

Dephasing time of disordered two-dimensional electron gas in modulated magnetic fields

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The dephasing time of disordered two-dimensional electron gas in modulated magnetic field $\mathbf{H} = \{0, 0, H/\cosh^2[(x-x_0)/\delta]\}$ is studied. In the weak inhomogeneity limit where δ is much larger than the linear size of the sample, τ_ϕ^{-1} is proportional to H . In the strong inhomogeneity limit, it is shown that the dependence is quadratic, $\tau_\phi^{-1} = D(e/\hbar c)^2 H^2 \delta^2$. In the intermediate regime, a crossover between these two limits occurs at $H_c = (\hbar c/4e)\delta^{-2}$. It is demonstrated that the origin of the dependence of τ_ϕ on H lies in the nature of corresponding single-particle motion. A semiclassical Monte Carlo algorithm is developed to study the dephasing time, which is of a qualitative nature but efficient in uncovering the dependence of τ_ϕ on H for arbitrarily complicated magnetic-field modulation. Computer simulations support analytical results. The crossover from linear to quadratic dependence is then generalized to the situation with magnetic field modulated periodically in one direction with zero mean, and it is argued that this crossover can be expected for a large class of modulated magnetic fields.

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I. INTRODUCTION

Dephasing is one of the key elements in the study of quantum coherent phenomena in mesoscopic systems. Coupling with the environment suppresses the quantum interference of electrons. The phase-breaking time, or dephasing time, is the characteristic time beyond which the phase coherence is lost. Although a static magnetic field does not destroy all quantum effects, it may introduce a cutoff to the interference effect. Dephasing of disordered electron gas by coupling with a uniform magnetic field has been studied¹⁻³ in the early stage of weak-localization theory. In the absence of spin-orbit scattering, the magnetic field suppresses the weak-localization effect and leads to a positive magnetoconductance, which has been observed (for a review, see Ref. 4). The dephasing rate due to coupling with uniform field turns out to be proportional to the field amplitude. These results can also be established by qualitative considerations according to Khmel'nitskii.⁵

In recent years there has been increasing interest⁶ in hybrid semiconductor systems both for the fundamental understanding and the potentiality of enhancing the functionality of the devices. The disposition of superconducting⁷ or magnetic⁸ microstructures on the surface of heterostructures with a two-dimensional electron gas (2DEG) may produce inhomogeneous magnetic field which influences electron motion locally. Some interesting consequences of the modulated magnetic fields have already been reported. It was shown⁹ that the effect of electron-electron umklapp scattering can be observed in a 2DEG at a GaAs/Ga_{1-x}Al_xAs interface by imposing a spatially alternating magnetic field normal to the 2DEG plane. Kubrak *et al.*¹⁰ fabricated a few different types of hybrid ferromagnetic-semiconductor devices, which allowed them to study how these different modulated magnetic fields influence the transport properties of 2DEG. And magnetoresistance oscillations due to the internal Landau band structure of a 2D electron system in periodic magnetic field have been observed.¹¹ Theoretically, Peng¹² calculated the transport properties in a parabolic

channel exposed to a periodically modulated magnetic field. Gumbs and Zhang¹³ developed a magnetotransport theory for the magnetoconductivity of a square lattice in a periodically modulated magnetic field, and predicted some anomalies due to commensurability effects. Recently Matulis and Peeters¹⁴ studied semiclassical magnetoresistance in weakly modulated magnetic fields. They considered the case when the field is periodic in one dimension with zero mean. In the limit of small magnetic-field amplitude, it is shown that the contribution of the magnetic modulation to magnetoresistance increases as $B^{3/2}$ in the diffusive limit, while increases linearly in B in the ballistic limit.

The magnetoresistance in the diffusive limit is well described by standard weak-localization theory. It arises from the suppression of magnetic field on the cooperon propagator which represents the interference of time-reversal trajectories. Taking the magnetic field into consideration, the cooperon propagator satisfies¹

$$\left[-i\omega + D \left(-i\nabla - \frac{2e}{c} \mathbf{A}(\mathbf{r}) \right)^2 \right] C(\mathbf{r}, \mathbf{r}'; \omega) = \frac{1}{\tau} \delta(\mathbf{r} - \mathbf{r}'), \quad (1)$$

which may be viewed as an imaginary-time Schrödinger equation with parameters properly replaced. For the uniform field situation, the solution of the corresponding Schrödinger equation is the well-known Landau levels. The dephasing time was found to be¹⁻³

$$\frac{1}{\tau_\phi^u} = \frac{4DeH}{\hbar c}, \quad (2)$$

where the superscript u represents result for uniform field.

In this paper we first study the dephasing time of disordered 2DEG due to coupling with the following modulated magnetic field:

$$\mathbf{H} = \{0, 0, H/\cosh^2[(x-x_0)/\delta]\}. \quad (3)$$

The corresponding Schrödinger equation has been solved exactly by Hudák,¹⁵ where it was shown that a proper transformation converts the equation into a form solved earlier by Morse and Feshbach.¹⁶ Making use of this solution, we construct the cooperon propagator and calculate the dephasing time of a disordered 2DEG in this modulated field. We find in the weak inhomogeneity limit, $\delta \gg L$, where L is the linear size of the sample, τ_ϕ^{-1} is proportional to H , as in the uniform field case. In the strong inhomogeneity limit, it is shown that

$$\frac{1}{\tau_\phi} = D \left(\frac{e}{\hbar c} \right)^2 H^2 \delta^2. \quad (4)$$

In the intermediate regime, a crossover between these two limits is expected to occur at $H_c = (\hbar c/4e) \delta^{-2}$. The dephasing rate dependence on the magnetic-field amplitude is shown to be related to the nature of corresponding single-particle motion. Bound states lead to linear dependence, while nearly free motion results in quadratic. A semiclassical Monte Carlo algorithm is developed to study the dephasing time, which is of a qualitative nature but efficient in uncovering the dependence of τ_ϕ on H for arbitrarily complicated magnetic-field modulation. Computer simulations support analytical results. The considerations are then generalized to the situation where the modulated magnetic field is periodic in one direction with zero mean, and it is argued that this crossover between linear and quadratic dependencies can be expected for a large class of modulated magnetic fields.

Before going to quantitative calculations, let's see how qualitative considerations of Khmelnitskii⁵ can predict Eq. (4). Let the magnetic field be nonzero only in a stripe of width δ . Consider a large loop of area $D\tau$; we are interested in the flux piercing it. When δ is small, the effective area where the field is nonzero is $\delta\sqrt{D\tau}$. Requiring the phase change due to this loop to be of order 1, $(e/\hbar c)H\delta\sqrt{D\tau} \sim 1$, one immediately obtains Eq. (4). In subsequent quantitative calculations, we will see that understanding the origin of this dependence allows us to generalize the result to more complicated situations.

This paper is organized as follows. In Sec. II, the solution of the Schrödinger equation by Hudák¹⁵ is briefly outlined. The construction of the cooperon propagator and calculation of dephasing time is presented in Sec. III. Section IV contains a description of the numerical algorithm and the simulation results. The generalization to the situation with modulated magnetic field periodic in one direction is presented in Sec. V. Conclusions are given in Sec. VI.

II. SOLUTION OF THE SCHRÖDINGER EQUATION IN $\mathbf{H} = \{0, 0, H/\cosh^2[(x-x_0)/\delta]\}$

For completeness, the solution of the Schrödinger equation in the modulated magnetic field $\mathbf{H} = \{0, 0, H/\cosh^2[(x-x_0)/\delta]\}$ is briefly outlined in this section. Interested readers are referred to the original article of Hudák¹⁵ and the book of Morse and Feshbach.¹⁶

Under the Landau gauge

$$\mathbf{A} = \{0, H \delta \tanh[(x-x_0)/\delta], 0\}, \quad (5)$$

the Schrödinger equation can be written as

$$\frac{1}{2m} \left[-\hbar^2 \frac{d^2}{dx^2} + \left[p_y - \frac{eH\delta}{c} \tanh\left(\frac{x-x_0}{\delta}\right) \right]^2 \right] \chi(x) = E\chi(x), \quad (6)$$

where we have separated variables

$$\psi(x, y) = e^{(i/\hbar)p_y y} \chi(x). \quad (7)$$

This Schrödinger equation describes the motion of a particle in the potential $V(x)$,

$$V(x) = \frac{\hbar^2}{2m\delta^2} \left(2\pi \frac{\Phi}{\Phi_0} \right)^2 \left(\frac{p_y \delta/\hbar}{2\pi\Phi/\Phi_0} - \tanh\left(\frac{x-x_0}{\delta}\right) \right)^2, \quad (8)$$

where $\Phi_0 = hc/e$ is the flux quantum, and $\Phi = H\delta^2$ is a measure of the magnetic flux by the external field. Introducing

$$F = 2\pi \frac{\Phi}{\Phi_0},$$

$$P = -p_y \delta/\hbar,$$

$$z = (x-x_0)/\delta, \quad (9)$$

the potential may be written as

$$V(z) = \frac{\hbar^2 F^2}{2m\delta^2} \left(\frac{P}{F} + \tanh z \right)^2. \quad (10)$$

For $p_y \neq 0$, this potential is an asymmetric well, with different limiting values of $V(+\infty)$ and $V(-\infty)$. Hudák¹⁵ observed that the transformation

$$\nu \cosh^2 2\mu = P^2 + F^2,$$

$$\nu \sinh^2 2\mu = 2FP \quad (11)$$

can convert the Schrödinger equation into a form that was solved by Morse and Feshbach¹⁶ earlier:

$$\frac{d^2 \chi(z)}{dz^2} + \left(\varepsilon - \nu \cosh 2\mu - \nu \sinh 2\mu \tanh z + \nu \frac{\cosh^2 \mu}{\cosh^2 z} \right) \chi(z) = 0, \quad (12)$$

where $\varepsilon = 2m\delta^2 E/\hbar^2$.

One may distinguish situations between $|P| \leq F$ and $|P| > F$. In the former case, there is a discrete part in the energy spectrum as well as a continuous one, while the latter leads to only a continuous part. The solution¹⁵ is as follows. If

$$|P| < F(F - |P|)^2, \quad (13)$$

then the energy spectrum for the motion in the x direction contains a discrete part given by

$$E_n(p_y) = \frac{p_y^2}{2m} \left[1 - \frac{F^2}{\left(\sqrt{F^2 + \frac{1}{4}} - (n + \frac{1}{2})\right)^2} \right] + \frac{\hbar^2}{2m\delta^2} \left[F^2 - \left(\sqrt{F^2 + \frac{1}{4}} - (n + \frac{1}{2}) \right)^2 \right], \quad (14)$$

with $n = 0, 1, \dots, [n_{\max}]$, and

$$n_{\max} = \sqrt{F^2 + \frac{1}{4}} - \frac{1}{2} - \sqrt{|P|F}. \quad (15)$$

The corresponding eigenfunction is

$$\chi_n(z) = N_n \exp(-a_n z) (e^{-z} + e^z)^{-b_n} \times F(-n, 2\sqrt{F^2 + 1/4} - n, a_n + b_n + 1, e^{-z}/(e^{-z} + e^z)), \quad (16)$$

where

$$a_n = \frac{|P|F}{\sqrt{F^2 + \frac{1}{4}} - (n + \frac{1}{2})},$$

$$b_n = \sqrt{F^2 + \frac{1}{4}} - (n + \frac{1}{2}).$$

Here $F(\alpha, \beta, \gamma, \delta)$ denotes the hypergeometric function, and N_n is a normalization constant.

Thus there is discrete part if

$$|P| < P_d(F) = F + \frac{1}{2F} - \sqrt{1 + \frac{1}{4F^2}}. \quad (17)$$

The continuous spectrum may be divided into two parts. For

$$(F - |P|)^2 < \varepsilon < (F + |P|)^2, \quad (18)$$

the energy is given by

$$E(k, p_y) = \frac{1}{2m} \left[\left(\frac{\hbar k}{\delta} \right)^2 + [|p_y| - p(F)]^2 \right], \quad (19)$$

where k is the momentum in the x direction, and $p(F) = F\hbar/\delta$. The corresponding wave function is

$$\chi_{k,p_y}^A(z) = N_{k,p_y} \exp(-az) (e^{-z} + e^z)^{-b} \times F(b - \gamma + \frac{1}{2}, b + \gamma + \frac{1}{2}, K_+ + 1, e^{-z}/(e^{-z} + e^z)), \quad (20)$$

with

$$a = (k_+ + ik)/2, \quad b = (k_+ - ik)/2,$$

$$\gamma = \sqrt{F^2 + \frac{1}{4}}, \quad k_+ = \sqrt{4F|P| - k^2}.$$

This wave function vanishes exponentially for $x \rightarrow \infty$ when $p_y < 0$, and is bound from above when $x \rightarrow -\infty$. For $p_y > 0$, the opposite is true. The inequality (18) is equivalent to

$$0 < k^2 < 4|P|F. \quad (21)$$

If the parameters are such that the inequality (21) is replaced by

$$k^2 \geq 4|P|F, \quad (22)$$

then there is another type of continuous energy spectrum. The wave functions are nonvanishing but bound as $|x| \rightarrow \infty$. They describe overbarrier motion. The energy is still given by Eq. (19), but the corresponding wave functions are

$$\chi_{k,p_y}^B(z) = N_{k,p_y} \exp[iz(k_+ - k)/2] (e^{-z} + e^z)^{i(k+k_+)/2} \times F[-i(k+k_+) + 1 - 2\gamma/2, (-i(k+k_+) + 1 + 2\gamma)/2, 1 - ik_+, e^{-z}/(e^{-z} + e^z)], \quad (23)$$

where

$$\gamma = \sqrt{F^2 + \frac{1}{4}}, \quad k_+ = \sqrt{k^2 - 4F|P|}.$$

The above results are valid for $|P| \leq F$. When $|P| > F$, an analytical continuation can be performed.¹⁵ And it was found that there is no discrete part in the spectrum; the wave functions and energy are of the same form as that for $|P| \leq F$, but now valid for this region of parameters.

III. DEPHASING TIME

In the weak inhomogeneity limit $\delta \gg L$, where L is the linear size of the sample, and $F \gg 1$ for not very weak field. Then the potential well [given by Eq. (8)] is deep enough to host many discrete levels, which are reminiscent of the Landau levels. Recall that low-energy states dominate in the cooperon propagator, and the continuous part (with energies higher than the barrier height) gives a negligible contribution, thus only the discrete part of the spectrum is important in the cooperon. It can be shown that in this limit, inhomogeneity brings a correction of order $O(1/F)$ to the usual uniform field weak-localization magnetoresistance. And the dephasing time has the same form as the situation of a uniform field. The opposite limit, $F \ll 1$, which we will focus on, is more interesting.

Assuming the field described by Eq. (3) can be realized in experiment, let's show that for realistic parameters it is possible to have the discrete part of the spectrum absent in the strong inhomogeneity limit. For $\delta \sim 100$ nm, $F \sim H\delta^2/\Phi_0 \sim 10H$ if H is in Tesla, thus up to $H \sim 100$ G, one can take F as a small quantity. Then we check if the discrete part of the spectrum exists in this limit. The criterion for its existence is $P = p_y \delta/\hbar < F^3 < 10^{-3}$. Since $p_y > \hbar/L_\phi$, one has $P = p_y \delta/\hbar > \delta/L_\phi$. For a system with $L_\phi < 10^4$ nm, this gives $P = p_y \delta/\hbar > 10^2$ nm/10⁴ nm $\sim 10^{-2}$. Thus the inequality for the existence of discrete levels does not hold in this situation.

One therefore concludes there is no discrete part in the spectrum. In this limit,

$$E = \frac{\hbar^2}{2m\delta^2} [p_x^2 + (|p_y| - F)^2], \quad (24)$$

and the cooperon propagator is

$$C(r, r'; \omega) = \left(\frac{\hbar}{\delta} \right)^2 \int dp_x dp_y \frac{\psi_{p_x, p_y}^*(x, y) \psi_{p_x, p_y}(x', y')}{-i\omega\tau + \frac{D\tau}{\delta^2} [p_x^2 + (|p_y| - F)^2]}, \quad (25)$$

where the integral should be done under the constraint

$$\frac{D\tau}{\delta^2} [p_x^2 + (|p_y| - F)^2] \ll 1, \quad (26)$$

or

$$p_x^2 + (|p_y| - F)^2 \ll q^2, \quad q^2 = \frac{\delta^2}{D\tau}, \quad (27)$$

as it is a condition for perturbation theory.

Before proceeding, we need to discuss the boundary condition. If we use the wave functions Eqs. (20) and (23) in their present form, then the cooperon propagator constructed naively in Eq. (25) does not satisfy the zero amplitude boundary condition

$$C(\pm L, y) = C(x, \pm L) = 0. \quad (28)$$

The boundary condition in the y direction can be taken care of in the same way as in the uniform field case, where the plane-wave wave functions are replaced by their linear combinations, and p_y takes only some discrete allowed values. The boundary condition in the x direction can be dealt with in the same manner. We note that from Eq. (19) the eigenstates with k and $-k$ are degenerate, thus a linear combination of them is also an eigenstate. The new eigenstate can be constructed as

$$\psi_{p_x, p_y}^{A,B}(x) = \alpha \chi_{p_x, p_y}^{A,B} + \beta \chi_{-p_x, p_y}^{A,B}, \quad (29)$$

which are required to satisfy the boundary condition

$$\psi_{p_x, p_y}^{A,B}(L) = \psi_{p_x, p_y}^{A,B}(-L) = 0. \quad (30)$$

As in the case of uniform field, only some discrete p_x will be allowed. Thus the continuous part of the spectrum becomes again discrete in this boundary condition. The integration over momentum will be replaced by summation over these discrete allowed values. We note that the dispersion relation is not altered in the construction, Eq. (29).

Introducing the dimensionless momenta $q_x = p_x/F$, $q_y = p_y/F$, $q' = q/F$, and $P' = P/F$, and going to a time domain, we find

$$\begin{aligned} C(r, r'; t, t') &= \frac{\hbar^2 F^2}{\delta^2 \tau} \sum_{q_x^2 + (|q_y| - 1)^2 \leq q'^2} \psi_{q_x, q_y}^*(x, y) \psi_{q_x, q_y}(x', y') \exp \left\{ -\frac{DF^2}{\delta^2} [q_x^2 + (|q_y| - 1)^2] (t' - t) \right\} \\ &= \frac{\hbar^2 F^2}{\delta^2 \tau} \left\{ \sum_{q_x^2 < 4|P'|} \psi_{q_x, q_y}^{A*}(x, y) \psi_{q_x, q_y}^A(x', y') + \sum_{q_x^2 \geq 4|P'|} \psi_{q_x, q_y}^{B*}(x, y) \psi_{q_x, q_y}^B(x', y') \right\}_{q_x^2 + (|q_y| - 1)^2 \leq q'^2} \\ &\quad \times \exp \left\{ -\frac{DF^2}{\delta^2} [q_x^2 + (|q_y| - 1)^2] (t' - t) \right\}. \end{aligned} \quad (31)$$

We see that the magnetic field results in a characteristic time scale beyond which the cooperon propagator or the interference effect is no longer important. Therefore

$$\frac{1}{\tau_\phi} = \frac{DF^2}{\delta^2}, \quad (32)$$

which leads to Eq. (4).

Thus the dephasing rate depends quadratically on the field amplitude in the strong inhomogeneity limit. In this calculation, we see that the dependence of $1/\tau_\phi$ on H is a result of the nature of corresponding single-particle motion. From Eq. (8), when F is large, the potential well is deep enough to

accommodate many bound states with discrete levels (which are reminiscent of Landau levels). The dominant contribution to the cooperon is from these low-lying levels which results in linear dependence. In contrast, if F is small, the potential well can be so shallow that a discrete level does not appear, and the particle executes overbarrier motion which is nearly free with a continuous spectrum. When this part of the spectrum is dominant in the cooperon, the dephasing rate is a quadratic function of the field amplitude. The quadratic dependence implies that the electrons are more slowly dephased compared with the uniform field case, where the dependence is linear. Physically this is because for a given diffusion time, electrons in this regime can visit a larger area than the constrained motion. Since the magnetic field is non-

zero only in a limited region, the phase change accumulated during this time is smaller than that of the uniform case. So the electron has to wander for a longer time before it gets dephased. Therefore the constrained motion of the single particle leads to a linear dephasing rate dependence on field amplitude, and nearly free overbarrier motion to quadratic.

Equation (4) is obtained in the limit of small F , which may be realized with tiny δ and small H . More realistic for experimental observation is the situation with moderate δ . In this case, one expects a crossover from linear (uniform limit) to quadratic (inhomogeneity limit) dependence. This point can be illustrated in the following way. Consider the electron which has diffused for a time τ . If $D\tau < \delta^2$, the inhomogeneity is not noticed by the electron. If during this time the electron has already been dephased, $\tau > \tau_\phi^\mu$, where τ_ϕ^μ is the dephasing time in the uniform field [given by Eq. (2)], then the inhomogeneity is not important at all. When τ is beyond this time scale, the inhomogeneity effect enters. Thus, for a given δ , there is a crossover field implied by

$$D\tau_\phi^\mu = \delta^2, \quad (33)$$

which yields

$$H_c = \frac{\hbar c}{4e\delta^2}. \quad (34)$$

This is equivalent to saying that the inhomogeneity effect enters when $F \sim 1$.

On the other hand, if we can adjust the width δ , then for a given H , there is a crossover length scale δ_c ,

$$\delta_c = \sqrt{\frac{\hbar c}{4eH}}, \quad (35)$$

which is the magnetic length for a uniform field.

Before proceeding, let's mention that from Eq. (31) one can see that the magnetoconductance is proportional to $H^2\delta^2$ in this small F limit. In the large F limit, a logarithmic dependence is expected as in the uniform field case. These can also be established by the qualitative argument of Khmel'nitskii.⁵

IV. MONTE CARLO SIMULATION

A Monte Carlo algorithm has been developed to simulate the dephasing process. In order to be consistent with that implied by Eq. (1), the simulations will be semiclassical in nature. Trajectories will be used and the only quantum-mechanical effect will be in the phases. In this approach, a particle performs a random walk in a square lattice. The value of the perpendicular field is assigned to a dual lattice. We trace all the closed loops that are formed. Once a loop is formed, all the inner points are picked up (for technical details, see Ref. 17), and we calculate and record the phase change ϕ_i due to this loop. Then the trajectory of this loop is erased, and the particle continues the random walk. The phase accumulated this way is

$$\phi = \sum_i \phi_i, \quad (36)$$

whose average is zero since the loop has an equal possibility to be clockwise or counterclockwise. Then

$$\langle \delta\phi^2 \rangle = \langle \phi^2 \rangle = \sum_i \phi_i^2. \quad (37)$$

The random walk stops as soon as

$$\langle \delta\phi^2 \rangle \geq 1, \quad (38)$$

and we specify the time (the total number of random-walk steps) that has been spent to reach this as the dephasing time. An ensemble of such walks is performed and the dephasing time is averaged. In the simulations, the mean free path l and the flux quantum Φ_0 have been set to 1, so that

$$\phi_i = \sum_j H_j, \quad (39)$$

where j runs over all the inner points in the dual lattice of loop i .

Several features make this simulation of a qualitative nature. First, the criterion, Eq. (38), is a qualitative one. Second, in the simulation, the impurities are assumed to be on regular lattice sites, with the mean free path as the lattice constant. Third, sometimes we choose the field that is not necessarily the original one, but is qualitatively the same. However, essential physics is not lost despite its qualitative nature, and this simulation is very efficient in uncovering *dependence* of the dephasing time on the field amplitude, which is particularly appropriate for situations where the field modulation is complicated so that it is difficult to make progress with analytical approaches.

As a check of the algorithm, we have simulated the uniform field case. Excellent linearity in the plot of the dephasing rate against H is observed, in agreement with Eq. (2). For comparison with the modulated field we studied in the previous section, the simulation is performed with the following field:

$$H(x) = \begin{cases} H, & |x| \leq \delta, \\ 0, & \text{otherwise,} \end{cases} \quad (40)$$

which is qualitatively the same as Eq. (1). The results plotted in Fig. 1 show a clear linear dependence for large δ , and a crossover from linear to quadratic dependence for moderate δ . The boundary is set at $L = 5000l$ in this simulation. Trajectories which touch the boundary are excluded. The results are obtained by averaging over 10^4 random walks. When the inhomogeneity effect enters, fluctuations become significant in the result. This is because there are extreme trajectories that wander for a long time in the zero-field region. However, by recording not only the average but also the standard deviation, we are able to get an estimation 2.0 ± 0.1 for the exponent in the inhomogeneous limit, in agreement with Eq. (4). Thus the numerics support analytical results.

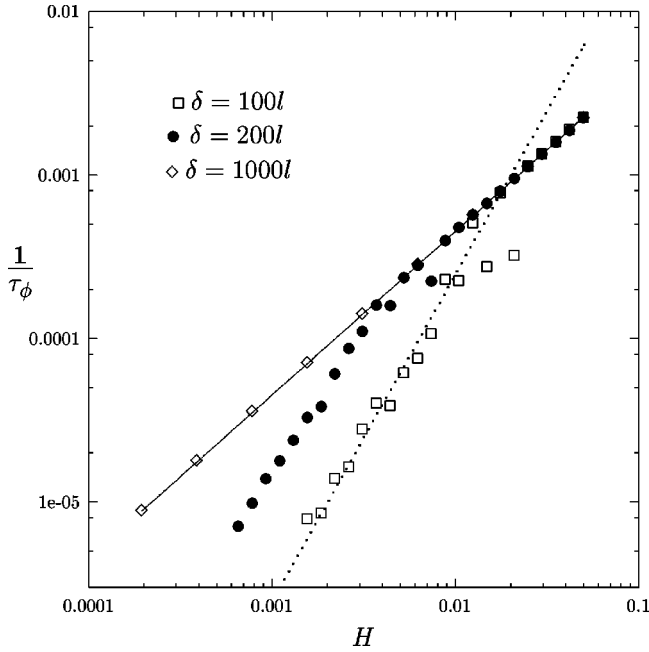


FIG. 1. Simulation results for the dephasing time in the modulated magnetic field, Eq. (40). The data are obtained by averaging over 10^4 random walks. The three curves are for $\delta=1000l$, $200l$, and $100l$, respectively. The boundary is $L=5000l$. The straight line is a fit to linear dependence, the dotted line that of quadratic dependence.

V. GENERALIZATION TO MODULATED MAGNETIC FIELD, PERIODIC IN ONE DIRECTION WITH ZERO MEAN

The qualitative feature of the results, linear dependence in the weak inhomogeneity limit and quadratic in the strong inhomogeneity limit, may be more general. Consider the situation with magnetic field modulated periodically in one direction with zero mean. While the weak inhomogeneity limit is easy to understand, we shall focus on the opposite limit to discuss possible quadratic dependence of $1/\tau_\phi$ on H .

Assume the magnetic-field profile is such that nearest-neighbor stripes of width δ have the same magnitude but opposite sign. Consider the phase change of a closed loop due to this magnetic field. Since the field is periodic with zero mean, what is important for the magnetic flux is the ratio of the linear size of the loop with the width of one stripe, $\sqrt{D}\tau_\phi/\delta$. If this ratio is an even integer, then the net flux is zero. If it is an odd integer, then the flux is equal to that piercing through a single stripe. In general this ratio may fluctuate, however it is clear that the average flux is proportional to $\sqrt{D}\tau_\phi H \delta$. Demanding the phase change due to this flux to be 1, one obtains Eq. (4), with probably an additional numerical coefficient.

For a general magnetic field periodic in one direction, the corresponding Schrödinger equation is often difficult to solve analytically. For example, for $\mathbf{H}=[0,0,H \sin(x/\delta)]$, one can choose $\mathbf{A}=[0,-H\delta \cos(x/\delta),0]$. Then the resulting Schrödinger equation is a Whittaker-Hill equation whose solution may be reduced to three-term recurrence relations,¹⁸ but an explicit analytical result for the spectrum is not known.

However some insight can be gained by analyzing the structure of the energy spectrum. In general, if the magnetic field is $[0,0,H(x)]$, in the Landau gauge $\mathbf{A}=[0,A(x),0]$. Separating variables in the wave function, the electron motion in the x direction is described by a Schrödinger equation with potential

$$V(x,p_y)=\left[p_y-\frac{e}{c}A(x)\right]^2, \quad (41)$$

which is also a periodic function. For given p_y , this potential is bound from above as that described by Eq. (8). The profile of this potential is a series of potential wells joining each other. The energy spectra of electrons in a family of such modulated magnetic fields are calculated by Ibrahim and Peeters.¹⁹ For electron energy less than the barrier height, the wave functions in neighboring wells overlap and spread. So the discrete levels in single wells now form minibands due to the periodic structure. When H and δ are large, the barrier can be quite high, and states in these minibands dominate in the cooperon propagator. According to the experience in dealing with the previous case, the dephasing rate is expected to be a linear function of H . When H and δ are small, however, the potential well is not deep enough to support these minibands, and the dominant contribution is expected from nearly free overbarrier motion, which leads to quadratic dependence.

To examine these considerations, computer simulations are performed for the following field:

$$H(x)=\begin{cases} H, & 2n\delta \leq x < (2n+1)\delta, \\ -H, & (2n-1)\delta \leq x < 2n\delta. \end{cases} \quad (42)$$

The results are shown in Fig. 2. The curves show much resemblance to that in Fig. 1, and a crossover from linear to quadratic dependence is evident. There are also some differences which will be noted here. First, for a given δ , the crossover field H_c is smaller in the field (42) than in the field in Eq. (40), which suggests when H is small, an electron in the field (42) is more quickly dephased than in the field (40). This may look strange at first sight, since the field (42) has zero mean; one may expect that the electrons in this field are more slowly dephased. However, this result can be understood by noticing what enters is the variance of the phase instead of the average, and the field (42) is nonzero everywhere in the plane. Another difference is that in the field (42) there is broader crossover region. This is associated with the fact that, in the periodic situation, states with energy lower than the barrier height are not really bound states. They share some feature of the plane waves according to Bloch's theorem. As discussed previously, this feature has the tendency to lead to quadratic dependence. When δ is moderate, the barrier is not very high for many values of p_y . Then these states can be dominant in the cooperon, resulting in a broad crossover region.

Numerical experiments are also performed for situations with some other magnetic-field modulations, for example, that with circular symmetry, or periodic in both directions. Similar crossover behaviors have been observed. These simulations as well as the qualitative understanding of the

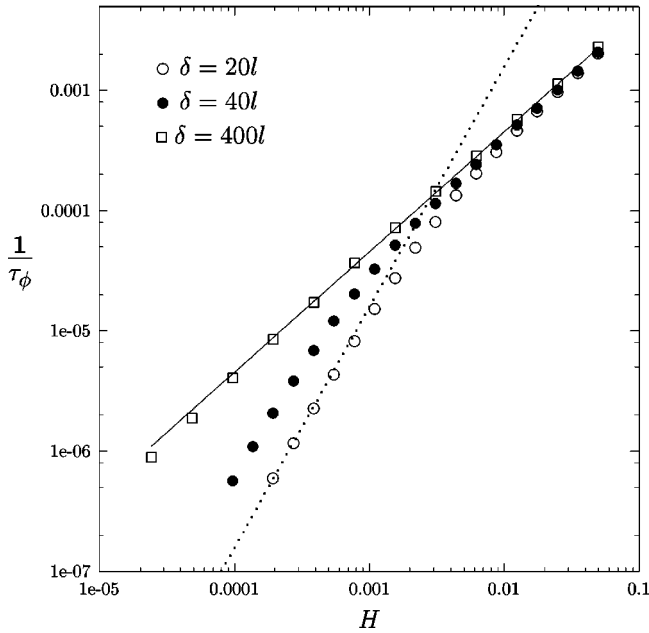


FIG. 2. Simulation results for the dephasing time in the modulated magnetic field periodic in one direction given by Eq. (42). The data are obtained by averaging over an ensemble of 10^3 random walks. The three curves are for $\delta=400l$, $40l$, and $20l$, respectively. The straight line is a fit to linear dependence, the dotted line that of quadratic dependence.

origin of the dependencies suggest that the crossover from linear to quadratic dependence in the dephasing rate on field amplitude can be quite general. For situations where the corresponding single-particle potential is bound from above, the particle motion can exhibit both bound and free natures, leading to a crossover from linear to quadratic dependence.

VI. CONCLUSIONS

We have studied the dephasing time of disordered two-dimensional electron gas in modulated magnetic field $\mathbf{H} = \{0, 0, H/\cosh^2[(x-x_0)/\delta]\}$. It is shown that in the weak inhomogeneity limit, $\delta \gg L$, where L is the linear size of the sample, τ_ϕ^{-1} is proportional to H . This happens when the bound states with a discrete spectrum of the corresponding Schrödinger equation dominate in the cooperon propagator. While in the strong inhomogeneity limit, the dependence is quadratic, $\tau_\phi^{-1} = D(e/\hbar c)^2 H^2 \delta^2$. In this case, the nearly free overbarrier motion gives the dominant contribution to the cooperon. In the intermediate regime, a crossover between these two limiting situations occurs at $H_c = (\hbar c/4e) \delta^{-2}$. A semiclassical Monte Carlo algorithm has been developed to study the dephasing time, which is of a qualitative nature but efficient in uncovering the dependence of τ_ϕ on H for arbitrarily complicated magnetic-field modulations. Computer simulations support analytical results. These considerations are generalized to the situation with magnetic field modulated periodically in one direction with zero mean, where a similar crossover is observed. We believe this crossover between linear and quadratic dependencies can be expected for a large class of modulated magnetic fields where the corresponding single particle potential is bound from above, so that the motion exhibits both bound and free natures depending on parameters.

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