Broken-symmetry states in quantum Hall superlattices

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We argue that broken-symmetry states with either spatially diagonal or spatially off-diagonal order are likely in the quantum Hall regime, for clean multiple-quantum-well (MQW) systems with small layer separations. We find that for MQW systems, unlike bilayers, charge order tends to be favored over spontaneous interlayer coherence. We estimate the size of the interlayer-tunneling amplitude needed to stabilize superlattice Bloch minibands by comparing the variational energies of interlayer-coherent superlattice miniband states with those of states with charge order and states with no broken symmetries. We predict that when coherent miniband ground states are stable, strong interlayer electronic correlations will strongly enhance the growth-direction tunneling conductance and promote the possibility of Bloch oscillations.

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I. INTRODUCTION

The strong-correlation physics of bilayer electron systems in the quantum Hall regime has been of interest since shortly after the discovery of the quantum Hall effect.[1,2] Of particular interest is the occurrence of broken-symmetry states with spontaneous interlayer phase coherence[3] (SILC) that continue to be a source of surprises and puzzles.[4–9] In the quantum Hall regime, Landau-level degeneracy leads to competing nascent broken symmetries; the SILC broken symmetry is driven by the strong interlayer electronic correlations that it produces, and competes subtly with a spatially diagonal broken-symmetry state in which electronic charge spontaneously occupies one of the two layers. (In the case of a bilayer system with small interlayer separations and low densities, the SILC state can occur in principle even at zero magnetic field.[10–12])

The energy of bilayer quantum Hall systems can be expanded in powers of the layer separation d, the parameter that most critically controls their properties. The competition between SILC and charge-ordered states is decided only by terms of second and higher order in d. The property that SILC and charge-ordered states in bilayers have the same energies to first order in d is not an accident and can be understood using the bilayer pseudospin language.[2] In this description, electrons in the top or bottom layers are eigenstates of the $\hat{z}$ component of the pseudospin, while electrons with interlayer coherence have pseudospin projections in the x-y plane. Similarly, the charge-ordered and SILC-ordered states correspond, respectively, to states with Ising and XY ferromagnetic order. The competition between these two states depends on the sign of the pseudospin anisotropy energy.[13] The pseudospin-dependent part of the interaction in bilayers is proportional to the difference between same-layer and different-layer electron-electron interactions, which for two-dimensions is proportional to $[1-\exp(-qd)]/q$ in reciprocal space, where q is the magnitude of the in-plane wave vector. The origin of the mysterious absence of a pseudospin anisotropy energy at first order in d is now clear, since the term in this interaction that is first order in d is independent of q; i.e., it is a delta-function interaction that has no effect on fully spin-polarized fermions because of the Pauli exclusion principle. The leading terms in the pseudospin magnetic anisotropy energy[13] in the bilayer case appears at second order in d and, as it turns out, leads to SILC rather than charge order.

In this paper, we generalize the investigation of order in quantum Hall systems at integer total filling factors to the case of multilayers, where the pseudospin analogy does not apply. We should then expect the energetic difference between charge (diagonal) and SILC (off diagonal) order to be settled at first order in d. We find that, in contrast to the bilayer case, charge order is favored in multilayers for small d. States with interlayer phase coherence can, however, be stabilized by relatively weak interlayer tunneling, and we predict that they will have unusual physical properties.

Our conclusions are based on a comparison of variational energies for states with charge order and states with SILC in multilayer quantum Hall systems.[14] The multilayer SILC states are characterized by spontaneous finite-width Bloch minibands. We compare the energies of these miniband wave functions to those of states with strong but independent correlations within each layer, including charge-ordered staggered states with unequal layer densities. Physically, this difference between bilayers and multiple-quantum-well systems occurs because the Hartree energy per volume for states with a given charge imbalance is far lower in the superlattice case, as much as four times lower compared to bilayers. In some cases, however, relatively small interlayer tunneling amplitudes are sufficient to stabilize uniform density miniband states. We propose that these states could be identified experimentally by a substantially-enhanced growth-direction conductance that is due to their strong interlayer correlations, and argue that they might also support Bloch oscillations.

Multilayer quantum Hall systems have been fabricated that exhibit the quantum Hall effect at integer filling factors per layer,[15] and an interesting body of recent work has focused on the chiral surface states that occur in this instance.[16,17] At fractional filling factors, most work[18,19] has

concentrated on the physics at the special values of filling factor per layer for which Laughlin-Jastrow[20] states with strong intralayer correlations occur. Our interest here is in nearly disorder-free multilayer systems at strong magnetic fields with general fractional filling factors per layer ν between zero and one. Other recent work on multilayer quantum Hall systems at fractional filling factor per layer has focused on the possible realization of the Josephson effect,[21] and on metal-insulator transitions[22] and critical exponents for localization in disordered systems.[23] The possibility of charge order in superlattice quantum Hall systems has been proposed previously,[24–26] but not confronted against competing SILC states. Related physics can in principle occur in other quasi-two-dimensional electron systems, for example, layered organic conductors.[27]

Our paper is organized as follows. In Sec. II, we present our SILC variational wave functions and compare their energies with those of states that have neither charge order nor interlayer correlations. The SILC states, which have better interlayer correlations but less favorable intralayer correlations, have lower energy when d is small. In Sec. III, we consider charged-ordered multiple-quantum-well states. These states achieve the objective of reducing the likelihood of close approaches between electrons in nearby layers simply by reducing the average charge density in the neighboring layer, rather than by improving interlayer correlations. In that section, we demonstrate that charge-ordered states are favored over SILC states. Section IV discusses the properties of the coherent Bloch miniband states, in the case where interlayer coherence is aided by tunneling between the electron layers; although the coherence in this case is not spontaneous, it is strongly enhanced by interactions. We conclude in Sec. V.

II. MINIBAND GROUND STATE

We consider a system with many coupled quantum wells separated by a distance d, and a Landau-level filling factor per layer $\nu \equiv N_e/(N_w N_\phi)$ smaller than one. Here N_e, N_w, N_ϕ are the total number of electrons, the number of quantum wells containing two-dimensional electron layers, and the number of flux quanta passing through each layer, respectively. (The Landau-level degeneracy is N_ϕ.) Unless the interlayer separation is quite small ($d \sim l$, where l is the magnetic length), independent strongly-correlated states will form in each two-dimensional layer. From theoretical and experimental studies of the fractional quantum Hall effect, the energies of these states,[28] whose character changes rapidly with ν, is relatively accurately known. In bilayer systems, as interactions between the two-dimensional layers increase in importance, the many-particle ground state develops SILC, which promotes interlayer correlations at the cost of partially disrupting correlations within the layers. The variational wave function we propose to accomplish this compromise in a superlattice is the following single Slater determinant,

$$|\Psi_0\rangle = \prod_{q,X} c^\dagger_{q,X}|0\rangle. \tag{1}$$

Here the single-particle miniband states are

$$|q,X\rangle = \frac{1}{\sqrt{N_w}} \sum_{j=1}^{N_w} e^{iqdj}|j,X\rangle, \tag{2}$$

where $|j,X\rangle$ is the state of a particle in the Landau-gauge lowest-Landau-level state $|X\rangle$ in the jth quantum well. Such a miniband state has strong built-in interlayer correlations. The variational wave function in Eq. (1) is obtained from the requirements that it have equal constant densities in each layer and a density matrix $\rho_{j_1 j_2}(X)$ that is translationally invariant (so that it depends only on the difference $|j_1 - j_2|$ of layer indices):

$$\rho_{j_1 j_2}(X) = \langle \Psi_0 | c^\dagger_{j_1 X} c_{j_2 X} | \Psi_0 \rangle = \rho_0(|j_1 - j_2|). \tag{3}$$

This form for the density matrix self-consistently solves the Hartree-Fock (HF) equations, and hence $|\Psi_0\rangle$ is a local extremum of the HF energy functional.

We focus here on the limit $N_w \to \infty$, appropriate to a system with a large number of coupled quantum wells. In Eq. (1), the product X goes over all N_ϕ states within the lowest Landau level, but the product q goes over only a fraction ν of the growth-direction Bloch miniband states in the interval $-\pi/d < q \leqslant \pi/d$. As we discuss in Sec. IV, the energy is minimized when the occupied wave vectors are contiguous, but, in the absence of interlayer tunneling, is invariant under a simultaneous translation of all occupied wave vectors. For $N_w \to \infty$, the ground-state density matrix for electrons separated by j layers is

$$\rho_0(j) = \frac{\sin(\pi j \nu)}{\pi j}, \tag{4}$$

so that there are, in general, correlations between layers at all separations.

The miniband wave function in Eq. (1) generalizes to a multiple quantum well the notion of SILC studied previously in bilayer[2,3] and trilayer[29–31] systems. This wave function will have a low energy at small interlayer separations because it establishes correlations between electrons in different layers: the pair distribution function for two electrons that are j layers apart and which have an in-plane separation r is

$$g_j(r) = 1 - e^{-r^2/2l^2}\left[\frac{\sin(\pi j \nu)}{\pi j \nu}\right]^2. \tag{5}$$

The decreased probability for finding pairs of electrons in nearby layers with small in-plane separations lowers the energy when the layers are close together.

The HF ground-state energy per particle of the miniband state in Eq. (1) may be obtained by summing over the interactions with correlation holes distributed over all layers:

$$\varepsilon(\nu) = \frac{E_0(\nu)}{N_e} = -2t\frac{\sin(\pi\nu)}{\pi\nu} - \frac{\nu}{2}\sum_{j=-\infty}^{\infty}\int \frac{d^2r}{2\pi l^2} \times [1 - g_j(r)]\frac{e^2/4\pi\epsilon}{\sqrt{r^2 + (dj)^2}}, \tag{6}$$

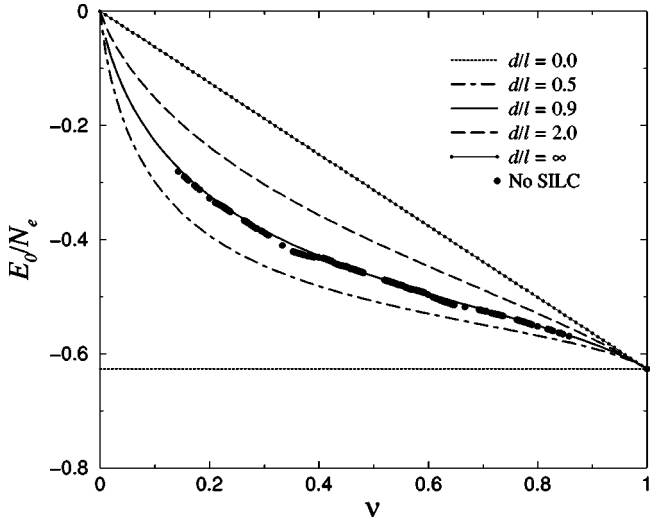

FIG. 1. Ground-state energies per particle $\varepsilon(\nu)$ in units of $e^2/4\pi\epsilon l$, for an $N_w\to\infty$ miniband quantum Hall state, versus the filling factor per layer ν, for $t=0$. The estimated ground-state energies per particle for states without correlations between the layers are shown as filled circles ("No SILC").

where $-t$ is the interlayer tunneling matrix element in the tight-binding approximation, $-e$ is the electronic charge, and ϵ is the dielectric constant of the host semiconductor. Note that interlayer tunneling ($t>0$) lowers the energy of the miniband state, and that when this term is present, the interlayer coherence in $|\Psi_0\rangle$ is not spontaneously generated. In Fig. 1, we plot the ground-state energy per particle, $\varepsilon = E_0/N_e$, versus ν for the miniband quantum Hall state at $t=0$ for different layer separations, to estimate the energy per particle of a uniform-density superlattice ground state possessing SILC. We also plot the estimated energy per particle for isolated-layer states without SILC.[32] For $d<d_c\sim 0.9l$, the superlattice state with SILC has a lower energy than independent-layer states with identical layer densities. The calculations used to produce Fig. 1 would seem to indicate that $\nu=1/2$ is the most favorable filling factor per layer for observing SILC. This is because the relatively high energy per particle of the $\nu=1/2$ independent-layer state leads to a larger critical distance d_c below which the $t=0$ miniband state is stable. Conversely, $\nu=1/3$ would appear to be among the least-favored filling factors for obtaining SILC, because the sizeable negative correlation energy per particle of the $\nu=1/3$ independent-layer Laughlin states[19] gives a smaller value of d_c.

The comparison of the energies of variational wave functions depicted in Fig. 1 shows that the $t=0$ (SILC) miniband state has a lower energy than identical uniform layers of independent states when $d<d_c\sim 0.9l$. In the case of balanced bilayer systems with filling factor $\nu=1/2$ per layer, the energy of the Slater-determinant variational SILC state is lower than that of two $\nu=1/2$ states without interlayer correlations for $d/l<d_c/l\sim 1.16$. This result for d_c for balanced bilayers is in close agreement with other theoretical estimates: the time-dependent HF collective-mode stability criterion[33] gives $d_c/l=1.18$, and finite-size exact diagonalization studies[9] give $d_c/l=1.175$. This is consistent with experimental findings[34,4] when the finite thickness[9] of the electrons layers and quantum fluctuations[35] are taken into account. In double-quantum-well systems, theoretical predictions[36,33] of SILC have been confirmed experimentally, most directly in recent experiments that have discovered a Josephson-like peak in the interlayer tunneling conductance.[4] Thus, Fig. 1, taken at face value, is evidence for the possibility of a miniband SILC state at sufficiently small layer separation. However, as explained in Sec. I, the competition between SILC and charge order is a subtle one even in the bilayer case, and there is good reason to expect that this competition plays out differently in a multiple-quantum-well system. We, therefore, turn our attention in the following section to variational wave functions with charge order.

III. STAGGERED STATES

We showed in the preceeding section that for sufficiently small interlayer separations, the miniband state is always energetically favored over independent-layer states of equal uniform density. However, we must also consider the possibility that at small interlayer separations, the exchange interaction could favor independent-layer states with unequal densities. Indeed, the possibility of staging transitions in multilayer systems was discussed in Ref. 25, where HF calculations predicted a series of staging transitions in which occupied layers were separated by an increasing number of empty layers as the particle density and interlayer separation were decreased. In the bilayer case, the predicted[37] staging transition is not expected to occur, even at zero magnetic field;[10–12] instead, it is preempted by a transition to a bilayer SILC state with equal layer densities.[3] Even in the case of the bilayer quantum Hall state, the SILC state is barely favored over the unbalanced state with $\nu=1$ in one layer and $\nu=0$ in the other layer for $d/l\to 0$: the two states are degenerate at $d=0$ and, as explained in the Sec. I, the SILC state is lower in energy only at order d^2. The bilayer SILC state is most stable at intermediate densities between 0 and d_c.

In the superlattice case, we show below that the ($t=0$) miniband SILC state loses energetically to staggered states at small d. One important reason for this is that the Hartree energy per volume of a staggered-density state is far lower in a superlattice than in a bilayer. This is understood qualitatively by considering the case of two-dimensional charge sheets of alternating areal charge density $\pm\sigma$. For a superlattice, the bulk-system requirement that the voltage drop due to the electric fields be zero across two-layer spacings produces an electric field of constant magnitude that alternates in sign across each layer: $E_\infty=\pm\sigma/2\epsilon$, from Gauss's law. The same calculation for a classical bilayer capacitor gives an electric field that is twice as large, $E_2=\sigma/\epsilon$. The Hartree energy per unit volume is given by $\epsilon E^2/2$, so it costs four times less energy per volume to have a staggered state in a superlattice as compared to a bilayer.

To make the comparison more precise, we have computed the energy of the SILC state analytically in the limit $d\to 0$ by summing up the interactions of an electron with exchange-hole contributions from remote layers. The two states compete by measuring the loss in exchange energy in the SILC

state that is due to removing a part of the exchange hole to remote layers against the Hartree energy of the charge-ordered state discussed above. From Eq. (6), we find that for the SILC state

$$\varepsilon(\nu)\to\varepsilon(1)\left[1-\frac{d/l}{4\nu}\left(\frac{2}{\pi}\right)^{5/2}\ln(l/d)+\frac{d(1-\nu)}{2l}\right] \quad (7)$$

for $d\to 0$, where $\varepsilon(1)=\varepsilon(\nu=1)=-(1/2)\sqrt{\pi/2}e^2/4\pi\epsilon l$. Because of the $d\ln(l/d)$ term above, the SILC state always has a higher energy in the limit $d\to 0$ than a staggered state that alternates filled ($\nu=1$) layers with completely empty layers, since the Hartree cost of such staggered states grows only linearly with d. However, the HF approximation underestimates the miniband energy, and it may be that correlation effects (and finite-thickness effects) improve the energetics of the miniband states. Unfortunately, we cannot provide a reliable estimate of the correlation-energy contribution to the miniband state, since the generalized random-phase approximation gives a logarithmically divergent (and negative) result.[35] We are not able to make a definitive conclusion on the possibility that fluctuation effects beyond the HF approximation stabilize the miniband state for small or even zero interlayer tunneling, or whether there are other superlattice SILC states besides the miniband state that could be realized, leaving this as an issue that must ultimately be decided experimentally. In Sec. IV, we discuss experimental signatures of the miniband state.

It is useful to compare the energy of the miniband state to those of more general staggered states. Consider a staging transition of order n to a staggered state that consists of repeating units of $n+1$ layers, with one layer having filling factor $\nu+n\delta$ and n depleted layers having filling factor $\nu-\delta$. The driving force for the staging transition is the intralayer exchange-correlation interaction, which favors concentrating the electrons in a single plane. The main energy cost of the staging transitions is the Hartree charging energy due to having charge-imbalanced layers. We note that the surface charge density of the n depleted layers is $e\delta/2\pi l^2$, so that the Hartree contribution to the energy per particle for the staggered states is[25]

$$\varepsilon_H(n)=\frac{n(n+2)}{12}\frac{e^2}{4\pi\epsilon l}\frac{\delta^2}{\nu}\frac{d}{l}, \quad (8)$$

which is always linear in d. At large separations, the Hartree cost for the staging transitions is prohibitive, so $n=0$. The exchange-correlation contribution to the total energy per particle for independent-layer states is

$$\varepsilon_{xc}(n)=\frac{(\nu+n\delta)\varepsilon_1(\nu+n\delta)+n(\nu-\delta)\varepsilon_1(\nu-\delta)}{(n+1)\nu}, \quad (9)$$

where $\varepsilon_1(\nu)$ is the energy per particle for a single-layer quantum Hall state at filling factor ν, which is indicated by the dots in Fig. 1. Note that $\varepsilon_1(\nu)$ decreases with increasing ν, which favors increasing the density of some layers at the expense of others. The total energy per particle of the staggered system is $\varepsilon_s=\varepsilon_H+\varepsilon_{xc}$. For the sake of definiteness, we consider two special cases below, $\nu=1/2$ and $\nu=1/4$.

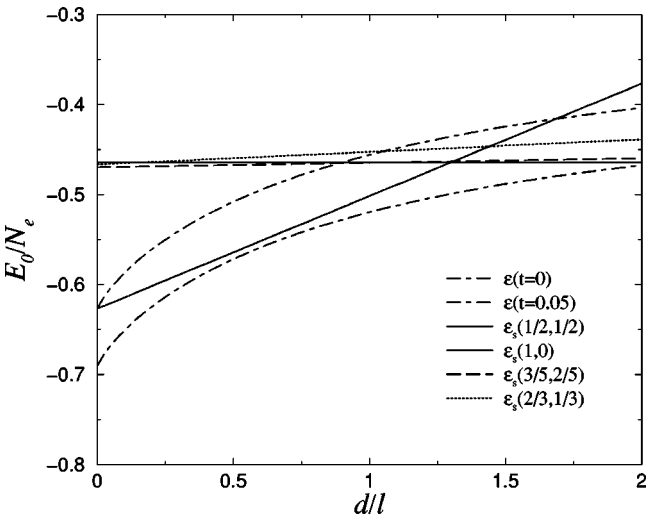

FIG. 2. Ground-state energies per particle ε in units of $e^2/4\pi\epsilon l$, for an $N_w\to\infty$ miniband quantum Hall state at $\nu=1/2$ versus the layer separation d/l. The estimated ground-state energies per particle ε_s for staggered states without correlations between the layers are also shown. Interlayer tunneling of order $t\sim 0.05e^2/4\pi\epsilon l$ is required to stabilize the miniband state.

For $\nu=1/2$, we consider an $n=1$ independent-layer staggered state with $\delta=(1/2)/(2k-1)$ for $k=1,2,3,\infty$, corresponding to a superlattice state with a two-layer unit cell with filling factors (1,0), (2/3,1/3), (2/5,3/5), and (1/2,1/2), respectively. The energy per particle for these states is plotted in Fig. 2, along with the energy per particle for the miniband state at $t=0$ and $t=0.05$ (in units of $e^2/4\pi\epsilon l$).

Figure 2 shows the energy per particle at $\nu=1/2$ for miniband states (dashed-dot curves) at $t=0$ (upper) and $t=0.05$ (lower), for $n=1$ staggered independent-layer states consisting of pairs of layers with alternating filling factors (1,0) (sloped solid line), (2/3,1/3) (dotted line), and (3/5,2/5) (dashed line), and for the $n=0$ independent-layer state with $\nu=1/2$ (horizontal solid line). Note that for $t=0$, the $n=1$ staggered state (1,0) has the lowest energy for $d/l<1.3$, after which the $n=0$ independent-layer state with $\nu=1/2$ has the lowest energy. Thus, the miniband state requires finite interlayer tunneling to be stabilized: for $t=0.05$, the miniband energy is always lower than that of the staggered (1,0) state, and is lower than that of the $n=0$ independent-layer state for $d/l<2$. Note that finite interlayer tunneling produces a constant downward shift of the miniband energy at fixed ν and t, but is expected to have a much smaller effect (of order t^2, from perturbation theory) on the independent-layer states.

For $\nu=1/4$, we consider an $n=3$ independent-layer staggered state corresponding to a superlattice state with a four-layer unit cell with filling factors (1,0,0,0), and a two-layer unit cell with filling factors (1/2,0). The energies per particle ε_s for these states are plotted in Fig. 3, along with the energies per particle ε for the miniband states at $t=0$ and $t=0.03$ (in units of $e^2/4\pi\epsilon l$).

Figure 3 shows the energy per particle at $\nu=1/4$ for miniband states (dashed-dot curves) at $t=0$ (upper) and $t=0.03$ (lower), for an $n=3$ staggered independent-layer states consisting of four layers with alternating filling factors (1,0,0,0)

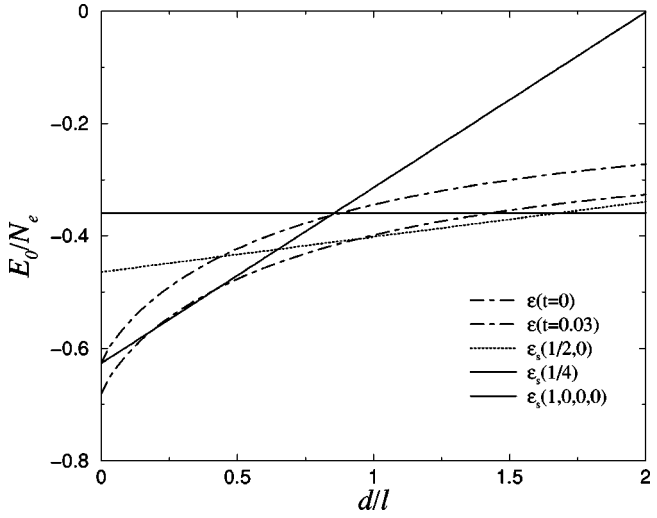

FIG. 3. Ground-state energies per particle ε in units of $e^2/4\pi\epsilon l$, for an $N_w\to\infty$ miniband quantum Hall state at $\nu=1/4$, versus the layer separation d/l. The estimated ground-state energies per particle ε_s for staggered states without correlations between the layers are also shown. Interlayer tunneling of order $t\sim 0.03e^2/4\pi\epsilon l$ is required to stabilize the miniband state.

(sloped solid line), for an $n=1$ staggered independent-layer states consisting of pairs of layers with alternating filling factors (1/2,0) (dotted line), and for the $n=0$ independent-layer state with $\nu=1/4$ (horizontal solid line). Note that for $t=0$, the $n=3$ staggered state (1,0,0,0) has the lowest energy for $d/l<0.6$, after which the $n=1$ staggered state (1/2,0) has a lower energy until $d/l\approx 1.6$, after which the $n=0$ independent-layer state with $\nu=1/4$ has the lowest energy. At $\nu=1/4$, the miniband state requires finite interlayer tunneling of order $t\sim 0.03$ to be stabilized. When the miniband state is stabilized by tunneling, the interlayer correlations of the miniband state are expected to have a strong effect on growth-direction transport, as we discuss in the following section.

IV. INTERLAYER TRANSPORT

Individual-particle transport between correlated electron layers in independent-layer quantum Hall states is strongly suppressed[38] because many-particle states containing the electron and hole created by a tunneling event have small overlaps with low-energy states. Miniband states can have a large conductivity because of the possibility of collective transport, much as the tunneling conductance in bilayer systems is increased by many orders of magnitude[4] when interlayer coherence is established.

The nature of disorder in multiple-quantum-well systems is important for transport considerations, and indeed for the formation of the miniband state. Unlike double-quantum-well systems that can be modulation doped from the sides, multiple-quantum-well systems must have layers of dopants between the quantum wells. These layers will create disorder within the quantum wells and in the interlayer tunneling amplitude. In the bilayer case, it is expected theoretically, and known experimentally, that interlayer correlations occur only in weakly disordered systems. The occurrence of the physics we propose will require, in all likelihood, special efforts to limit disorder due to modulation doping between the layers. One possible strategy is to place the interlayer dopants in a deep quantum well, creating carriers that can screen lateral disorder but which do not contribute to growth-direction transport.[39] The following discussion assumes that conditions of relatively modest disorder can be achieved.

To describe coherent transport, we separate the tunneling amplitude between the layers into a nonrandom part that is diagonal in Landau-level state indices, $-t$, and a fluctuating part, $\delta t_{X',X}$. In the absence of disorder and interactions, a constant tunneling amplitude would lead to a Bloch miniband with dispersion $\varepsilon_0(q)=-2t\cos(qd)$. In a semiclassical approximation, the effect of an electric field $\mathcal{E}$ on the spontaneous miniband state is to move it rigidly in the reduced-zone k-space at the rate $\dot{K}=-e\mathcal{E}/\hbar$. When δt and the disorder within the layers are ignored, the macroscopic current density carried by the miniband state is given by

$$j=\frac{e}{\hbar}\frac{\nu}{2\pi l^2 d}\frac{\partial\varepsilon_K}{\partial K}, \tag{10}$$

where ε_K is the energy per particle for the miniband state when the center of the occupied region is located at K in k-space. It is easy to show that only the band energy $\varepsilon_0(q)$ contributes to the K-dependence of $E_{\rm tot}$. Hence

$$j=\frac{e}{\hbar}\frac{\nu}{2\pi l^2}\frac{\sin(\pi\nu)}{\pi\nu}2t\sin(eVT/\hbar), \tag{11}$$

where $V=\mathcal{E}d$ is the interlayer voltage difference, and T in Eq. (11) denotes time. These time-dependent oscillating currents are just the Bloch oscillations[40] that are expected to occur for noninteracting electrons in a low-disorder limit that has never been approached in any degenerate electron system. Related effects have, however, been seen in multiple quantum wells with optically-excited carriers.[41] The role of strong interactions and miniband formation in the quantum Hall regime is to allow this physics to appear in samples where the transport at zero magnetic field is incoherent.

That interactions support the robustness of the disorder-free quasiparticle bands can be seen in Fig. 4, where we plot the quasiparticle bands for the maximum-current state with $K=\pi/2d$, for $t=0.05e^2/4\pi\epsilon l$ and $\nu=1/2$. The HF eigenvalues are given by

$$\varepsilon(q)=-2t\cos(qd)-\frac{1}{N_w}\sum_p f(q-p)X(p), \tag{12}$$

where $f(q)=\langle c^\dagger_{qX}c_{qX}\rangle$ is the wave-vector occupation function, and $X(p)$ is the HF self-energy

$$X(p)=\frac{e^2}{4\pi\epsilon l}\int_0^\infty dx e^{-x^2/2}\left[\frac{\sinh(xd/l)}{\cosh(xd/l)-\cos(pd)}\right]. \tag{13}$$

Equation (12) shows that if there were no interlayer tunneling, the quasiparticle bands would shift rigidly with K. For

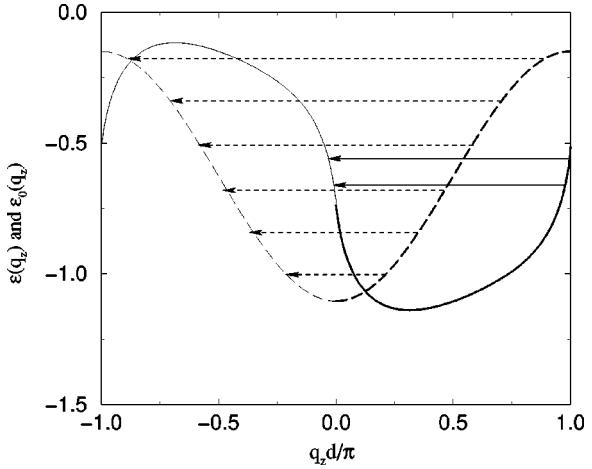

FIG. 4. Bare single-particle dispersion $\varepsilon_0(q) = -2t\cos(qd)$ for $t = 0.24e^2/4\pi\epsilon l$ (dashed), and the HF dispersion $\varepsilon(q)$ for $\nu = 1/2$ and $t = 0.05e^2/4\pi\epsilon l$ (solid). The dark portions of the curves, on the right-hand side, indicate the occupied states, whose momenta we have shifted to the right by $K = \pi/2d$ to produce a current-carrying state. The miniband states shift to the right with K, and have a much smaller phase space for scattering.

noninteracting electrons, shifting the occupied momenta by $K = \pi/2d$ would open up an enormous phase space for scattering (dashed arrows of Fig. 4), because for every occupied state, there would be an unoccupied state of opposite momentum. Compared to the interaction-free case, very few of the occupied states in the nonequilibrium finite-K state are degenerate with the unoccupied quasiparticle states (solid arrows of Fig. 4). Scattering is further reduced because the available scattering states are all near k_F, where the density of states is greatly suppressed, even for the maximum current-carrying state. The system is *not* a superconductor because the quasiparticles in the current-carrying state are *not* in equilibrium; nevertheless, the phase space for current relaxation by disorder scattering is immensely reduced.

The basic physics that we believe will lead to suppressed current relaxation in miniband quantum Hall states is explained in the above paragraph. The linear conductivity in the growth direction can be evaluated more formally, following the lines of a recent calculation of bilayer interlayer conductance.[42] When applied to superlattices, these considerations imply that the conductance is approximately that of the auxiliary single-particle system with interaction-enhanced growth-direction quasiparticle velocities and suppressed quasiparticle scattering rates. This suggests that the growth-direction conductivity will be enhanced by orders of magnitude upon entering the miniband state.

V. CONCLUSION

We have investigated the stability of uniform-density interlayer-coherent quantum Hall miniband states, comparing their energies to those of states without interlayer correlations, including staggered states with diagonal order. Our calculations demonstrate that SILC does not occur in superlattices within the HF approximation, although coherent miniband states with strongly-enhanced band widths can be stabilized by weak interlayer tunneling. For small interlayer separations, independent-layer staggered states with diagonal order are preferred because they have larger exchange energies and because the Hartree energy cost of inhomogeneity is not large: the Hartree energy per volume for a staggered state can be four times lower in a superlattice as compared to a bilayer. The superlattice case differs in this regard from that of a bilayer, which does exhibit SILC. At large layer separations, intralayer correlations become more important energetically than interlayer correlations, and independent-layer states of uniform density and constant filling factor are favored, just as for bilayers.

The quantum Hall miniband state, when formed (e.g., with the help of interlayer tunneling), is expected to have a strongly enhanced growth-direction conductivity. This is because, in the miniband state as in the SILC bilayer state, electrons in different layers are arranged so as to accommodate tunneling electrons from other layers. This greatly increased growth-direction conductivity may be the most definitive experimental signature of the superlattice miniband state. We emphasize that the tunneling conductance we describe is strongly enhanced by interlayer exchange and correlations over the value expected by single-particle tunneling alone.

We also suggested that a quantum Hall miniband state would constitute a promising candidate for a Bloch oscillator, without the need for optically excited carriers. The period of Bloch oscillations is $T_B = 2\pi\hbar/(e\mathcal{E}d)$ for an applied dc electric field $\mathcal{E}$,[40] where d is the distance between the layers. For typical samples at room temperature, the scattering relaxation time is about 10^9 times smaller than the Bloch period T_B, which precludes the possibility of observing Bloch oscillations. Quantum Hall miniband states are expected to have greatly reduced scattering rates because of their many-body interlayer correlations.

We caution that our proposals must be regarded as uncertain, since they are based on single-Slater-determinant variational wave functions. For example, interlayer correlations can be established by quantum fluctuations not included in this HF theory, without breaking any symmetries. Additionally, the fact that interlayer correlations establish a charge gap in bilayer systems, but not in multiple-quantum-well systems, may make mean-field-theory considerations less reliable in the present case. We therefore feel that the true nature of the ground state can only be established experimentally and, with this in mind, have argued that enhanced growth-direction conductivity due to collective transport is a reliable signature of spontaneous or interaction-enhanced interlayer coherence.

Aside from a large miniband-enhanced growth-direction conductivity and the possibility of producing Bloch oscillations, there may also be other many-body effects that are analogous to those proposed for bilayer SILC systems.[3] These includes novel effects produced by in-plane magnetic fields, enhanced Coulomb drag at zero temperature, and the existence of charged topological excitations. The collective modes of miniband states are very different in character from those of a superlattice system with independent-layer states.[24–26] The collective-mode spectrum of the miniband

state could be explored by tunneling conductivity measurements made in a parallel magnetic field,[43] or by resonant inelastic light scattering.[44]

We also note that while finite tunneling is needed to stabilize the miniband state within the HF approximation, it is still an open question whether SILC becomes favorable when quantum fluctuations, which lower the miniband energy, are included. Finite thickness is also known to enhance SILC in bilayer systems.[9] It is also likely that the uniform-density (in the plane) staggered states that we have considered have a higher energy than states with nonuniform densities in the plane. For example, the (1,0) staggered state may break up into domains of $\nu=1$ and $\nu=0$ within a plane, so that the average filling factor of the plane remains $\nu=1/2$. This would reduce the Hartree energy without too large a sacrifice in exchange energy. Such a nonuniform state might also possess interlayer correlations at the boundaries between $\nu=1$ and $\nu=0$ regions. It is also possible that at some filling factors, in particular, those at which Laughlin-Jastrow states occur, other types of states with SILC could be realized, including states that possess both charge order and SILC. While this work is necessarily incomplete in exploring all of these possibilities, and unable to reach definitive conclusions concerning the possibilities that have been explored, it does demonstrate that the interaction physics of multiple-quantum-well systems at fractional filling factors per layer will be even richer than that of single-layer systems, if systems with sufficiently weak disorder can be fabricated.

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