Experimental implications of quantum phase fluctuations in layered *d*-wave superconductors

Arun Paramekanti

Dept. of Theoretical Physics, Tata Institute of Fundamental Research, Mumbai 400 005, India

I study the effect of quantum and thermal phase fluctuations on the in-plane and *c*-axis superfluid stiffness of layered *d*-wave superconductors. First, I show that quantum phase fluctuations in the superconductor can be damped in the presence of external screening of Coulomb interactions, and suggest an experiment to test the importance of these fluctuations, by placing a metal in close proximity to the superconductor to induce such screening. Second, I show that a combination of quantum phase fluctuations and the linear temperature dependence of the in-plane superfluid stiffness leads to a linear temperature dependence of the c-axis penetration depth, below a temperature scale determined by the magnitude of in-plane dissipation.

I. INTRODUCTION

The linear temperature dependence of the low temperature in-plane penetration depth $\lambda_{ab}(T)$ is well established now in nearly all the high-T_c superconductors.[1,2] This behavior is most simply explained in terms of nodal quasiparticle excitations in a *d*-wave superconductor.[3] These low energy nodal quasiparticles then determine the low temperature behavior of thermodynamic properties and in-plane response functions. However, since the high-T_c superconductors (SC's) have a low superfluid density and a short coherence length compared to conventional superconductors, it is plausible that in addition to quasiparticle excitations, quantum and thermal phase fluctuations could also be important. The importance of such quantum phase fluctuations, especially for *c*-axis properties, has been emphasized earlier, from an analysis of the c-axis optical conductivity and sum rules.[4]

Based on our study of an effective quantum XY phase action in the superconducting state, we had argued[5,6] that quantum phase fluctuations are important in the cuprate SC's at low temperature. With increasing temperature, in the presence of dissipation, the system gradually crosses over from a quantum fluctuation regime to a regime of classical phase fluctuations[6,7] at a crossover scale, $T_{cl} \lesssim T_c$. (This has been observed in recent experiments on thin films of conventional dirty *s*-wave superconductors[8] where $T_{cl} \sim 0.94 \ T_c$.) It has been suggested,[9] that this crossover scale $T_{cl} \ll T_c$ in the high-T_c systems, and thus classical phase fluctuations should dominate the low temperature superconducting state properties. However, for parameter values relevant to the cuprate SC's, and even *overestimating* the magnitude of dissipation, which should lead to a *smaller* crossover temperature, we found[6] that $T_{cl} \sim 20$ K. Since a linear T behavior of the superfluid stiffness has been observed in many high-T_c SC's from ~ 30 K down to ~ 1 K, with no evidence of such a thermal crossover, we conclude that the crossover temperature is high $\gtrsim 30$ K, and classical effects are irrelevant for in-plane properties at low temperatures $T \ll T_c$. The low-temperature regime is thus described entirely by quasiparticle excitations around a ground state with strong quantum phase fluctuations. Recently, an experiment has been proposed[10] to observe these quantum phase fluctuations by

studying the excess current along the *c*-axis, after suppressing Josephson tunneling using an in-plane magnetic field.

The presence of strong quantum phase fluctuations in the low-temperature superconducting (SC) state raises several questions: Can the magnitude of low-temperature quantum phase fluctuations be directly probed? What is the effect of such phase fluctuations on the *c*-axis superfluid stiffness? We shall study these questions in this paper. The principal results of this paper are as follows. (1) External screening of the Coulomb interactions in a superconductor damps quantum phase fluctuations. Based on this I suggest an experiment where a metal is placed in close proximity to the superconductor to induce such external screening, similar to experiments which have been carried on gated Josephson junction arrays.[11] In the quantum fluctuation regime, relevant to our case, this screening reduces the fluctuations and leads to observable changes in the superfluid stiffness and its temperature dependence. In the classical regime, the screening would not lead to any change in the superfluid stiffness, since the fluctuations are already overdamped. Thus, this experiment would serve as a way to directly measure the magnitude of quantum fluctuations in the superconductor. (2) I show that quantum phase fluctuations can lead to a linear T increase of the *c*-axis penetration depth. This linear T slope arises from the linear T dependence of the *in-plane superfluid stiffness* and the Josephson coupling between layers. The magnitude of this effect is sensitive to the in-plane dissipation. The linear temperature dependence may be consistent with some low-temperature *c*-axis penetration depth data[12–15] in YBa$_2$Cu$_3$O$_{7-\delta}$ (YBCO) and Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ (BSCCO) systems, thus serving as indirect evidence of quantum phase fluctuations in the SC state. More generically however, the *c*-axis penetration depth does not appear to have a linear temperature dependence. I discuss possible implications of this towards the end.

II. EFFECTIVE PHASE ACTION

I begin by briefly reviewing the formalism of Refs. 5,6 to study phase fluctuations in the low-temperature superconducting state. We model the dynamics of the phase variables $\theta_{\mathbf{R}}(\tau)$ defined on a coarse-grained lattice (with lattice spac-

ing equal to the coherence length ξ_0) by a quantum XY action. We have derived the following coarse-grained action for the layered d-wave SC's:

$$S[\theta] = \frac{1}{8T} \sum_{\mathbf{Q},\omega_n} \left[\frac{\omega_n^2 \xi_0^2 d_c}{\widetilde{V}(\mathbf{Q})} + \frac{\bar{\sigma}}{2\pi} |\omega_n| \gamma_\parallel(\mathbf{Q}) \right] |\theta(\mathbf{Q},\omega_n)|^2$$
$$+ \frac{1}{4} \int_0^{1/T} d\tau \sum_{\mathbf{R},\alpha} J_\alpha \{1 - \cos[\theta(\mathbf{R},\tau) - \theta(\mathbf{R}+\alpha,\tau)]\},$$
$$(1)$$

where d_c is the interplane separation, ξ_0 is the in-plane coherence length, $\alpha = x,y,z$, and $\gamma_\parallel(\mathbf{Q}) = (4 - 2\cos Q_x - 2\cos Q_y)$. The internal dissipation in the superconductor arises from the conductivity of the electronic degrees of freedom which have been integrated out, and is parametrized by $\bar{\sigma}$ where, for simplicity, I assume a constant (Ohmic) layer conductance $\sigma d_c = (e^2/h)\bar{\sigma}$. The couplings J_α are given by $J_{x,y} \equiv J_\parallel = D_\parallel^0 d_c$ and $J_z \equiv J_\perp = D_\perp^0 d_c(\xi_0/d_c)^2$, where $D_\parallel^0, D_\perp^0$ denote the bare in-plane and c-axis stiffnesses respectively, which are related to the penetration depths through $\lambda_{\parallel,\perp}^{-2} = 4\pi e^2 D_{\parallel,\perp}/\hbar^2 c^2$. $\widetilde{V}(\mathbf{Q}) \equiv V(\mathbf{Q}_\parallel a/\xi_0, \mathbf{Q}_\perp)$ denotes the scaled Coulomb interaction which arises from the coarse graining procedure, with

$$V(\mathbf{Q}) = \frac{2\pi e^2 a d_c}{Q_\parallel \epsilon_b} \left[\frac{\sinh(Q_\parallel d_c/a)}{\cosh(Q_\parallel d_c/a) - \cos Q_\perp} \right] \quad (2)$$

being the Coulomb interaction appropriate for layered systems. Here ϵ_b is some background dielectric constant, a is the original in-plane lattice spacing, and $Q_\parallel, Q_\perp$ are the in-plane and c-axis components of the momentum, measured in units of $1/a$ and $1/d_c$, respectively.

I analyze the quantum XY action within the self-consistent harmonic approximation[16] (SCHA). The SCHA replaces the above action by a trial harmonic theory with the renormalized stiffness $D_{\parallel,\perp}$ chosen to minimize the free energy. It thus takes into account the renormalization of the stiffness due to anharmonic longitudinal phase fluctuations but ignores vortices and vortex-antivortex pairs (transverse fluctuations). The validity of this approximation at low temperature, in the classical case where only $n=0$ Matsubara frequency is retained, has been confirmed in recent Monte Carlo simulations of the classical XY model.[9] Carrying out the SCHA for our anisotropic case leads to

$$D_{\parallel,\perp} = D_{\parallel,\perp}^0 \exp(-\langle \delta\theta_{\parallel,\perp}^2 \rangle/2), \quad (3)$$

where $\delta\theta_{\parallel,\perp} \equiv (\theta_{\mathbf{r},\tau} - \theta_{\mathbf{r}+\alpha,\tau})$ with $\alpha = x,y$ for the in-plane ($\parallel$) case and $\alpha = z$ for the c-axis ($\perp$) case. The expectation values are evaluated in the renormalized harmonic theory, which yields

$$\langle \delta\theta_\parallel^2 \rangle = 2T \int_{-\pi}^{\pi} \frac{d^3\mathbf{Q}}{(2\pi)^3} \sum_n \gamma_\parallel(\mathbf{Q}) G(\mathbf{Q},\omega_n),$$

$$\langle \delta\theta_\perp^2 \rangle = 4T \int_{-\pi}^{\pi} \frac{d^3\mathbf{Q}}{(2\pi)^3} \sum_n \gamma_\perp(\mathbf{Q}) G(\mathbf{Q},\omega_n), \quad (4)$$

where $\gamma_\parallel(\mathbf{Q}) = (4 - 2\cos Q_x - 2\cos Q_y)$ and $\gamma_\perp(\mathbf{Q}) = (2 - 2\cos Q_z)$. Here, $G(\mathbf{Q},\omega_n)$ is the phase propagator given by

$$G^{-1}(\mathbf{Q},\omega_n) = \frac{\omega_n^2 \xi_0^2 d_c}{\widetilde{V}(\mathbf{Q})} + \left(J_\parallel + \frac{\bar{\sigma}}{2\pi} |\omega_n| \right) \gamma_\parallel(\mathbf{Q}) + J_\perp \gamma_\perp(\mathbf{Q}).$$
$$(5)$$

In what follows, I analyze the above equations in various limits, at low temperature, and also compare with a full self-consistent numerical solution[17] of Eqs. (3)–(5).

III. AN EXPERIMENTAL PROBE OF IN-PLANE QUANTUM PHASE FLUCTUATIONS

In this section, I study the damping of in-plane quantum phase fluctuations due to *external* screening of Coulomb interactions. This leads us naturally to a suggestion for an experiment to estimate the magnitude of these fluctuations in the cuprate SC's. Previous work on the damping of quantum phase fluctuations include studies on resistively shunted Josephson junctions arrays with short range charging energy[18] and on a related model for superconductors with a low superfluid density.[19] These works focussed on the quantum phase transition from an insulator to a superconductor, driven by increasing the strength of dissipation. Subsequently, such a quantum phase transition was observed experimentally in gated Josephson junction arrays,[11] where the quantum phase fluctuations were damped by external screening from a (gate-tuned) metallic bath. I show below within our phase fluctuation action, which differs in important respects from the earlier models, that such damping of quantum fluctuations through an external metallic bath leads to *observable consequences* for the low-temperature superfluid stiffness. This may be used to study the importance of quantum fluctuations in the high-T_c superconductors, with a setup similar to the one used in the above experiment.

Let us confine ourselves to the case of a thin superconducting film capacitively coupled to a metallic bath, and work in the two-dimensional (2D) limit of the phase action. For electrons situated at an interface between vacuum, with a dielectric constant of unity, and a (metallic) substrate with $\epsilon_{\text{sub}}(\omega) = (\epsilon_\infty + 4\pi i\sigma_{\text{ext}}/\omega)$, the Coulomb interaction is given by $V_{2D}(\mathbf{Q}) = 4\pi e^2 a d_c/[1 + \epsilon_{\text{sub}}(\omega)]Q_\parallel$. Thus, the metallic bath provides dynamical screening of the Coulomb interactions,[19,20] and leads to an additional term in the phase action in Eq. (1), such that

$$(\bar{\sigma}/2\pi)\gamma_\parallel(\mathbf{Q})|\omega_n| \rightarrow [\bar{\sigma}\gamma_\parallel(\mathbf{Q}) + \bar{\sigma}_E \mathbf{Q}_\parallel] \frac{|\omega_n|}{2\pi}, \quad (6)$$

where $\bar{\sigma}_E \equiv (\xi_0/d_c)(\sigma_{\text{ext}} d_c)/(e^2/h)$, and $\epsilon_b \rightarrow (1+\epsilon_\infty)/2$ in the Coulomb interaction in Eq. (2). Thus, external dissipation due the metallic bath (σ_{ext}) appears together with the internal dissipation from the electronic degrees of freedom of the superconductor which have been integrated out ($\bar{\sigma}$), and they can both lead to damping of quantum fluctuations in a similar manner. Note that our approach differs from the earlier work of Ref. 19 in two important ways. (i) We retain the dynamical term $\omega_n^2 \xi_0^2 d_c/\widetilde{V}(\mathbf{Q})$ in the action in Eq. (1), and
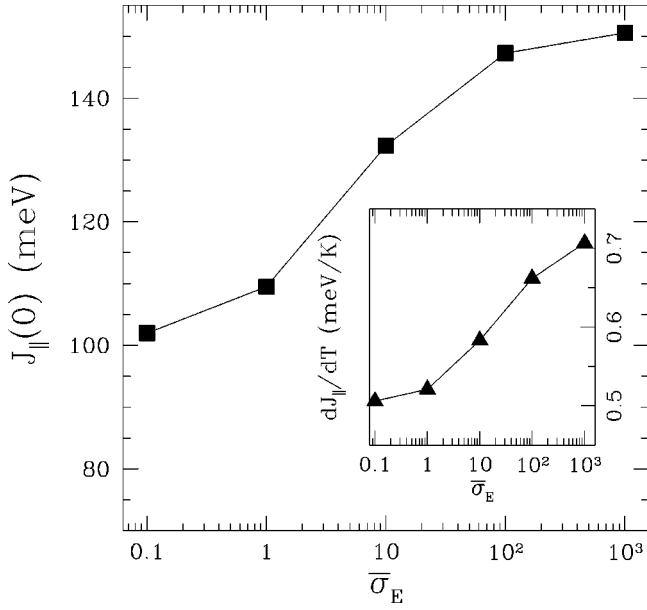
FIG. 1. The $T=0$ in-plane superfluid stiffness for various values of external dissipation $\bar{\sigma}_E$. The inset shows the behavior of the linear T slope of $J_\parallel(T)$. The above results have been obtained by solving the Eqs. (3)–(5) for a two-dimensional case, assuming bare bilayer stiffness $J_\parallel(0) \approx 150$ meV with a slope ~ 0.65 meV/K, and an internal dissipation $\bar{\sigma}=10$. These values have been chosen such that, for $\bar{\sigma}_E=0$ we obtain a penetration depth $\lambda_\parallel(0) \approx 1600$ Å, and a slope $d\lambda_\parallel/dT \approx 4$ Å/K, in agreement with experiment[1].

the renormalized propagator in Eq. (5). (ii) We make a distinction between the dissipation which arises from the degrees of freedom internal to the superconductor and that from external screening. This is reflected in the fact that the conductivities $\bar{\sigma}$ and $\bar{\sigma}_E$ appear with different **Q**-dependent coefficients in the above action.[21]

To quantitatively study the effect of the external screening, let us consider parameter values relevant to the cuprate SC's. I choose $\epsilon_\infty \approx 10$, $\xi_0/a \approx 10$ and $\bar{\sigma} \approx 10$ as representative of the YBCO system[22] and any typical substrate material. We model YBCO as a system of strongly coupled bilayers, with the bilayer stiffness being twice the single layer stiffness, and an interbilayer spacing $d_c/a \approx 3$ being twice the mean layer spacing. This will simplify our calculation, since we do not have to introduce an additional parameter to distinguish intrabilayer and interbilayer couplings; a more sophisticated calculation would not lead to any qualitative or significant quantitative change. For the present analysis, we will use a bare *bilayer* stiffness and its linear T slope such that the renormalized stiffness $J_\parallel(T)$, for the above parameters and with $\bar{\sigma}_E=0$, leads to a penetration depth $\lambda_\parallel(0) \approx 1600$ Å and $d\lambda_\parallel/dT \approx 4$ Å/K in agreement with experiment.[1,14] We can then vary the external dissipation $\bar{\sigma}_E$, and study its effect on $J_\parallel(T)$. In Fig. 1, I plot the behavior of the the stiffness $J_\parallel(0)$, and its slope $dJ_\parallel/dT$, for various values of $\bar{\sigma}_E$ corresponding to differing levels of external dissipation.

It is clear that the stiffness $J_\parallel(0)$ increases with increasing dissipation, as quantum fluctuations are damped, leading to a

more ordered state. With increasing dissipation (classical) thermal phase fluctuations also contribute to the normal fluid density, in addition to the ''bare'' quasiparticle contribution, leading to an enhancement in the slope of $J_\parallel(T)$. A measurement of the stiffness and its temperature dependence in the presence of external screening would thus serve as a test of quantum phase fluctuations. This could be probed in low frequency optical experiments, or penetration depth experiments, and it might also be reflected in a systematic increase in the superconducting transition temperature with external dissipation. A minor caveat concerning penetration depth experiments is that the effective (Pearl) penetration depth $\lambda_{\text{eff}} = \lambda_\parallel^2/\delta$ where δ is the film thickness, and hence one cannot use the strictly 2D limit in the above calculation but δ has to be kept finite. This is not expected to lead to any qualitative changes in our results for $J_\parallel(T)$.

IV. QUANTUM PHASE FLUCTUATIONS AND THE *c*-AXIS PENETRATION DEPTH

Having shown that quantum phase fluctuations may affect the in-plane superfluid stiffness, I next turn to examine the effect of these fluctuations on the *c*-axis superfluid stiffness. Many earlier studies of the *c*-axis stiffness have focussed on the quasiparticle contribution to the normal fluid density, emphasizing the role of tunneling matrix elements,[23] or effects of disorder and pair tunneling on the temperature dependence of the critical current.[24,25] Here, I study the effect of phase fluctuations on the c-axis superfluid stiffness within the SCHA, and find that phase fluctuations can lead to a linear temperature dependence of the *c*-axis superfluid stiffness and penetration depth. This linear T slope arises from the linear T dependence of the *in-plane stiffness* and the Josephson coupling between layers, and it is a *generic* effect in such models.

We shall begin by assuming that the bare *c*-axis stiffness is T independent at low temperature. This assumption may be justified as follows. Quasiparticles may be important for in-plane properties, but the interplane tunneling matrix element[26] being proportional to $\sim(\cos k_x - \cos k_y)^2$, leads to a very small depletion of the superfluid density, since matrix element vanishes for in-plane nodal quasiparticles. In our calculation we shall neglect *c*-axis dissipation since the sharp Josephson plasmon seen in experiments on BSCCO implies a very small *c*-axis conductivity, and the conductivity has been measured to be small over a wide frequency range.[27] In YBCO, there are added complications due to the chains, but I will ignore this.

To study the effect of phase fluctuations, let us analyze the fluctuation integral for $\langle \delta\theta_\perp^2 \rangle$ in Eq. (4) for the physically relevant case of $J_\perp/J_\parallel \ll 1$. Since we are interested only in $J_\perp(T)$ we fix the in-plane stiffness, $J_\parallel(T)$, from experiment, and study the *c*-axis fluctuations $\langle \delta\theta_\perp^2 \rangle(T)$. I will first present results for the case with no in-plane dissipation ($\bar{\sigma}=0$) and later consider the effect of dissipation to see how these results are affected by a finite $\bar{\sigma} \neq 0$. Finally, I will compare our results with experimental data on BSCCO and YBCO.

A. Dissipationless case ($\bar{\sigma}=0$)

For $\bar{\sigma}=0$, we can first do the Matsubara summation in the fluctuation integral in Eq. (4). This leads to

$$\langle\delta\theta_\perp^2\rangle = 2\int_{-\pi}^{\pi}\frac{d^3\mathbf{Q}}{(2\pi)^3}\gamma_\perp\sqrt{\frac{\tilde{V}}{(J_\parallel\gamma_\parallel+J_\perp\gamma_\perp)}}$$

$$\times\coth\left(\frac{1}{2T}\sqrt{\tilde{V}(J_\parallel\gamma_\parallel+J_\perp\gamma_\perp)}\right),\quad (7)$$

where we have suppressed the $\mathbf{Q}$ dependence of $\gamma_{\perp,\parallel}$ and $\tilde{V}$. We next note that the temperature dependence of $\langle\delta\theta_\perp^2\rangle$ arises from two sources: (i) in the cotanh factor, which corresponds to thermal excitations of the plasmon mode and (ii) in the prefactor, through the temperature dependence of $J_\parallel$. I have numerically checked that the cotanh factor may be set to unity at low temperature for the cases of interest, since the in-plane plasmon energy is very large. Even when the c-axis Josephson plasmon energy is very low, as in case of BSCCO, the phase space for this low-energy excitation is small in the fluctuation integral and the characteristic plasmon energy is very high (see Fig. 1 of Ref. 5). The crossover associated with this factor then only leads to large power laws for temperatures above the c-axis plasmon scale. Further, $J_\perp/J_\parallel \ll 1$ means we can now safely set $J_\perp=0$ in the prefactor. Thus we are led to

$$\langle\delta\theta_\perp^2\rangle \approx \frac{2}{\sqrt{J_\parallel(T)}}\int_{-\pi}^{\pi}\frac{d^3\mathbf{Q}}{(2\pi)^3}\gamma_\perp(\mathbf{Q})\sqrt{\frac{\tilde{V}_\mathbf{Q}}{\gamma_\parallel(\mathbf{Q})}}$$

$$\equiv C_1\left[\left(\frac{2\pi e^2}{\epsilon_b\xi_0}\right)\frac{1}{J_\parallel(T)}\right]^{1/2},\quad (8)$$

where C_1 is a constant of order unity, which depends only on ξ_0/d_c, and which can be determined numerically for a given system. It is now easy to see that the linear T dependence of $J_\parallel$ directly leads to a linear T dependence of $\langle\delta\theta_\perp^2\rangle$ and hence of $J_\perp,\lambda_\perp$. To relate the slope of $\lambda_\perp$ to the slope of $J_\parallel$, we set $J_\parallel(T)=J_\parallel(0)-\alpha T$. Using Eq. (3) we then get

$$\frac{d\lambda_\perp}{dT} = C_1\left(\frac{\lambda_\perp(0)}{8}\right)\left(\frac{\alpha}{J_\parallel(0)}\right)\sqrt{\left(\frac{2\pi e^2}{\epsilon_b\xi_0}\right)\frac{1}{J_\parallel(0)}}.\quad (9)$$

B. Nonzero dissipation ($\bar{\sigma}\neq 0$)

In the presence of dissipation, we can again analyze the Matsubara summation in the fluctuation integral to obtain the asymptotic low temperature behavior, similar to our earlier analysis for the in-plane fluctuations.[6] In this case, for $\bar{\sigma} \gg 1$ and $T\to 0$, we find

$$\langle\delta\theta_\perp^2\rangle \approx \frac{8}{\bar{\sigma}}\ln\left(\frac{\bar{\sigma}}{2\pi}\sqrt{\frac{2\pi e^2}{\epsilon_b}\frac{1}{J_\parallel(T)}}\right)$$

$$+T^2\frac{2\bar{\sigma}}{3}\int_{-\pi}^{\pi}\frac{d^3\mathbf{Q}}{(2\pi)^3}\frac{\gamma_\perp\gamma_\parallel}{(\gamma_\parallel J_\parallel+\gamma_\perp J_\perp)^2}.\quad (10)$$

While the leading linear T dependence of $\langle\delta\theta_\perp^2\rangle$ arises from the linear T dependence of $J_\parallel$ in the first term in the above equation, a T^2 dependence arises, primarily from the second term for $\bar{\sigma}\gg 1$. With $J_\parallel(T)=J_\parallel(0)-\alpha T$, as before, the leading temperature dependence of the fluctuations is now given by

$$\langle\delta\theta_\perp^2\rangle(T)-\langle\delta\theta_\perp^2\rangle(0) = \frac{4T}{\bar{\sigma}}\frac{\alpha}{J_\parallel(0)}+C_2\frac{2\bar{\sigma}}{3}\left(\frac{T}{J_\parallel(0)}\right)^2,\quad (11)$$

where

$$C_2 = \int_{-\pi}^{\pi}\frac{d^3\mathbf{Q}}{(2\pi)^3}\frac{\gamma_\parallel\gamma_\perp}{\{\gamma_\parallel+[J_\perp(0)/J_\parallel(0)]\gamma_\perp\}^2}\quad (12)$$

is a constant depending on the anisotropy $J_\perp/J_\parallel$ at $T=0$, which may be easily determined numerically. This leads to a crossover scale $T_2=4\alpha J_\parallel(0)/(C_2\bar{\sigma}^2)$ beyond which temperature the linear temperature dependence crosses over to a T^2 behavior. For $T\ll T_2$, we find a linear T behavior in $\lambda_\perp(T)$ with a slope

$$\frac{d\lambda_\perp}{dT} = \frac{\alpha}{\bar{\sigma}J_\parallel(0)}\lambda_\perp(0).\quad (13)$$

However, since the linear T to quadratic T^2 crossover temperature depends very sensitively on the dissipation, large dissipation might lead to a T^2 behavior down to the lowest observed temperatures. We therefore make estimates of this temperature scale T_2 for BSCCO and YBCO$_{6.95}$; we then compare our results for the temperature dependence of $\lambda_\perp(T)$ with some experimental data on BSCCO and YBCO$_{6.95}$.

C. Comparison with experiments

I begin by fixing the parameters connected to the in-plane stiffness in the phase action, and $\lambda_\perp(0)$ since we are only interested in our predictions for the temperature dependence of $\lambda_\perp(T)$. I then compare the slope $d\lambda_\perp(T)/dT$ with existing experiments in BSCCO and YBCO systems.

For BSCCO, I use $\xi_0/a\approx 10$ and $d_c/a\approx 4$, and set the bilayer stiffness $J_\parallel(0)\approx 80$ meV with its linear T slope $\alpha\approx 0.8$ meV/K. These values correspond[28] to a penetration depth $\lambda_\parallel=2100$ Å with $d\lambda_\parallel/dT\approx 10$ Å/K. I then set $\lambda_\perp(0)\approx 150$ μm and choose a reasonable value ($\bar{\sigma}=20$) for the internal dissipation. This gives $C_1\approx 0.6$, $C_2\approx 1.35$, and $T_2\approx 60$ K below which we expect to see linear T behavior; the low temperature slope $d\lambda_\perp/dT\approx 0.075$ μm/K is somewhat smaller than some experimentally reported values[12,15] of ~ 0.25–0.3 μm/K.

For YBCO$_{6.95}$, I use $\xi_0/a\approx 10$, $d_c/a\approx 3.2$, and set the bilayer stiffness $J_\parallel(0)\approx 100$ meV with its slope $\alpha\approx 0.5$ meV/K. This leads to a penetration depth $\lambda_\parallel$

≈ 1600 Å with $d\lambda_\parallel/dT \approx 4$ Å/K in agreement with experiment.[1,14] Finally, I set $\lambda_\perp(0) \approx 1.1$ μm, and use a reasonable value $\bar{\sigma} = 10$ for the intrinsic dissipation. This gives $C_1 \approx 0.5$, $C_2 \approx 0.4$, and $T_2 \gtrsim T_c$, below which we expect to see linear T behavior. Thus, the linear T behavior from phase fluctuations is expected to persist over a larger temperature scale in YBCO$_{6.95}$. I then find the slope $d\lambda_\perp/dT \approx 5$ Å/K. This is somewhat smaller than some reports[13] of $d\lambda_\perp/dT \sim 15$–$20$ Å/K on this system. While this behavior was attributed[13] to the effect of the chains in YBCO, the role of phase fluctuations could clearly also be important. The fluctuations at $T = 0$ lead to a $\sim 30\%$ renormalization of the stiffness $J_\perp(0)$, in reasonable agreement with an earlier c-axis conductivity sum rule analysis[4] carried out for YBCO$_{6.6}$.

More generically, the experimentally observed c-axis penetration depth in BSCCO and YBCO$_{6.95}$ is reported to have a weaker temperature dependence (see Ref. 14, for instance), possibly $\sim T^2$ at low temperature. One possible reason for this discrepancy between the prediction of the quantum phase fluctuation model and the experiments could be the effects of disorder between the planes which deserves a more careful investigation.

V. CONCLUSIONS

We have seen that it is possible to directly probe the importance of in-plane quantum phase fluctuations in the high-T_c superconductors for $T \ll T_c$ through measurements of the in-plane superfluid stiffness or the transition temperature in the presence of external screening. I have also shown that an indirect measure of quantum phase fluctuations may be obtained from studying the temperature dependence of the c-axis penetration depth $\lambda_\perp(T)$. It is possible that disorder between the planes could affect our prediction, leading to a weaker temperature dependence more consistent with the experimental data; we leave this issue for future work. However, the linear T behavior might still be observable in some clean materials. Experimental tests of these would lead to a better understanding of the superconducting ground state and its low energy excitations.

ACKNOWLEDGMENTS

[1] W.N. Hardy, D.A. Bonn, D.C. Morgan, R. Liang, and K. Zhang, Phys. Rev. Lett. **70**, 3999 (1993).

[2] C. Panagopoulos and T. Xiang, Phys. Rev. Lett. **81**, 2336 (1998), and references therein.

[3] P.J. Hirschfeld and N. Goldenfeld, Phys. Rev. B **48**, 4219 (1993); P.A. Lee and X.G. Wen, Phys. Rev. Lett. **78**, 4111 (1997); A.J. Millis, S.M. Girvin, L.B. Ioffe, and A.I. Larkin, J. Phys. Chem. Solids **59**, 1742 (1998); J. Mesot, M.R. Norman, H. Ding, M. Randeria, J.C. Campuzano, A. Paramekanti, H.M. Fretwell, A. Kaminski, T. Takeuchi, T. Yokoya, T. Sato, T. Takahashi, T. Mochiku, and K. Kadowaki, Phys. Rev. Lett. **83**, 840 (1999).

[4] L.B. Ioffe and A.J. Millis, Science **285**, 1241 (1999); L.B. Ioffe and A.J. Millis, Phys. Rev. B **61**, 9077 (2000).

[5] A. Paramekanti, M. Randeria, T.V. Ramakrishnan, and S.S. Mandal, Phys. Rev. B **62**, 6786 (2000).

[6] L. Benfatto, S. Caprara, C. Castellani, A. Paramekanti, and M. Randeria, Phys. Rev. B **63**, 174513 (2001).

[7] T.R. Lemberger, A.A. Pesetski, and S.J. Turneaure, Phys. Rev. B **61**, 1483 (2000).

[8] S.J. Turneaure, T.R. Lemberger, and J.M. Graybeal, Phys. Rev. Lett. **84**, 987 (2000).

[9] E.W. Carlson, S.A. Kivelson, V.J. Emery, and E. Manousakis, Phys. Rev. Lett. **83**, 612 (1999).

[10] H-J. Kwon, A.T. Dorsey, and P.J. Hirschfeld, Phys. Rev. Lett. **86**, 3875 (2001).

[11] A.J. Rimberg, T.R. Ho, C. Kurdak, J. Clarke, K.L. Campman, and A.C. Gossard, Phys. Rev. Lett. **78**, 2632 (1997).

[12] T. Shibauchi, N. Katase, T. Tamegai, and K. Uchinokura, Physica C **264**, 227 (1996).

[13] C. Panagopoulos, J.R. Cooper, T. Xiang, G.B. Peacock, I. Gameson, and P.P. Edwards, Phys. Rev. Lett. **79**, 2320 (1997).

[14] K. Zhang, D.A. Bonn, S. Kamal, R. Liang, D.J. Baar, W.N. Hardy, D. Basov, and T. Timusk, Phys. Rev. Lett. **73**, 2484 (1994); D.A. Bonn, S. Kamal, A. Bonakdarpour, R. Liang, W.N. Hardy, C.C. Homes, D.N. Basov, and T. Timusk, Czech. J. Phys. **46**, 3195 (1996); A. Hosseini, S. Kamal, D.A. Bonn, R. Liang, and W.N. Hardy, Phys. Rev. Lett. **81**, 1298 (1998).

[15] D.V. Shovkun, M.R. Trunin, A.A. Zhukov, Yu.A. Nefyodov, H. Enriquez, N. Bontemps, A. Buzdin, M. Daumens, and T. Tamegai, JETP Lett. **71**, 92 (2000); H. Enriquez, N. Bontemps, A.A. Zhukov, D.V. Shovkun, M.R. Trunin, A. Buzdin, M. Daumens, and T. Tamegai, Phys. Rev. B **63**, 144525 (2001); M.R. Trunin, Yu.A. Nefyodov, D.V. Shovkun, A.A. Zhukov, N. Bontemps, A. Buzdin, M. Daumens, H. Enriquez, and T. Tamegai, cond-mat/0006371 (unpublished).

[16] D.M. Wood and D. Stroud, Phys. Rev. B **25**, 1600 (1982).

[17] In the numerics, the Matsubara sum in Eq. (4) is cutoff at finite n. While this affects the magnitude of quantum fluctuations at $T = 0$ to a small extent, the results for the temperature dependence are not sensitive to this cutoff.

[18] S. Chakravarty, G.-L. Ingold, S. Kivelson, and A. Luther, Phys. Rev. Lett. **56**, 2303 (1986); S. Chakravarty, G.-L. Ingold, S. Kivelson, and G. Zimanyi, Phys. Rev. B **37**, 3283 (1988).

[19] V.J. Emery and S.A. Kivelson, Phys. Rev. Lett. **74**, 3253 (1995).

[20] S. De Palo, C. Castellani, C. Di Castro, and B.K. Chakraverty, Phys. Rev. B **60**, 564 (1999).

[21] This difference arises from the fact that the external (bath) electrons have been treated in a gauge invariant fashion, while gauge invariance is not recovered for the superconductor unless the phase fluctuations are also integrated out. See Ref. 5 for a detailed discussion of this last point. This distinction between the two sources of dissipation was missed in earlier work (Ref. 19).

[22] The damping of phase fluctuations arises from the real part of the optical conductivity over a wide frequency range ~ 0–50 meV. Over this range, we use a reasonable value for the bilayer conductivity $\bar{\sigma} = 10$ for $YBCO_{6.95}$, corresponding (Ref. 29) to $\sigma \approx 2000$ $(\Omega \ cm)^{-1}$. We are not aware of conductivity data on BSCCO over a similar frequency range; however the $T \rightarrow 0$ conductivity at the highest terahertz frequencies (Ref. 30) ($\omega \sim 0.5$ meV) is not inconsistent with $\bar{\sigma} \approx 20$. We use this in our calculations for BSCCO, noting that the real dissipation is likely *smaller*. We have checked that an additional (large) low frequency Drude conductivity, experimentally observed (Ref. 30) for $\omega \lesssim 0.5$ meV, does not significantly affect our results.

[23] T. Xiang and J.M. Wheatley, Phys. Rev. Lett. **77**, 4632 (1996).

[24] R.A. Klemm and S.H. Liu, Phys. Rev. Lett. **74**, 2343 (1995).

[25] R.J. Radtke, V.N. Kostur, and K. Levin, Phys. Rev. B **53**, R522 (1996).

[26] S. Chakravarty, A. Sudbo, P.W. Anderson, and S. Strong, Science **261**, 337 (1994); O.K. Andersen, A.I. Liechtenstein, O. Jepsen, and F. Paulsen, J. Phys. Chem. Solids **56**, 1573 (1995); O.K. Andersen, O. Jepsen, A.I. Liechtenstein, and I.I. Mazin, Phys. Rev. B **49**, 4145 (1994).

[27] K. Tamasaku, Y. Nakamura, and S. Uchida, Phys. Rev. Lett. **69**, 1455 (1992); S. Uchida, K. Tamasaku, and S. Tajima, Phys. Rev. B **53**, 14 558 (1996).

[28] S.-F. Lee, D.C. Morgan, R.J. Ormeno, D.M. Broun, R.A. Doyle, J.R. Waldram, and K. Kadowaki, Phys. Rev. Lett. **77**, 735 (1996).

[29] D.N. Basov, B. Dabrowski, and T. Timusk, Phys. Rev. Lett. **81**, 2132 (1998).

[30] J. Corson, J. Orenstein, Seongshik Oh, J. O'Donnell, and J.N. Eckstein, Phys. Rev. Lett. **85**, 2569 (2000).