) = \int \frac{dk}{2\pi} \delta F_k = -\frac{\sqrt{3}(1+i)}{4}\frac{D}{\sqrt{\epsilon\tau}}. \tag{28}$$

From Eqs. (7), (24)–(28), and the fact that all the functions are continuous at $x = 0$, one can determine the constant D

$$D = \frac{\Delta}{\epsilon}\frac{4\sqrt{\epsilon\tau}}{\sqrt{3}(1+i)}. \tag{29}$$

We see that the constant D does not depend on the strength of the exchange field h, and therefore the condensate function in the superconductor is not h dependent. Thus, we come to the following conclusion: in the limit $h\tau \gg 1$ and if $d_F > l$ the thermodynamic quantities of the superconductor do not depend on the strength of the exchange field h; in par-

ticular, the critical temperature of the bilayer does not depend on the thickness of the ferromagnetic layer. This result is in qualitatively agreement with experimental data presented in Ref. 3, in which the dependence $T_c^*(d_{Gd})$ of Nb/Gd systems was determined and a saturation of T_c^* for large d_{Gd} was observed.

In order to estimate the critical temperature T_c^* of the system, we need to calculate the pair potential $\Delta(x)$. The latter satisfies the self-consistency equation

$$\Delta = \lambda \int d\epsilon \tanh(\epsilon\beta)(F_2^R - F_2^A). \tag{30}$$

We notice that all equations presented above are for the retarded component. The advanced Green function F_2^A can be obtained using the relation $(F_2^R)^* = -F_2^A$. One can easily check that at $x=0$ the integral in Eq. (30) diverges as $1/\sqrt{\epsilon}$ and therefore the function $\Delta(x)$ must vanish at $x=0$. Thus, the boundary condition for the pair potential at the S/F interface can be written as

$$\Delta(0) = 0. \tag{31}$$

Using the Ginsburg-Landau equation

$$\xi_{GL}^2 \Delta'' = -\Delta, \tag{32}$$

where $\xi_{GL} = \xi_0/\sqrt{1 - T_c^*/T_c}$, and the boundary condition Eq. (31) we obtain

$$\Delta = A \sin(x + d_s)/\xi_{GL}. \tag{33}$$

At the boundary with the vacuum $(x = -d_s)$ $\partial_x\Delta = 0$. From this condition we obtain an expression which determines T_c^*

$$1 - T_c^*/T_c = (\pi\xi_0/2d_s)^2. \tag{34}$$

We remind the reader that Eq. (34) is only valid in the case $|T_c^* - T_c| < T_c$. It gives the asymptotic value of T_c^* in the limit $h\tau \gg 1$ and $d_F \gg l$. This formula describes the usual proximity effect in a S/N structure: T_c^* coincides with T_c if $d_s \gg \xi_0$ and decreases with increasing d_s Ref. 21. The same expression for T_c^* is valid in the dirty limit ($h\tau \ll 1$, see Ref. 7).

VI. CONCLUSION

In conclusion, we have considered different S/F structures in the case of a strong exchange field h, when the condition $h\tau \gg 1$ is satisfied. The results of our analysis differ considerably from those obtained in the "dirty"[5–10] and in the pure ballistic limit.[12] We have shown that the thermodynamic properties (such as local DOS and critical temperature) of a S/F bilayer do not depend on h in the main approximation with respect to the small parameter $(h\tau)^{-1}$. It is worth mentioning that in this approximation the LDOS does not change in the ferromagnet below T_c. The superconducting gap is however suppressed near the S/F interface (gapless state) and restored at distances of the order of $\xi_s \sim v_F/\Delta$. In a $S/F/S$ sandwich the correction to the LDOS in the ferromagnet is nonzero, spatially constant, and depends on the phase difference φ between the superconductors. The different behavior of the LDOS in a S/F and $S/F/S$ structure is due to the interference of the induced condensate functions created at each S/F interface. Thus, the changes of the LDOS ν due to the proximity effect may be observed more clearly in a $S/F/S$ structure by measuring the dependence of ν on the phase difference φ between the superconductors.

Note added in proof. A similar problem was analyzed by Baladie and Buzdin in a recently published paper.[22] The authors of Ref. 22 also came to the conclusion that in the lowest order in the parameter $(h\tau)^{-1}$, there is no change in the DOS in the ferromagnet F. Unlike our approach which allows one to obtain an exact solution for the condensate function [Eq. (14)], the authors of Ref. 22 obtained an approximate solution making an expansion in the parameter $(h\tau)^{-1}$. The system $S/F/S$, where the DOS is changed in the lowest order of $(h\tau)^{-1}$, was not studied in Ref. 22.

[1] T. Kontos, M. Aprili, J. Lesueur, and X. Grison, Phys. Rev. Lett. **86**, 304 (2001).

[2] P. Koorevar, Y. Suzuki, R. Coehoorn, and J. Aarts, Phys. Rev. B **49**, 441 (1994).

[3] C. Strunk, C. Sürgers, U. Paschen, and H.v. Löhneysen, Phys. Rev. B **49**, 4053 (1994).

[4] J.S. Jiang, D. Davidović, D.H. Reich, and C.L. Chien, Phys. Rev. Lett. **74**, 314 (1995).

[5] Z. Radovic, M. Ledvij, L. Dobrosavljevic, A.I. Buzdin, and J.R. Clem, Phys. Rev. B **44**, 759 (1991).

[6] A.I. Buzdin, B. Bujicic, and M.Yu. Kupriyanov, Sov. Phys. JETP **74**, 124 (1992); A.I. Buzdin and M.Yu. Kupriyanov, JETP Lett. **52**, 487 (1990).

[7] J. Aarts, J.M. Geers, E. Brück, A.A. Golubov, and R. Coehoorn, Phys. Rev. B **56**, 2779 (1997).

[8] R. Fazio and C. Lucheroni, Europhys. Lett. **45**, 707 (1999).

[9] L. Lazar, K. Westerholt, H. Zabel, L.R. Tagirov, Yu.V. Goryunov, N.N. Garif'yanov, and I.A. Garifullin, Phys. Rev. B **61**, 3711 (2000).

[10] A. Buzdin, Phys. Rev. B **62**, 11 377 (2000).

[11] M. Zareyan, W. Belzig, and Yu.V. Nazarov, Phys. Rev. Lett. **86**, 308 (2001).

[12] K. Halterman and O.T. Valls, cond-mat/0107232 (unpublished).

[13] F.S. Bergeret, K.B. Efetov, and A.I. Larkin, Phys. Rev. B **62**, 11 872 (2000).

[14] A.I. Buzdin, L.N. Bulaevskii, and S.V. Panyukov, JETP Lett. **35**, 178 (1982).

[15] F.S. Bergeret, A.F. Volkov, and K.B. Efetov, Phys. Rev. B **64**, 134506 (2001).

[16] A.V. Zaitsev, Sov. Phys. JETP **59**, 863 (1984).

[17] F.S. Bergeret, A.F. Volkov, and K.B. Efetov, Phys. Rev. Lett. **86**, 4096 (2001).

[18] A. Kadigrobov, R.I. Shekhter, and M. Jonson, Europhys. Lett. **54**, 394 (2001).

[19] This assumption is justified when $|T_c^* - T_c| < T_c$, where T_c^* is the transition temperature of the bilayer and T_c the critical temperature of the bulk superconductor.

[20] We continue δF for positive x in a symmetric way; however the solution we need corresponds to negative values of x.

[21] P.G. de Gennes, *Superconductivity of Metals and Alloys* (Benjamin, New York, 1966).

[22] I. Baladie and A. Buzdin, Phys. Rev. B **64**, 224514 (2001).