

Theory of double magnetophonon resonance in a two-dimensional electron gas in a tilted magnetic field

V. V. Afonin,^{1,2} V. L. Gurevich,^{1,2} and R. Laiho¹

¹*Wihuri Physical Laboratory, Department of Physics, University of Turku, FIN-20014 Turku, Finland*

²*Solid State Physics Division, A. F. Ioffe Institute, 194021 Saint Petersburg, Russia*

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A theory of double magnetophonon resonance (MPR) in quantum wells in magnetic field is developed. The magnetic field is assumed to be tilted at an angle θ to the perpendicular to the plane of quantum well. The resonance is due to the resonant interaction of two-dimensional conduction electrons with the longitudinal optic phonons. The electrons are assumed to be nondegenerate. The θ dependence of MPR maxima is investigated. The existence of a double resonance, i.e., two resonant peaks for each value of \mathcal{N} (the number of resonance) and sharpness of the θ dependence is explained by the screening in conjunction with the combined role of the phonon and electron damping and variation of electron concentration in the well with magnetic field.

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I. INTRODUCTION

Magnetophonon resonance (MPR) in semiconductors is an internal resonance that is reached when the limiting frequency of a longitudinal optic phonon equals the cyclotron frequency of an electron Ω times some small integer \mathcal{N} (for instance, Ref. 1). Since its theoretical prediction² and subsequent experimental observation^{3,4} in three-dimensional (3D) semiconductor structures MPR, along with cyclotron resonance, has become one of the main instruments of semiconducting compound spectroscopy.

The advances in semiconductor nanofabrication in recent years have made available materials of great crystalline perfection and purity. The electrical conduction and some other transport phenomena in such nanoscale structures has been a focus of numerous investigations, both theoretical and experimental, with a number of important discoveries. In particular, the discovery of MPR in the quantum wells took place in the pioneering paper by Tsui, Englert, Cho, and Gossard.⁵ After this publication a number of papers has appeared where various aspects of this physical phenomenon have been investigated. The most detailed experimental investigation has been done by Nicholas with co-workers (see Ref. 6, and references therein). It has been shown that the 2D MPR qualitatively differs from the same phenomenon in the 3D structures. As first steps in developing theory of 2D MPR in the perpendicular magnetic field we can quote the theoretical papers.^{7,8} They consider the MPR within the perturbation theory approximation with regard of collisional broadening of the electron state. A theory in tilted magnetic field is developed in Ref. 9 where transitions to a higher band of spatial quantization are taken into consideration. In our opinion, these theories do not provide detailed interpretation of the existing experimental data. Our purpose is to give an interpretation of experimental findings,⁶ such as the double resonance, by simultaneously taking into account the screening of the phonon potential by 2D conduction electrons as well as the phonon and electron damping. For such a program, as we have indicated in Ref. 10, the lowest approximation of the perturbation theory is not sufficient. As is shown in the

present paper, the relative role of screening is determined not by temperature (as has been suggested in Ref. 11) but by the interplay between the screening and the electron and phonon collisional damping that is usually weakly temperature dependent.

There are two main groups of the MPR experiments in quantum wells. The first group deals with the MPR in the perpendicular (to the plane of 2DEG) magnetic field. The main features of the findings in this case are (i) the fact that the resonance is determined by the transverse optic frequency ω_t (rather than the longitudinal frequency ω_l) and (ii) a rather narrow interval of electron concentrations where the MPR is observable. The second group concerns with the experiments in a magnetic field tilted at an angle θ to the perpendicular. Its characteristic feature may be called a *double resonance*. For small values of θ the MPR is determined by ω_t . Then, for somewhat larger values of θ its amplitude sharply goes down within a narrow angular interval typically of the order of 10° . For even bigger values of θ there is another maximum, this time determined by ω_l .¹² These two types of resonance may be called the ω_t and ω_l resonances as their positions are determined by the frequencies ω_t and ω_l , respectively.

In Ref. 10 we have given an interpretation for the first group of experiments. In the present paper we offer an interpretation of the second group. It is shown that the angular and concentration dependencies of the MPR amplitudes are deeply interrelated.

The magnetic field \mathbf{B} is assumed to be in the (y, z) plane, the z axis being perpendicular to the 2DEG, while the external electric field is oriented along the y axis. The ρ_{yy} ($= \rho_{xx}$) component of the resistivity tensor will be calculated. This is the transport coefficient expressed through the 2D conductivity $\sigma_{\mu\nu}$ (averaged over the width of the well) as

$$\rho_{yy} = \sigma_{xx} / (\sigma_{xy})^2.$$

As usual, it is assumed that $\sigma_{xy}^2 \gg \sigma_{xx}\sigma_{yy}$. We consider the situation where the well is so narrow that only one electron band of spatial quantization is filled. Hence one can assume that the z component of electron velocity vanishes. Then

$$\dot{p}_x = eE_x + \frac{eB}{c}v_y \cos \theta, \quad \dot{p}_y = eE_y - \frac{eB}{c}v_x \cos \theta, \quad (1.1)$$

where p_x, p_y and v_x, v_y are, respectively, the components of the electron quasimomentum and velocity. These classical equations illustrate the physics describing a 2D circular motion of an electron with the angular frequency

$$\Omega \cos \theta = \frac{eB}{mc} \cos \theta \quad (1.2)$$

where m is the effective mass, so that $p_{x,y} = mv_{x,y}$. It follows from Eq. (1.1) that

$$\sigma_{xy} = \frac{enc}{B \cos \theta}, \quad (1.3)$$

where e and n are the electron charge and concentration. It means that within one miniband approximation the magnetic field enters the problem only in the combination $B \cos \theta$.

II. GENERAL EQUATIONS

To calculate the x component of the dc current it is convenient to consider the motion of a center of Landau oscillator. The conductivity σ_{xx} averaged over the width of the well is given by (see Ref. 13)

$$\sigma_{xx} = \frac{e^2}{2k_B T S} \int_{-\infty}^{\infty} dt \langle \dot{X}(0) \dot{X}(t) \rangle, \quad (2.1)$$

where S is the area of the 2DEG, T is the temperature, X is the operator of coordinate of the center of Landau oscillator in the Heisenberg representation. According to Eq. (1.1), in the Schrödinger representation

$$X = -\frac{ic}{eB \cos \theta} \frac{\partial}{\partial y} + x$$

(cf. Ref. 14, Sec. 112). It commutes with the free electron Hamiltonian \mathcal{H} in magnetic field \mathbf{B} as well as with the operator of Coulomb electron-electron interaction. This is a consequence of the quasimomentum conservation in electron-electron collisions. Here $\langle \dot{X}(0) \dot{X}(t) \rangle$ is the ensemble averaged correlation function of velocities of the centers of Landau oscillators. In the present and the following sections we will usually assume $\hbar = 1, k_B = 1$ and will restore these symbols in the resulting formulas.

Now

$$\begin{aligned} \dot{X}(t) &= \sum_{\sigma} \int \psi^{\dagger}(\mathbf{r}, \sigma) i[\mathcal{H}, X] \psi(\mathbf{r}, \sigma) d^3 r \\ &= \sum_{\sigma} \frac{c}{eB \cos \theta} \int \psi^{\dagger}(\mathbf{r}, \sigma) \frac{\partial \hat{U}}{\partial y} \psi(\mathbf{r}, \sigma) d^3 r. \end{aligned} \quad (2.2)$$

The summation is over the spin variable. Here ψ is the operator of the electron wave function while $\hat{U}(t, \mathbf{r})$ is the operator of phonon field interacting with electrons. For the time being, we consider it as an external random field; later on we will average over all its realizations introducing the optic

phonons. The expression for σ_{xx} can be presented in such a form (we remind the reader that we calculate the conductivity averaged over the width of the well)

$$\begin{aligned} \sigma_{xx} &= \frac{e^2}{2TS} \left(\frac{c}{eB \cos \theta} \right)^2 \int_{-\infty}^{\infty} dt \int d^3 r \int d^3 r' \\ &\times \left\langle \hat{n}(0, \mathbf{r}') \frac{\partial}{\partial y'} \hat{U}(0, \mathbf{r}') \hat{n}(t, \mathbf{r}) \frac{\partial}{\partial y} \hat{U}(t, \mathbf{r}) \right\rangle, \end{aligned} \quad (2.3)$$

where $\hat{n}(t, \mathbf{r})$ is the electron density operator. Representing the ensemble average in Eq. (2.3) as a sum over the exact quantum states of the system (see Ref. 15, Sec. 36) we get¹⁶

$$\begin{aligned} \sigma_{xx} &= \frac{e^2}{2T} \left(\frac{c}{eB \cos \theta} \right)^2 \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int \frac{d^2 q}{(2\pi)^2} \int dz \int dz' \\ &\times \frac{q_y^2 N(\omega)}{1 - \exp(-\omega/T)} [D_R(-\omega; \mathbf{q}; z, z') \\ &- D_A(-\omega; \mathbf{q}; z, z')] [\Pi_R^{(3)}(\omega; \mathbf{q}; z', z) \\ &- \Pi_A^{(3)}(\omega; \mathbf{q}; z', z)] \end{aligned} \quad (2.4)$$

(see details of the derivation in Ref. 10). We made use of the quasimomentum conservation along the plane of the quantum well; \mathbf{q} is a 2D wave vector parallel to the plane of the well. $N(\omega)$ is the Bose function. $\Pi_R^{(3)}(\omega; \mathbf{q}; z', z)$ is the exact 3D electron polarization operator.

Now, $D_R(\omega; \mathbf{q}; z, z')$ is the phonon propagator with regard of the direct Coulomb electron-electron ($e-e$) interaction [see below Eq. (2.7)]. In the present paper we consider the magnetophonon resonant contribution to σ_{xx} . This means that the phonon contribution to the Green function D_R is determined by the optic phonons. Further on we will assume that one can neglect the difference between the lattice properties within and outside the well. This assumption should not affect the qualitative results of the theory. [Equation (2.4) permits us to consider also a more general (nonhomogeneous) case.] Without regard of $e-e$ interaction the phonon propagator has the form

$$D_{R,A}^{(0)}(\omega) = \frac{4\pi e^2}{(q^2 + k^2)\epsilon_c} \frac{\omega_l}{2} \left(\frac{1}{\omega - \omega_l \pm i\delta} - \frac{1}{\omega + \omega_l \pm i\delta} \right), \quad (2.5)$$

where k is the z component of the wave vector while ϵ_c is given by

$$\frac{1}{\epsilon_c} = \frac{1}{\epsilon_{\infty}} - \frac{1}{\epsilon_0}. \quad (2.6)$$

Here ϵ_0 and ϵ_{∞} are the dielectric susceptibilities for $\omega \rightarrow 0$ and $\omega \rightarrow \infty$, respectively. We have included the Frölich electron-phonon interaction¹⁷ into the definition of the zero-order phonon Green function.

When calculating the exact phonon propagator it will be necessary to insert along with the phonon lines $D_{R,A}^{(0)}$ the direct Coulomb interaction lines

$$V^{(C)}(\mathbf{q}, k) = \frac{4\pi e^2}{\varepsilon_\infty(q^2 + k^2)}. \quad (2.7)$$

One should, however, observe the following important point. Both ends of the exact phonon propagator should be ordinary phonon lines $D^{(0)}$ without Coulomb interaction lines. This is due to the fact that the operator X commutes with the electron-electron interaction operator [see Eq. (2.2)] as the latter conserves the electron quasimomentum.

Further we assume for the electrons a parabolic confining potential $m\omega_0^2 z^2/2$ with the following gauge for the vector potential $\mathbf{A} = (-By \cos \theta + Bz \sin \theta, 0, 0)$. It is also assumed that

$$\hbar\omega_0 \gg \hbar\Omega, k_B T \quad (2.8)$$

(where $\Omega = eB/mc$ is the cyclotron frequency). In other words, we assume the confining potential to be strong. The diagonalization of quadratic Hamiltonian is a well-known procedure (for instance, Ref. 18). We wish, however, to emphasize that the actual form of the confining potential is not essential for the present theory provided that $\hbar\omega_l$ is much smaller than the distance to the bottom of the second miniband [see below — Eq. (2.12)].

The energy of confined electron in the magnetic field defined by the vector potential \mathbf{A} is

$$\begin{aligned} \mathcal{U} = & \frac{1}{2} m \Omega^2 \cos^2 \theta (y - y_0)^2 - \frac{1}{2} m \Omega^2 \sin(2\theta) z (y - y_0) \\ & + \frac{1}{2} m (\omega_0^2 + \Omega^2 \sin^2 \theta) z^2, \end{aligned} \quad (2.9)$$

where $y_0 = -cp_x/eB \cos \theta$ while p_x is the electron quasimomentum component that is conserved. We will see in the Appendix that in the leading order in $(\Omega/\omega_0)^2$ one can retain in Eq. (2.9) only the terms describing the electron motion in the magnetic field $B \cos \theta$ perpendicular to the plane of the well (cf. Ref. 19). This can be visualized in the following way. One can obviously neglect the magnetic field correction to the confinement potential, i.e., $\Omega^2 \sin^2 \theta$ as compared to ω_0^2 . This means that the characteristic values of z are of the order of $l = \sqrt{\hbar/\omega_0 m}$. The mixed term, i.e., the second term on the right-hand side of Eq. (2.9) for the typical values of z can be also discarded provided that $\hbar\omega_0$ is the biggest energy in our problem. Therefore the 2D polarization operator (calculated in detail in the Appendix) for the case of Boltzmann statistics and a small gas parameter we are interested in has the same structure as the polarization operator of Ref. 10 in the perpendicular field B with the replacement $B \rightarrow B \cos \theta$:

$$\begin{aligned} \Pi_R = & -2n_s \exp \left[-\frac{(qa_B)^2 \coth \alpha}{2 \cos \theta} \right] \\ & \times \sum_{\mathcal{N}=-\infty}^{\infty} \frac{\sinh \mathcal{N} \alpha}{\omega - \mathcal{N} \Omega \cos \theta + i\delta} I_{\mathcal{N}} \left(\frac{q^2 a_B^2}{2 \cos \theta \sinh \alpha} \right). \end{aligned} \quad (2.10)$$

Here $I_{\mathcal{N}}$ is the modified Bessel function, $\alpha = \hbar \Omega \cos \theta / 2k_B T$, $a_B^2 = c\hbar/eB$, n_s is the 2D electron concentration. The polarization operator $\Pi^{(3)}$ of Eq. (2.4) differs from Π_R by the factor $\psi(z)^2 \psi(z')^2$ due to the electron motion along the z axis. Here $\psi(z)$ is the wave function of the lowest level of transverse quantization.

The $e-e$ interaction can take place both via exchange of a phonon and as a direct interaction described by $V^{(C)}$, Eq. (2.7). The sum of two interactions is

$$\begin{aligned} V_{R,A}(\mathbf{q}, k) &= \frac{4\pi e^2}{(q^2 + k^2) \varepsilon_{R,A}(\omega)}, \\ \varepsilon_{R,A}(\omega) &= \varepsilon_\infty \frac{\omega_l^2 - (\omega \pm i\delta)^2}{\omega_l^2 - (\omega \pm i\delta)^2}. \end{aligned} \quad (2.11)$$

Here $\omega_l^2 = \omega_l^2 (1 - \varepsilon_\infty / \varepsilon_c)$.

Equation (2.10) shows that the electron-electron interaction cannot be treated within the perturbation theory. Let the frequency ω in Eq. (2.10) be close to the frequency $\mathcal{N}\Omega \cos \theta$ so that only one term of the series is important. The higher orders of the perturbation theory (without regard of the electron damping Γ_e) give powers of an extra factor $1/(\omega - \mathcal{N}\Omega \cos \theta + i\delta)$. Therefore, the $e-e$ interaction must be a sum of chains of loop diagrams (see Ref. 10). Physically this means taking into account the screening of phonon polarization potential by the conduction electrons. Thus in 2D case in a resonance the screening can be very important. The reason as to why one does not need to take the screening into account in 3D case has been discussed in Ref. 10. The only point demanding some attention is taking into account the spatial nonhomogeneity. However, the procedure is essentially facilitated by the fact that $\Pi(z_1, z_2)$ depends on $\psi(z_1)^2 \psi(z_2)^2$ as factors. Then the index of the progression generated by the loops is proportional to

$$\int \int dz_1 dz_2 V_R(\mathbf{q}, z_1 - z_2) \psi^2(z_1) \psi^2(z_2) \quad (2.12)$$

that can be presented in the form

$$\int \frac{dk}{2\pi} V_R(\mathbf{q}, k) \left[\int \frac{ds}{2\pi} \psi_s \psi_{s+k} \right]^2,$$

where ψ_s is the Fourier component of $\psi(z)$. This result is valid for a well of any form so far as the the distance to the bottom of the second miniband remains much bigger than $\hbar\omega_l$. It is only necessary to insert into Eq. (2.12) the appropriate wave function $\psi(z)$.

For a quadratic confining potential we get a theory of 2D electrons with the interaction potential

$$V_R(\omega, \mathbf{q}) = \frac{4\pi e^2}{\varepsilon_R(\omega)} \int_{-\infty}^{\infty} \frac{dk}{2\pi} \frac{\exp(-k^2 l^2/2)}{k^2 + q^2}. \quad (2.13)$$

Further on we will also need the expression

$$V^{(C)}(\mathbf{q}) = \frac{4\pi e^2}{\varepsilon_\infty} \int_{-\infty}^{\infty} \frac{dk}{2\pi} \frac{\exp(-k^2 l^2/2)}{k^2 + q^2} \quad (2.14)$$

as well as the equation for the exact phonon Green function

$$D_R(\omega, \mathbf{q}) = \frac{V_R(\omega, \mathbf{q})}{1 + V_R(\omega, \mathbf{q})\Pi_R(\omega, \mathbf{q})}. \quad (2.15)$$

Now we take into account the aforementioned point that both ends of the chain in Eq. (2.4) should be ordinary phonon lines (without the Coulomb interaction). We have

$$D_R(\omega, \mathbf{q}) = D_R^{(0)}(\omega, \mathbf{q}) + D_R^{(0)}(\omega, \mathbf{q}) \frac{1}{\Pi_R^{-1} - V_R(\omega, \mathbf{q})} D_R^{(0)}(\omega, \mathbf{q}), \quad (2.16)$$

where

$$D_R^{(0)}(\omega, \mathbf{q}) = \int \frac{dk}{2\pi} D_R^{(0)}(\omega, \mathbf{q}, k) \exp(-k^2 l^2 / 2) \quad (2.17)$$

and a purely 2D equation for the MPR

$$\sigma_{xx} = \frac{1}{4T} \left(\frac{c}{B \cos \theta} \right)^2 \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int \frac{d^2 q}{(2\pi)^2} \frac{q^2 N(\omega)}{1 - \exp(-\omega/T)} \times [D_R(-\omega; \mathbf{q}) - D_A(-\omega; \mathbf{q})] [\Pi_R(\omega; \mathbf{q}) - \Pi_A(\omega; \mathbf{q})]. \quad (2.18)$$

Here we made use of the fact that for $\hbar\omega_0 \gg \hbar\Omega$ the integrand in Eq. (2.18) is symmetric in q_x and q_y .

We will not insert directly Eq. (2.16) into Eq. (2.18) as it seems to have poles at $\omega = \omega_l$ that in fact disappear after integration and some algebra. It is convenient instead to present Eq. (2.16) in the form

$$D_R(\omega, \mathbf{q}) = \mathcal{D}_R(\omega, \mathbf{q}) - V^{(C)} + \frac{2V^{(C)}V_R(\mathbf{q})}{\Pi_R^{-1} + V_R(\mathbf{q})} - \frac{(V^{(C)})^2}{\Pi_R^{-1} + V_R(\mathbf{q})}. \quad (2.19)$$

One can see that neither of these terms has a pole $\omega = \omega_l$ [see Eq. (2.13) in combination with Eq. (2.11)]. This is a manifestation of the influence of screening. It means that the screening may play a certain role even for relatively small electron concentrations.

The MPR is, as we will see, determined by the last term while the contribution of all the rest terms in Eq. (2.18) vanishes (provided that one neglects the electron and phonon damping). As a result, we have for the \mathcal{N} th resonance of σ_{xx}

$$\sigma_{xx} = \frac{2n_s c^2 \hbar^2}{\varepsilon_\infty k_B T B^2 \cos^2 \theta} \frac{N(\omega_l) \sinh(\hbar\omega_l / 2k_B T)}{1 - \exp(-\hbar\omega_l / k_B T)} \times \mathcal{J}_N \delta[1/\varepsilon(\mathcal{N}\Omega \cos \theta)], \quad (2.20)$$

where

$$\mathcal{J}_N = \int_0^\infty dq q^3 V^{(C)}(q) I_N \left(\frac{a_B^2 q^2}{2 \cos \theta \sinh \alpha} \right) \times \exp \left(- \frac{a_B^2 q^2 \coth \alpha}{2 \cos \theta} \right). \quad (2.21)$$

As is indicated in Sec. I, it is natural that in the lowest approximation in Ω^2/ω_0^2 only the combination $B \cos \theta$ enters the equations describing the 2D motion of an electron in the quantum well. The integrals in the rest terms in Eq. (2.19) are either real or proportional to $\varepsilon^{-1} \delta(\varepsilon^{-1})$. This can be easily checked if one takes into consideration that $\delta(\varepsilon^{-1})$ comes from the pole $[\Pi_R^{-1} + V_R(\mathbf{q})]^{-1}$ and the terms in question in Eq. (2.19) have the factor ε^{-1} .

Further we will be interested in the case $ql \ll 1$ when the effective e - e interaction does not depend on the form of potential and is equal to

$$V^{(C)}(q) = \frac{2\pi e^2}{\varepsilon_\infty q}. \quad (2.22)$$

Then one can present \mathcal{J}_N as

$$\mathcal{J}_N = \frac{4i\sqrt{\pi}e^2}{\varepsilon_\infty a_B^3} \cos^{3/2} \theta \sinh^{1/2} \left(\frac{\hbar\Omega \cos \theta}{2k_B T} \right) \times Q_{\mathcal{N}-1/2}^1 \left(\cosh \frac{\hbar\Omega \cos \theta}{2k_B T} \right), \quad (2.23)$$

where $Q_{\mathcal{N}-1/2}^1(z)$ is the associated Legendre function of the second kind (we remind the reader that $Q_{\mathcal{N}-1/2}^1$ is an imaginary function of a real argument). We will consider the case $\alpha \ll 1$. Then the characteristic values of q are of the order of $q_T = \sqrt{2mk_B T/\hbar}$ and one can present \mathcal{J}_N in the following form:

$$\mathcal{J}_N = \frac{4\sqrt{2\pi}e^2 \cos \theta}{\varepsilon_\infty a_B^3} \sqrt{\frac{k_B T}{\hbar\Omega}}. \quad (2.24)$$

As $\varepsilon^{-1}(\omega)$ [Eq. (2.11)] has a zero at $\omega = \omega_l$, σ_{xx} exhibits in this approximation an infinitely narrow magnetophonon resonance at

$$\mathcal{N}\Omega \cos \theta = \omega_l. \quad (2.25)$$

Physically this is due to the fact that the e - e interaction without regard of the damping is very strong in the resonance.

III. ANGULAR DEPENDENCE OF MPR MAXIMA

In the present section we will investigate dependence of the positions of the MPR maxima on the angle θ . As we have indicated, in the limit of vanishing phonon and electron damping (Γ and Γ_e , respectively) the screening in the resonance is very strong. If we take into account that the dampings are finite one can calculate the critical concentration n_s where the screening ceases to play a role. As the interaction depends also on θ for each value of n_s one can indicate the corresponding critical value(s) of θ .

We will start with taking into account the phonon damping. Finite optic phonon damping is due to the decay of an optic phonon into two acoustic ones (see Ref. 10). Technically it can be taken into account by replacement $\omega \rightarrow \omega \pm i\Gamma$ in the retarded and advanced phonon Green functions, respectively. One can easily see that in such a case the MPR acquires a finite width which one can take into account by the following replacement in Eq. (2.20)

$$\delta(\varepsilon^{-1}) \rightarrow \frac{1}{\pi} \text{Im} \varepsilon_R. \quad (3.1)$$

Here

$$\frac{1}{\pi} \text{Im} \varepsilon_R = \frac{\omega_l^2 - \omega_t^2}{2\pi\omega_t} \frac{\varepsilon_\infty \Gamma}{(\mathcal{N}\Omega \cos \theta - \sqrt{\omega_t^2 + \Gamma^2})^2 + \Gamma^2}. \quad (3.2)$$

In what follows we will assume that

$$\Gamma_e, \Gamma \ll \omega_l - \omega_t \ll \omega_t. \quad (3.3)$$

One needs the first inequality to be able to discriminate between frequencies ω_l and ω_t . The second inequality is fulfilled for such systems as GaAs/GaAlAs and facilitates the calculations.

Now we will discuss the role of the electron damping. Good examples of importance of the damping for the MPR are given in Refs. 8,20. For us it is important as it may both destroy the resonance and determine the angular interval for its existence we are looking for. We assume that $\Gamma_e \ll \Omega \cos \theta$. The electron Green function in magnetic field has been investigated by Ando and Uemura²¹ and in more detail by Laikhtman and Altshuler²² for Γ_e determined by an elastic short range scattering. In the case we are particularly interested in, i.e., GaAs the temperature variation of mobility from the liquid helium temperature to the temperature of experiment (about 200 K) is substantial (in the typical cases, at least, by several times). It means that the acoustic phonon scattering (that can be considered as short range elastic) should be predominant. It was shown in Refs. 21,22 that in this case the electron Green function has a non-Lorentzian form with the characteristic width Γ_e given by

$$\Gamma_e = \Gamma_e^{(0)} \sqrt{\cos \theta}, \quad \Gamma_e^{(0)} = \sqrt{\Omega/2\pi\tau}, \quad (3.4)$$

where τ is the relaxation time for $B=0$ obtained by assuming the same scatterers as for finite B .

Further on our formulas should be considered as order-of-magnitude estimates giving the correct dependencies on the parameters, though not the parameter-independent numerical coefficients of the order of unity. For such estimates it will be sufficient to use the Lorentzian form of $\Pi(\omega, \mathbf{q})$. As has been indicated, in the resonance approximation one should retain only the resonant term of all the series (2.10) for $\Pi_R(\omega, \mathbf{q})$

$$\Pi_R(\omega, \mathbf{q}) = - \frac{\mathcal{R}_N(\omega, \mathbf{q})}{\omega - \mathcal{N}\Omega \cos \theta + i\Gamma_e}, \quad (3.5)$$

where \mathcal{R}_N is the residue at the pole $\omega = \mathcal{N}\Omega \cos \theta - i\Gamma_e$. Calculating σ_{xx} one can evaluate the integral over the frequen-

cies taking the residues in the poles $\omega = \mathcal{N}\Omega \cos \theta \pm i\Gamma_e$. To get the result one should remove the factor $\delta(\varepsilon^{-1})$ in Eq. (2.20) and insert instead into the integrand of Eq. (2.21)

$$\Delta \equiv \frac{1}{\pi} \frac{\text{Im} \varepsilon_A^{-1} + 2\gamma}{(2\gamma + \text{Im} \varepsilon_A^{-1})^2 + (\text{Re} \varepsilon_A^{-1})^2}, \quad (3.6)$$

where

$$\gamma = \frac{\Gamma_e}{\omega}; \quad \bar{\omega} = \frac{2\pi e^2}{q} \mathcal{R}_N(\mathcal{N}\Omega \cos \theta, \mathbf{q})$$

while ε_A should be calculated at $\omega = \mathcal{N}\Omega \cos \theta + i(\Gamma + \Gamma_e)$. One can see that integral (2.21) is dominated by the values of q where the asymptotic expansion of the Bessel function is valid, so that

$$\mathcal{R}_N = \frac{n_s}{\sqrt{2\pi}} \frac{\mathcal{N}\Omega \cos^2 \theta}{Tqa_B} \sqrt{\frac{\Omega}{T}} \exp\left(-\frac{q^2}{4q_T^2}\right). \quad (3.7)$$

Note added. We remind the reader that Eq. (2.20) is derived within the so-called RPA (loop) approximation [see Eq. (2.16)] as it takes into account the resonant interaction of electrons with optic phonons. Such resonant terms should probably be added also to the vertex parts describing the polaron effects. These terms may be essential in the magnetophonon maxima. However, we are looking for the critical values of the parameters such as electron concentration where the resonant interaction disappears, so that the MPR signal sharply goes down. In such a situation the polaron effects should be also suppressed. Thus we believe that our theory, though probably not permitting the exact calculation of the MPR amplitude, still gives the correct characteristic values of the electron concentrations and the angles where the MPR signal rapidly decreases.

In the present paper we limit ourselves with the sufficiently low concentrations n_s where the MPR maxima are well defined. As is shown in Ref. 10, an additional mechanism of electron level broadening due to the electron-electron interaction appears at high electron concentration [see Eq. (6.10) of Ref. 10]. Here we consider the concentrations that are not so large that one would have to take into consideration this effect.

For small values of γ Eq. (3.6) turns into $\text{Im} \varepsilon_R(\omega_t)$ as one can neglect the terms 2γ . When γ has reached the critical value $(1/2)|\varepsilon_A^{-1}(\omega_t)|$ the amplitude of the maximum begins to go down. This condition can be written as

$$2\gamma \approx \text{Im}(\varepsilon_A)^{-1}. \quad (3.8)$$

for

$$\mathcal{N}\Omega \cos \theta = \omega_t. \quad (3.9)$$

Under the MPR condition (3.9) $\text{Im}(\varepsilon_A)^{-1}$ is small. Therefore even a small variation of the quantities determining γ can violate Eq. (3.8). The screening is important provided that the parameter

$$\beta \equiv \text{Im}[\varepsilon_A(\mathcal{N}\Omega \cos \theta + i\Gamma_e + i\Gamma)]^{-1}/2\gamma \quad (3.10)$$

is bigger than (or of the order of) unity. If it is much bigger than unity the screening is strong and the result is independent of β . The MPR maxima become β -dependent when β is of the order of unity. Then a relatively small variation of β might drastically change the result (see Figs. 4 and 5 of Ref. 10). To achieve such a change one needs a variation of β that need not be large.

Let us follow a variation of the MPR signal as a function of θ . One can write for an arbitrary magnetic field $B(\theta)$ that is near the \mathcal{N} th resonance value $B_{\mathcal{N}}(\theta)$

$$B(\theta) = B_{\mathcal{N}}(\theta) + \Delta B. \quad (3.11)$$

$B_{\mathcal{N}}(\theta) = \hbar \omega_c mc / e \mathcal{N}$ is the position of the \mathcal{N} th MPR maximum for $\theta=0$. The field corresponding to the \mathcal{N} th resonance, varies for finite values of θ according to

$$B_{\mathcal{N}}(\theta) \cos \theta = B_{\mathcal{N}}(0). \quad (3.12)$$

Thus the MPR amplitudes for various angles θ (and the same resonance number \mathcal{N}) correspond to the same value of β and therefore should coincide. (Only the width of the maximum should enhance with decrease of θ). This conclusion of the theory is in drastic disagreement with the experiment.⁶

We think that this is due to the assumption that the electron concentration n_s is a constant independent of B . It is known, however, that in the course of temperature variation from 77 to 300 K at $B=0$ the variation of the carrier concentration may comprise several tens per cent. This means that the energy variation of some donors on the scale of the order of hundred K, or so noticeably shifts the electron concentration balance between the donors and the well.

Let us give a rough estimate of the variation of impurity level positions as a function of B . If one assumes a hydrogen-like states the variation of $\delta\epsilon$ of their lowest level position $\epsilon(B)$ under the shift of magnetic field δB is (for instance, Ref. 14, section 112)

$$\delta\epsilon \approx - \frac{\delta B}{B} \frac{|\epsilon(B)|}{2 \ln[\hbar \Omega(B) / \epsilon_0]} \quad (3.13)$$

where $\epsilon_0 = m e^4 / 2 \hbar^2 \epsilon_\infty^2$ is the Bohr energy while $\epsilon(B)$ is the position of the level in magnetic field B , mark that $\epsilon(B) < 0$. This is a lower estimate as with the overlap of atomic orbits the influence of the magnetic field should enhance. As in our case $\ln[\hbar \Omega(B) / \epsilon_0]$ is, roughly, of the order of unity we will accept the estimate $\delta\epsilon / \epsilon \approx - \delta B / B$. This should result in redistribution of electrons between the donors and the well. In other words, the electron concentration in the well will decrease (see below). The chemical potential μ can be obtained from the equation

$$N_i = \exp\left(\frac{\mu}{k_B T}\right) \left\{ N_i \exp\left[-\frac{\epsilon_i(B)}{k_B T}\right] + \frac{S}{2 \pi a_B^2 \cos^2 \theta} \sum_L \exp\left[-\frac{\hbar \Omega_L(B \cos \theta)}{k_B T}\right] \right\}, \quad (3.14)$$

where N_i is the total number of donors. Further on we will assume that the number of electrons bound to the donors is

much bigger than in the well. This seems to be a rather typical experimental situation. Practically the actual form of the impurity states and the distribution of impurity levels may be (and usually is) much more complicated. What is actually relevant to bring about a variation of the electron concentration n_s in the well is the magnetic field dependence of the positions of impurity levels in the proper interval of energies.

The variation of electron concentration for the values of B given by Eq. (3.12) is

$$\frac{\delta n_s}{n_s(B)} = \xi \frac{\delta B}{B} \quad \text{with} \quad \xi = \frac{\epsilon(B) / k_B T}{1 + n_s(B) / n_i(B)} \quad (3.15)$$

where $n_i(B)$ is the 2D concentration of the electrons bound by the donors. It will be natural to assume below for the estimates that $|\xi|$ is of the order of unity.

It is convenient to present β as a ratio of the electron concentrations to some characteristic value

$$\beta = n_s(B) / n_{\text{down}} \quad (3.16)$$

where

$$\frac{1}{n_{\text{down}}} = \frac{\sqrt{2} \pi \hbar e^3 \omega_t B_{\mathcal{N}}(0)}{\epsilon_\infty q_T T^2 \Gamma_e m c} \cdot \frac{\Gamma_e + \Gamma}{\omega_l - \omega_t}. \quad (3.17)$$

n_{down} is the lower critical concentration where the screening ceases to play a role. For $\mathcal{N}=3$ in GaAs its characteristic value is about 10^{10} cm^{-2} . Equation (3.17) differs from the equation for n_{down} given in Ref. 10 as we have assumed there $q_T \approx a_B^{-1}$ and $\hbar \omega_t \approx 2 k_B T$ that is valid for GaAs under certain conditions and may differ for other situations.

The parameter β depends on the electron concentration n_s . This fact permits one to compare the dependence of the MPR amplitudes in perpendicular magnetic field under the variation of concentration (i.e., in different samples) and the angular dependence of the MPR maxima under variation of θ . This will permit to check as to whether the magnetic field induced variation of n_s is sufficient to explain the decrease of the MPR amplitude as a function of θ on the one hand and the existence of the subsidiary (side) maximum in the angular dependence of the MPR amplitude on the other hand (see Ref. 6).

We start with discussion of the behavior of the MPR amplitude under magnetic field rotation at small angles $\theta \ll 1$. We will consider the concentration interval $n_s \approx n_{\text{down}}$ where there is a sharp dependence of the MPR amplitude on β . According to Eq. (3.12) the MPR maximum will shift by

$$\delta B_{\mathcal{N}} \approx B_{\mathcal{N}}(0) (1 - \cos \theta). \quad (3.18)$$

This will result in the relative variation of concentration $\delta n_s / n_s \approx \xi (1 - \cos \theta)$ that manifests itself in the angular dependence of the MPR amplitude. As is known from the experiment in the perpendicular magnetic field, in this region of concentrations when n_s goes down it brings about a sharp decrease of the amplitude.

As a reasoning supporting our view we will consider the following numerical example. According to Ref. 6, Fig. 6 the variation of the MPR amplitude is about 60% provided the

concentration varies within the interval from $1.8 \times 10^{10} \text{ cm}^{-2}$ to $2 \times 10^{10} \text{ cm}^{-2}$. The same decrease of the amplitude induced by magnetic field should take place for $\delta n_s/n_s \approx 0.1$, i.e., for θ of the order of 20° . This shows an order-of-magnitude correspondence with the results of Ref. 6. Further increase of θ gets the system into the region where $n < n_{\text{down}}$ and a well defined resonant peak with the resonant condition (3.9) disappears.

The behavior of this sort takes place provided that $\beta \gg 1$, for such concentrations where $\beta \gg 1$ small variation of this parameter does not play an essential role. This means that for the samples that have a maximal MPR amplitude ($n_s \approx 5 \times 10^{10} \text{ cm}^{-2}$) its θ dependence should be absent.

Now we will treat the region of large angles. Consider the samples with the concentration of the order of n_{down} . The angle θ going up, n_s can become so small that the perturbation approach becomes applicable. In other words, one observes the resonance determined by the condition

$$\mathcal{N}\Omega \cos\theta = \omega_l. \quad (3.19)$$

The screening is not important provided that $\beta < 1$. In view of Eq. (3.19) one should insert in (3.10) $\epsilon_A(\omega_l + i\Gamma_e + i\Gamma)$. As a result, one can write

$$\beta_l = \frac{n_s(\theta)}{n_{\text{down}}} \left(\frac{\omega_l - \omega_l}{\Gamma_e + \Gamma} \right)^2. \quad (3.20)$$

If the last factor in this equation is large enough one needs rather small concentrations $n_s(\theta)$. Let us estimate the angles where they can be achieved. It follows from Eq. (3.14) under the same assumptions provided that $\delta B/B$ is not small

$$n_s(\Delta B + B) = n_s(B) \exp(\delta\epsilon/k_B T). \quad (3.21)$$

Very rough estimates in the spirit of Ref. 23 give [the dependence $\epsilon(B)$ is rather smooth and we believe that its expansion up to the linear term can be justified]

$$\delta\epsilon \approx -\zeta \epsilon_0 \left(\frac{\hbar\Omega}{\epsilon_0} \right) = -\zeta \hbar\Omega \quad (3.22)$$

where ζ is a number of the order of unity. This estimate is based on the idea that the level shift depends on a single parameter, i.e., the ratio of the magnetic energy to the Coulomb one (cf., the analytical treatment of the hydrogen atom in magnetic field in Ref. 14, Sec. 112).

Thus the discussed ω_l maximum exists for the angles bigger than θ_c given by the equation

$$\sec\theta_c \approx \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{2\mathcal{N}k_B T}{\zeta \hbar \omega_l} \ln \left(\frac{\omega_l - \omega_l}{\Gamma_e + \Gamma} \right)}. \quad (3.23)$$

The second term under the sign of root is, most probably, of the order of unity. In this case θ is somewhere in the interval between 40° and 70° , or so. Thus in the regions of small and large angles we have well-defined ω_l and ω_l resonances, respectively. Their positions depend on $B \cos\theta$. In the intermediate interval of angles there are no well-defined MPR maxima.

It is interesting to note that in the region of small angles the width of a maximum goes up with the number of resonance \mathcal{N} . It can be easily seen if one takes into consideration enhancement of n_s with decrease of magnetic field as well as enhancement of n_c with \mathcal{N} . All these conclusions are in a qualitative agreement with the results described in Ref. 6.

IV. CONCLUSION

To summarize, we would like to stress that the interpretation of behavior of the MPR in a tilted magnetic field has been a long-standing problem.^{5,6,12} Two types of resonant maxima have been discovered on experiment, i.e., the ω_l and ω_l resonances as their positions are determined by the frequencies ω_l and ω_l , respectively.

Important points to provide theoretical interpretation of these resonances are the dependence of all quantities characterizing the 2D motion of electrons on the combination $B \cos\theta$ whereas their total concentration in the well depends on B . Had only the $B \cos\theta$ dependence existed, the amplitude of the resonances, i.e., the heights of the maxima would have been independent of the angle θ . We think that the only way to preserve the theory of 2D electron gas describing the concentration dependence of the MPR is to assume variation of the electron concentration with B . It manifests itself in the angular dependence of the MPR amplitude. In this way we have been able to give a qualitative interpretation of the results given in Ref. 6.

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APPENDIX: CALCULATION OF POLARIZATION OPERATOR

As indicated in Sec. II, we assume for the electrons a parabolic confining potential $m\omega_0^2 z^2/2$ with the gauge for the vector potential $\mathbf{A} = (-By \cos\theta + Bz \sin\theta, 0, 0)$. In spite of the first inequality (2.8), it is convenient to look for the exact transformation of the Hamiltonian and solution of the Schrödinger equation and only then go to the limit

$$\Omega/\omega_0 \ll 1. \quad (A1)$$

Applying a standard procedure of diagonalization we get for the bigger eigenvalue

$$\Omega_2^2 = \omega_0^2 + \Omega^2 - \Omega_c^2 \quad (A2)$$

while the smaller eigenvalue is

$$\Omega_c^2 = \frac{\Omega^2 + \omega_0^2}{2} \left[1 - \sqrt{1 - \frac{4\Omega^2 \omega_0^2 \cos^2\theta}{(\Omega^2 + \omega_0^2)^2}} \right]. \quad (A3)$$

The variables (Y, Z) diagonalizing the Hamiltonian are expressible through the initial variables (y, z) as

$$\begin{pmatrix} Y \\ Z \end{pmatrix} = \frac{1}{C} \begin{pmatrix} y - y_0 - rz \\ r(y - y_0) + z \end{pmatrix}. \quad (\text{A4})$$

Here $C = 1/\sqrt{1+r^2}$,

$$r = \frac{\Omega_c^2 - \Omega^2 \cos^2 \theta}{\Omega^2 \cos \theta \sin \theta}.$$

As a result, we get two noninteracting oscillators, i.e., the Y oscillator and the Z oscillator

$$\mathcal{H} = -\frac{1}{2m} \frac{\partial^2}{\partial Y^2} - \frac{1}{2m} \frac{\partial^2}{\partial Z^2} + \frac{m}{2} \Omega_c^2 Y^2 + \frac{m}{2} \Omega_2^2 Z^2. \quad (\text{A5})$$

We are interested in the eigenfrequency of the Y oscillator that is

$$\Omega_c = \Omega \cos \theta \quad (\text{A6})$$

plus small terms proportional to $\Omega^2/\omega_0^2 \ll 1$. The eigenfrequency of the Z oscillator is equal to ω_0 (with the same accuracy).

Now we turn to calculation of the polarization operator for nondegenerate free electrons in magnetic field by a slight modification of the method proposed by Sondheimer and Wilson.²⁴ The method is based on the spectral representation (see Sec. 36 of Ref. 15).

$$\begin{aligned} \Pi_R(\omega, z', z) &= \sum_{m,l} w_l \frac{\langle m | \hat{n}(0, z') | l \rangle \langle l | \hat{n}(0, z) | m \rangle}{\omega - \omega_{lm} + i\delta} \\ &\times [1 - \exp(-\omega_{lm}/T)]. \end{aligned} \quad (\text{A7})$$

Sondheimer and Wilson introduced a Green function of a complex time argument γ

$$G(\mathbf{r}, \mathbf{r}', \gamma) = \sum_{\beta} \psi_{\beta}^*(\mathbf{r}') \psi_{\beta}(\mathbf{r}) \exp(-\varepsilon_{\beta} \gamma), \quad \text{Re } \gamma > 0. \quad (\text{A8})$$

Here β is the set of all quantum numbers of an electron, while ε_{β} is the energy of the quantum state β . One can express the polarization operator through the Green functions of a complex time argument.

The retarded polarization operator for Hamiltonian (A5) is

$$\begin{aligned} \Pi_R^{(3)}(\mathbf{r}, \mathbf{r}', t) &= 2i \cosh \frac{\Omega_2 + \Omega_c}{2T} \exp\left(\frac{\mu}{T}\right) \Theta(t) \\ &\times \left[G\left(\mathbf{r}', \mathbf{r}, \frac{1}{T} - it\right) G(\mathbf{r}, \mathbf{r}', it) \right. \\ &\left. - G(\mathbf{r}', \mathbf{r}, -it) G\left(\mathbf{r}, \mathbf{r}', \frac{1}{T} + it\right) \right]. \end{aligned} \quad (\text{A9})$$

Here $\psi_{\beta}(\mathbf{r})$ is a product of the eigenfunctions of the Y and Z oscillators, $\Theta(t)$ is the step function.

In order to sum up the series in Eq. (A8) over the Y - and Z -oscillator quantum numbers we will use the following relation:²⁵

$$\begin{aligned} \sum_0^{\infty} \frac{1}{2^n n!} \exp\left[-\frac{1}{2}(y^2 + y'^2) - n\gamma\omega\right] H_n(y) H_n(y') \\ = (1 - e^{-2\gamma\omega})^{-1/2} \exp\left[-\frac{1}{4}(y + y')^2 \frac{1 - e^{-\gamma\omega}}{1 + e^{-\gamma\omega}} \right. \\ \left. - \frac{1}{4}(y - y')^2 \frac{1 + e^{-\gamma\omega}}{1 - e^{-\gamma\omega}}\right], \end{aligned} \quad (\text{A10})$$

where $H_n(y)$ is the Hermite polynomial. It is convenient to replace the summation over the quantum number p_x by integration over y_0 .

Now,

$$\begin{aligned} G(\mathbf{r}, \mathbf{r}', \gamma) &= \frac{a_1 C e B \cos \theta}{\sqrt{\pi a_2}} (\sinh \gamma \Omega_c \sinh \gamma \Omega_0)^{-1/2} \\ &\times \left(\tanh \frac{\gamma \Omega_c}{2} + \frac{a_1^2 r^2}{a_2^2} \tanh \frac{\gamma \Omega_0}{2} \right)^{-1/2} \\ &\times \exp\left\{ -\frac{1}{4} \coth\left(\frac{\gamma \Omega_c}{2}\right) (\Delta Y)^2 - \frac{a_1^2}{4a_2^2} \right. \\ &\times \coth\left(\frac{\gamma \Omega_0}{2}\right) (\Delta Z)^2 - \frac{1}{4} b_1^2 \tanh\left(\frac{\Omega_0 \gamma}{2}\right) (z + z')^2 \\ &\left. - \frac{1}{4} \frac{C^2 [\Delta x + ib \tanh(\Omega_0 \gamma/2) (z + z')]^2}{\tanh(\gamma \Omega_c/2) + (a_1^2 r^2/a_2^2) \tanh(\gamma \Omega_0/2)} \right\}. \end{aligned} \quad (\text{A11})$$

Here

$$b = \frac{a_1^2 r}{a_2^2}, \quad b_1 = \frac{b}{r}; \quad (\text{A12})$$

the variables in Eq. (A11) are made dimensionless by the transformations

$$\begin{aligned} \Delta Y &= (Y - Y')/a_1, \quad \Delta Z = (Z - Z')/a_1, \quad z \rightarrow z/a_1, \\ z' &\rightarrow z'/a_1, \end{aligned} \quad (\text{A13})$$

where

$$a_1^2 = 1/m\Omega_c, \quad a_2^2 = 1/m\Omega_0. \quad (\text{A14})$$

To calculate the polarization operator it is convenient to use the Green functions in the momentum representation. After rather involved but straightforward calculations we get

$$\Pi^{(3)}(\mathbf{q}, q_z, Q_z, t) = P(\mathbf{q}, q_z, Q_z, t) + P^*(-\mathbf{q}, -q_z, Q_z, t), \quad (\text{A15})$$

where the asterisk means a complex conjugation

$$\begin{aligned}
P(\mathbf{q}, q_z, Q_z, t) &= \frac{2iCa_1B^2 \cosh \frac{\Omega_c + \Omega_0}{2T} \Theta(t) e^{\mu/T}}{\pi a_2 \sinh(\Omega_c/2T) \sinh(\Omega_0/2T)} \\
&\times \exp \left\{ - \left(q_y^2 + \frac{q_x^2}{C^2} \right) g_1(\Omega_c) \right. \\
&\quad - \left(q_z^2 + \frac{r^2 q_x^2}{C^2} \right) \frac{a_2^2}{a_1^2} g_1(\Omega_0) \\
&\quad - \frac{Q_z^2}{4C^2} \left[\frac{a_2^2}{a_1^2} g_2(\Omega_0) + \frac{r^2}{2} g_2(\Omega_c) \right] \\
&\quad \left. + \frac{r q_x Q_z}{2C^2} [g_3(\Omega_c) - g_3(\Omega_0)] \right\}, \tag{A16}
\end{aligned}$$

where

$$\begin{aligned}
g_1(\Omega) &= \frac{1}{2} \left[\coth \frac{\Omega}{2T} - \cos \left(\Omega t + \frac{i\Omega}{2T} \right) \right] / \sinh \frac{\Omega}{2T}, \\
g_2(\Omega) &= \frac{1}{2} \left[\coth \frac{\Omega}{2T} + \cos \left(\Omega t + \frac{i\Omega}{2T} \right) \right] / \sinh \frac{\Omega}{2T}, \\
g_3(\Omega) &= \frac{\sin \Omega(t + i/2T)}{\sinh(\Omega/2T)}.
\end{aligned}$$

Here \mathbf{q} and q_z refer to the Fourier components over the differences $\mathbf{r} - \mathbf{r}'$ and $z - z'$ while Q_z is related to $(z + z')/2$.

We are looking for a frequency representation of the polarization operator. We are going to take into consideration only the lowest level of transverse quantization, i.e., the lowest miniband. At the first sight one could try to average Eq. (A16) over the time interval bigger than $1/\omega_0$ but smaller than $1/\Omega \cos \theta$. However, some spurious terms can appear as a result of the direct averaging due to the even powers of $\sin \omega_0 t$ and $\cos \omega_0 t$. Therefore we will look for the frequency representation of the whole expression Eq. (A16). One can find it with the help of the identities

$$\begin{aligned}
\exp(-z \cos A) &= \sum_{n=-\infty}^{\infty} e^{inA} I_n(z), \\
\exp(-z \sin A) &= \sum_{n=-\infty}^{\infty} e^{inA} J_n(iz). \tag{A17}
\end{aligned}$$

where J_m are the ordinary Bessel functions. Combining them with Eq. (A16) one can see that A is a linear function of time, so that one can easily calculate the Fourier components. Discarding the poles describing the transitions to the higher minibands one gets

$$\begin{aligned}
\Pi_R^{(3)} &= -2Cn_s \exp \left[- \left(\frac{q_y^2}{2} + \frac{q_x^2}{2C^2} \right) \coth \frac{\Omega_c}{2T} \right. \\
&\quad \left. - \frac{Q_z^2}{C^2} \left(\frac{a_2^2}{8a_1^2} + \frac{r^2}{8} \coth \frac{\Omega_c}{2T} \right) - \frac{a_2^2}{a_1^2} \left(\frac{q_z^2}{2} + \frac{r^2 q_x^2}{2C^2} \right) \right] \\
&\times \sum_{lmn} I_{n+l} \left(\frac{q_y^2 + q_x^2/C^2}{2 \sinh(\Omega_c/2T)} \right) I_{l-m} \left(\frac{r^2 Q_z^2}{8C^2 \sinh(\Omega_c/2T)} \right) \\
&\times J_m \left(\frac{ir q_x Q_z}{2C^2 \sinh(\Omega_c/2T)} \right) \frac{\sinh(\Omega_c n/2T)}{\omega - \Omega_c n + i\delta}. \tag{A18}
\end{aligned}$$

There are elegant formulas to calculate this sum. However, as we are interested in the limiting case $\Omega^2/\omega_0^2 \ll 1$ one can see the result immediately from Eq. (A18). Indeed, the Bessel functions having in their arguments as a factor the small parameter r can be discarded unless $l - m = m = 0$ and we are left with

$$\begin{aligned}
\Pi_R^{(3)} &= -2n_s \exp \left[- \frac{q_x^2 + q_y^2}{2} \coth \frac{\Omega_c}{2T} - \frac{a_2^2}{a_1^2} \frac{Q_z^2}{8} - \frac{a_2^2}{a_1^2} \frac{q_z^2}{2} \right] \\
&\times \sum_n I_n \left(\frac{q_y^2 + q_x^2}{2 \sinh(\Omega_c/2T)} \right) \frac{\sinh(n\Omega_c/2T)}{\omega - \Omega_c n + i\delta}. \tag{A19}
\end{aligned}$$

One can see that $\Pi^{(3)}$ Eq. (A19) coincides with $\Pi_R^{(3)}$ of Eq. (2.4) and (2.10) if one takes into consideration that within the accepted approximation $\Omega_c = \Omega \cos \theta$ and that we use in Eq. (A19) dimensionless variables.

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