

Solitons in polarized double-layer quantum Hall systems

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(Received 15 October 2001; published 13 March 2002)

A new manifestation of interlayer coherence in strongly polarized double-layer quantum Hall systems with total filling factor $\nu=1$ in the presence of a small or zero tunneling is theoretically predicted. It is shown that moving (for small tunneling) and spatially localized (for zero tunneling) stable pseudospin solitons develop which could be interpreted as mobile or static charge-density excitations. The possibility of their experimental observation is also discussed.

DOI: 10.1103/PhysRevB.65.121310

PACS number(s): 73.43.Lp, 11.15.-q, 05.45.-a

Much study has been devoted to the dynamical properties of charge-density excitations in double-layer quantum Hall systems with total Landau filling factor $\nu=1$. An effective field-theoretical pseudospin model was developed in order to explain the experimentally observed spontaneous phase coherence even in the absence of tunneling¹⁻³ as well as for nonzero tunneling and in the presence of an applied in-plane magnetic field.⁴ The main prediction⁵⁻⁷ of this model, the existence of the broken symmetry Goldstone mode⁸, has been confirmed by the observation⁹ of split off peaks in the tunneling conductance in the presence of in-plane magnetic field.

However, the study of double layer (pseudo) ferromagnets is usually restricted to the charge balanced state (see, e.g., Ref. 10, and references therein), although very recently the existence of a stable unbalanced state (the so-called canted phase^{11,12}) in the presence of in-plane magnetic field was predicted. To date, static structures have dominated theoretical and experimental investigations. Only recently was the creation of a slowly moving pseudospin soliton investigated,¹³ but again this was done near the balanced state.

In this communication we study the dynamical problem of a strongly unbalanced double layer system in the presence of small or zero tunneling. Under such conditions the system is in highly nonequilibrium state but it is hard to reach the ground state if the interlayer tunneling amplitude is small, or even impossible if the amplitude is zero. On the other hand, the nonlinear coupling induced by interlayer coherence is still present and thus the appearance of different exotic nonlinear creations might be expected. Indeed, as we will show below, stable, mobile, or spatially localized pseudospin textures arise which could be described in terms of ac driven nonlinear Schrödinger (NLS) equation widely studied in the literature (see, e.g., Refs. 14 and 15). This circumstance relates quantum Hall (pseudo)ferromagnets with completely different physical systems, such as yttrium iron garnet thin magnetic films,¹⁶ long Josephson junctions,¹⁷ relativistic electron-positron plasma,¹⁸ etc., which are governed by the same NLS equation and where similar localized excitations are well studied both experimentally and theoretically.

The phenomenological model for double-layer quantum Hall (pseudo) ferromagnets, which is based upon microscopic considerations^{1,2}, is effectively described by the following Hamiltonian:^{1-3,10}

$$\mathcal{H} = -\frac{eV}{2}n_z + \frac{\rho_E}{2} \left\{ \left(\frac{\partial n_x}{\partial x} \right)^2 + \left(\frac{\partial n_y}{\partial x} \right)^2 \right\} + \beta n_z^2 - \Delta_{SAS} \{ n_x \cos(Qx) + n_y \sin(Qx) \}, \quad (1)$$

where ρ_E is in-plane spin stiffness; β is the a hard axis anisotropy; Δ_{SAS} is a tunneling amplitude, $Q \sim B_{\parallel}$ is the “wave vector” associated with the tumbling of the electronic phase due to the in-plane magnetic field; V describes the gate voltage; e is an electron charge; $\hbar=1$ and $\hat{n}(x,t)$ is an order parameter unit vector. The component $n_z(x,t)$ is the local charge imbalance between the layers and can be expressed in terms of local filling factors of top and bottom layers, $n_z = \nu_1 - \nu_2$; the local filling factors ν_1 and ν_2 are proportional to the local electric densities N_1 and N_2 in the corresponding layers. For example, in the fully balanced case, $\nu_1 = \nu_2 = 1/2$ (i.e. $n_z = 0$) and $N_1 = N_2 \equiv N$. In typical experiments⁹ on double-layer systems $N = 3.0 \times 10^{10} \text{ cm}^{-2}$. In the strongly unbalanced situation which is a subject of study in the present paper, one has $n_z \approx 1$ and thus the densities of top and bottom layers are close to $2N$ and 0 , respectively.

The time-space behavior of ordering vector could be described in terms of Landau-Lifshitz equation:¹⁹

$$\frac{\partial \hat{n}}{\partial t} = (\hat{n} \times \mathbf{H}_{eff}), \quad (2)$$

$$\mathbf{H}_{eff} = -2 \left\{ \frac{\partial \mathcal{H}}{\partial \hat{n}} - \frac{\partial}{\partial x} \left[\frac{\partial \mathcal{H}}{\partial \frac{\partial \hat{n}}{\partial x}} \right] \right\},$$

where \mathbf{H}_{eff} is the effective (pseudo)magnetic field. We introduce the variables

$$n^{\pm} \equiv n_x \pm i n_y \quad (3)$$

so that Eq. (2) has the following form:

$$\frac{\partial n^+}{\partial t} = -ieVn^+ + 4i\beta n^z n^+ + 2i\rho_E n^z \frac{\partial^2 n^+}{\partial x^2} - 2i\Delta_{SAS} n^z e^{iQx}. \quad (4)$$

We will seek for the solution of this equation around $n_z = 1$. This state corresponds to the highest energy value static solution. Any dynamical fluctuation around this state is the

precession around n_z which is allowed by the equations of motion. In the limit of small deviations from this state and small tunneling it is possible to apply the multiple scale approach:^{20,21}

$$n^+ = \varepsilon m^+(\xi, \tau) e^{i(Qx - \omega t)},$$

$$n_z = \sqrt{1 - |n^+|^2} = 1 - \varepsilon^2 \frac{|m^+|^2}{2}, \quad (5)$$

$$\Delta_{SAS} \equiv \varepsilon^3 \tilde{\Delta}_{SAS}$$

where $m^\pm(\xi, \tau)$ is a slowly varying functions of the variables

$$\xi = \varepsilon(x - vt); \quad \tau = \varepsilon^2 t \quad (6)$$

and ε is a formal small parameter expressing the smallness or “slowness” of the object before which it appears, allowing for a multiple scale analysis of the problem. The range of acceptable ε will be determined in the course of the calculation. We are working in the regime

$$|n^+| = |m^+| \ll 1. \quad (7)$$

To study the perturbative solution we substitute Eq. (5) into Eq. (4), and collect terms of the same order of ε . In the first order of ε the following equality is obtained:

$$\omega = eV + 2(\rho_E Q^2 - 2\beta), \quad (8)$$

while to second order we get the expression for the group velocity v :

$$v = 4\rho_E Q. \quad (9)$$

Finally, to third order in ε we come to the following nonlinear equation:

$$i \frac{\partial m^+}{\partial \tau} + 2\rho_E \frac{\partial^2 m^+}{\partial \xi^2} + (\rho_E Q^2 - 2\beta) |m^+|^2 m^+ = -2\tilde{\Delta}_{SAS} e^{i\omega t}, \quad (10)$$

which in the case of

$$\rho_E Q^2 > 2\beta; \quad \frac{eV + 2\rho_E Q^2 - 4\beta}{\rho_E Q^2 - 2\beta} \sim \varepsilon^2 \quad (11)$$

reduces to the exactly solvable ac driven NLS equation considered in Refs. 14 and 15.

The physical meaning of the second expression from (11) together with definition (8) is that one has a near resonance driving of pseudospin mode with a wave number Q . The possibility of soliton, chaotic, or other exotic solutions of the NLS equation (10) is studied in details in Refs. 14 and 15 and we direct the reader there for more information. Here we only mention that the stable solutions of Eq. (10) could be interpreted as envelope solitons of order parameter \hat{n} moving with a group velocity proportional to applied in-plane magnetic field [see expression (9)] and characterized by carrier wave vector Q which is also proportional to in-plane magnetic field. Note that according to the restriction of Eq. (11), the in-plane magnetic field should be nonzero.

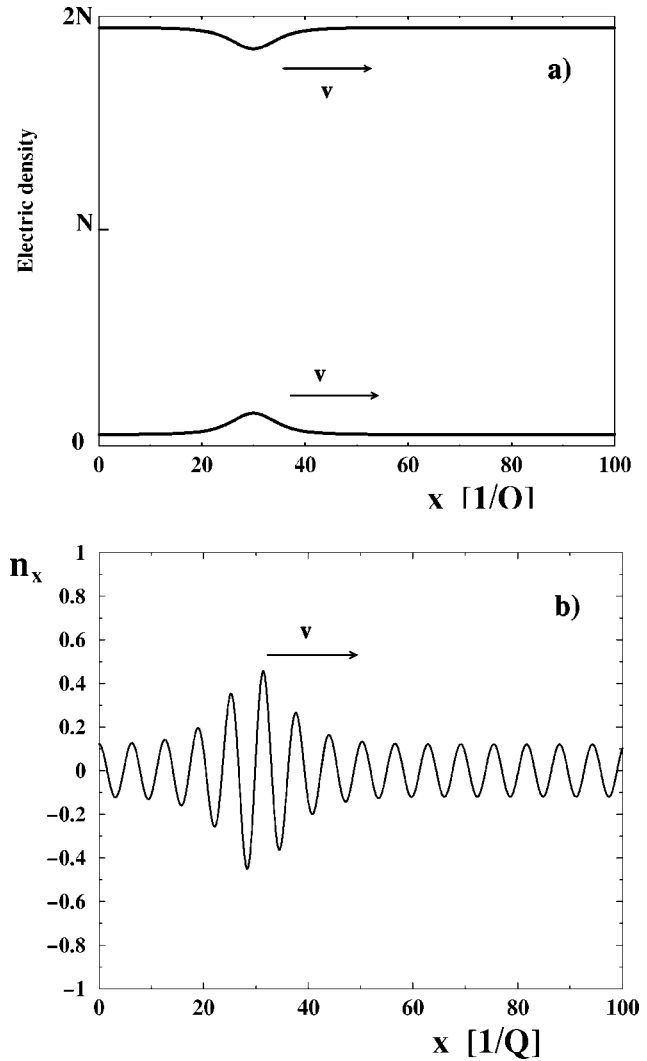


FIG. 1. (a) Local electric density of each layer versus x in the case of small tunneling. Top and bottom graphs correspond to the local electric densities of top and bottom layer, respectively. N is an electric density of each layer in the fully balanced state. (b) One of the transverse components of ordering vector (the second one is shifted in phase by 90°) with carrier wave number Q . This envelope soliton moves with velocity v proportional to the applied in-plane magnetic field [see also Eq. (9)].

In Fig. 1 this excitation is presented as a moving bump of electric density through each layer. Note that envelope soliton nature only characterizes the transverse component of the order parameter, while the physically measurable quantity—local charge imbalance—is just an ordinary soliton.

According to the general results of Refs. 14 and 15 the stability of the mentioned soliton is restricted to the following condition:

$$h \equiv \frac{\Delta_{SAS} \sqrt{\rho_E Q^2 - 2\beta}}{(eV + 2\rho_E Q^2 - 4\beta)^{3/2}} < \left(\frac{1}{27}\right)^{1/2}. \quad (12)$$

This quantity stands for the driving force in the NLS equation¹⁴ and therefore defines the soliton amplitude.

Here we also present the solutions for simple, but physically very important case of zero tunneling ($\Delta_{SAS}=0$), which do not directly follow from the exact solutions for nonzero Δ_{SAS} presented in Ref. 14. In the case of zero tunneling the in-plane magnetic field has no effect on the system. For simplicity we assume also $V=0$. Then the solution of the initial equation of motion (4) is sought for in the following simple form:

$$n^+ = \varepsilon m^+(x,t)e^{4i\beta t}, \quad n_z = 1 - \varepsilon^2 \frac{|m^+|^2}{2}. \quad (13)$$

Substituting this expression into Eq. (4) we come to the ordinary NLS equation:

$$i \frac{\partial m^+}{\partial t} + 2\rho_E \frac{\partial^2 m^+}{\partial x^2} - 2\beta |m^+|^2 m^+ = 0, \quad (14)$$

which has a stable “dark soliton” solution.²² Since the physically measurable variable is n_z (the local charge imbalance between the layers) we present here only an expression for its profile:

$$n_z = 1 - \frac{D^2}{2} \left| \frac{\sqrt{1-A^2}}{A} + i \cdot \tanh\left\{\frac{x}{\Lambda}\right\} \right|^2, \quad (15)$$

where D is a soliton amplitude; A denotes the contrast of dark soliton (if $A=1$ one has a “black” dark soliton and “gray” dark for $0 < A < 1$) and soliton width Λ is defined as follows:

$$\Lambda = \left(\frac{2\rho_E}{\beta} \right)^{1/2} \frac{1}{D}. \quad (16)$$

The difference between gray dark and black dark solitons can be expressed in terms of the charge densities in the top and bottom layers (see also Fig. 2): In a black dark soliton solution there exists some point along the x axis where the local electric density of top layer reaches the value $2N$, while the local density of bottom layer in the same point approaches zero. On the other hand, there always exists some deviation of the charge densities from $2N$ and 0 for top and bottom layers, respectively, for the gray dark soliton solution.

Unlike the previous case of small tunneling where the soliton is characterized by nonzero carrier wave number Q , in the case of zero tunneling that is not the case and consequently transverse components of ordering vector have the similar form to the z component and are not presented in Fig. 2.

We should emphasize again one more difference between two considered cases: In the case of nonzero (but small)

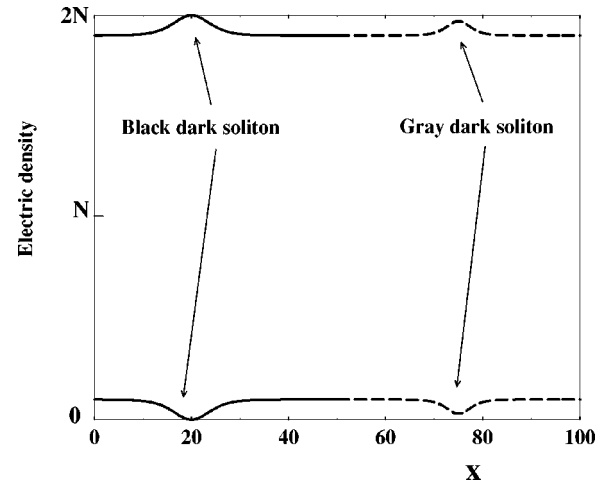


FIG. 2. Static “bumps” of electric density for zero tunneling in a strongly unbalanced double-layer system. Two types of solutions are presented: the solid line corresponds to black dark soliton solution, while the dashed line indicates gray dark soliton. Both soliton solutions are stable.

tunneling the envelope soliton amplitude is linked with the driving coefficient Δ_{SAS} , while in the absence of tunneling the dark soliton amplitude is arbitrary within the restriction (7).

The above considerations are only valid for quasi-one-dimensional samples, i.e., quantum Hall bars. Thus the main requirement for stability of soliton solutions is that the transverse dimension of the double-layer system should be less than soliton width. In this case the large wavelength modulations do not grow (which destabilize the soliton) and solitons remain stable.^{23,24} Future work will investigate their stability.

In conclusion, we have found stable (mobile or static) charge-density excitations in strongly unbalanced double-layer systems. For their observation the double layer quantum Hall system should be prepared so as $n_z \approx 1$. Then any perturbation will cause the appearance of either standing (zero tunneling) or moving (for small tunneling and applied in-plane magnetic field) charge-density train of “bumps,” which could be easily detected experimentally. The perturbation could be induced by temporary application of local electric field perpendicular to the layers.

The work at the University of Oklahoma was supported by the NSF under Grant No. EPS-9720651 and a grant from the Oklahoma State Regents for higher education. R. Kh. was supported by NSF-NATO Grant No DGE-0075191. R.A. was supported by NSF MRSEC Grant No. DMR-0080054.

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