$SU_M(2)\times U_C(1)$ gauge symmetry in high-T_c superconductivity

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The square lattice structure of CuO_2 layers and the strongly correlated property of electrons indicate that the high-T_c superconductivity in cuprates can be described by an intrinsic $SO_M(5)$ coherent pairing theory in which a $SU_M(2)\times U_C(1)$ gauge symmetry is embedded. Besides the usual charge order, this $SU_M(2)\times U_C(1)$ gauge symmetry is also related to three new magnetic-charge orders—the local AF magnet, the local spin current, and the *d*-wave charge order. These magnetic-charge orders are completely determined by the $SO_M(5)$ coherent pairing state. The magnetic and charge fluctuations that characterize the low-energy excitations in cuprates are then described by this gauge symmetry. Thus, the coexistence of antiferromagnetism and superconductivity can be realized naturally in a unified framework.

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I. INTRODUCTION

The study of low-energy quantum fluctuations in strongly correlated systems is one of the most difficult problems in theoretical physics. In copper oxides, the low-energy properties of electrons are indeed the central issue in the study of the high-T_c superconducting mechanism. Up to date, there exist no proper skills and tools to deal with low-energy quantum fluctuations of strongly correlated electrons. However, solving a strongly correlated problem crucially depends on how to extract the relevant degrees of freedom that characterize low-energy excitations observed in experiments. Determining the low-energy degrees of freedom mainly relies on intrinsic symmetry breakings of the system. In this paper, I show that the low-lying magnetic properties observed over a large doping range ($0.05\leqslant\delta\leqslant0.25$) in cuprates[1–4] may be controlled by a $SU_M(2)\times U_C(1)$ gauge symmetry, here the subscripts M and C representing magnet and charge, respectively.

In accordance with experimental observations, high-T_c superconductivity (SC) arises as a consequence of hole (or electron) dopings from the parent copper-oxide compounds which are antiferromagnetic (AF) Mott insulators.[5] In the AF phase, which is very close to half-filling ($\delta\leqslant0.03$), the low-lying excitations are mainly the spin-density-wave (SDW) fluctuation with respect to the $SU_S(2)$ spin rotational symmetry.[6] In the optimal dopings ($0.15\leqslant\delta\leqslant0.3$), the *d*-wave SC (dSC) order[7] implies that the low-lying excitations should be dominated by phase fluctuations associated with the $U_C(1)$ charge symmetry, based on Anderson's resonant valence bond (RVB) theory.[8] In this picture, the AF and dSC phases are considered to be well separated. Between the AF and dSC phases, there is a pseudogap phase which may be described by an intrinsic $SU_C(2)$ gauge symmetry.[9,10]

However, the optimally doped $YBa_2Cu_3O_{6+\delta}$ in SC phase displays a sharp magnetic resonance centered at (π,π) in reciprocal space[1] which obviously cannot be explained by the breaking $SU_C(2)$ charge gauge symmetry. Zhang proposed[11] that the magnetic π resonance may imply the existence of a breaking SO(5) [denoted specifically hereafter as $SO_Z(5)$] superspin symmetry between the AF and dSC phases.[12] Furthermore, in the underdoping region, especially around $\delta\sim1/8$, neutron-scattering experiments to $La_{2-x}Sr_xCuO_4$ show clear evidence of incommensurate magnetic excitations disposed symmetrically about (π,π).[2] The discovery of incommensurate peaks brings another important issue, i.e., the possible existence of a stripe phase.[14] Very recently, many experiments confirmed that commensurate and incommensurate magnetic excitations exist in both the LSCO and YBCO copper oxides in the underdoping as well as the optimal doping regions,[3] and a weak magnetic ordering can coexist with the dSC phase in a certain doping range.[4] Such universal properties naturally lead one to ask whether there exists an intrinsic symmetry to underlying these low-lying magnetic and charge degrees of freedom over a large range of doped cuprates.

Based on the square lattice structure of CuO_2 layers and the strongly correlated property of electrons, I investigate various intrinsic dynamical symmetries in cuprates. I find that the low-energy physics of high-T_c cuprates may be described by an intrinsic $SO_M(5)$ coherent pairing theory in which a $SU_M(2)\times U_C(1)$ gauge symmetry is embedded. The $SO_M(5)$ coherent pairing state consists of the singlet *s*-wave and the *d*-wave pairs plus triplet π pairs so that it can describe the coexistence of antiferromagnetism and superconductivity. The magnetic and charge fluctuations that characterize the low-energy excitations in cuprates are then controlled by the $SU_M(2)\times U_C(1)$ gauge symmetry which is related to three new magnetic-charge orders—the local AF magnet, the local spin current, and the *d*-wave charge order—plus the usual charge order.

This paper is organized as follows. In Sec. II, I present a general theory of many-electron square lattice systems, based on the coherent-state theory of the dynamical group SO(8).[13] Then in Sec. III, I discuss the dominated low-lying degrees of freedom in cuprates. Also, I analyze various possible gauge symmetries and show that the best candidate for high-T_c superconductivity is the magnetic-charge mixed $SU_M(2)\times U_C(1)$ gauge symmetry embedded in the $SO_M(5)$ coherent pairing states. In Sec. IV, a conclusion is given and perspectives are discussed.

II. COHERENT-STATE MANY-BODY THEORY AND THE ASSOCIATED GAUGE SYMMETRY

The existence of an intrinsic symmetry in condensed matter systems depends not only on basic interacting properties of electrons, but also on crystal structures of the corresponding materials. In general, in terms of the 8-dim basis of charge, spin, and crystal wave vector (in the reduced first Brillouin zone), the noninteracting electrons in lattice space have a maximum U(8) symmetry.[15] For the high-T_c superconductors, the conductivity is mainly in the CuO_2 layers. In such a two-dimensional square lattice plane, the symmetry between $\mathbf{k}$ and $-\mathbf{k}$ reduces the general U(8) group to a smaller subgroup SO(8). The RVB $U_C(1)$ and $SU_C(2)$ charge gauge symmetries[8–10] as well as the $SO_Z(5)$ superspin symmetry[11] are all subgroups of SO(8). However, neither the RVB $U_C(1)$ and $SU_C(2)$ gauge symmetries nor the $SO_Z(5)$ superspin symmetry can encompass the low-energy physics of cuprates.

To be more explicit, I introduce a Nambu basis $\Psi_{\mathbf{k}}^\dagger = (\alpha_{\mathbf{k}}^\dagger, \alpha_{-\mathbf{k}})$, where $\alpha_{\mathbf{k}}^\dagger = (c_{\mathbf{k}\uparrow}^\dagger, c_{\mathbf{k}\downarrow}^\dagger, c_{\mathbf{k}+\mathbf{Q}\uparrow}^\dagger, c_{\mathbf{k}+\mathbf{Q}\uparrow}^\dagger)$ and $\alpha_{-\mathbf{k}} = (c_{-\mathbf{k}\uparrow}, c_{-\mathbf{k}\downarrow}, c_{-\mathbf{k}+\mathbf{Q}\uparrow}, c_{-\mathbf{k}+\mathbf{Q}\downarrow})$. The SO(8) is then a transformation group with respect to the Nambu basis $\Psi_{\mathbf{k}}$. It is generated by the following 28 composite operators:

$$\Psi_{\mathbf{k}}^\dagger \mathcal{O}_{SO(8)} \Psi_{\mathbf{k}} = \begin{pmatrix} \alpha_{\mathbf{k}}^\dagger \mathbf{b}_l \alpha_{\mathbf{k}} & \alpha_{\mathbf{k}}^\dagger \mathbf{\Delta}_i^\dagger \alpha_{-\mathbf{k}}^\dagger \\ \alpha_{-\mathbf{k}} \mathbf{\Delta}_i \alpha_{\mathbf{k}} & -\alpha_{-\mathbf{k}} \mathbf{b}_l^t \alpha_{-\mathbf{k}}^\dagger \end{pmatrix}, \tag{1}$$

where $\mathbf{k}$ is restricted in the reduced first Brillouin zone, $\mathbf{b}_l$ and its transposition $\mathbf{b}_l^t$ are 4×4 Hermitian matrices, and $\mathbf{\Delta}_i$ and its Hermitian conjugate $\mathbf{\Delta}_i^\dagger$ are 4×4 antisymmetric matrices. The 28 generators in Eq. (1) consists of 12 pair operators and 16 particle-hole operators. The 12 pair operators include the isotropic s-wave[16] (or the d_{xy}-wave[17]) pair denoted by Δ_s, the extended s-wave or $d_{x^2-y^2}$-wave pair Δ_d,[8,9] the quasispin η pair Δ_η,[18] and the three triplet π pairs $\mathbf{\Delta}_{\boldsymbol{\pi}}$,[12] plus their Hermitian conjugates. These pair operators can be represented by $\Delta_{i\mathbf{k}} = \alpha_{-\mathbf{k}} \mathbf{\Delta}_i \alpha_{\mathbf{k}}$ and $\Delta_{i\mathbf{k}}^\dagger = (\Delta_{i\mathbf{k}})^\dagger = \alpha_{\mathbf{k}}^\dagger \mathbf{\Delta}_i^\dagger \alpha_{-\mathbf{k}}^\dagger$, with $i=s,d,p,\boldsymbol{\pi}$, and $\mathbf{\Delta}_i = \frac{1}{2}(\gamma^z\gamma^x, i\gamma^5\gamma^y, -i\gamma^0\gamma^y, \gamma^x\gamma^z\boldsymbol{\gamma})$; here, the γ matrix is given in the standard Dirac representation in field theory:

$$\gamma^0 = \begin{pmatrix} I_2 & 0 \\ 0 & -I_2 \end{pmatrix}, \quad \boldsymbol{\gamma} = \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ -\boldsymbol{\sigma} & 0 \end{pmatrix}, \tag{2}$$

and $\gamma^5 = i\gamma^0\gamma^x\gamma^y\gamma^z$. The 16 particle-hole operators are the spin $\mathbf{S}$, the charge Q, the SDW $\mathbf{S}_{\mathbf{Q}}$, and the charge density wave (CDW) $C_{\mathbf{Q}}$ plus their d-wave partners, namely, the d-wave spin and charge operators denoted by $\mathbf{A}$ and Y, the spin current $\mathbf{J}_s$, and the charge current J_c, respectively. These 16 operators can be written explicitly by $B_{l\mathbf{k}} = \alpha_{\mathbf{k}}^\dagger \mathbf{b}_l \alpha_{\mathbf{k}} - \alpha_{-\mathbf{k}} \mathbf{b}_l^t \alpha_{-\mathbf{k}}^\dagger$, where $\mathbf{b}_l = \frac{1}{2}(I_4, \gamma^0, \boldsymbol{\gamma}/2i, \gamma^0\boldsymbol{\gamma}/2, \gamma^5, i\gamma^5\gamma^0, \boldsymbol{\gamma}\gamma^5/2, \gamma^5\gamma^0\boldsymbol{\gamma}/2)$ with $l = Q, Y, \mathbf{J}_s, \mathbf{S}_{\mathbf{Q}}, C_{\mathbf{Q}}, J_c, \mathbf{A}, \mathbf{S}$. Physically, the 28 generators of SO(8) correspond to 28 order parameters that encompass all possible low-energy degrees of freedom of a square lattice interacting Hamiltonian. However, because of the further constraint of some intrinsic symmetries, not all these 28 order parameters can independently coexist.

Explicitly, to determine the low-energy physics of a many-electron system, it is very useful to start with the many-body coherent-state theory.[19] Using the generalized coherent-state theory,[13,20] I can define a general quasiparticle picture in terms of the SO(8) Nambu basis as follows:

$$\alpha_{\mathbf{k}}|0\rangle = 0 \overset{SO(8)}{\rightarrow} \beta_{\mathbf{k}}|\Omega_{SO(8)}\rangle = 0. \tag{3}$$

The quasiparticle vacuum state (i.e., physical ground state) $|\Omega_{SO(8)}\rangle$ is given by

$$|\Omega_{SO(8)}\rangle = \Omega(\eta)|0\rangle, \tag{4}$$

where $\Omega(\eta) \in SO(8)/U(4)$, and it can be expressed as

$$\Omega(\eta) = \prod_{\mathbf{k}}{}' \exp\{\eta_s(\mathbf{k})\Delta_{s\mathbf{k}}^\dagger + \eta_d(\mathbf{k})\Delta_{d\mathbf{k}}^\dagger + \eta_p(\mathbf{k})\Delta_{p\mathbf{k}}^\dagger + \boldsymbol{\eta}_\pi(\mathbf{k})\cdot\mathbf{\Delta}_{\pi\mathbf{k}}^\dagger - \text{H.c.}\}. \tag{5}$$

In Eq. (5), the pairing wave functions $\eta_{s,d,p,\pi}(\mathbf{k})$ are generally *link-dependent* complex parameters with an additional constraint $\eta_{s,d,p,\pi}(\mathbf{k}) = \eta_{s,d,p,\pi}(-\mathbf{k})$ due to the parity symmetry.

The state $|\Omega_{SO(8)}\rangle$ is nothing but the SO(8)/U(4) coherent pairing state which is the underlying pairing state I proposed recently to describe high-T_c superconductivity.[22] As I have discussed in Ref. 22, $|\Omega_{SO(8)}\rangle$ consists of all electron pairs concerned in the study of superconductivity. These pairs can be classified according to the symmetric property of the pairing wave function under the transformation of $\mathbf{k}$ to $\mathbf{k}+\mathbf{Q}$ as follows: $\eta_s(\mathbf{k}) = \eta_s(\mathbf{k}+\mathbf{Q})$ represents the isotopic s-wave and d_{xy}-wave ($\sim \sin k_x \sin k_y$) singlet pairs, etc., $\eta_d(\mathbf{k}) = -\eta_d(\mathbf{k}+\mathbf{Q})$ describes the extended singlet pairs (including the extended s-wave $[\sim\gamma(\mathbf{k}) = \cos k_x + \cos k_y]$, the d-wave $[\sim d(\mathbf{k}) = \cos k_x - \cos k_y]$ and the $(s+id)$-wave ($\sim \cos k_x + i\cos k_y$) pairs, etc.), while $\eta_p(\mathbf{k}) = \eta_p(\mathbf{k}+\mathbf{Q})$ corresponds to the pseudospin pairs, and finally, $\boldsymbol{\eta}_\pi(\mathbf{k}) = -\boldsymbol{\eta}_\pi(\mathbf{k}+\mathbf{Q})$ describe the triplet π pairs.

Meanwhile, apart from a phase factor, $|\Omega_{SO(8)}\rangle$ is directly obtained by acting SO(8) on the trivial vacuum $|0\rangle$,[20] while the associated phase factor contains the freedom of $U_g(4)$ gauge transformations that describe quantum fluctuations of all the pairing wave functions $\eta_{s,d,p,\pi}(\mathbf{k})$. Explicitly, let $g(a)$ be a general SO(8) unitary transformation and $a =: \{a_i, i=1,\ldots,28\}$ are the corresponding transformation parameters. The group theory tells us[23] that $g(a)$ can be uniquely decomposed as $g(a) = \Omega(\eta)h(b)$, where $h(b)$ is a U(4) unitary transformation [generated by the 16 particle-hole operators in the SO(8) group] that keeps the trivial vacuum $|0\rangle$ invariant up to a phase factor, and $\Omega(\eta)$ is an element of the coset space SO(8)/U(4) defined explicitly by Eq. (5). Then,

$$g(a)|0\rangle = \Omega(\eta)h(b)|0\rangle = \Omega(\eta)|0\rangle e^{i\phi}. \tag{6}$$

Obviously, any further SO(8) unitary transformation can be reduced to a U(4) transformation in the coset space SO(8)/U(4) which is in one-to-one correspondence to the coherent pairing states $|\Omega_{SO(8)}\rangle$ (Ref. 13):

$$g(a')\Omega(\eta)|0\rangle=\Omega(\eta')|0\rangle e^{i\phi'}. \tag{7}$$

This indicates that the subgroup U(4) in the dynamical group SO(8) is a gauge group embedded in the coherent pairing state $|\Omega_{SO(8)}\rangle$. Once the coherent pairing state $|\Omega_{SO(8)}\rangle$ is specified, the gauge freedom is fixed. The nonzero expectation values of the U(4) generators in $|\Omega_{SO(8)}\rangle$ correspond to a spontaneous gauge symmetry breaking of the associated gauge degrees of freedom. Thus, this U(4) gauge symmetry can describe indeed all the low-energy excitations induced by the quantum fluctuations of the pairing wave functions $\eta_{s,d,p,\pi}(\mathbf{k})$.

On the other hand, from Eqs. (4) and (5), one can easily show that the quasiparticle operators $(\beta^\dagger_{\mathbf{k}},\beta_{\mathbf{k}})$ are determined by the corresponding SO(8) Bogoliubov transformation with respect to $|\Omega_{SO(8)}\rangle$,[21]

$$\begin{pmatrix}\beta_{\mathbf{k}}\\ \beta^\dagger_{-\mathbf{k}}\end{pmatrix}=\Omega(\eta)\begin{pmatrix}\alpha_{\mathbf{k}}\\ \alpha^\dagger_{-\mathbf{k}}\end{pmatrix}\Omega^{-1}(\eta)=\begin{pmatrix}W(\mathbf{k}) & -Z(\mathbf{k})\\ Z^\dagger(\mathbf{k}) & W^t(\mathbf{k})\end{pmatrix}\begin{pmatrix}\alpha_{\mathbf{k}}\\ \alpha^\dagger_{-\mathbf{k}}\end{pmatrix}. \tag{8}$$

It is also not difficult to find that the Bogoliubov transformation matrix is given by

$$Z(\mathbf{k})=\eta(\mathbf{k})\frac{\sin\sqrt{\eta^\dagger(\mathbf{k})\eta(\mathbf{k})}}{\sqrt{\eta^\dagger(\mathbf{k})\eta(\mathbf{k})}},$$

$$W(\mathbf{k})=\sqrt{I_4-Z(\mathbf{k})Z^\dagger(\mathbf{k})}, \tag{9}$$

with

$$\eta(\mathbf{k})=\begin{pmatrix}0 & \eta_1(\mathbf{k}) & \eta_2(\mathbf{k}) & \eta_3(\mathbf{k})\\ -\eta_1(\mathbf{k}) & 0 & \eta_4(\mathbf{k}) & \eta_5(\mathbf{k})\\ -\eta_2(\mathbf{k}) & -\eta_4(\mathbf{k}) & 0 & \eta_6(\mathbf{k})\\ -\eta_3(\mathbf{k}) & -\eta_5(\mathbf{k}) & -\eta_6(\mathbf{k}) & 0\end{pmatrix} \tag{10}$$

and the elements in $\eta(\mathbf{k})$ are related to the pairing wave functions $\eta_{s,d,p,\pi}(\mathbf{k})$ of Eq. (5) by $\eta_{1,6}(\mathbf{k})=\eta_s(\mathbf{k})\pm\eta_d(\mathbf{k})$, $\eta_{4,3}(\mathbf{k})=\eta^z_\pi(\mathbf{k})\pm\eta_p(\mathbf{k})$, $\eta_2(\mathbf{k})=\eta^-_\pi(\mathbf{k})$, and $\eta_5(\mathbf{k})=-\eta^+_\pi(\mathbf{k})$. Similarly, we can write the Bogoliubov transformation matrix $Z(\mathbf{k})$ as

$$Z(\mathbf{k})=\begin{pmatrix}0 & z_1(\mathbf{k}) & z_2(\mathbf{k}) & z_3(\mathbf{k})\\ -z_1(\mathbf{k}) & 0 & z_4(\mathbf{k}) & z_5(\mathbf{k})\\ -z_2(\mathbf{k}) & -z_4(\mathbf{k}) & 0 & z_6(\mathbf{k})\\ -z_3(\mathbf{k}) & -z_5(\mathbf{k}) & -z_6(\mathbf{k}) & 0\end{pmatrix}, \tag{11}$$

where $z_{1,6}(\mathbf{k})=z_s(\mathbf{k})\pm z_d(\mathbf{k})$, $z_{4,3}(\mathbf{k})=z^z_\pi(\mathbf{k})\pm z_p(\mathbf{k})$, $z_2(\mathbf{k})=z^-_\pi(\mathbf{k})$, and $z_5(\mathbf{k})=-z^+_\pi(\mathbf{k})$. This is another expression of the pairing wave functions, and they are in one-to-one correspondence to $\eta_{s,d,p,\pi}(\mathbf{k})$ of Eq. (5) by Eq. (9). Thus, either the pairing wave functions $\eta_{s,d,p,\pi}(\mathbf{k})$ or $z_{s,d,p,\pi}(\mathbf{k})$ can equivalently determine the low-energy properties of the many-electron square lattice systems. In practice, as one will see in the next section, it is more convenient to express physical observables in terms of the pairing wave functions $z_{s,d,p,\pi}(\mathbf{k})$. Meantime, the above discussion shows that the coherent-state theory provides a gauge theory realization of many-body quantum systems.

III. DOMINATED LOW-LYING DEGREES OF FREEDOM AND THE $SU_M(2)\times U_C(1)$ GAUGE SYMMETRY

Since high-T_c superconductors are obtained by doping the parent copper oxides which are AF insulators, to demonstrate how can this picture be realized by $|\Omega_{SO}(8)\rangle$ and what must the necessary low-lying degrees of freedom be involved, I should first check the AF order parameter and the hopping dynamics contained in $|\Omega_{SO(8)}\rangle$. Without loss generality, I define $\mathbf{z}_\pi(\mathbf{k})=z_\pi(\mathbf{k})\boldsymbol{\alpha}_{\mathbf{k}}$ where $\boldsymbol{\alpha}_{\mathbf{k}}$ is the normalized Néel spin order, $|\boldsymbol{\alpha}_{\mathbf{k}}|=1$. Then the AF order is given by the matrix element of $\mathbf{S}_{\mathbf{Q}}$ in $|\Omega_{SO(8)}\rangle$:

$$\mathbf{M}_{AF}\equiv\frac{1}{N}\langle\Omega_{SO(8)}|\frac{1}{2}\sum_{\mathbf{k}}c^\dagger_{\mathbf{k}\alpha}\boldsymbol{\sigma}_{\alpha\beta}c_{\mathbf{k}+\mathbf{Q}\beta}|\Omega_{SO(8)}\rangle$$

$$=\frac{2}{N}\sum_{\mathbf{k}}{}'\,[z_d(\mathbf{k})z^*_\pi(\mathbf{k})+z^*_d(\mathbf{k})z_\pi(\mathbf{k})]\boldsymbol{\alpha}_{\mathbf{k}}, \tag{12}$$

where N is the total number of lattice sites. Hoppings are described by $H_{t-t'}=\Sigma_{\mathbf{k}\sigma}\varepsilon_{t-t'}(\mathbf{k})c^\dagger_{\mathbf{k}\sigma}c_{\mathbf{k}\sigma}$, where $\varepsilon_{t-t'}(\mathbf{k})=-2t(\cos k_x+\cos k_y)-4t'\cos k_x\cos k_y$. For the leading hopping Hamiltonian, its expectation value in $|\Omega_{SO(8)}\rangle$ is

$$\langle H_t\rangle=\langle\Omega_{SO(8)}|\sum_{\mathbf{k}\sigma}\varepsilon_t(\mathbf{k})c^\dagger_{\mathbf{k}\sigma}c_{\mathbf{k}\sigma}|\Omega_{SO(8)}\rangle=4\sum_{\mathbf{k}}{}'\,\varepsilon_t(\mathbf{k})$$

$$\times[z_s(\mathbf{k})z^*_d(\mathbf{k})+z^*_s(\mathbf{k})z_d(\mathbf{k})]. \tag{13}$$

Equation (12) simply tells us that the AF state mixes the extended singlet pairs $\Delta_{d\mathbf{k}}$ and the triplet pairs $\Delta_{\pi\mathbf{k}}$, while Eq. (13) shows that the hopping requires the simultaneously presence of the singlet pairs of $\Delta_{s\mathbf{k}}$ and $\Delta_{d\mathbf{k}}$ when all electrons are paired in the low-energy state of Eq. (4). As a result, a realization of the AF to dSC phase transition via the state $|\Omega_{SO(8)}\rangle$ involves at least the singlet pairs of $\Delta_{d\mathbf{k}}$ and $\Delta_{d\mathbf{k}}$ plus the triplet pairs $\Delta_{\pi\mathbf{k}}$.

In order to dynamically determine the necessary low-lying degrees of freedom in the high T_c, one must start with the basic interactions of electrons in cuprates. It has been commonly believed that the Hubbard model serves as a paradigm for strongly correlated electrons on a lattice: $H=H_{t-t'}+U\Sigma_i n_{i\uparrow}n_{i\downarrow}$. In the space of SO(8), the leading contribution of the U term is

$$\left\langle\frac{H_U}{UN}\right\rangle=\frac{n^2}{4N^2}+\frac{1}{4}\mathcal{C}^2_{\mathbf{Q}}+|\Delta_s|^2+|\Delta_p|^2-\mathbf{M}^2_{AF}, \tag{14}$$

where n is the total number of electrons, $\mathcal{C}_{\mathbf{Q}}$ the CDW order parameter,

$$\mathcal{C}_{\mathbf{Q}}\equiv\frac{1}{N}\langle\Omega_{SO(8)}|\sum_{\mathbf{k}\sigma}c^\dagger_{\mathbf{k}\sigma}c_{\mathbf{k}+\mathbf{Q}\sigma}|\Omega_{SO(8)}\rangle$$

$$=\frac{4}{N}\sum_{\mathbf{k}}{}'\,[z_s(\mathbf{k})z^*_p(\mathbf{k})+z^*_s(\mathbf{k})z_p(\mathbf{k})], \tag{15}$$

and Δ_s and Δ_p are the order parameters of the isotropic s-wave pairing (or the d_{xy}-wave pair) and the quasispin η pairing, respectively. Equation (14) shows that for a positive U the Hubbard interaction suppresses the formation of the s-wave pairing and the η pairing as well as the CDW order but favors forming AF (Néel) order $\mathbf{M}_{\rm AF}$ in which all electrons are paired as the singlet pairs $\Delta_{d\mathbf{k}}$ mixed with the triplet pairs $\Delta_{\pi\mathbf{k}}$ [see Eq. (12)]. In contrast, for a negative U, the Hubbard interaction favors forming the s-wave and η pairs but suppresses the AF order. This manifests exactly the particle-hole symmetry with respect to the positive and negative U at half-filling where the hopping Hamiltonian has no contribution.

When cuprates are doped, the hopping Hamiltonian plays an important role. Equation (13) has shown that a nonvanishing of the leading hopping in a pairing state requires the presence of the isotropic s-wave (or d_{xy}-wave) singlet pairs even though such pairs are suppressed by the U term. The next leading hopping contribution $\langle H_{t'}\rangle = 4\Sigma'_{\mathbf{k}}[\varepsilon_{t'}(\mathbf{k})\Sigma_i|z_i(\mathbf{k})|^2]$ $(i=s,d,p,\pi)$ does not favor forming any specific pairing. Hence, the low-energy pairing state of the Hubbard model that gives the best kinetic energy should be the state $|\Omega_{\rm SO(8)}\rangle$ consisting only of the singlet pairs $\Delta_{s\mathbf{k}}$ and $\Delta_{d\mathbf{k}}$ plus the triplet π pairs $\Delta_{\pi\mathbf{k}}$ but no η pairs $\Delta_{p\mathbf{k}}$, i.e., $\eta_p(\mathbf{k})=0$ in Eq. (5).

Zhang's $\mathrm{SO}_Z(5)$ superspin theory[11] is built with the singlet pair Δ_d plus the triplet pairs Δ_π only. The corresponding coherent pairing state can be generally expressed as

$$
\begin{aligned}
|\Omega_{\rm SO(6)}\rangle &= \prod_{\mathbf{k}}{}' \exp\{\eta_d(\mathbf{k})\Delta^\dagger_{d\mathbf{k}} + \boldsymbol{\eta}_\pi(\mathbf{k})\cdot\boldsymbol{\Delta}^\dagger_{\pi\mathbf{k}} - \mathrm{H.c.}\}|0\rangle \\
&= \begin{cases} \prod_{\mathbf{k}}{}' \exp\{\boldsymbol{\eta}'_\pi(\mathbf{k})\cdot\boldsymbol{\Delta}^\dagger_{\pi\mathbf{k}} - \mathrm{H.c.}\}|\mathrm{AF}\rangle \\ \text{or} \\ \prod_{\mathbf{k}}{}' \exp\{\boldsymbol{\eta}''_\pi(\mathbf{k})\cdot\boldsymbol{\Delta}^\dagger_{\pi\mathbf{k}} - \mathrm{H.c.}\}|^{d\ \text{wave}}_{\rm BCS}\rangle \end{cases} \\
&\equiv |\mathrm{SO}_Z(5)\rangle,
\end{aligned} \tag{16}
$$

which gives a realization of the picture of how the π operator continuously rotates the AF state into a d-wave SC and vice versa. However, Eq. (13) shows that the $\mathrm{SO}_Z(5)$ theory cannot directly describe the hopping dynamics because of the lack of the isotropic s-wave (or d_{xy}-wave) type pairing. This is why the doping has to be addressed through a chemical potential in Zhang's $\mathrm{SO}_Z(5)$ theory. The importance of the singlet pairs $\Delta_{s\mathbf{k}}$ was also naively ignored in my previous consideration.[22]

On the other hand, if the triplet π pairs are not included, the state $|\Omega_{\rm SO(8)}\rangle$ is deduced to a $\mathrm{SO}_C(5)/\mathrm{SU}_C(2)\times\mathrm{U}_C(1)$ coherent pairing state:

$$
\begin{aligned}
|\Omega_{\mathrm{SO}_C(5)}\rangle = \prod_{\mathbf{k}}{}' \exp\{&\eta_s(\mathbf{k})\Delta^\dagger_{s\mathbf{k}} + \eta_d(\mathbf{k})\Delta^\dagger_{d\mathbf{k}} \\
&+ \eta_p(\mathbf{k})\Delta^\dagger_{p\mathbf{k}} - \mathrm{H.c.}\}|0\rangle.
\end{aligned} \tag{17}
$$

Here the $\mathrm{SO}_C(5)$ group is different from the $\mathrm{SO}_Z(5)$ group in Zhang's theory. The subgroup $\mathrm{SU}_C(2)\times\mathrm{U}_C(1)$ (generated by $\{C_Q, J_c, Y, Q\}$) is a gauge symmetry embedded in $|\Omega_{\mathrm{SO}_C(5)}\rangle$ that describes quantum fluctuations of the s- and d-wave pairs and the η pairs in terms of the CDW $C_{\mathbf{Q}}$, the charge current J_c, the d-wave type charge Y, and the usual charge Q. Note that in $|\Omega_{\mathrm{SO}_C(5)}\rangle$, the CDW order $\mathcal{C}_{\mathbf{Q}}$ must be accompanied by the charge current order $\mathcal{J}_c$ because of the constraint of the $\mathrm{SU}_C(2)$ gauge group. Meantime, the charge current order $\mathcal{J}_c$ is physically manifested by the staggered flux χ_s or the d-density-wave (DDW) order χ_d. Explicitly, these charge-oriented order parameters given in the state $|\Omega_{\mathrm{SO}_C(5)}\rangle$ are the same as that in the state $|\Omega_{\rm SO(8)}\rangle$:

$$
\begin{aligned}
\chi_s &\equiv \frac{1}{N}\langle\Omega_{\rm SO(8)}|\sum_{\mathbf{k}\sigma} \gamma(\mathbf{k}) c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}+\mathbf{Q}\sigma}|\Omega_{\rm SO(8)}\rangle \\
&= \frac{4i}{N}\sum_{\mathbf{k}}{}' \gamma(\mathbf{k})[z_p(\mathbf{k})z_d^*(\mathbf{k}) - z_p^*(\mathbf{k})z_d(\mathbf{k})],
\end{aligned} \tag{18}
$$

$$
\begin{aligned}
\chi_d &\equiv \frac{1}{N}\langle\Omega_{\rm SO(8)}|\sum_{\mathbf{k}\sigma} d(\mathbf{k}) c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}+\mathbf{Q}\sigma}|\Omega_{\rm SO(8)}\rangle \\
&= \frac{4i}{N}\sum_{\mathbf{k}}{}' d(\mathbf{k})[z_d(\mathbf{k})z_p^*(\mathbf{k}) - z_d^*(\mathbf{k})z_p(\mathbf{k})],
\end{aligned} \tag{19}
$$

respectively, and the CDW order parameter $\mathcal{C}_{\mathbf{Q}}$ has been calculated in Eq. (15). Hence, the coherent pairing state $|\Omega_{\mathrm{SO}_C(5)}\rangle$ covers the $\mathrm{SU}_C(2)$ RVB gauge theory[9,10] and the recently proposed DDW order.[24] The staggered flux χ_s and the DDW order χ_d are just two different representations of the charge current $\mathcal{J}_c$. Recently, the DDW state was proposed to be the observed weak magnetic order in the dSC state.[4,24] However, the DDW and staggered flux phase must coexist with the CDW [see Eq. (19)] if the s-wave and d-wave singlet pairs coexist. But the CDW has not been observed in the dSC state. Also $|\Omega_{\mathrm{SO}_C(5)}\rangle$ has no active spin degrees of freedom so that it cannot describe the observed AF magnetic peaks in doped cuprates.[3] Therefore, most likely there is no DDW state in cuprates based on the present theory.

From the above analysis, one can see that the low-lying degrees of freedom in cuprates are dominated by the singlet pairs of $\Delta_{s\mathbf{k}}$ and $\Delta_{d\mathbf{k}}$ plus the triplet pairs $\Delta_{\pi\mathbf{k}}$ and the associated particle-hole orders contained in the U(4) gauge symmetry. The low-energy ground state determined from the Hubbard model and the experiments is favorable to let $\eta_p(\mathbf{k})=0$ in Eq. (5). To highlight the low-lying degrees of freedom, I can further rewrite Eq. (4) [with $\eta_p(\mathbf{k})=0$] as

$$
|\Omega_{\rm SO(8)}\rangle_{\eta_p=0} = \exp\left\{\sum_{\mathbf{k}} \boldsymbol{\theta}(\mathbf{k})\cdot\mathbf{S}_{\mathbf{k}}\right\}|\Omega_{\mathrm{SO}_M(5)}\rangle, \tag{20}
$$

where $|\Omega_{\mathrm{SO}_M(5)}\rangle$ is a $\mathrm{SO}_M(5)/\mathrm{SU}_M(2)\times\mathrm{U}_C(1)$ coherent pairing state with the Néel spin order $\boldsymbol{\alpha}_{\mathbf{k}}$ being rotated to along the easy z axis:

$$
\begin{aligned}
|\Omega_{\mathrm{SO}_M(5)}\rangle = \prod_{\mathbf{k}}{}' \exp\{&\eta_s(\mathbf{k})\Delta^\dagger_{s\mathbf{k}} + \eta_d(\mathbf{k})\Delta^\dagger_{d\mathbf{k}} \\
&+ \eta^z_\pi(\mathbf{k})\Delta^\dagger_{\pi^z\mathbf{k}} - \mathrm{H.c.}\}|0\rangle.
\end{aligned} \tag{21}
$$

This corresponds to picking up a specific group state from $|\Omega_{SO(8)}\rangle$ that spontaneously breaks the spin rotational symmetry. The subgroup $SU_M(2)\times U_C(1)$ which is generated by $\{S^z_{\mathbf{Q}},J^z_s,Y,Q\}$ is the gauge symmetry embedded in $|\Omega_{SO_M(5)}\rangle$ that describes the quantum fluctuations of the s- and d-wave pairs and the π pairs.

The decomposition in Eq. (20) separates the quantum fluctuations of the spin rotational freedom from other low-energy degrees of freedom. This is very similar to the description of a rotor's intrinsic motion in terms of a rotating coordinates instead of the usual laboratory coordinates. Hence $|\Omega_{SO_M(5)}\rangle$ is called as an intrinsic pairing state of Eq. (20). Now, the dynamics of the Néel spin order $\boldsymbol{\alpha}_{\mathbf{k}}$ is described by $\boldsymbol{\theta}_{\mathbf{k}}$ which characterizes a continuous manifold of degenerate ground states in $|\Omega_{SO(8)}\rangle$ with regard to the $SU_S(2)$ spin rotational symmetry. Under the decomposition (20), the conventional SDW arising from spin fluctuations is determined by varying the orientation of $\boldsymbol{\theta}_{\mathbf{k}}$. It can be shown that the SDW does not explicitly depend on dopings and hoppings because both the hopping [see Eq. (13)] and the doping

$$\delta=1-\langle\Omega_{SO(8)}|\frac{1}{N}\sum_{\mathbf{k}\sigma}c^\dagger_\sigma(\mathbf{k})c_\sigma(\mathbf{k})|\Omega_{SO(8)}\rangle$$

$$=1-4\sum_{i\mathbf{k}}|z_s(\mathbf{k})|^2\quad(i=s,d,p,\pi)\tag{22}$$

are independent of $\boldsymbol{\alpha}_{\mathbf{k}}$ [and $\boldsymbol{\theta}(\mathbf{k})$]. I can thereby conclude that the observed magnetic excitations that linearly depend on dopings[3] cannot originate from the spin fluctuation of SDW.

In fact, it is not the Néel spin vector $\boldsymbol{\alpha}_{\mathbf{k}}$ but the local (short-range) staggered AF ordering [associated with the generator $S^z_{\mathbf{Qk}}$ in the $SU_M(2)\times U_C(1)$ gauge group]

$$\mathcal{M}_{AF}(\mathbf{k})\equiv\langle\Omega_{SO(8)}|S^z_{\mathbf{Qk}}|\Omega_{SO(8)}\rangle=z_d(\mathbf{k})z^*_\pi(\mathbf{k})+z^*_d(\mathbf{k})z_\pi(\mathbf{k})\tag{23}$$

that sensitively depends on dopings. Experimentally, no long-range AF order $\mathbf{M}_{AF}=(2/N)\Sigma'_{\mathbf{k}}\mathcal{M}_{AF}(\mathbf{k})\boldsymbol{\alpha}_{\mathbf{k}}$ [see Eq. (12)] is directly observed in doped cuprates because it is smeared out by the quantum fluctuation of the Néel order $\boldsymbol{\alpha}_{\mathbf{k}}$. But the local AF ordering $\mathcal{M}_{AF}(\mathbf{k})$ itself still exists in doped cuprates once the triplet π pairs mix with the d-wave singlet pairs in $|\Omega_{SO_M(5)}\rangle$. In other words, the local AF ordering $\mathcal{M}_{AF}(\mathbf{k})$ of Eq. (23) represents a local AF magnet. Furthermore, accompanied with the local AF magnet, there must also exist a local spin current $\mathcal{J}_s(\mathbf{k})$ by the $SU_M(2)\times U_C(1)$ gauge symmetry [the matrix element of another generator $J^z_{s\mathbf{k}}$ in $SU_M(2)\times U_C(1)$],

$$\mathcal{J}_s(\mathbf{k})\equiv\langle\Omega_{SO(8)}|J^z_{\mathbf{k}}|\Omega_{SO(8)}\rangle=i[z_s(\mathbf{k})z^*_\pi(\mathbf{k})-z^*_s(\mathbf{k})z_\pi(\mathbf{k})],\tag{24}$$

which hides in the spin current order $\vec{\mathcal{J}}_s=(2/N)\Sigma'_{\mathbf{k}}\mathcal{J}_s(\mathbf{k})\boldsymbol{\alpha}_{\mathbf{k}}$. Similar to the AF order $\mathbf{M}_{AF}$, a long-range spin current order $\vec{\mathcal{J}}_s$ cannot be observed in doped cuprates because it is also smeared out by quantum fluctuations of the Néel order $\boldsymbol{\alpha}_{\mathbf{k}}$. But the local spin current $\mathcal{J}_s(\mathbf{k})$ can be manifested in terms of a d-density-spin wave. Because of the magnetic properties of the local AF magnet $\mathcal{M}_{AF}(\mathbf{k})$ and the local spin current $\mathcal{J}_s(\mathbf{k})$, I argue that the observed AF magnetic excitations[3] and the short-range or fluctuating AF order[4] in both the pseudogap phase and the dSC phase should be the quantum fluctuation effects of $\mathcal{M}_{AF}(\mathbf{k})$ and $\mathcal{J}_s(\mathbf{k})$, rather than the CDW dynamics of the Néel spin field $\boldsymbol{\alpha}_{\mathbf{k}}$.

Besides, there are also other two possible orders in the $SU_M(2)\times U_C(1)$ gauge symmetry, a d-wave charge order $\mathcal{Y}$ and the usual charge order Q of SC states. The d-wave charge order $\mathcal{Y}$ is a new order parameter that has not been discussed in the literature,

$$\mathcal{Y}_d=\langle\Omega_{SO(8)}|\sum_{\mathbf{k}\sigma}d(\mathbf{k})c^\dagger_{\mathbf{k}\sigma}c_{\mathbf{k}\sigma}|\Omega_{SO(8)}\rangle$$

$$=\frac{4}{N}\sum_{\mathbf{k}}{}'\,d(\mathbf{k})[z_s(\mathbf{k})z^*_d(\mathbf{k})+z^*_s(\mathbf{k})z_d(\mathbf{k})].\tag{25}$$

I find that this order parameter can lower the superexchange interaction energy of the t-J model.[25]

The above analysis shows that the state $|\Omega_{SO_M(5)}\rangle$ is magnetic-charge codominated. It can simultaneously describe the coexistence of intrinsic AF orders and dSC order with real hopping processes. The associated $SU_M(2)\times U_C(1)$ gauge symmetry can dynamically determine the quantum fluctuations of the s- and d-wave singlet pairs and π triplet pairs in terms of the local AF magnet $\mathcal{M}_{AF}(\mathbf{k})$, the local spin current $\mathcal{J}_s(\mathbf{k})$, and the d-wave charge order $\mathcal{Y}$ plus the usual charge order Q. As a result, a $SU_M(2)\times U_C(1)$ low-energy gauge theory can be established based on the $SO_M(5)$ coherent pairing state when a practice Hamiltonian of strongly correlated electrons, such as the t-J model, is considered, similar to the construction of the nonlinear σ model for the Heisenberg antiferromagnets via the SO(3) spin coherent state.[26] The dynamics of the pairing amplitudes $z_s(\mathbf{k})$, $z_d(\mathbf{k})$, and $z_\pi(\mathbf{k})$ as a function of the doping δ and the temperature T can then be determined, and a quantitative comparison to experiments can be carried out. Further work on this subject is in progress.[25]

IV. CONCLUSIONS AND PERSPECTIVES

In conclusion, using coherent-state many-body theory, I have shown that the low-energy degrees of freedom in cuprates that compasses SDW, CDW, staggered flux order, DDW, and associated magnetic excitations and various pairing (including the s- and d-wave singlet and π triplet pairs) orders can be determined by the SO(8)/U(4) coherent pairing state $|\Omega_{SO(8)}\rangle$ [i.e., Eqs. (4) and (5)] in terms of the four pairing wave functions $z_{s,d,p,\pi}(\mathbf{k})$. The SO(8) coherent pairing state contains three different SO(5) subgroup pairing states

$$|\Omega_{SO(8)}\rangle\rightarrow\begin{cases}|\Omega_{SO_Z(5)}\rangle,\\|\Omega_{SO_C(5)}\rangle,\\|\Omega_{SO_M(5)}\rangle.\end{cases}$$

The first two states, given by Eqs. (16) and (17), cover Zhang's $SO_Z(5)$ theory and $SU_C(2)$ RVB gauge theory, respectively, and the last one, the intrinsic $SO_M(5)$ coherent pairing state defined by Eq. (21) is a new discovery.

The above three SO(5) coherent pairing states are generated by different pair operators, and they describe different physical properties of strongly correlated electrons. In doped cuprates, only the $SO_M(5)$ symmetry is capable of describing the low-lying magnetic excitations incorporating with the hopping dynamics. Specifically, all the three pairing states contains the d-wave superconducting order. However, they carry different gauge degrees of freedom associated with different quantum fluctuations:

$$\frac{SO(8)}{U(4)} \rightarrow \begin{cases} \dfrac{SO_Z(5)}{SU_S(2)\times U_C(1)}, \\ \dfrac{SO_C(5)}{SU_C(2)\times U_C(1)}, \\ \dfrac{SO_M(5)}{SU_M(2)\times U_C(1)}. \end{cases}$$

In Zhang's $SO_Z(5)$ theory, the gauge symmetry is represented by the spin rotational $SU_S(2)$ group plus the charge $U_C(1)$ group. It separately describes the SDW quantum fluctuation and the $U_C(1)$ charge fluctuation. However, these quantum fluctuations are not essential to the magnetic excitations. Also the doping can only be added artificially through a chemical potential in this theory. In $SU_C(2)$ RVB gauge theory, the $SU_C(2)$ gauge symmetry describes the CDW and the staggered flux phase as well as the recently proposed DDW order but it has no AF magnetic feature.

Only in the $SO_M(5)$ coherent pairing state, the $SU_M(2) \times U_C(1)$ gauge symmetry can simultaneously and dynamically address quantum fluctuations of the AF amplitude, hoppings as well as the d-wave pairing. This $SU_M(2)\times U_C(1)$ gauge symmetry is determined by three new orderings: the local AF magnet $\mathcal{M}_{AF}(\mathbf{k})$, the local spin current $\mathcal{J}_s(\mathbf{k})$, and the d-wave charge order $\mathcal{Y}$. These orderings have not been realized in the previous study of high-T_c theories. Because of the magnetic properties and charge properties of these three new orders, most likely it is this $SU_M(2)\times U_C(1)$ gauge symmetry that controls various magnetic excitations in the dCS state as well as in the pseudogap phase. Hence, the intrinsic $SO_M(5)$ coherent pairing state $|\Omega_{SO_M(5)}\rangle$ with the $SU_M(2)\times U_C(1)$ gauge symmetry is a good candidate to describe the observed low-energy degrees of freedom in cuprates. Certainly, experimentally examining the existence of the local AF magnet $\mathcal{M}_{AF}(\mathbf{k})$, the local spin current $\mathcal{J}_s(\mathbf{k})$, and the d-wave charge ordering $\mathcal{Y}_d$ is crucial to demonstrating the practical value of the present theory.

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