Dynamic correlations of the spinless Coulomb Luttinger liquid

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The dynamic density response function, structure factor, and spectral function of a Luttinger liquid with Coulomb interaction are studied with the emphasis on the short-range electron correlations. The Coulomb interaction changes dramatically the density response function as compared to the case of the short-ranged interaction. The coordinate dependence of the density response is smoothing with time, and the oscillatory structure appears. However, the spectral functions remain qualitatively the same. The dynamic structure factor contains the δ peak in the long-wave region, corresponding to one-boson excitations. In addition, the multi-boson-excitations band exists in the wave-number region near to $2k_F$, where k_F is the Fermi wave number. We develop a method to analyze the asymptotic behavior of the spectral functions near to the edges of the multi-boson-excitations band at zero temperature. The dynamic structure factor diverges at the edges of this band, while the dielectric function goes to zero there. Hence, a new collective charge mode appears in the one-dimensional electron liquid as a consequence of short-range dynamic correlations. The mode energy goes to zero at $2k_F$, which means that the $2k_F$ mode is soft.

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I. INTRODUCTION

Electron-electron (e-e) interaction is known to produce the most pronounced effects in one-dimensional (1D) systems, where a strongly correlated state appears even if the interaction is weak.[1] The dynamic electron correlations are now the most fascinating topic in this field.

Many attempts were made to treat correlations in 1D using the approaches, devised for 2D and 3D. They are based on the random phase approximation (RPA) with various versions of the local field corrections, both static and dynamic.[2–5] These methods give no qualitatively new results as compared to higher dimensions, leading to the picture of charge excitations that is exhausted by common long-wave plasmons.[6–8] However, this way to consider electron correlations has obvious shortcomings, especially in what concerns the dynamic short-range correlations, inherent in 1D. The flaws of the RPA-like approaches become worse as the dimensionality is reduced.[9] In 1D a number of unphysical results appear such as the negative dynamic structure factor,[10] the violation of the compressibility sum rule and negative pair correlation function, the latter two being also present in 2D and 3D.[4,5]

Another way to treat the dynamic correlations uses the Hartree-Fock decoupling scheme.[11,12] It takes into consideration only the exchange-correlation hole but ignores the Coulomb one. This approach leads to the nonrealistic conclusion that the 3D electron gas is unstable against the $2k_F$ fluctuations due to e-e interaction.[13] This was not confirmed by subsequent theories and experiments. What concerns 1D systems, the HF approach, on the contrary, did not give any sharp peculiarities of the susceptibility near to $2k_F$.[14]

Presently, the most advanced way to treat dynamic electron correlations in 1D is based on the Luttinger liquid (LL) theory.[1,15–18] The short-range correlations[19–21] appear here in a natural way, because this model takes properly into account the nesting of the Fermi surface for the wave number equal to $2k_F$. Owing to the short-range correlations, the LL density response contains the $2k_F$ component, in addition to the usual long-wave one. In what follows the $2k_F$ term is also referred to as the charge-density wave (CDW).

The $2k_F$ susceptibility $\chi_{\text{CDW}}(q,\omega)$ was considered by Luther and Peschel[17] for the LL with short-ranged e-e interaction. They have found that $\chi_{\text{CDW}}(q,\omega)$ diverges as $|q-2k_F|\to\omega/v$, where v is the velocity of the LL bosonic excitations. We stress that the divergency of the susceptibility is very important because it corresponds to the presence of a collective mode in the system.

In real 1D structures e-e interaction is, generally speaking, long ranged, unless it is screened by a metallic gate. It is well known that Coulomb interaction can highly modify the ground state and transport properties of 1D systems. Thus, in a Luttinger liquid with Coulomb interaction (CLL) the static correlations become much stronger than in the short-ranged LL, with the short-range $2k_F$ ($4k_F$ in a spinful case) component being the dominating one.[22]

How the dynamic LL correlations are affected by Coulomb interaction is the highly intriguing question. However, considering only the long-wave density component does not lead to the qualitatively new behavior of the dynamic structure factor and collective modes.[1,23–26] The results are very close to those obtained in RPA.

The present paper concentrates on the CDW contribution to the dynamic response functions and spectral characteristics of the spinless Coulomb Luttinger liquid at zero temperature. We show analytically that Coulomb interaction strengthens the divergency of the dynamic density susceptibility in comparison with that of the short-ranged LL. The singularity of the CDW susceptibility leads to the new behavior of the collective charge mode that is immanent only in 1D. The strong spatial dispersion arises in the vicinity of $q=2k_F$, with the mode frequency going to zero as $q\to 2k_F$. This means that the soft mode appears. Such soft mode is absent in 2D and 3D systems, because the short-range dynamic electron correlations are much weaker there. Note that the $2k_F$ mode cannot be obtained within the RPA approach, even with local field corrections.

Our results concerning the CLL spectral function diverge from those of Ref. 27, where it was argued that the nonlinear dispersion of bosons, which appears in the presence of Coulomb interaction, kills the spectral function singularities. Instead, a flattened maximum was found in a spectral function, the position of the maximum being shifted from the resonant frequency. We show that the Coulomb interaction, on the contrary, only strengthens the divergency, which is a rather general result for the CLL correlation functions.

The outline of the paper is as follows. In Sec. II we investigate the dynamic density response function, structure factor, spectral function, dissipative conductance, and dielectric function of a CLL, comparing them with the short-ranged LL results. The methods to calculate the dynamic structure factor of a short-ranged LL and CLL are presented in Appendixes A and B, respectively.

II. DYNAMIC CORRELATIONS

We start with a bosonized spinless LL Hamiltonian[1]

$$H=\Sigma_p \hbar \omega_p b_p^+ b_p ,$$

where $b_p^+(b_p)$ are boson creation (annihilation) operators. The boson frequency is given by $\omega_p=|p|v(p)$, where the velocity of excitations is $v(p)=v_F/g(p)$, with the interaction parameter $g(p)$ and the Fermi velocity v_F. The interaction parameter equals $g(p)=[1+V(p)/\pi\hbar v_F]^{-1/2}$, $V(p)$ being the Fourier transformed e-e interaction potential. For the short-range interaction g is constant ($0<g<1$ for e-e repulsion). For Coulomb interaction $V(r)=e^2/\sqrt{r^2+d^2}$, the interaction parameter in the long-wave limit $|pd|\ll 1$ is $g(p)=\beta|\ln|pd||^{-1/2}$, d being the quantum wire diameter, $\beta=[\pi\hbar v_F/2e^2]^{1/2}$.

The electron-density-fluctuation operator in the Luttinger model is written as[20,21]

$$\rho(x)=-\frac{1}{\pi}\partial_x\phi+\frac{1}{2\pi}\partial_x \sin(2k_F x-2\phi), \qquad (1)$$

$\phi(x)$ being the bosonic phase. The first component of the density operator ρ_{lw} describes long-wave excitations and represents the sum of the densities of the right- and left-moving electrons. The corresponding excitations come about separately within each branch of electron spectrum and have momentum $q\ll k_F$. The second component ρ_{CDW}, which rapidly oscillates in space, is due to the interference of the right- and left-moving electrons. This corresponds to excitations with momentum $q\approx 2k_F$. It is this term that describes the short-range electron correlations. Note that the presented form of ρ_{CDW} differs from the conventional one[1] in that the former is the exact differential. The exact differential form of the density fluctuation operator guarantees the particle number conservation in an isolated 1D system. The conventional form of ρ_{CDW} does not conserve the number of particles and thus violates the electroneutrality of the 1D system.[21]

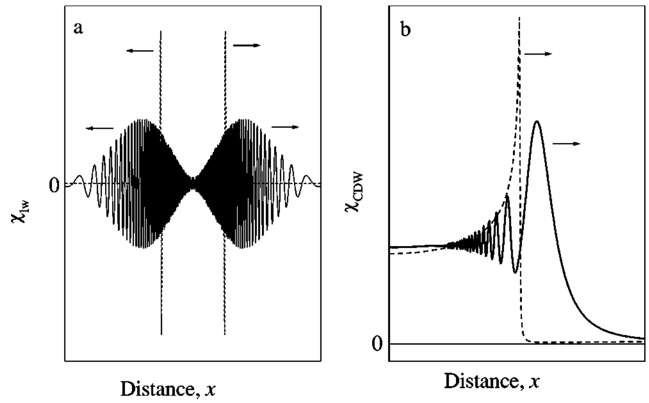

FIG. 1. The long-wave (a) and CDW (b) components of the density response function for the short-ranged LL (dash line) and CLL (solid line). In the CDW response function the $2k_F$ filling is not shown. Arrows show the direction of wave propagation.

A. The density response function

For the two density components of Eq. (1), the density response function (DRF) is calculated via the Kubo formula to give

$$\chi(x,t)=\chi_{\mathrm{lw}}(x,t)+\chi_{\mathrm{CDW}}(x,t),$$

where

$$\chi_{\mathrm{lw}}(x,t)=\frac{\theta(t)}{\pi h}\partial_x^2 f_2(x,t),$$

$$\chi_{\mathrm{CDW}}(x,t)=\frac{\theta(t)}{2\pi h}\partial_x^2\{e^{-f_1(x,t)}\sin[f_2(x,t)]\cos(2k_F x)\}.$$

The functions $f_1(x,t)$ and $f_2(x,t)$ are as follows:

$$f_1(x,t)=v_F\int_0^{+\infty}\frac{dp}{\omega_p}[2-\cos(\omega_p t+px)-\cos(\omega_p t-px)]e^{-\alpha p}, \qquad (2)$$

$$f_2(x,t)=v_F\int_0^{+\infty}\frac{dp}{\omega_p}[\sin(\omega_p t+px)+\sin(\omega_p t-px)]e^{-\alpha p},$$

$$\alpha=k_F^{-1}. \qquad (3)$$

For the short-ranged e-e interaction these functions can be calculated exactly:[17,21,23]

$$e^{-f_1(x,t)}=\alpha^{2g}[v^2t^2-x^2]^{-g},$$

$$f_2(x,t)=\pi g\,\theta(v^2t^2-x^2).$$

The DRF behavior is illustrated by Fig. 1 for both short-ranged and Coulomb LL's.

As is seen, the DRF of a short-ranged LL is presented by two wave fronts, propagating in opposite directions with constant velocity v. The Coulomb DRF, as distinct from the short-ranged one, has no sharp fronts. The wave form is smoothing because of the nonlinear dispersion of CLL boson excitations. The characteristic space scale of the Coulomb DRF depends on time as $x \approx t\sqrt{\ln t}$, where dimensionless x and t are normalized, respectively, by d and $\beta d/v_F$. The nonperiodic oscillatory structure of the DRF arises in the CLL because the phase velocity diverges at wave number $q=0$. This divergency leads to the appearance of the stationary phase $\phi_{\rm st}$ in integrals (2),(3). The oscillatory structure, owing to the strong Coulomb dispersion, complements the smoothing of the wave form, usual for the wave propagation in weakly dispersive media. The asymptotic decay of the Coulomb DRF with the time is extremely slow, and has the form similar to the decay of the static CLL correlator with the distance.[22] Devoid of a sharp front, the Coulomb DRF contains a slowly decaying tail at large distance $x \gg t\sqrt{\ln t}$.

Introducing the characteristic distance $x_f = t\sqrt{\ln t}$ and stationary phase

$$\phi_{\rm st} = -\frac{t}{2\sqrt{e}}\left[\sqrt{2} - 2\frac{x}{t} + \frac{x^2}{\sqrt{2}t^2} + O\left(\frac{x^3}{t^3}\right)\right],$$

the asymptotic behavior of f_1 and f_2 can be presented as follows:

(i) $x \gg x_f$

$$f_1(x,t) \sim 2\beta(\sqrt{\ln(x-x_f)} + \sqrt{\ln(x+x_f)}),$$

$$f_2(x,t) \sim \frac{\pi\beta}{2}\frac{x_f^3}{x^3},$$

(ii) $x \ll x_f$

$$f_1(x,t) \sim 2\beta\left[\sqrt{\ln(x_f - x)} + \sqrt{\ln(x_f + x)} - \sqrt{\frac{\pi\sqrt{2e}}{2t}}\cos\left(\phi_{\rm st} + \frac{\pi}{4}\right)\right],$$

$$f_2(x,t) \sim 2\beta\left[\frac{1}{\sqrt{\ln(x_f - x)}} + \frac{1}{\sqrt{\ln(x_f + x)}} - \sqrt{\frac{\pi\sqrt{2e}}{2t}}\sin\left(\phi_{\rm st} + \frac{\pi}{4}\right)\right].$$

We draw attention to the fact that in 1D there are two different mechanisms of the density evolution.[21] First of all, there exist soundlike waves, caused by forward scattering and described by the long-wave DRF component. In this kind of motion neighboring electrons move almost in the same phase, so that the corresponding correlations are almost static. Secondly, electrons suffer the backward scattering from the nearest particles and interfere, which gives rise to $2k_F$ density oscillations. Electron correlations, related to the $2k_F$ mechanism of electron density response, are essentially dynamic. Therefore, taking into account the short-range electron correlations, the two-particle Wigner function $f(x_1,p_1,x_2,p_2,t)$ in no way can be represented as a product of two single-particle Wigner functions and the *static* pair correlation function $g(x)$, that is,

$$f(x_1,p_1,x_2,p_2,t) \neq f(x_1,p_1,t)f(x_2,p_2,t)g(x_1-x_2)$$

in 1D. Thus the basic assumption of the Singwi-Tosi-Land-Sjölander theory[4,5] is violated in 1D, which explains why the $2k_F$ collective mode is overlooked in the excitation spectrum within this approach.

B. The dynamic structure factor

Now we turn to the LL dynamic structure factor $S(q,\omega)$, which is the Fourier transform of the density-density correlator $R(x,t) = \langle \rho(x,t)\rho(0,0)\rangle$. At zero temperature the structure factor coincides with the imaginary part of the susceptibility $S(q,\omega) = -2\hbar\chi''(q,\omega)$. Although the form of the Coulomb DRF $\chi_{\rm lw}(x,t)$ differs dramatically from the short-ranged one, the long-wave part of the structure factor $S_{\rm lw}(q,\omega)$ has the universal expression[1]

$$S_{\rm lw}(q,\omega) = |q| g(q)\,\delta(\omega - \omega_q).$$

Thus, the boson dispersion results only in the shift of the δ-peak position. This is clear physically, since $S_{\rm lw}(q,\omega)$ determines the probability to create a *single* boson when absorbing the quantum $\hbar\omega$, which fixes the singularity position.

The CDW structure factor $S_{\rm CDW}(q,\omega)$ describes the excitation of several bosons and is much more interesting. For the short-range interaction $S_{\rm CDW}(q,\omega)$ can be exactly calculated:[17,21,23]

$$S_{\rm CDW}(q,\omega) = \frac{1}{v_F}\frac{g}{4^{g+1/2}\Gamma^2(g)}\left(\frac{q}{k_F}\right)^2 \times \sum_{r=\pm 1}\left[\left(g\frac{\hbar\omega}{\varepsilon_F}\right)^2 - \left(\frac{q}{k_F} - 2r\right)^2\right]^{g-1}, \quad (4)$$

ε_F being the Fermi energy. The entire complex susceptibility is

$$\chi_{\rm CDW}(q,\omega) = -\frac{1}{\hbar v_F}\frac{g}{4^{g+1}\Gamma^2(g)\sin(\pi g)}\left(\frac{q}{k_F}\right)^2 \times \sum_{r=\pm 1}\left[\left(\frac{q}{k_F} - 2r\right)^2 - \left(g\frac{\hbar\omega + i0}{\varepsilon_F}\right)^2\right]^{g-1}. \quad (5)$$

The details of the calculation are presented in Appendix A. Notice that the CDW structure factor is zero out of the q band $||q| - 2k_F| < g\omega/v_F$ and diverges at the band edges. For simplicity, in what follows we consider $q > 0$.

For the CLL the CDW structure factor can not be calculated exactly, but its general properties are easily understood, using the formula

$$S_{\rm CDW}(q,\omega) = (2\pi)^2 \sum_m |\langle m|\rho_{\rm CDW}|0\rangle|^2 \times \delta(\omega - \omega_m)\,\delta(q - q_m - 2k_F),$$

which can be obtained directly from the expression (A1) for the CDW density correlator. This formula differs from the conventional one[28] in that the wave number argument of the δ function is additionally shifted by $2k_F$. The conventional derivation of the structure factor representation via δ functions pays no attention to the $2k_F$ modulation of the density operator of Eq. (1).

The sum in the last expression is taken over all stationary $|m\rangle$ states of the system, $\hbar\omega_m$ being the state energy and $\hbar q_m$ the state momentum. The state consists of a number of bosons, excited above the vacuum. The specific form of the matrix element is not of interest now, but what is important is that $\langle m|\rho_{\rm CDW}|0\rangle$, in contrast to $\langle m|\rho_{\rm lw}|0\rangle$, is nonzero for excited states, containing more than one boson. Thus all possible boson systems of the total energy $\hbar\omega$ and momentum $\hbar(q-2k_F)$ contribute to $S_{\rm CDW}(q,\omega)$.

The boson dispersion curve $\omega=\omega_p$ is convex, i.e., $\omega_p''<0$. Therefore, if the boson system has the total momentum $\hbar p$, then its energy can not be less than $\hbar\omega_p$. Whence, the structure factor $S_{\rm CDW}(q,\omega)$ is zero when $\omega<\omega_{q-2k_F}$. When $\omega=\omega_{q-2k_F}$, only one boson can be excited. As one increases ω from the threshold value ω_{q-2k_F}, the number of different boson systems of the given total energy $\hbar\omega$ and momentum $\hbar(q-2k_F)$ increases rapidly, and their contribution to the structure factor is increasing too. The formation of boson systems occurs when ω is being shifted from the threshold on the scale v_F/L, where L is the length of the system. Hence, taking the thermodynamic limit, i.e., $L\to\infty$, we find the nonzero values of $S_{\rm CDW}(q,\omega)$ as $\omega\to\omega_{q-2k_F}+0$. Moreover, we show in Appendix B that $S_{\rm CDW}(q,\omega)$ diverges as $\epsilon=\omega-\omega_{q-2k_F}\to+0$, just like in the short-ranged LL:

$$\omega S(q,\omega)\sim\frac{e^{-4\beta|\ln\epsilon|^{1/2}}}{\epsilon|\ln\epsilon|^{1/2}}. \tag{6}$$

The CLL dynamic structure factor, containing both CDW and long-wave components, is shown in Fig. 2 as a function of q at fixed ω.

In a similar way we find that the CLL spectral function[1] is zero when $\omega<\omega_q$ and diverges as $\delta=\omega-\omega_q\to+0$:

$$\omega\,\rho(q,\omega)\sim\frac{e^{-A\beta|\ln\delta|^{1/2}}}{\delta|\ln\delta|^{1/2}}, \tag{7}$$

with A being $[g^{-1}(q)-1]^2$. The last result contradicts to the one obtained in Ref. 27, where it was claimed that the nonlinear boson dispersion flattens the singularity of $\rho(q,\omega)$ at $\omega=\omega_q+0$, producing a maximum instead, with $\rho(q,\omega)$ going to zero at the resonant frequency. The approach of Ref. 27, if applied to the CDW structure factor, would force us to conclude that the CLL structure factor has a cusp instead of a singularity. In the following subsection we argue that such conclusion is physically incorrect.

Notice that since the expression (6) represents the exact differential, the structure factor singularity is integrable, as it should be, because the integral of $S_{\rm CDW}(q,\omega)$ with respect to ω gives the static structure factor $S_{\rm CDW}(q)$, which is finite at nonzero wave numbers. The static structure factor was calculated via direct integration of the static density-density correlator to give the following asymptotic behavior for $Q=q-2k_F\to 0$

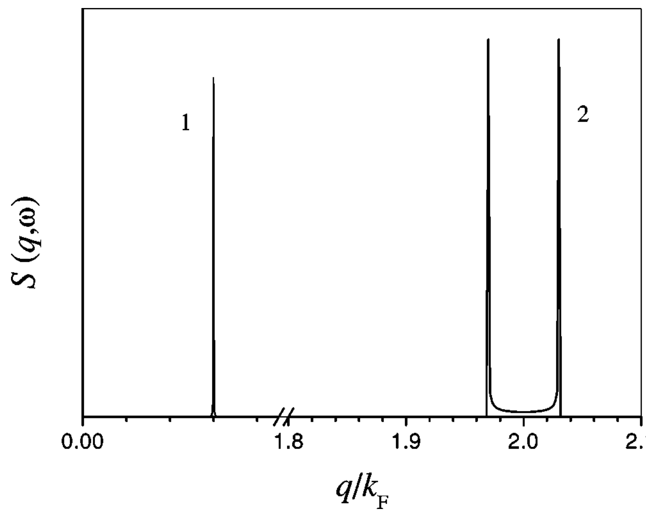

FIG. 2. The CLL dynamic structure factor as a function of wave number q. The δ peak in the long-wave region corresponds to one-boson excitations. The multi-boson-excitations band exists in the vicinity of $q=2k_F$.

$$S_{\rm CDW}(q)=\frac{k_F^2}{4\pi^2}\,\frac{e^{-4\beta|\ln Q|^{1/2}}}{Q|\ln Q|^{1/2}}.$$

The dynamic structure factor determines the power P that is dissipated in a LL, disturbed by external electric potential φ:

$$P(\omega)=\frac{e^2}{4h}\int_{-\infty}^{+\infty}\omega S(q,\omega)|\varphi(q)|^2dq.$$

The contribution of the long-wave density response to the dissipated power was investigated in detail in Ref. 24. It was shown there that the dissipated power determines the conductance of a LL, providing that no current-carrying leads are taken into account. The CDW contribution to the dissipated power was calculated in Ref. 21 for the short-ranged LL. In a CLL, the frequency dependence of $P_{\rm CDW}(\omega)$ is as follows:

$$P_{\rm CDW}(\omega)\sim\frac{e^{-4\beta|\ln\omega|^{1/2}}}{|\ln\omega|^{1/2}}|\varphi(2k_F)|^2,$$

where ω is normalized by $v_F/\beta d$, so that the dimensionless $\omega\ll 1$. For the short-ranged LL the frequency dependence is

$$P_{\rm CDW}(\omega)\sim\omega^{2g}|\varphi(2k_F)|^2.$$

In a CLL the power $P_{\rm CDW}$ diminishes very slowly with ω, so that in the low-frequency regime the CDW dominates in the dissipation,[21] since the power dissipated due to the long-wave density component behaves as $P_{\rm lw}\sim\omega^2$.

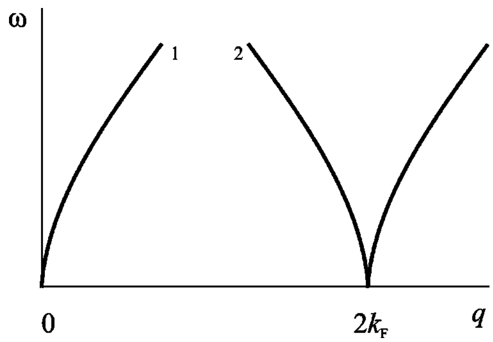

FIG. 3. The dispersion of the collective modes in a CLL. Line (1) is a plasmon mode. Line (2) is the $2k_F$ mode, related to the short-range electron correlations.

C. The dielectric function

The dielectric function $\varepsilon(q,\omega)$ is connected with the dynamic susceptibility via

$$\varepsilon^{-1}(q,\omega)=1+V(q)\chi(q,\omega).$$

When the structure factor goes to infinity, the dielectric function evidently turns to zero. The collective modes of the system under consideration are determined by zeros of $\varepsilon(q,\omega)$. Since the structure factor has singularities in two wave number regions, we find two regions, where the collective modes can propagate. The dispersion of the collective modes in a LL is illustrated by Fig. 3.

The first region corresponds to the singularity of the long-wave structure factor component $S_{\rm lw}(q,\omega)$, which occurs at $\omega=\omega_q$. This mode is just long-wave plasmons, almost identical to the RPA ones.[2,24,26]

The singularities of the CDW structure factor $S_{\rm CDW}(q,\omega)$ give the new mode, situated near to $q=2k_F$. The dispersion of this mode is as follows: $\omega=\omega_{q-2k_F}$. Here ω is pure real, which means that the $2k_F$ mode is nondecaying. This is because the structure factor $S_{\rm CDW}(q,\omega)$ has the true divergency. The cusp instead of a singularity in $S_{\rm CDW}(q,\omega)$ would lead to the complex frequency solutions of the equation $\varepsilon(q,\omega)=0$ and thus to the strong damping of the collective mode. We believe that such damping is physically absurd. Indeed, the boson excitations are noninteracting in the frame of the Luttinger model. They cannot decay into Landau quasiparticles either. Hence, there is no possibility for a collective mode in a CLL to transfer its energy to some other excitations and thus to damp. This is clear with the bosonization approach, which explains also why the usual long-wave plasmons are not damping in 1D.

An important conclusion is that the mode frequency goes to zero at $q\to 2k_F$, in other words, the mode is soft. It is the presence of the $2k_F$ mode that principally distinguishes the LL picture of collective excitations from the RPA one.

III. CONCLUSION

In the present work we have investigated the response functions of a spinless Luttinger liquid with Coulomb interaction at zero temperature. We have found the following.

(i) The CLL density response function is qualitatively different from that of a short-ranged LL. The nonlinear dispersion of bosonic excitations in a CLL results in that the DRF has no sharp front, and as the wave propagates, its form is smoothing with the appearance of the oscillatory structure.

(ii) The dynamic CDW structure factor is nonzero only in a region where $\omega_{q-2k_F}<\omega$, which is a consequence of energy and momentum conservation laws, applied to LL bosons. The CDW structure factor is diverging as $\epsilon=\omega-\omega_{q-2k_F}\to +0$ similar to $S(q,\omega)\sim\exp(-4\beta|\ln\epsilon|^{1/2})/\epsilon|\ln\epsilon|^{1/2}$. The similar result was obtained for the spectral function.

(iii) Owing to the CDW contribution, the dielectric function goes to zero as $\omega\to\omega_{q-2k_F}+0$. This means that the nondecaying mode appears in the region near to $q=2k_F$. The frequency of the mode tends to zero as $q\to 2k_F$, which means that the mode is soft. This $2k_F$ mode exists because the short-range dynamic electron correlations are highly pronounced in 1D, which is due to the $2k_F$ nesting of the Fermi surface of 1D electrons. RPA-like approaches are unable to describe the $2k_F$ mode since the dynamic nature of the short-range correlations is neglected there.

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APPENDIX A: THE CDW STRUCTURE FACTOR OF THE SHORT-RANGED LL

In this section we calculate the CDW structure factor and the entire complex CDW susceptibility of a short-ranged LL. Consider the CDW density correlator

$$R_{\rm CDW}(x,t)=-\frac{1}{8\pi^2}\partial_x^2\left(\exp\left[-v_F\int_{-\infty}^{+\infty}\frac{dp}{\omega_p}(1-e^{-i\omega_p t-ipx})\times e^{-\alpha|p|}\right]\cos(2k_Fx)\right). \tag{A1}$$

The structure factor is, by definition, the Fourier tranform of $R(x,t)$:

$$S(q,\omega)=\int_{-\infty}^{+\infty}dx\, e^{iqx}\int_{-\infty}^{+\infty}dt\, e^{i\omega t}R(x,t). \tag{A2}$$

First of all, we reduce the calculation of the $S_{\rm CDW}(q,\omega)$ to the calculation of an auxiliary function $F(q,\omega)$. Substituting Eq. (A1) into Eq. (A2), we get

$$S_{\rm CDW}(q,\omega)=\frac{q^2}{16\pi^2}[F(q-2k_F,\omega)+F(q+2k_F,\omega)], \tag{A3}$$

where

$$F(q,\omega)=\int_{-\infty}^{+\infty}dx\,e^{iqx}\int_{-\infty}^{+\infty}dt\,e^{i\omega t}F(x,t), \tag{A4}$$

and

$$F(x,t)=\exp\left(-v_F\int_{-\infty}^{+\infty}\frac{dp}{\omega_p}(1-e^{-i\omega_p t-ipx})e^{-\alpha|p|}\right). \tag{A5}$$

For the short-range interaction the dispersion is linear: $\omega_p=v|p|$, and the function $F(x,t)$ is easily calculated to give

$$F(x,t)=\alpha^{2g}[\alpha+i(vt-x)]^{-g}[\alpha+i(vt+x)]^{-g}.$$

Denote $\xi=vt-x$, $\zeta=vt+x$ to get

$$F(q,\omega)=\frac{\alpha^{2g}}{2v}\int_{-\infty}^{+\infty}d\xi e^{ia\xi}(\alpha+i\xi)^{-g}\int_{-\infty}^{+\infty}d\zeta e^{ib\zeta}(\alpha+i\zeta)^{-g},$$

where

$$a=\frac{1}{2}\left(\frac{\omega}{v}-q\right)\quad\text{and}\quad b=\frac{1}{2}\left(\frac{\omega}{v}+q\right).$$

Thus we have reduced the double Fourier-tranform to the single one. The integrals are easily calculated by closing contours upwards or downwards, depending on *a* and *b* signs, to give

$$F(q,\omega)=\frac{8\pi^2}{v}\left(\frac{\alpha^g}{2^g\Gamma(g)}\right)^2\left(\frac{\omega^2}{v^2}-q^2\right)^{g-1}\Theta\left(\frac{\omega^2}{v^2}-q^2\right).$$

Substituting the last expression into Eq. (A3) finally leads to Eq. (4). Now we have the imaginary part of the susceptibility. How can we restore the entire complex $\chi(q,\omega)$, avoiding the direct use of Kramers-Kronig relations? We propose to guess it:

$$\chi_{\rm CDW}(q,\omega)=-\frac{1}{\hbar v_F}\frac{g}{4^{g+1}\Gamma^2(g)\sin(\pi g)}\left(\frac{q}{k_F}\right)^2\times\sum_{r=\pm1}\left[\left(\frac{q}{k_F}-2r\right)^2-\left(g\frac{\hbar\omega+i0}{\varepsilon_F}\right)^2\right]^{g-1}+(\text{unknown}\ \chi_1).$$

Indeed, the first term on the right-hand side correctly gives the imaginary part of $\chi(q,\omega)$. Thus the unknown function χ_1 is a pure real function. On the other hand, χ_1 is analytic in the upper half-plane of ω (because both the LHS and the first term on the RHS are analytic functions in the upper half-plane of ω, and χ_1 is their difference). Whence, $\chi_1=0$ as a consequence of Kramers-Kronig relations. This way we get Eq. (5).

APPENDIX B: THE CDW STRUCTURE FACTOR OF THE COULOMB LL

Here we present a method to find $S_{\rm CDW}(q,\omega)$ for the CLL in the region near the threshold $\omega=\omega_{q-2k_F}$. In the Coulomb case it is not possible to calculate exactly the double Fourier transform of CDW density correlator, given by Eq. (A1). Instead, we propose to formulate the equation on $S(q,\omega)$, which would contain only the spectral parameters q and ω. Then we solve this equation near the threshold $\omega=\omega_{q-2k_F}$.

First of all, just as in Appendix A, we reduce the calculation of the $S_{\rm CDW}(q,\omega)$ to the calculation of an auxiliary function $F(q,\omega)$. Substituting Eq. (A5) into Eq. (A4) and performing once integration by parts with respect to *t*, we get our main equation:

$$\frac{\omega}{v_F}F(q,\omega)=\int_{-\infty}^{+\infty}dQ\,F(q-Q,\omega-\omega_Q). \tag{B1}$$

We stress that this equation contains only spectral-parameter dependence. The advantage of this equation is that the resonant frequencies turn out to be specified here, and it is much easier to extract the information about the spectral dependence of the structure factor from this equation, rather than from the direct expression of Eq. (A4).

Let us shift the integration variable Q by q, so that

$$\frac{\omega}{v_F}F(q,\omega)=\int_{-\infty}^{+\infty}dQ\,F(Q,\omega-\omega_{q+Q}). \tag{B2}$$

Here we used the fact that $F(q,\omega)$ is an even function of q [since $R(x,t)$ is an even function of x]. Now let us expand the RHS of Eq. (B2) with respect to Q that is contained in the frequency argument $(\omega-\omega_{q+Q})$:

$$\frac{\omega}{v_F}F(q,\omega)=\int_{-\infty}^{+\infty}dQ\left[F(Q,\omega-\omega_q)+\frac{Q^2}{2!}F_{qq}(Q,\omega-\omega_q)+\cdots\right]. \tag{B3}$$

This expansion gives us all the necessary information.

(i) Since the function $F(x,t)$ is analytic in the lower half-plane of complex time t [which is seen from Eq. (A5)], we find that $F(q,\omega)=0$ when $\omega<0$. (It is, of course, clear physically, why the structure factor at zero temperature is zero when $\omega<0$: there is no possibility to create an excitation below the ground state.) So, the RHS of Eq. (B3) is zero when $\epsilon=\omega-\omega_q<0$.

Whence, the LHS, i.e., $F(q,\omega)$ has the threshold: when $\epsilon>0$, $F(q,\omega)$ is nonzero, whereas for $\epsilon<0$, $F(q,\omega)=0$. The existence of this threshold was also explained from the physical background in the main text.

(ii) The first term on the RHS, i.e., the integral $\int_{-\infty}^{+\infty}dQ\,F(Q,\epsilon)$ equals $2\pi F_{\rm static}(\epsilon)$, where

$$F_{\rm static}(\epsilon)=\int_{-\infty}^{+\infty}dt\,e^{i\epsilon t}F(x=0,t). \tag{B4}$$

Using integration by parts, all the terms on the RHS of Eq. (B3) can be expressed via the Fourier-transform of the functions, depending on *t* only. So, the expansion (B3) allows one to reduce the calculation of the dynamic, two-argument-dependent function $F(q,\omega)$ to the calculation of static functions, depending on ϵ only. The calculation of these static functions takes *one* integration only, and is much more easily performed. Using the direct calculation, it can be shown that the function $F_{\mathrm{static}}(\epsilon)$ diverges as $\epsilon\to 0$, and so do all the terms on the RHS of Eq. (B3). For a short-ranged LL, all the terms diverge as a power law of ϵ with the same exponent, therefore it would take us to sum all the expansion to get the correct exponent on the LHS In a CLL, the first term divergency $F_{\mathrm{static}}(\epsilon)$ is the strongest one, and the following terms divergencies were found to be weaker. So, in the leading order of divergency, $F(q,\omega)$ in the CLL is given by

$$\frac{\omega}{v_F}F(q,\omega)\sim 2\pi F_{\mathrm{static}}(\omega-\omega_q),\quad \omega-\omega_q\to 0. \tag{B5}$$

Using the asymptotic form of $F(x=0,t)$ at large t, we can find (via the direct integration) the form of $F_{\mathrm{static}}(\epsilon)$ as $\epsilon\to 0$. But we prefer to get the result by another way. It is easy to show that $F_{\mathrm{static}}(\epsilon)$ satisfies the integral equation, similar to Eq. (B1):

$$\frac{\omega}{v_F}F_{\mathrm{static}}(\omega)=2\int_0^{+\infty}dQ\,F_{\mathrm{static}}(\omega-\omega_Q). \tag{B6}$$

Denote $\xi=\omega-\omega_Q$ to rewrite the last equation in the form

$$\frac{\omega}{v_F}F_{\mathrm{static}}(\omega)=2\int_0^{\omega}\frac{\beta d\xi}{v_F\sqrt{\ln\frac{\omega-\xi}{v_F}\beta d}}F_{\mathrm{static}}(\xi). \tag{B7}$$

The limits of integration are dictated by the property that $F_{\mathrm{static}}(\omega)=0$ at $\omega<0$. It is a good approximation to replace the kernel

$$\sqrt{\ln\frac{\omega-\xi}{v_F}\beta d}\quad\text{with}\quad\sqrt{\ln\frac{\omega}{v_F}\beta d},$$

because the singularity of $F_{\mathrm{static}}(\xi)$ at $\xi=0$, which gives the major contribution to the RHS of Eq. (B7), is then integrated with a correct weight. Then, multiplying both sides of Eq. (B7) on

$$\sqrt{\ln\frac{\omega}{v_F}\beta d}$$

and then differentiating with respect to ω, we get the following differential equation for $F_{\mathrm{static}}(\omega)$:

$$\frac{d}{d\omega}\left(\omega\sqrt{\ln\frac{\omega}{v_F}\beta d}F_{\mathrm{static}}(\omega)\right)=2\beta F_{\mathrm{static}}(\omega),$$

whose solution is

$$F_{\mathrm{static}}(\omega)=C\frac{e^{-4\beta\sqrt{\ln\frac{\omega}{v_F}\beta d}}}{\omega\sqrt{\ln\frac{\omega}{v_F}\beta d}}, \tag{B8}$$

C being a constant. Of course, this result can also be found by direct integration in Eq. (B4).

Substituting the expression for $F_{\mathrm{static}}(\omega)$ into Eq. (B5), we get Eq. (6) of the main text. We see, that the CDW structure factor is diverging near the threshold, and the divergency in a CLL is very strong, almost $\sim 1/\epsilon$, which would correspond to the limit $g\to 0$ in the short-ranged case. The divergency in a CLL is, nevertheless, integrable [since the expression in Eq. (6) is the exact differential].

The LL spectral function satisfies the integral equation, similar to Eq. (B1). Using the above procedure, we solve it and get the Eq. (7) of the main text.

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