Stable magnetostatic solitons in yttrium iron garnet film waveguides for tilted in-plane magnetic fields

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The possibility of nonlinear pulses generation in yttrium iron garnet thin films for arbitrary direction between waveguide and applied static in-plane magnetic field is considered. Up to now only the cases of in-plane magnetic fields either perpendicular or parallel to the waveguide direction have been studied both experimentally and theoretically. In the present paper it is shown that also for other angles (besides 0 or 90°) between a waveguide and static in-plane magnetic field stable bright or dark (depending on magnitude of magnetic field) solitons can be created.

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I. INTRODUCTION

The investigation of magnetostatic envelope solitons in yttrium iron garnet thin magnetic films is one of the ''hot topics'' in physics nowadays. Advanced instrumentation for microwave pulse generation, detection, and analysis together with a solid theoretical base has led to a growing interest in studying such localized objects. The definition ''magnetostatic soliton'' refers to a propagating pulse formed by large wavelength spin excitations which do not ''feel'' the exchange interaction. Therefore only the dipolar interactions could be taken into account. Thus the processes are characterized by the Landau-Lifshitz and magnetostatic equations.

The linearized solutions of these equations were obtained by Damon and Eshbach[1] 40 years ago for an arbitrary direction between the wave vector of spin excitations and in-plane magnetic field. The nature of those excitations has been also studied experimentally.[2] The weakly nonlinear limit for the mentioned equations also was considered for the particular cases when the wave vector of spin excitations is either parallel (backward volume waves) or perpendicular (surface waves) to the direction of the in-plane magnetic field. It was found[3] that the envelope of spin excitations in both cases satisfies the two-dimensional (2D) nonlinear Schrödinger (NLS) equation which permits well-known 1D soliton solutions depending on the relative sign of dispersion and nonlinear terms.

In full accordance with the theoretical predictions bright solitons have been observed for the nonlinear backward volume wave case[4–7] (the in-plane field is directed parallel to the carrier wave vector and propagation velocity of the envelope soliton), while dark solitons are created in the case of nonlinear surface waves[8–10] (the carrier wave vector and group velocity is perpendicular to the magnetic field). It should be especially noted that all the mentioned solitons are observed in narrow strips. In such geometries transverse instabilities do not develop and experiments show the stable propagation of 1D solitons along the waveguides. At the same time, in wide samples 1D solitons are in general unstable[11,12] and form metastable spin-wave bullets which decay either after edge reflection or mutual interaction.[13,14]

We emphasize that the solitons in in-plane magnetized films are studied both theoretically and experimentally only for two particular cases when the pulse propagates along or perpendicular to the magnetic field. Only very recently was the general case of linear and nonlinear magnetostatic wave propagation in wide samples investigated[15] for a wide range of angles between the propagation velocity and magnetic field. In this connection the natural question arises: why one does not consider the nonlinear pulses in waveguides which are not either parallel or perpendicular to the in-plane static magnetic field. As we show below for each magnitude of the internal magnetic field it is possible to choose the direction (besides 0 or 90°) of the waveguide respect to the magnetic field direction for which the stable propagation of envelope solitons is allowed (see the inset of Fig. 1 for a geometry of the problem). We determine the limits for the magnitude of the magnetic field, angle between waveguide, and magnetic field vector and pulse frequency necessary for the creation of dark or bright envelope solitons. We also calculate their widths and propagation velocities and claim that such solitons can be experimentally observed.

II. BASIC CONSIDERATION: LINEAR MAGNETOSTATIC WAVES

The linear consideration is based upon the Damon-Eshbach formulation[1] and its generalization by Hurben and Patton[2] for the case of arbitrary angles between a wave vector $\vec{k}$ and static internal magnetic field $\vec{H}_0$. Further we will examine only the so-called ''near-uniform'' case ($kd \ll 1$, d stands for film thickness) and derive the dispersion expansion over the parameter kd up to second order. Therefore we present here only the steps necessary for this purpose. Consideration of the mentioned wave number range sufficiently simplifies the calculations and, besides that, most of the ex-

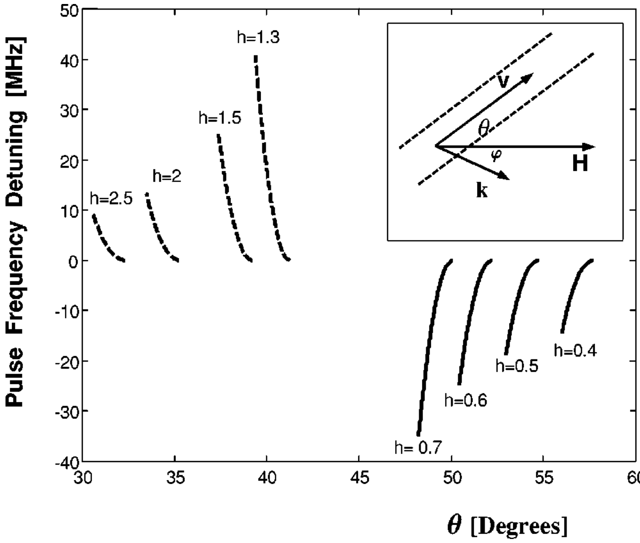

FIG. 1. Detuning of the pulse frequency ω-ω_0 vs an angle θ between waveguide and static in-plane magnetic field for its different magnitude $h=\omega_H/\omega_M$. Dashed and solid lines correspond to the dark and bright soliton cases, respectively. The inset shows the geometry of the problem and dashed lines indicate the direction of a waveguide. The following film parameters are used in the calculations: film thickness $d=10$ μm and value of the demagnetizing field $H_M=1750$ Oe.

periments on the magnetostatic envelope solitons are made having such carrier wave numbers.

Examining an in-plane magnetized ferromagnetic film with unpinned surface spins let us make the following definitions: z is the direction of the internal static magnetic field, $\vec{r}$ indicates the radius vector lying in the sample plane $(y,\ z)$, and x is a coordinate along the direction perpendicular to the film plane. Then one can write down the Landau-Lifshitz and magnetostatic equations

$$\frac{d\vec{M}}{dt}=-g[\vec{M}\times\vec{H}], \quad \mathrm{div}\ (\vec{H}+4\pi\vec{M})=0, \quad \mathrm{rot}\ \vec{H}=0. \tag{1}$$

Here g is a modulus of the gyromagnetic ratio for electrons, $\vec{M}$ is a magnetization density vector, and $\vec{H}$ is an internal total magnetic field. Introducing the dynamical dimensionless quantities $\vec{m}=(\vec{M}-\vec{M}_0)/M_0$ and $\vec{h}=(\vec{H}-\vec{H}_0)/H_0$ the following equations are obtained in the linear limit over $|\vec{m}|$:

$$\frac{dm_x}{dt}=\omega_H(m_y-h_y), \quad \frac{dm_y}{dt}=-\omega_H(m_x-h_x), \tag{2}$$

$$\frac{\partial}{\partial x}(\omega_H h_x+\omega_M m_x)+\frac{\partial}{\partial y}(\omega_H h_y+\omega_M m_y)+\frac{\partial}{\partial z}(\omega_H h_z)=0,$$

where $\omega_H=gH_0$ and $\omega_M=4\pi g M_0$. Defining $\vec{h}=\mathrm{grad}\Phi$ and searching for the solution of Eq. (2) in periodical form over t [i.e., Φ and $m_{x,y}$ being proportional to $\exp(-i\omega t)$] we get

$$\frac{\partial^2}{\partial z^2}\Phi+(\chi_1+1)\left(\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial x^2}\right)\Phi=0 \quad \text{for} \quad |x|<\frac{d}{2},$$

$$\left(\frac{\partial^2}{\partial z^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial x^2}\right)\Phi=0 \quad \text{for} \quad |x|>\frac{d}{2}, \tag{3}$$

and

$$m_x=\frac{\omega_H}{\omega_M}\left(i\chi_2\frac{\partial\Phi}{\partial y}+\chi_1\frac{\partial\Phi}{\partial x}\right),$$

$$m_y=-\frac{\omega_H}{\omega_M}\left(i\chi_2\frac{\partial\Phi}{\partial x}-\chi_1\frac{\partial\Phi}{\partial y}\right),$$

where

$$\chi_1=\frac{\omega_H\omega_M}{\omega_H^2-\omega^2}, \qquad \chi_2=\frac{\omega\omega_M}{\omega_H^2-\omega^2}.$$

Thus from Eqs. (3) we can write down a linear solution of Eqs. (1) in the form[2]

$$\Phi=(Ae^{\kappa x}+Be^{-\kappa x})e^{-i(\omega t-\vec{k}\vec{r})} \quad \text{for} \quad |x|<\frac{d}{2},$$

$$\Phi=Ce^{-k(x-d/2)}e^{-i(\omega t-\vec{k}\vec{r})} \quad \text{for} \quad x>\frac{d}{2}, \tag{4}$$

$$\Phi=De^{k(x+d/2)}e^{-i(\omega t-\vec{k}\vec{r})} \quad \text{for} \quad x<-\frac{d}{2},$$

where A, B, C, D are arbitrary constants at the present stage,

$$\kappa^2=\frac{k^2+\chi_1 k_y^2}{1+\chi_1},$$

and let us recall that the two-dimensional vectors $\vec{r}=(y,\ z)$ and $\vec{k}=(k_y,\ k_z)$ lie in the film plane and $k\equiv\sqrt{k_y^2+k_z^2}$.

If $\kappa^2>0$, we deal with a so-called surface mode; otherwise $\kappa=i\sqrt{-\kappa^2}$ and a volume mode exists. However, note that due to the condition of "near uniformity" $kd\ll 1$ the difference between these two modes is negligible.

The dispersion relation can be obtained from Eq. (4) if we remember about the boundary conditions. Particularly the functions Φ and $h_x+4\pi m_x$ should be continuous on the boundaries $-d/2$ and $d/2$ of the film. The dispersion relation for both modes could be written as follows:

$$2k(\chi_1+1)\kappa\frac{e^{\kappa d}+e^{-\kappa d}}{e^{\kappa d}-e^{-\kappa d}}-k_y^2\chi_2^2+k^2+\kappa^2(\chi_1+1)^2=0. \tag{5}$$

Working in the limit $kd\ll 1$ and keeping only the terms up to second order of this parameter the following expression is obtained:

$$\omega=\omega_0+\frac{\omega_M}{4\omega_0}\frac{d}{k}(\omega_M k_y^2-\omega_H k_z^2)-\frac{\omega_M^2}{32\omega_0^3}\frac{d^2}{k^2}(\omega_M k_y^2-\omega_H k_z^2)^2$$
$$+\frac{\omega_M}{4\omega_0}d^2\left(\omega_H\frac{k_z^2}{3}-\omega_M k_y^2\right), \tag{6}$$

where $\omega_0\equiv\omega(k=0)=\sqrt{\omega_H(\omega_H+\omega_M)}$. Then we get from Eq. (6) the following expressions for the derivatives of ω over k_y and k_z:

$$v_y=\frac{\partial\omega}{\partial k_y}=\frac{\omega_M}{4\omega_0}d\left\{\frac{k_y}{k}\left[\omega_M\frac{k_y^2}{k^2}+(2\omega_M+\omega_H)\frac{k_z^2}{k^2}\right]+\mathcal{O}_1(kd)\right\},$$

$$v_z=\frac{\partial\omega}{\partial k_z}=-\frac{\omega_M}{4\omega_0}d\left\{\frac{k_z}{k}\left[\omega_H\frac{k_z^2}{k^2}+(2\omega_H+\omega_M)\frac{k_y^2}{k^2}\right]+\mathcal{O}_2(kd)\right\},$$

$$\omega''_{yy}=\frac{\partial^2\omega}{\partial k_y^2}=\frac{\omega_M}{4\omega_0}\frac{d}{k}\left\{\frac{k_z^2}{k^2}\left[(2\omega_M+\omega_H)\frac{k_z^2}{k^2}-(2\omega_H+\omega_M)\frac{k_y^2}{k^2}\right]+\mathcal{O}'_1(kd)\right\},$$

$$\omega''_{zz}=\frac{\partial^2\omega}{\partial k_z^2}=\frac{\omega_M}{4\omega_0}\frac{d}{k}\left\{\frac{k_y^2}{k^2}\left[(2\omega_M+\omega_H)\frac{k_z^2}{k^2}-(2\omega_H+\omega_M)\frac{k_y^2}{k^2}\right]+\mathcal{O}'_2(kd)\right\},$$

$$\omega''_{yz}=\frac{\partial^2\omega}{\partial k_z k_y}=-\frac{\omega_M}{4\omega_0}\frac{d}{k}\left\{\frac{k_y k_z}{k^2}\left[(2\omega_M+\omega_H)\frac{k_z^2}{k^2}-(2\omega_H+\omega_M)\frac{k_y^2}{k^2}\right]+\mathcal{O}'_3(kd)\right\}. \tag{7}$$

The higher approximation terms $\mathcal{O}_j(kd)$ and $\mathcal{O}'_j(kd)$ are not presented here because of their rather cumbrous form, but we use them in the calculation as far as the leading terms in expressions (7) vanish in the vicinity of some points, e.g., $k_y=0$ or $k_z=0$.

III. WEAKLY NONLINEAR LIMIT: SOLITON SOLUTIONS

Defining a wave envelope u,

$$m_x+im_y=u\cdot e^{-i(\omega t-\vec{k}\vec{r})},$$

and following the well-known modulation approach[16,3] the nonlinear equation for wave envelope u is derived (we redirect the reader for details of obtaining this equation to the recent paper, Ref. 4 where the full procedure is well described):

$$i\left(\frac{\partial u}{\partial t}+v_y\frac{\partial u}{\partial y}+v_z\frac{\partial u}{\partial z}\right)+\frac{\omega''_{yy}}{2}\frac{\partial^2 u}{\partial y^2}+\frac{\omega''_{zz}}{2}\frac{\partial^2 u}{\partial z^2}+\omega''_{yz}\frac{\partial^2 u}{\partial y\partial z}$$
$$-N|u|^2u=0. \tag{8}$$

All of the coefficients are defined by formulas (7) except the nonlinear coefficient N which could be easily calculated taking into account that in the nonlinear case we have the following identity:

$$\omega_M=4\pi gM_0 m_z.$$

Then substituting in Eq. (6) the expansion of m_z in a weakly nonlinear limit $m_z=1-|u|^2/2$ and using the expression for N from Refs. 16,3, and 4 we get in the large wavelength limit $(kd\ll 1)$

$$N=\left.\frac{\partial\omega}{\partial|u|^2}\right|_{k\to 0,\quad |u|\to 0}=\left.\frac{\partial\omega_0}{\partial|u|^2}\right|_{|u|\to 0}=-\frac{\omega_H\omega_M}{4\omega_0}. \tag{9}$$

Let us mention that if the carrier wave vector is parallel or perpendicular to the static internal magnetic field, the coefficients v_y and ω''_{yz} are equal to zero.[3,4] But for arbitrary angles between $\vec{k}$ and $\vec{H}_0$ that is not the case. Therefore we should introduce a new frame of reference in order to vanish the nondiagonal term with coefficient ω''_{yz}. This could be done rotating the frame of reference yz by the angle ϑ:

$$\xi=z\cos\vartheta+y\sin\vartheta,\qquad \eta=y\cos\vartheta-z\sin\vartheta, \tag{10}$$

where

$$tg2\vartheta=2\frac{\omega''_{yz}}{\omega''_{zz}-\omega''_{yy}}. \tag{11}$$

Then from Eqs. (8)–(11) we obtain the following nonlinear equation:

$$i\left(\frac{\partial u}{\partial t}+v_1\frac{\partial u}{\partial\xi}+v_2\frac{\partial u}{\partial\eta}\right)+\frac{1}{2}R\frac{\partial^2 u}{\partial\xi^2}+\frac{1}{2}S\frac{\partial^2 u}{\partial\eta^2}-N|u|^2u=0, \tag{12}$$

where

$$R=\omega''_{zz}\cos^2\vartheta+2\omega''_{zy}\cos\vartheta\sin\vartheta+\omega''_{yy}\sin^2\vartheta,$$
$$S=\omega''_{zz}\sin^2\vartheta-2\omega''_{zy}\cos\vartheta\sin\vartheta+\omega''_{yy}\cos^2\vartheta, \tag{13}$$

$$v_1=v_z\cos\vartheta+v_y\sin\vartheta,\qquad v_2=v_z\sin\vartheta-v_y\cos\vartheta. \tag{14}$$

Afterwards in the moving frame of reference

$$\xi_1=\xi-v_1 t,\qquad \eta_1=\eta-v_2 t,$$

we come to the 2D nonlinear Schrödinger equation

$$i\frac{\partial u}{\partial t}+\frac{1}{2}R\frac{\partial^2 u}{\partial\xi_1^2}+\frac{1}{2}S\frac{\partial^2 u}{\partial\eta_1^2}-N|u|^2u=0 \tag{15}$$

and can write down its 1D bright or dark soliton solutions assuming that the soliton envelope is a function only of the variables ξ_1 and t (thus the soliton propagates along a spatial axis ξ with a velocity v_1). If $NR<0$, we have bright soliton with envelope

$$|u|=|u|_{max}\,\mathrm{sech}\left\{\frac{\xi_1}{\Lambda}\right\}, \tag{16}$$

while in the case $NR>0$, a dark soliton solution is permitted:

$$|u|=|u|_{max}\left|\frac{\sqrt{1-A^2}}{A}+i\tanh\left\{\frac{\xi_1}{\Lambda}\right\}\right|, \tag{17}$$

where A denotes the contrast of dark soliton (if $A=1$, one has a black dark soliton and gray dark otherwise) and the soliton width Λ is defined for both cases in the same way:

$$\Lambda=\left|\frac{R}{N}\right|^{1/2}\frac{1}{|u|_{max}}. \tag{18}$$

Now we shall discuss the question of the stability of these 1D solitons.

IV. STABLE SOLITONS IN WAVEGUIDES FOR TILTED MAGNETIC FIELDS

As is well known 1D soliton solutions (16) and (17) of the 2D NLS equation are not stable to the transverse modulations with wave numbers $0<\kappa<\kappa_c$. According to recent results (see, e.g., Ref. 12), $\kappa_c\sim 1/\Lambda$; thus, if the limits of the transverse variable η_1 are less than the soliton width, instabilities do not develop and 1D soliton solutions (16) and (17) would be stable. When one has a fully spatial transverse variable η_1 the above condition means that narrow samples should be used. In our case we have the mixed variable $\eta_1=\eta-v_2t$, and therefore also the condition for the time-dependent part has to be introduced, $v_2t<\Lambda$, and in the case

$$v_2=0, \tag{19}$$

transverse instabilities do not develop even for an infinite time. Thus besides the condition (11) we get from Eqs. (14) and (19) an additional condition on the stable soliton parameters:

$$\tan g\,\vartheta=\frac{v_y}{v_z}. \tag{20}$$

Afterwards, in view of both conditions (14) and (20) we finally obtain the following equality:

$$\frac{\omega''_{yz}}{\omega''_{zz}-\omega''_{yy}}=\frac{v_yv_z}{v_z^2-v_y^2}. \tag{21}$$

Solving Eq. (21) as an expansion over the small parameter kd we simply come to the following expression for the angle φ between the carrier wave vector $\vec{k}$ and static magnetic field:

$$\sin\varphi=-\sqrt{\frac{\omega_H}{\omega_H+\omega_M}}\left(1+\frac{\omega_M(3\omega_H+\omega_M)}{3(\omega_H^2-\omega_M^2)}kd\right). \tag{22}$$

Obviously there exist also trivial solutions $\varphi=0$ and $\varphi=90°$ which will not be considered as long as they correspond to the well-studied cases of bright backward volume wave and dark surface wave solitons, respectively. Further using the condition (20) and definitions for the dispersion coefficient (13) we get expressions for ϑ and R as functions of the expansion parameter kd:

$$\sin\vartheta=\sqrt{\frac{\omega_M}{\omega_M+\omega_H}}\left(1-\frac{\omega_H}{\omega_H+\omega_M}kd\right),$$
$$R=\frac{\omega_M(\omega_M-\omega_H)}{2\sqrt{\omega_H(\omega_M+\omega_H)}}\frac{d}{k}. \tag{23}$$

As long as the nonlinear coefficient N according to Eq. (9) is always negative the possibility of the appearance of dark or bright solitons depends on the sign of the dispersion coefficient R. In view of the second relation in Eq. (23) we can conclude that bright solitons appear if $\omega_H<\omega_M$ while dark solitons could be created for larger magnetic fields $\omega_H>\omega_M$. From Eqs. (18) and (6) we are also able to get the expressions for the soliton width and detuning of the pulse frequency, respectively:

$$\Lambda=\frac{d}{|u|_{max}}\sqrt{\frac{2|\omega_M-\omega_H|}{\omega_Hkd}},$$
$$\omega-\omega_0=\frac{\omega_M}{3}\sqrt{\frac{\omega_M}{\omega_H+\omega_M}}\frac{\omega_M}{\omega_H-\omega_M}(kd)^2, \tag{24}$$

while the soliton propagation velocity could be given by simple approximate formula

$$v\equiv v_1\simeq\frac{\omega_Md}{2}\sqrt{\frac{\omega_M}{\omega_M+\omega_H}}. \tag{25}$$

It should be noted that our perturbative approach is violated if $\omega_H\to\omega_M$. Besides that, we have the following restriction on the internal static magnetic field: $\omega_H>0.3\omega_M$. Otherwise the threshold for three-magnon processes is reached and the localized nonlinear wave will decay rapidly.[3]

As we see all of the quantities ϑ, ω-ω_0, and Λ specifying the soliton are functions of kd and $h=\omega_H/\omega_M$. Thus it is possible to plot ω-ω_0 and Λ as functions of ϑ for different h (see Figs. 1 and 2).

In Fig. 1 we present how to choose the sample geometry (in other words how to choose an angle ϑ between the waveguide and static magnetic field) and frequency of the applied pulse for various $h\equiv\omega_H/\omega_M$ in order to create a bright or dark soliton, while in Fig. 2 we show the dependence of the soliton width on the geometry of the problem and static magnetic field. In both cases the curves are limited because of the restriction of "near uniformity" $kd\ll 1$ and the following parameters for the YIG film are used: $d=10\ \mu$m and $\omega_M=1750$ Oe. Note that bright solitons appear when the angle between the waveguide and static magnetic field is less than 45°. Besides that, the detuning of the pulse should be posi-

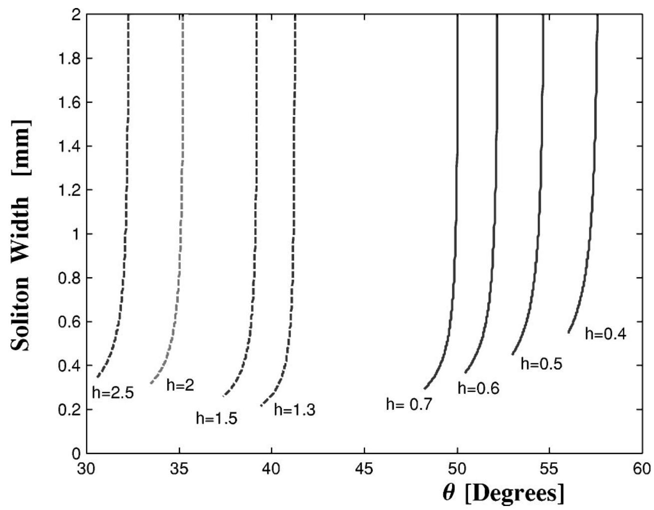

FIG. 2. Dependence of the envelope soliton width on the angle θ between waveguide direction and in-plane magnetic field for its different value. Dashed and solid lines indicate dark and bright soliton cases, respectively. As in the previous figure $h=\omega_H/\omega_M$, $H_M=1750$ Oe, film thickness is equal to $d=10$ μm, and, besides that, the relative amplitude of the soliton is taken as follows: $|u_{max}|=0.1$.

tive in order to create dark solitons. For the purpose of creating bright solitons the angles should be larger than 45° and detuning has to be negative.

V. CONCLUSIONS AND POSSIBLE EXPERIMENTAL SETUP

Summarizing we can declare that the possibility of stable magnetostatic soliton propagation in in-plane magnetized ferromagnetic films in the presence of tilted (from waveguide direction) static in-plane magnetic fields is proved. The widths and velocities as well as the range of angles between the waveguide and magnetic field is obtained for which stable soliton propagation is allowed.

However, the waveguide border (see the dashed line in the inset of Fig. 1) could cause a reflection of the carrier wave (wave vector $\vec{k}$), which will change the group velocity, thus destroying the soliton. To avoid such a possibility we propose to use tube like magnetic waveguides (see Fig. 3). Then the carrier wave will not be reflected and, besides that, the condition of quasi one dimensionality still holds. Let us make the following choice of parameters of the problem: tube diameter $L=0.5$ mm, film thickness $d=10$ μm, and $1/L \ll k \ll 1/d$. Thus the near-uniformity condition $kd \ll 1$ is still valid and simultaneously consideration of the sample as locally flat is allowed, proving thus the approximate validity of solutions like Eq. (4).

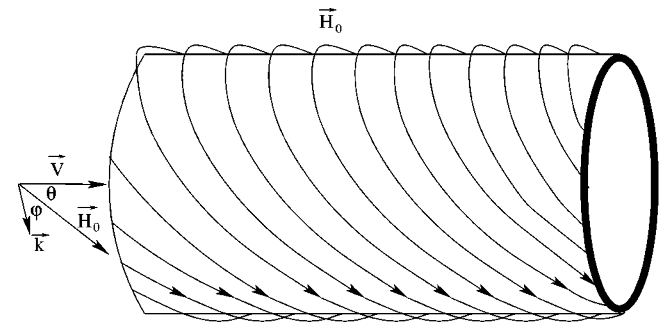

FIG. 3. The possible experimental setup for the observation of stable magnetostatic solitons. In each point of the sample the magnetic field is parallel to the tube surface and tilted by the angle θ from the tube symmetry axis which is parallel to the envelope soliton propagation direction.

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[1] R. W. Damon and J. R. Eshbach, J. Phys. Chem. Solids **19**, 308 (1961).
[2] M. J. Hurben and C. E. Patton, J. Magn. Magn. Mater. **139**, 263 (1995).
[3] A. K. Zvezdin and A. F. Popkov, Zh. Eksp. Teor. Fiz. **84**, 606 (1983).
[4] J. W. Boyle, S. A. Nikitov, A. D. Boardman, J. G. Booth, and K. Booth, Phys. Rev. B **53**, 12 173 (1996).
[5] M. Chen, M. A. Tsankov, J. M. Nash, and C. E. Patton, Phys. Rev. B **49**, 12 773 (1994).
[6] N. G. Kovshikov, B. A. Kalinikos, C. E. Patton, E. S. Wright, and J. M. Nash, Phys. Rev. B **54**, 15 210 (1996).
[7] H. Xia, P. Kabos, C. E. Patton, and H. E. Ensle, Phys. Rev. B **55**, 15 018 (1997).
[8] M. Chen, M. A. Tsankov, J. M. Nash, and C. E. Patton, Phys. Rev. Lett. **70**, 1707 (1993).
[9] A. N. Slavin, Yu. S. Kivshar, E. A. Ostrovskaya, and H. Benner, Phys. Rev. Lett. **82**, 2583 (1999).
[10] B. A. Kalinikos, M. M. Scott, and C. E. Patton, Phys. Rev. Lett. **84**, 4697 (2000).
[11] V. E. Zakharov and A. M. Rubenchik, Zh. Éksp. Teor. Fiz. **65**, 997, (1973) [Sov. Phys. JETP **38**, 494 (1974)].
[12] Yu. S. Kivshar and D. E. Pelinovsky, Phys. Rep. **331**, 117 (2000).
[13] M. Bauer, O. Büttner, S. O. Demokritov, B. Hillebrands, V. Grimalsky, Yu. Rapaport, and A. N. Slavin, Phys. Rev. Lett. **81**, 3769 (1998).
[14] O. Büttner, M. Bauer, S. O. Demokritov, B. Hillebrands, M. P. Kostilev, B. A. Kalinikos, and A. N. Slavin, Phys. Rev. Lett. **82**, 4320 (1999).
[15] O. Büttner, M. Bauer, S. O. Demokritov, B. Hillebrands, Yu. S. Kivshar, V. Grimalsky, Yu. Rapoport, and A. N. Slavin, Phys. Rev. B **61**, 11 576 (2000).
[16] V. I. Karpman, *Nonlinear Waves in Dispersive Media* (Nauka, Moskow, 1970).