# **Charge and spin collective excitations in a coupled spin-polarized bilayer system**

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We investigate the occurrence of charge and spin density collective excitations in a bilayer quasi-twodimensional spin polarized system in the presence of interlayer tunneling. The inclusion of the self-consistent magnetization interaction among the electron spins, in addition to the Coulomb interaction, permits the resolution of coupled in-phase and out-of-phase electric and magnetic modes associated with the charge and spin densities in the two layers. While the in-phase modes are not affected by tunneling, the out-of-phase modes, both electric and magnetic, are gaped, the width of the gap being proportional to the tunneling amplitude. We analyze the existence of these modes outside the electron-hole continuum within the random phase approximation and suggest that they can be relevant for materials with large gyromagnetic factors, for example, dilute magnetic semiconductors.

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## **I. INTRODUCTION**

In an interacting electron system, charge and spin density excitations (CDE and SDE, respectively) are triggered by those components of a weak electromagnetic perturbation electric potential and *z ˆ* component of a magnetic field—that locally distort the single particle distribution function, without changing the spin direction. Since the departure of their resonant frequencies from the single particle threshold is a measure of the strength of the many-body interaction, the theoretical modeling of the collective excitations has always received a great deal of attention.

The leading terms of the dispersion laws describing charge density collective modes in two-dimensional  $(2D)$ single and uncoupled multilayer systems have been obtained within the time-dependent Hartree-Fock (HF) or random phase approximations (RPA) quite some time ago. $1-4$  The validity of these results was confirmed, mostly in the case of Ga-As single layer structures, by infrared spectroscopy experiments.<sup>5</sup>

In the absence of tunneling and magnetic effects, a 2D bilayer structure exhibits in-phase and out-of-phase CDE. The in-phase mode retains the usual 2D plasma dispersion law,  $\omega_+ \sim \sqrt{q}$ , while the out-of-phase CDE is characterized by a linear dispersion law in the long wavelength limit,  $\omega_+$  $\sim q$ , such as an acoustic oscillation. The latter was detected in inelastic light scattering spectroscopy performed on GaAs-AlGaAs heterostructures and has remained the only observable occurrence of an acoustic plasmon in a solid state system.<sup>6</sup> At the close interlayer separation involved in this problem, the inclusion of tunneling supports a more interesting experimental situation. It was found, that in this situation the in-phase CDE dispersion is preserved, while the out-ofphase mode is significantly changed, by acquiring a gap proportional to the tunneling probability,<sup>7</sup>  $\omega$ <sub>-</sub> ~ ( $\Delta_0^2$ +Aq  $+ Bq^{2})^{1/2}$ .

The application of an external magnetic field puts the whole problem of the collective excitations in a bilayer system in a new perspective. First, a new energy scale determined by the Zeeman interaction between the electron spin and the external magnetic field is introduced. The interplay between the energies involved, interlayer tunneling, intralayer kinetic energy, Zeeman splitting, and Coulomb interaction, determines the characteristics of a greatly enlarged spectrum of oscillations that includes magnetic and spin effects. Secondly, the spin polarization factor in each layer  $\xi$  $\sigma = (n_{\sigma} - n_{\sigma})/(n_{\sigma} + n_{\sigma}), n_{\sigma}$  and  $n_{\sigma}$  being the density of spin parallel and antiparallel to the static magnetic field, respectively, is a free parameter of the problem, whose magnitude can be varied continuously as a function of the magnetic field between  $-1$  and 1.

In a spin imbalanced single layer, is found that CDE and SDE are coupled when a spin dependent component of the interaction is considered. $8$  This can be either the selfconsistent magnetization or the exchange and correlation effects, associated with the short range Coulomb interaction, introduced via spin dependent local field corrections. $9-11$  In a double layer configuration preliminary results obtained in the RPA, when only self-consistent magnetization is assumed, suggest that the coupling is preserved, while at the same time, on account of the interlayer Coulomb interaction, each mode acquires in-phase and out-of-phase components. The out-of-phase modes are acoustic for both charge and spin oscillations, following a linear dispersion law in the long wavelength limit. It is important to point out, that within the RPA, the existence of the out-of-phase magnetic modes outside the electron-hole continuum is realized only for materials with large gyromagnetic factors, such as dilute magnetic semiconductors.<sup>12</sup>

The incorporation of tunneling in this context is expected to generate qualitatively new physics, in the same way it did in the case of the unpolarized problem. Our results, which will be presented in detail below, show that in this situation, gaping occurs for both spin and charge acoustic modes with gaps proportional to the interlayer tunneling amplitude. This outcome gives hope for the experimental observation of an acoustic spin mode (magnetoplasmon) by conveniently engineering the gap, such that the mode falls outside the electronhole continuum.

#### **II. THE MODEL**

We consider two identical quantum wells of width *L*, situated in the *xy* plane, separated by a distance *d* in the  $\hat{z}$ 



FIG. 1. Schematic representation of the bilayer structure.

direction (see Fig. 1). A constant magnetic field  $\vec{B}$  is applied along the  $\hat{z}$  axis, such that in equilibrium there is a nonzero polarization factor  $\xi$ . This model, in which the spin imbalance is realized by an external dc magnetic field, can serve as a basis for the more general case when the finite equilibrium polarization is created by a self-consistent magnetization. Since the exchange and correlations corrections associated to the electron spins depend only on the particle density and are independent of the cause of the spin polarization, we expect that the many body effects are the same in the two problems.<sup>13,14</sup>

Considering a finite thickness for the component layers allows us to include in our calculation the self-consistent magnetization. In general, for a finite thickness *L*, each layer develops a subband structure of the energy spectrum, $15,16$ making the problem of collective modes more complicated. In the following, we assume that  $L \le d$ , such that inside each layer only the first subband level is occupied, the layers being considered with a good approximation a 2D spin polarized electron gas with *n* electrons per unit area, such that the total electron density in the system is  $N=2n$ . The single particle states, in the absence of tunneling, are described by momentum  $\vec{k}$ , spin  $\sigma$ , and energy, in the parabolic band approximation  $\varepsilon_{\sigma}(\vec{k}) = \hbar^2 \vec{k}^2/(2m^*) + \gamma B \text{ sgn}(\sigma)$ . Here  $\gamma$  is the gyromagnetic factor that describes the interaction of the electron spin with the magnetic field, while the function signum, sgn( $\sigma$ ), is 1 for spin parallel to the field, and  $-1$  for spin antiparallel to the field. The electron effective mass, *m*\* is considered independent of spin, equal to the band value.

The interlayer tunneling is introduced, for simplicity, through a constant matrix element,  $t_{\vec{k}, \vec{k}}$ ,  $\vec{t} = t \delta_{\vec{k}\vec{k}}$ , In the leading order in perturbation theory, in the presence of tunneling, the single particle eigenfunctions are symmetric (S) and antisymmetric  $(A)$  linear combinations of states, those occupied by an electron before and after tunneling.<sup>7</sup> The corresponding energy eigenstates are, respectively,  $\varepsilon_{\sigma}^{S(A)}(\vec{k})$  $= \hbar^2 \vec{k}^2/(2m^*) + \gamma B \, \text{sgn}(\sigma) \pm t$ . (+ is for *S* and - for *A*.) For a given spin  $\sigma$ , the two levels are separated by an energy gap  $\Delta_{SAS} = E_S - E_A = 2t$ . The nonzero Coulomb interaction terms in the presence of interlayer tunneling are  $v_+(q)$  $v = v_{SS}^{SS}(q) = v_{AA}^{AA}(q) = v_1(q) + v_2(q)$  and  $v_-(q) = v_{SA}^{SA}(q)$ 

 $=v_{AS}^{AS}(q) = v_1(q) - v_2(q)$ , with  $v_1(q) = 2 \pi e^2/(\kappa q)$  and  $v_2(q)$  $=v_1(q) \exp(-qd)$ , respectively, being the intralayer and interlayer Coulomb interaction matrix elements. Excitations are generated when, under the effect of the external perturbation, electrons undergo collective transitions between these energy levels. The intralevel transitions lead to in-phase dielectric and magnetic oscillations in the two layers, while interlevel transitions are responsible for the out-of-phase ones. In the first order of perturbation theory, the induced density fluctuations associated with a collective transition from state  $\alpha$  into state  $\beta$  ( $\alpha$ ,  $\beta = A$ , *S*) are proportional to an effective single particle interaction potential, given by

$$
\Delta \rho_{\sigma}^{\alpha \beta}(\vec{q}, \omega) = \Pi_{\sigma}^{\alpha \beta}(\vec{q}, \omega) V_{\sigma}^{\text{eff}}(\vec{q}, \omega), \qquad (1)
$$

where the proportionality coefficient is the polarization function  $\Pi_{\sigma\sigma}^{\alpha\beta}(\vec{q},\omega)$ :

$$
\Pi_{\sigma}^{\alpha\beta}(\vec{q},\omega) = \sum_{\vec{k}} \frac{n_{\vec{k}-\vec{q}/2,\sigma}^{\alpha} - n_{\vec{k}+\vec{q}/2,\sigma}^{\beta}}{\hbar \omega - \left[\varepsilon_{\beta\sigma}(\vec{k}+\vec{q}/2) - \varepsilon_{\alpha\sigma}(\vec{k}-\vec{q}/2)\right]}.
$$
\n(2)

 $n_{\vec{k},\sigma}^{\alpha}$  is the usual Fermi distribution function.

### **III. CHARGE AND SPIN DENSITY EXCITATIONS**

The effective potential experienced by an electron of spin  $\sigma$  that undergoes an intralevel transition is

$$
V_{\sigma}^{\text{eff}}(\vec{q}, \omega) = -e \phi + \gamma b_z \text{sgn}(\sigma)
$$
  
+  $v_{+}(q) [\Delta \rho_{\sigma}^{SS} + \Delta \rho_{\sigma}^{SS} + \Delta \rho_{\sigma}^{AA} + \Delta \rho_{\sigma}^{AA}]$   
+  $\frac{4 \pi \gamma^2}{L} [\Delta \rho_{\sigma}^{SS} - \Delta \rho_{\sigma}^{SS} + \Delta \rho_{\sigma}^{AA} - \Delta \rho_{\sigma}^{AA}].$  (3)

 $\Delta \rho_{\sigma}^{\alpha\beta}$  is the density fluctuation in the number of electrons of spin  $\sigma$  that transition inside level *i* (*i* stands for *A* and *S*). The dependence on  $\tilde{q}$  and  $\omega$  of all variables in Eq. (3) is assumed. In addition to the Coulomb interaction, the selfconsistent magnetization can be significant in the case of materials with large gyromagnetic factors, such as dilute magnetic semiconductors where  $\gamma$  can be as high as a  $100\mu_B$ . The density fluctuations  $\Delta \rho_{\sigma}^{ii}$  are connected selfconsistently to the effective potentials as in Eq.  $(1)$ . The collective modes of the system occur for those values of the frequency for which the system of equations generated by Eqs.  $(1)$  and  $(3)$  admits a nontrivial solution in the absence of the external perturbation. The secular equation obtained in this circumstance is

$$
1 - \left(v_{+}(q) + \frac{4\pi\gamma^{2}}{L}\right)(P_{+}^{SS} + P_{+}^{AA}) + \frac{4\pi\gamma^{2}}{L}v_{+}(q)[(P_{+}^{SS} + P_{+}^{AA})^{2} - (P_{-}^{SS} + P_{-}^{AA})^{2}] = 0,
$$
\n(4)

where we introduced

$$
P_+^{\alpha\beta}(\vec{q}, \omega) = \Pi_{\uparrow\uparrow}^{\alpha\beta}(\vec{q}, \omega) + \Pi_{\downarrow\downarrow}^{\alpha\beta}(\vec{q}, \omega),
$$
  
(5)  

$$
P_-^{\alpha\beta}(\vec{q}, \omega) = \Pi_{\uparrow\uparrow}^{\alpha\beta}(\vec{q}, \omega) - \Pi_{\downarrow\downarrow}^{\alpha\beta}(\vec{q}, \omega).
$$

General solutions of Eq.  $(4)$  are difficult to obtain even in RPA, but in the long-wavelength limit, when  $\omega \ge qv_F$  ( $v_F$ being the Fermi velocity associated with the unpolarized 2D electron gas) the dispersion  $\omega(q)$  can be extracted. In this limit it is easy to show that  $P_+^{SS}(\vec{q}, \omega) + P_+^{AA}(\vec{q}, \omega)$  $\approx Nq^2/(m^*\omega^2)$  and  $P^{SS}_{-}(\vec{q},\omega) + P^{AA}_{-}(\vec{q},\omega) \approx Nq^2 \xi/(m^*\omega^2)$ , results which do not depend on the level occupancy. Note that these approximations consider the effective mass of electrons with different spin orientation to be the same, an assumption valid only for small values of the polarization factor  $\xi$ . With these considerations, the asymptotic solutions of Eq.  $(4)$  are

$$
\omega_1^2(q) = \frac{4\pi e^2 N}{\kappa m^*} q \left[ 1 + \xi^2 \frac{\kappa \gamma^2}{L e^2} q \right]
$$
 (6)

and

$$
\omega_2^2(q) = \frac{4\,\pi\,\gamma^2}{L} \frac{N(1-\xi^2)}{m^*} q^2,\tag{7}
$$

where only terms up to the second order in the exchanged momentum *q* are considered. The high frequency solution  $\omega_1(q)$  corresponds to the usual 2D plasmon (CDE), including correction terms generated by the self-consistent magnetization and the initial polarization. The second value,  $\omega_2(q)$ describes the in-phase spin (magnetic) oscillation in the two layers. The existence of this mode is conditioned by the presence of a magnetic interaction, be that the self-consistent magnetization, as in the present case, or the short range Coulomb interaction. The results of Eqs.  $(6)$  and  $(7)$  reaffirm the independence of tunneling of the in-phase charge and spin excitations, their resonant frequencies depending only on the level of occupancy and the initial spin polarization factor.

The direct experimental observation of the spin density mode is conditioned by its presence outside of the electronhole continuum, the region where single particle excitations (SPE) occur. To this purpose, the slope of the dispersion curve, Eq.  $(7)$ , at the origin needs to be higher than the higher of the two Fermi velocities associated to the up and down spins,  $v_F^{\sigma} = v_F [1 + \xi \text{ sgn}(\sigma)]$  ( $v_F$  is the Fermi velocity for a free unpolarized electron gas). Therefore, in terms of the dimensionless parameter  $\Gamma = \kappa \gamma^2 k_F / (L e^2)$ , for  $\xi > 0$  $(v_F^{\perp} > v_F^{\perp})$ , we can write

$$
\Gamma > \Gamma_c = \frac{k_F}{q_{\rm TF}} \frac{1+\xi}{1-\xi},\tag{8}
$$

where  $k_F$  represents the Fermi momentum and  $q_{\text{TF}}$  $=2m*e^2/\kappa$  the usual Thomas-Fermi wave number. This criterion limits the number of real situations in which the spin density mode can be observed, the condition being satisfied



FIG. 2. The critical value  $\Gamma_c$  as function of the spin polarization factor  $\xi$  for different values of  $k_F/q_{\text{TF}}$ .

only for large values of the gyromagnetic factor  $\gamma$ . For  $\zeta$ approaching 1, the mode is soft, and the system undergoes a phase transition to a ferromagnetic state. In Fig. 2,  $\Gamma_c$  is represented as function of the spin polarization factor for different values of the total electron density in the system,  $N=10^9$  cm<sup>-2</sup> ( $k_F/q_{\text{TF}}$   $\approx$  0.01),  $N=10^{10}$  cm<sup>-2</sup> ( $k_F/q_{\text{TF}}$ )  $\approx$  0.03), and *N* = 10<sup>11</sup> cm<sup>-2</sup> ( $k_F / q_{\text{TF}} \approx$  0.12).

The interlevel excitations,  $A \rightarrow S$  or  $S \rightarrow A$ , occur under the effect of

$$
V_{\sigma}^{\text{eff}}(\vec{q}, \omega) = -e \phi + \gamma b_z \text{sgn}(\sigma)
$$
  
+  $v_{-}(q) [\Delta \rho_{\sigma}^{SA} + \Delta \rho_{\sigma}^{SA} + \Delta \rho_{\sigma}^{AS} + \Delta \rho_{\sigma}^{AS}]$   
-  $\frac{4 \pi \gamma^2}{L} [\Delta \rho_{\sigma}^{SA} - \Delta \rho_{\sigma}^{SA} + \Delta \rho_{\sigma}^{AS} - \Delta \rho_{\sigma}^{AS}],$  (9)

where we introduced  $\Delta \rho_{\sigma}^{ij}$  for the density fluctuations generated by the transition of an electron of spin  $\sigma$  between levels *i* and *j*, with  $i \neq j$ . Following the algorithm outlined in the previous case, the excitation frequencies of the collective excitations that can be supported by the system are obtained as solutions of the following equation:

$$
1 + \left(v_{-}(q) + \frac{4\pi\gamma^{2}}{L}\right)(P_{+}^{SA} + P_{+}^{AS}) + \frac{4\pi\gamma^{2}}{L}v_{-}(q)\left[(P_{+}^{SA} + P_{+}^{AS})^{2} - (P_{-}^{SA} + P_{-}^{AS})^{2}\right] = 0, (10)
$$

where  $P_{\pm}^{SA}$  and  $P_{\pm}^{AS}$  were introduced according to Eq. (5). As before, we inspect the long wavelength limit form of the dispersion relation. In this approximation, the polarization function acquire the forms given below

$$
P_{+}^{SA}(\vec{q}, \omega) + P_{+}^{AS}(\vec{q}, \omega)
$$
  
\n
$$
\approx \frac{2\Delta_{SAS}(n_S - n_A) + 2nq^2/m^*}{\omega^2 - \Delta_{SAS}^2} + \frac{4nq^2}{m^*} \frac{\Delta_{SAS}^2}{[\omega^2 - \Delta_{SAS}^2]^2}
$$
  
\nand (11)

$$
P_{-}^{SA}(\vec{q}, \omega) + P_{-}^{AS}(\vec{q}, \omega)
$$
  

$$
\approx \frac{2nq^{2}\xi}{m^{*}} \left( \frac{1}{\omega^{2} - \Delta_{SAS}^{2}} + \frac{2\Delta_{SAS}^{2}}{[\omega^{2} - \Delta_{SAS}^{2}]^{2}} \right). \quad (12)
$$

In Eq. (11),  $n_{S(A)} = n \pm n_c$  denotes the electron densities for the symmetric and antisymmetric level, where  $n_c$  $=$ (*m*/2 $\pi$ ) $\Delta$ <sub>SAS</sub> when both levels are occupied ( $E_F$ > $\Delta$ <sub>SAS</sub>) and  $n_c = n$  when only the symmetric level is occupied ( $E_F$  $\langle \Delta_{SAS} \rangle$ . With these results, the solutions of Eq. (10) can be easily obtained. The frequency of the CDE mode is

$$
\omega_3^2(q \to 0) \simeq \Delta_1^2 + C_1 q + C_2 q^2, \tag{13}
$$

an expression that supports the result of Ref. 7. Coupled to the CDE is a spin mode whose dispersion law is described by

$$
\omega_4^2(q \to 0) \simeq \Delta_2^2 + C_3 q + C_4 q^2. \tag{14}
$$

 $[C_i \ (i=1,2,3,4)$  are constants whose values are not shown here for brevity.] The two energy gaps  $\Delta_1$  and  $\Delta_2$  are given below:

$$
\Delta_1^2 = \Delta_{SAS}^2 + \frac{2\pi}{m^*} \Delta_{SAS}(n_S - n_A) q_{\text{TF}} d \left[ 1 + \frac{\left(\frac{\Gamma}{k_F d}\right)^2}{1 + \frac{\Gamma}{k_F d}} \right],
$$
  

$$
\Delta_2^2 = \Delta_{SAS}^2 + \frac{2\pi}{m^*} \Delta_{SAS}(n_S - n_A) q_{\text{TF}} d \frac{\Gamma}{1 + \frac{\Gamma}{k_F d}}.
$$
 (15)

Our results show that in the presence of tunneling, gaps open up in the spectra of both charge and spin density out-ofphase modes, with  $\Delta_1 > \Delta_2$  as shown in Eq. (15). In the strong tunneling case, at  $q=0$ , both gaps  $\Delta_1$  and  $\Delta_2$ , depend linearly on the single particle symmetric-antisymmetric gap  $\Delta_{SAS}$ . In the opposite situation, at weak tunneling, a twosubband description is more appropriate as  $\Delta_{SAS} \rightarrow 0$ . In this case, in the absence of the self-consistent magnetization term, it was found that  $\Delta_1 \rightarrow \Delta_0$  maintains its linear dependence on  $\Delta_{SAS}$ , but a logarithmic correction to the linear term appears as the two-subband situation develops.<sup>17</sup> The value of the four constants  $C_i$  ( $i=1,2,3,4$ ) entering Eqs. (13) and  $(14)$  is also dependent on the self-consistent magnetization term. However, in its absence only the charge density



FIG. 3. Collective modes characteristic frequencies for single level occupancy  $(\Delta_{SAS}/E_F \sim 20)$  for a bilayer system with *n*  $=10^9$  cm<sup>-2</sup>,  $\Gamma \sim 1$ ,  $d=200$  Å, and  $\xi=0.25$ .

mode survive and it is easy to prove that  $C_1 \rightarrow A$  and  $C_2$  $\rightarrow$ *B*. Both first order correction coefficients ( $C_1$  and  $C_3$ ) are negative. In the absence of interlayer tunneling  $(\Delta_{SAS}=0)$  a more careful analysis of Eq.  $(4)$  shows that both charge and spin density modes are acoustic in the long wavelength limit  $(\omega_{3,4}^{\circ}\sim q)^{12}$ 

The occurrence of gaps makes possible the observation of the acoustic modes outside the electron-hole continuum for different nonzero values of the tunneling probability. This situation does not happen in the absence of tunneling when certain conditions have to be satisfied by the magnetic interaction.<sup>12</sup>

In Fig. 3 we plot the dispersion relations for both intraand inter-level excitations in the case of a single level occupancy. For the considered parameters  $(\Delta_{SAS}/E_F \sim 20, n$  $=10^9$  cm<sup>-2</sup>,  $\Gamma$  ~ 1,  $d=200$  Å, and  $\xi$  = 0.25) the magnetoplasmon mode ( $\omega_2$ ) remains outside the electron-hole continuum and is expected to be observable in direct experiments. This mode is of particular interest in understanding the many body mechanism that supports the self-consistent magnetization giving rise to it. The inter-level charge density mode  $(\omega_3)$  is expected to be more stable than the similar magnetic mode  $(\omega_4)$ , as the out-of-phase magnetic oscillations fall rapidly into the electron-hole continuum as the exchange momentum *q* increases.

#### **IV. CONCLUSIONS**

The results derived in this paper extend the conclusion of Ref. 7 on the occurrence of gaping in the dispersion law of out-of-phase charge density modes, to spin density excitations. Here, we show that a bilayer system with broken spin symmetry exhibits coupled both charge and magnetic oscillations, the strength of the coupling being determined by the self-consistent magnetization. The SDE follow the same pattern as the CDE. Those associated with the in-phase oscillation of the magnetization in the two layers retain their dispersion law independent of coupling, while the out-of-phase fluctuations acquire a gap proportional to the tunneling probability.

The existence of the SDE is conditioned by the presence of a self-consistent magnetization among the electrons of the system. This occurs in materials with a large electron effective gyromagnetic factor, such as DMS. The incorporation of the self-consistent magnetization can provide an important guideline in solving the more complicated problem in which the itinerant magnetization appears as a consequence of the spin dependent component of the short range Coulomb interaction.

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