Chirality tunneling in a ferromagnetic spin chain

José A. Freire
Departamento de Física, Universidade Federal do Paraná, Caixa Postal 19081, Curitiba-PR 81531-990, Brazil
(Received 20 September 2001; published 1 March 2002)

The quantum-mechanical oscillations between two domain walls of opposite chirality in a ferromagnetic spin chain with easy and hard axis anisotropies are studied. The equations of motion of the instantons that connect the degenerate ground states in imaginary time are obtained and it is shown that there are two equivalent instantons that contribute to the expression of the splitting of the degenerate energy level. The instanton solutions and the energy splittings are obtained numerically for different values of the anisotropy parameters. It is found that the hard axis anisotropy inhibits chirality tunneling, in contrast to what happens in the single spin problem where the hard axis anisotropy favors tunneling. This behavior is explained as a crossover, driven by the increase of the hard axis anisotropy, from the ferromagnetic spin chain model to the sine-Gordon model.

DOI: 10.1103/PhysRevB.65.104436 PACS number(s): 75.45.+j, 75.60.Ch, 75.10.Jm

I. INTRODUCTION

Macroscopic quantum phenomena have been observed in molecular magnets such as Mn_{12}ac and Fe_8. The hysteresis loop in these systems show the magnetization relaxing via quantum mechanical tunneling of the effective spin $s=10$ through a barrier produced by the anisotropy field.[1] The theoretical description of the phenomena is usually made using a semiclassical approach and the instanton formalism.[2] It was pointed out by Loss and co-workers[3] that more than one instanton contributes to the expression of the energy splitting of the degenerate vacua at zero applied field, and that, for semi-integer s, these two instantons interfere destructively giving rise to a quench of the energy splitting. This was an interesting result since it provided an illustration of Kramer's degeneracy in terms of interfering instantons. The effect was observed[4] with Fe_8 with the aid of a field applied along the hard axis that introduces a phase difference in the contributions of the two instantons.[5]

As for extended spin systems, there have been reports on the observation of macroscopic quantum effects in thin rare earth magnetic films,[6] domain wall junctions,[7] and magnetic nanowires.[8] Calculations have used the semiclassical approximation and the instanton formalism,[2] as in the single spin problem, but with the added twist of reducing the many degrees of freedom of the spin system to a single collective coordinate.[9] An analog of the quantum oscillations of the molecular spin at zero field would be, in an extended system, the quantum oscillations of the chirality of a ferromagnetic domain wall.

Braun and Loss[10] have studied the dynamics of a one-dimensional ferromagnetic domain wall in a periodic potential and observed that the wall chirality plays the role of a gauge potential in the effective Lagrangian of the wall center coordinate. For a potential with the periodicity of the underlying spin chain the resulting energy bands for each chirality will be degenerate only if the spin quantum number s is integer (here I consider the case of a single chain). In the nondegenerate case (s semi-integer) an interesting effect arises when the states of opposite chirality are connected by tunneling. In this case the bands are mixed close to the points in the reciprocal space where band crossing occurs and a wall driven by an external field in k space will alternate its chirality as it moves. In this same work Braun and Loss calculated the level splitting due to chirality tunneling in the limit of weak transverse anisotropy (easy axis anisotropy much larger than hard axis anisotropy). Takagi and Tatara[11] have studied the effect, also using a collective coordinate approach and in the same limit of weak transverse anisotropy, and have discussed the role played by the pinning potential.

Here I discuss two other aspects of the problem. The first is the existence, as in the single spin case, of two instantons that contribute to the expression of the energy splitting, in Sec. III it is indicated under what conditions these solutions may interfere destructively. The other is the actual nature of the spin chain instanton and the dependence of the energy splitting on the anisotropy ratio, these are revealed by the numerical solution of the imaginary time field equations of motion in Sec. IV. In Sec. V the observed decrease in the energy splitting with increasing hard axis anisotropy is explained as a crossover from the ferromagnetic spin chain model to the sine-Gordon model and a contrast is made with the behavior found in the single spin tunneling problem. I conclude with general remarks on the parameter values that favor the observation of the quantum oscillations.

II. THE MODEL

Consider a spin chain with nearest neighbor Heisenberg ferromagnetic interaction, in addition to an easy x axis and a hard z-axis anisotropies. The Hamiltonian of the lattice model is

$$\mathcal{H}=-J\sum_i \mathbf{S}_{i+1}\cdot\mathbf{S}_i-\frac{K_e}{2}\sum_i S_{xi}^2+\frac{K_h}{2}\sum_i S_{zi}^2. \qquad (1)$$

I will deal with this model in the large-s and long wavelength limits. The spins are assumed to be locally aligned and the low energy excitations of the system correspond to smooth changes in the direction of the magnetization field. I introduce the fields θ and φ so that, in the continuum limit, the energy of a spin texture is (take x as the coordinate along the chain)

$$H[\theta,\varphi]=\int\left[\frac{\mathcal{J}}{2}(\theta'^2+\sin^2\theta\varphi'^2)-\frac{\mathcal{K}_e}{2}\sin^2\theta\cos^2\varphi+\frac{\mathcal{K}_h}{2}\cos^2\theta\right]dx. \quad (2)$$

The correspondence between the constants of the continuum model and those of the lattice model is $\mathcal{J}=Ja(\hbar s)^2$ and $\mathcal{K}=K(\hbar s)^2/a$. a is the lattice parameter and s is the individual spin quantum number.

The matrix element of the imaginary-time evolution operator between two coherent states can be written as a path integral using the complex fields z and $\bar{z}$ of the stereographic projection $z=\tan(\theta/2)e^{i\varphi}$ and $\bar{z}=\tan(\theta/2)e^{-i\varphi}$:

$$\langle z_f|e^{-\mathcal{H}T/\hbar}|z_i\rangle=\int \exp\{-S_{\rm E}[z,\bar{z}]/\hbar\}\mathcal{D}[z,\bar{z}]. \quad (3)$$

The Euclidean action is

$$S_{\rm E}[z,\bar{z}]=\frac{\hbar s}{a}\int_0^T\int\frac{\bar{z}\dot{z}-z\dot{\bar{z}}}{1+z\bar{z}}dx\,d\tau+\int_0^T H[z,\bar{z}]d\tau, \quad (4)$$

where the energy of a field configuration is now rewritten as

$$H[z,\bar{z}]=\int\left[\frac{\mathcal{J}}{2}\frac{4z'\bar{z}'}{(1+z\bar{z})^2}-\frac{\mathcal{K}_e}{2}\frac{(z+\bar{z})^2}{(1+z\bar{z})^2}+\frac{\mathcal{K}_h}{2}\frac{(1-z\bar{z})^2}{(1+z\bar{z})^2}\right]dx. \quad (5)$$

The first integral in the action is the Wess-Zumino term coming from the overlap of spin coherent states.[12] In terms of the original θ and φ fields it reads

$$S_{\rm WZ}[\theta,\varphi]=i\frac{\hbar s}{a}\int_0^T\int[(1-\cos\theta)\dot{\varphi}]dx\,d\tau. \quad (6)$$

As for the boundary conditions on the fields one has[13]

$$z(x,0)=z_i(x),\qquad \bar{z}(x,T)=z_f^*(x). \quad (7)$$

In looking for paths that extremize the action above one often encounters solutions such that $z(x,T)\neq z_f(x)$ and $\bar{z}(x,0)\neq z_i^*(x)$. In such cases a boundary term in the action, omitted in Eq. (4), play an important role.[14] This term reads

$$(S_{\rm E})_{\rm bound}=-\frac{\hbar s}{a}\int\left[\ln\left(\frac{1+z_i(x)\bar{z}(x,0)}{1+z_i(x)z_i^*(x)}\right)+\ln\left(\frac{1+z(x,T)z_f^*(x)}{1+z_f(x)z_f^*(x)}\right)\right]dx. \quad (8)$$

Domain walls of opposite chiralities are degenerate extrema of the energy functional. For walls centered at the origin and for a chain that extends over the entire x axis, these states are

$$z_+(x)=n[\tanh(x/\lambda)+i\,\mathrm{sech}(x/\lambda)], \quad (9)$$

$$z_-(x)=n[\tanh(x/\lambda)-i\,\mathrm{sech}(x/\lambda)]. \quad (10)$$

$\lambda=\sqrt{\mathcal{J}/\mathcal{K}_e}$ is the natural length in the problem that measures the size of the wall, $n=\pm1$ is the wall charge and the inner signal determines the wall chirality. Both walls are confined to the $\theta=\pi/2$ plane. In what follows I will consider finite chains with ferromagnetic boundary conditions at the chain ends, and it will be assumed that the wall is pinned, due to some inhomogeneity in the anisotropy constants, at an arbitrary point away from the ends. The fields $z_\pm$ will differ from the ones above but I will assume that the pinning potential does not move the spins away from the $\theta=\pi/2$ plane, so that $|z_\pm|=1$ and $z_+=z_-^*$.

To study the problem of macroscopic quantum coherence (MQC) between these states one computes the matrix element $\langle z_+|e^{-\mathcal{H}T/\hbar}|z_-\rangle$ using the saddle point approximation. The problem is analogous to a particle in a double well potential. The solutions that extremize the Euclidean action, called instantons, connect the two degenerate states in imaginary time.[15]

The instanton equations of motion are

$$\dot{z}=+\left[z''-\frac{2\bar{z}(z')^2}{(1+z\bar{z})}+\frac{z+\bar{z}-z^2\bar{z}-z^3+2\alpha z(1-z\bar{z})}{2(1+z\bar{z})}\right], \quad (11)$$

$$\dot{\bar{z}}=-[z\leftrightarrow\bar{z}]. \quad (12)$$

Here I use $\sqrt{\mathcal{J}\mathcal{K}_e}$ as unit of energy, λ as unit of length and $\hbar s/a\mathcal{K}_e$ as unit of time. In these units the only parameter left is the ratio $\alpha=\mathcal{K}_h/\mathcal{K}_e$.

One must solve these equations using as boundary conditions $z(x,0)=z_-(x)$ and $\bar{z}(x,T)=z_+^*(x)=z_-(x)$. This constitutes a boundary value problem for two independent complex fields since $\bar{z}\neq z^*$, meaning that the fields θ and φ, at the saddle point of the path integral (3), are complex.

The instanton solution allows one to determine the splitting of the degenerate energy level. This is obtained, in the limit $T\rightarrow\infty$, from value of the Wess-Zumino part of the Euclidean action

$$\Delta\propto\exp\left[-\frac{s\lambda}{a}\int\int\frac{\bar{z}\dot{z}-z\dot{\bar{z}}}{1+z\bar{z}}dx\,d\tau\right]. \quad (13)$$

The boundary term, Eq. (8), makes no contribution since, in the limit of large T, z becomes continuous at $\tau=T$ and $\bar{z}$ becomes continuous at $\tau=0$, meaning that $z(x,T)\rightarrow z_+(x)$ and $\bar{z}(x,0)\rightarrow z_-^*(x)$. The double integral above is in terms of dimensionless variables, the importance of quantum effects is measured by the value of s times the number of spins in

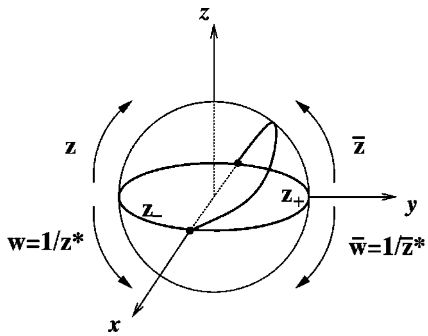

FIG. 1. The instanton $z(x,\tau)$ viewed as a line in the spin sphere that evolves from z_- to z_+ over the north pole. The instanton $w=1/z^*$ is the instanton that goes over the south pole. The field $\bar{z}$ is obtained from z through the relation $\bar{z}(x,\tau)=z(x,-\tau)$ (same for w and $\bar{w}$).

the wall region, λ/a. This number represents the spins that actually participate in the tunneling event. I shall be interested in the dependence of Δ on the anisotropy parameter α defined above.

III. TWO INSTANTONS

From the form of the equations of motion (11) and (12), and the assumption that $|z_\pm|=1$, it is easy to show that if $\{z,\bar{z}\}$ are solutions then $\{w=1/z^*,\bar{w}=1/\bar{z}^*\}$ are also solutions. Figure 1 shows these solutions projected in the spin sphere. From the form of the Wess-Zumino action, Eq. (13), one finds the following relation between the two actions:

$$S_{\rm WZ}[w,\bar{w}]=S^*_{\rm WZ}[z,\bar{z}]+i\,\frac{2\hbar s\lambda}{a}\int[\varphi(x,T)-\varphi(x,0)]dx. \tag{14}$$

The initial and final values of the φ field are obtained from $z(x,0)=z_-=e^{i\varphi_-}$ and $z(x,T)=z_+=e^{i\varphi_+}$. For a general φ_+ there is not much to be said about the integral above. If, however, the wall is symmetrically pinned at the chain center, one obtains

$$\int_0^{L/\lambda}[\varphi_+(x)-\varphi_-(x)]dx=-\frac{\pi L}{\lambda}\quad(\mathrm{mod}\;2\pi L/\lambda). \tag{15}$$

Adding the two contributions to the energy splitting gives

$$\Delta\propto K_z e^{-S_{\rm WZ}[z,\bar{z}]/\hbar}+K_w e^{-i\Delta\varphi}e^{-S^*_{\rm WZ}[z,\bar{z}]/\hbar}, \tag{16}$$

where K_w is the determinant of the fluctuations above the $\{w,\bar{w}\}$ solution (same for K_z) and $\Delta\varphi$ is the term proportional to the φ field integral in Eq. (14). In the appendix it is shown that the determinants are related, in the general case, by complex conjugation, $K_z=K_w^*$. In the very particular case of walls symmetrically pinned at the chain center, the boundary conditions on the instanton fields imply that $z(x,\tau)=-z^*(-x,\tau)$ (same for $\bar{z}$), leading to a purely real $S_{\rm WZ}[z,\bar{z}]$ (if $\{z,\bar{z}\}$ is the pair that goes over the north pole in Fig. 1). Moreover, in the appendix it is shown that in this case the determinant K_z is also real. These results, combined with the special value of the integral in Eq. (15), give the following energy splitting for walls symmetrically pinned:

$$\Delta_{\rm sym}\propto K_z e^{-S_{\rm WZ}[z,\bar{z}]/\hbar}(1+e^{i2\pi Ns}), \tag{17}$$

where $N=L/a$ is the total number of spins in the chain. The phase factor is $+1$ if Ns is integer and -1 if Ns is semi-integer. In the later case there is a quenching of the energy splitting due to instanton interference.

This effect is analogous to the destructive interference between instantons in the single spin problem[3] of MQC between the states $|S_x=\pm s\rangle$ in a Hamiltonian containing an easy x axis and a hard z axis. In that case, when the spin quantum number s is semi-integer, the energy splitting is zero and the degeneracy of the equivalent states is not removed, as it should not be by Kramer's theorem since the degenerate states are related by time reversal. However, in the case of the MQC between two symmetrically pinned walls, the states z_+ and z_- are not related by time reversal, the quenching of the energy splitting is accidental and cannot be attributed to Kramer's theorem.

A numerical calculation was performed to solve the instanton equations aimed at finding the nature of these spin instantons and the dependence of the Wess-Zumino action on the anisotropy parameter α. The study was restricted to walls symmetrically pinned at the chain center, thus yielding a real $S_{\rm WZ}$. In real systems the walls are pinned at random positions along the chain and this produces imaginary parts in the action and in the determinant of fluctuations. Since the energy splitting must be taken as the modulus of Δ in Eq. (16), it is the real part of $S_{\rm WZ}$ and the modulus of K_z that will play the major role,

$$\Delta\propto|K_z|e^{-S^R_{\rm WZ}[z,\bar{z}]/\hbar}. \tag{18}$$

The imaginary part of $S_{\rm WZ}$, the phase of K_z and $\Delta\varphi$ will only contribute with a numerical factor of order unity. One expects that the value of the real part of the action should be relatively insensitive to the actual location of the pinning site as long as it is away from the chain ends by a distance much greater than λ. This is because the existence of a larger ferromagnetic region in one side of the pinning site than in the other does not change the inhomogeneous part of the instanton, which is what determines the magnitude of the action.

IV. THE NUMERICAL SOLUTION

The numerical method employed to solve Eqs. (11) and (12) is the same used in Ref. 16 to solve the imaginary time equations that govern the quantum nucleation of a phase slip in a one-dimensional model of a superfluid. As in that case one has two independent complex fields $z(x,\tau)$ and $\bar{z}(x,\tau)$, the first with its initial value fixed, equal to $z_-(x)$, the other with its final value fixed, equal to $z_+^*(x)=z_-(x)$.

I used a space-time grid and wrote the field equations using finite differences. A typical grid had 50 points in the space coordinate axis, with a uniform grid spacing of $dx=0.03$ (in units of λ) and 40 points in the imaginary time

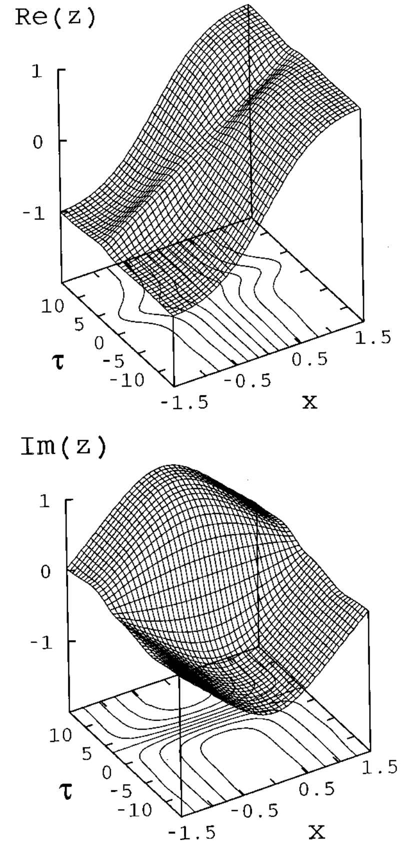

FIG. 2. The surfaces of the real and imaginary parts of $z(x,\tau)$ for $\alpha=0.6$. The field starts with negative chirality, as z_-, and ends with positive chirality, as z_+. The isolines run from -0.8 to 0.8 in steps of 0.2. x is in units of λ and τ is in units of $\hbar s/a\mathcal{K}_e$.

axis, with a grid spacing that ranged from $d\tau=0.80-0.25$ (in units of $\hbar s/a\mathcal{K}_e$). The choice of the chain length, $L=3\lambda$, was dictated by computational convenience and serves the purpose of illustrating the behavior of the instantons to be found in larger chains.

In principle one would have to work in the range $-L/2\leqslant x\leqslant L/2$ and $-T/2\leqslant\tau\leqslant T/2$, however some symmetries allow us to restrict our attention to one fourth of this domain. The fields $z_-(x)$ and $z_+(x)$ used as temporal boundary conditions are stationary solutions of Eqs. (11) and (12). They had to be obtained numerically and are the finite chain analogs of Eqs. (9) and (10). Ferromagnetic boundary conditions were used at the ends of the chain. The instantons retain the symmetry of the chirality states upon spatial inversion, $z(x,\tau)=-z^*(-x,\tau)$ (same for $\bar{z}$).

Another existing symmetry is $z(x,\tau)=\bar{z}(x,-\tau)$, that follows, even for finite T, from Eqs. (11) and (12) and the boundary conditions used. These two symmetries allowed us to restricted ourselves to the range $0\leqslant x\leqslant L/2$ and $-T/2\leqslant\tau\leqslant 0$.

The use of a finite T, when in fact the instanton one is seeking only exists in the limit $T\rightarrow\infty$, is justified since the temporal dependence of the instantons show that they have a sharp structure close to $\tau=0$ connected to the degenerate states by smooth tails. This implies that the main contribution to the value of S_{WZ} comes from the central portion of

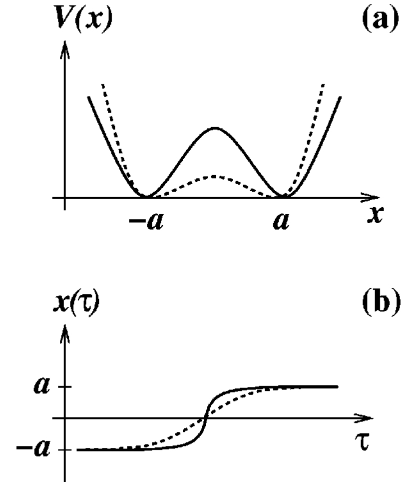

FIG. 3. (a) Two double wells with the same stationary states at $x=\pm a$ but with different barriers. (b) An illustration of the corresponding instantons. The potential with a larger barrier (solid line) has an instanton with a smaller "imaginary tunneling time."

the instanton [recall the dependence of S_{WZ} on the time derivative of the fields, see Eq. (4)].

The numerical algorithm used was a combination of Newton's method[17] and Powell's hybrid method[18] to enlarge the radius of convergence. An initial trial state was given and a sequence of steps was taken to bring the equations to zero. For the typical grid size one has 4000 complex variables and equations.

Figure 2 shows an instanton in the (x,τ) plane. In this figure the z field is shown evolving from z_- to z_+, the field $\bar{z}$ is not represented since it can be obtained from the relation $\bar{z}(x,\tau)=z(x,-\tau)$. Upon changing the value of the anisotropy parameter the instanton retains its general form but the "imaginary tunneling time" (the imaginary time size of the nonuniform central portion of the instanton) decreases with increasing α. This effect is analogous to what happens in the tunneling of a particle between the minima of a symmetric double well. In this case the instanton is the zero-energy solution of Newton's equation in the inverted potential. As the barrier increases the "imaginary tunneling time" (defined as above by the size of the transition region from one minima to the other) decreases. This is illustrated in Fig. 3. One should view the two chirality states as analogous to the equilibrium positions $x=\pm a$ and the instanton of Fig. 2 as the imaginary time trajectory $x(\tau)$.

The Euclidean action, being dependent on the time derivative of the instanton (also true in the quantum-mechanical analog), increases in value with decreasing "imaginary tunneling time." This leads to a smaller value of the energy splitting.

I performed the calculation for several values of the anisotropy ratio α. The dependence of the Wess-Zumino action on α is shown in Fig. 4. There one observes an increase of the action with increasing α, the line represents the power-law fit $S_{\mathrm{WZ}}=4.7(\hbar s\lambda/a)\alpha^{0.7}$. Values of α larger than 0.65 were not obtained due to convergence problems in the Newton method employed.

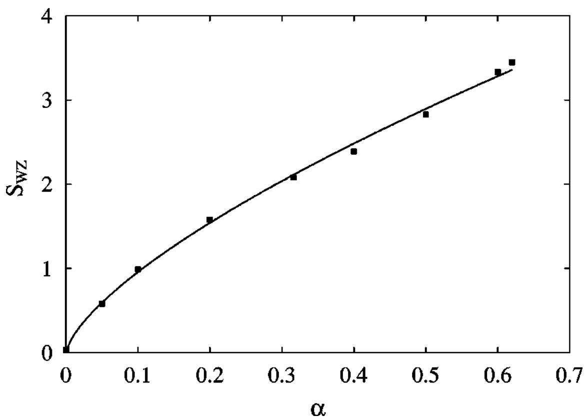

FIG. 4. The Wess-Zumino action for the instanton solution in units of $\hbar s\lambda/a$ versus $\alpha=\mathcal{K}_h/\mathcal{K}_e$. The solid line corresponds to $S_{\rm WZ}=4.7\alpha^{0.7}$.

In Ref. 10 Braun and Loss have found, in the limit of small α, $S_{\rm WZ}\sim\alpha^{0.5}$. The slight difference in the exponents may be due to their use of a collective coordinate approach and to the fact that their analysis is not applicable in the whole range of values of α studied here.

The lesson from the dependence of the instantons and of the action on α is that the hard axis anisotropy *inhibits* chirality tunneling in the same way that a larger barrier inhibits tunneling in the double well potential. This is in contrast to the role played by K_h in the single spin tunneling problem. This is discussed in the next section.

V. THE LARGE α LIMIT

For a single spin in an anisotropy field of the type used here, see Eq. (1),

$$\mathcal{H}=-\frac{K_e}{2}S_x^2+\frac{K_h}{2}S_z^2, \tag{19}$$

one observes that a large hard axis anisotropy *favors* spin tunneling between the states $|S_x=\pm s\rangle$. To understand why the single spin behaves so differently from the ferromagnetic chain I analyze the tunneling process in the large-s limit using θ and φ as coordinates. The Euclidean action analog to Eq. (4) reads

$$S_{\rm E}[\theta,\varphi]=\int_0^T\left[i\hbar s(1-\cos\theta)\dot{\varphi}-\frac{K_e\hbar^2s^2}{2}\sin^2\theta\cos^2\varphi+\frac{K_h\hbar^2s^2}{2}\cos^2\theta\right]d\tau. \tag{20}$$

In the limit of large $\alpha=K_h/K_e$ one may integrate out the θ variable to obtain (here time is in units of $1/K_e\hbar s$):

$$S_{\rm E}[\varphi]=\hbar s\int_0^T\left[i\dot{\varphi}+\frac{\dot{\varphi}^2}{2\alpha}-\frac{1}{2}\cos^2\varphi\right]d\tau. \tag{21}$$

The first term is responsible for the instanton interference,[3] the remaining terms are the Euclidean action of a particle of dimensionless mass α^{-1} in a double well potential independent of the anisotropy. Increasing α makes the particle mass smaller and has the effect of enhancing quantum effects and tunneling thereby. One should also note that if K_h were exactly zero the S_x operator would commute with the Hamiltonian and one could not have transitions between its two eigenvectors.

One can make a similar analysis of the chirality tunneling problem. To compute the matrix element $\langle z_+|e^{-\mathcal{H}T/\hbar}|z_-\rangle$ in the limit of large α I chose to work with the original θ and φ fields. In terms of these the Euclidean action is (x and τ are dimensionless):

$$S_E[\theta,\varphi]=\frac{\hbar s\lambda}{a}\int\int\left[i(1-\cos\theta)\dot{\varphi}+\frac{1}{2}(\theta'^2+\sin^2\theta\varphi'^2)-\frac{1}{2}\sin^2\theta\cos^2\varphi+\frac{\alpha}{2}\cos^2\theta\right]dx\,d\tau. \tag{22}$$

When $\alpha\gg1$ one can make an expansion $\theta=\pi/2+\delta\theta$ and keep only $\alpha\delta\theta^2/2$ among the terms of $o(\delta\theta^2)$. After integrating out the field $\delta\theta$ one gets the following effective action:

$$S_{\rm E}[\varphi]=\frac{\hbar s\lambda}{a}\int\int\left[i\dot{\varphi}+\frac{\varphi'^2}{2}+\frac{\dot{\varphi}^2}{2\alpha}-\frac{\cos^2\varphi}{2}\right]dx\,d\tau. \tag{23}$$

This is the Euclidean action of the Sine-Gordon model,[19] with an extra time derivative term already derived in Ref. 10. The two states of opposite chirality are also stationary solitons or kinks of the sine-Gordon model. They are solutions of

$$\varphi''=\cos\varphi\sin\varphi, \tag{24}$$

and correspond to the fields $\varphi_\pm$ of Sec. III ($z_\pm=e^{i\varphi_\pm}$). The two fields, $\varphi_+(x)$ and $\varphi_-(x)$, have different topological charges from the point of view of the sine-Gordon model, $\varphi_+:\pi\to0$ and $\varphi_-:-\pi\to0$. Since topological charge is a conserved quantity[19] tunneling between these objects is forbidden. Therefore the observed decrease of the chirality tunneling rate with increasing α can be understood as a crossover from the ferromagnetic spin model to the sine-Gordon model.

VI. CONCLUSION

The problem of chirality tunneling in a ferromagnetic domain wall was treated with the instanton formalism. A chain geometry was used which implies that the results obtained are applicable to three-dimensional magnets as long as the domain walls form a planar boundary region. Assuming uniformity in the direction perpendicular to the chains, one may use the above results after multiplying the argument of the WKB exponent by $N_{\rm ch}$, the number of chains in the cross sectional plane.

A comparison was made with the problem of a single spin tunneling in an anisotropic environment described by an easy and a hard axis. In both cases a pair of instantons contribute to the tunneling rate and under certain circumstances interfere leading to a quench of the energy splitting. In the single spin case the interference depends solely on the spin quantum number whereas in the case of the chain the interference is not of topological origin and happens only for a very special type of degenerate chirality states, one where the wall is pinned exactly at the chain center. Differently from the single spin problem the energy splitting *decreases* with increasing hard axis anisotropy, this was explained as a crossover from the ferromagnetic spin chain model to the sine-Gordon model.

The parameter that controls the size of the WKB tunneling exponent is $s\lambda/a$, the number of spins in the central portion of the wall times the individual spin quantum number. This indicates that, in addition to a small hard axis anisotropy, a hard ferromagnet, one with a small value of $\lambda = \sqrt{\mathcal{J}/\mathcal{K}_e}$, also favors the observation of the MQC effect.

The calculation presented here ignored the translational degrees of freedom of the wall, this assumed the presence of a strong pinning potential, which in fact also favors the observation of MQC.[11]

The oscillations of chirality could in principle be probed with a magnetic field in the easy plane, perpendicular to the easy axis. Very similar to the case of nanomagnets a resonance in the ac susceptibility would point to a MQC effect.[20] As pointed out in Ref. 10, a static field along the hard axis can tune the barrier height that separates the two wall chiralities and make the effect more easy to observe.

In comparison with the same problem in an antiferromagnetic spin chain[21] one observes that the parameter that controls the WKB exponent does not involve λ there, being simply proportional to s, indicating that antiferromagnetic chains are better suited to the observation of MQC than ferromagnetic chains. An interesting application of the approach followed here would be to numerically solve the antiferromagnetic spin chain instanton equations and contrast the results with the qualitative description put forth in Ref. 21.

ACKNOWLEDGMENTS

The author wishes to thank Daniel P. Arovas for fruitful discussions and FUNPAR for financial support.

APPENDIX: THE DETERMINANT OF FLUCTUATIONS

Here it is shown that the determinant of fluctuations about the solutions $\{z,\overline{z}\}$ and $\{w=1/z^*,\overline{w}=1/\overline{z^*}\}$ are related by complex conjugation. First note that if $\{\theta,\varphi\}$ correspond to the solution $\{z,\overline{z}\}$, then $\{\pi-\theta^*,\varphi^*\}$ correspond to the solution $\{1/z^*,1/\overline{z^*}\}$.

Insert $\theta+\xi$ and $\varphi+\eta$ in the expression for the Euclidean action, Eq. (22). The second order term reads

$$S_{\mathrm{E}}^{(2)}[\xi,\eta]=\frac{\hbar s\lambda}{a}\int\int(\xi\quad\eta)\begin{pmatrix}a & b\\ c & d\end{pmatrix}\begin{pmatrix}\xi\\ \eta\end{pmatrix}dx\,d\tau, \tag{A1}$$

where

$$a=\frac{i}{2}\dot{\varphi}\cos\theta-\frac{1}{2}\partial_{xx}+\frac{1}{2}\cos 2\theta\varphi'^2 - \frac{1}{2}\cos 2\theta\cos^2\varphi-\frac{\alpha}{2}\cos 2\theta, \tag{A2}$$

$$b=i\sin\theta\partial_\tau+\varphi'\sin 2\theta\partial_x-\frac{1}{2}\sin 2\theta\cos 2\varphi, \tag{A3}$$

$$c=0, \tag{A4}$$

$$d=-\frac{1}{2}\sin^2\theta\partial_{xx}-\frac{1}{2}\theta'\sin 2\theta\partial_x+\frac{1}{2}\cos 2\varphi\sin^2\theta. \tag{A5}$$

For the other solution use $\pi-\theta^*+\xi$ and $\varphi^*+\eta$ in the action to obtain

$$S_{\mathrm{E}}^{(2)}[\xi,\eta]=\frac{\hbar s\lambda}{a}\int\int(\xi\quad\eta)\begin{pmatrix}a^* & -b^*\\ -c^* & d^*\end{pmatrix}\begin{pmatrix}\xi\\ \eta\end{pmatrix}dx\,d\tau, \tag{A6}$$

where it was used that $\cos\theta^*=(\cos\theta)^*$ etc. Comparing Eqs. (A1) and (A6), leads one to conclude that $K_z=K_w^*$.

The results above apply whenever there are pairs of solutions such as $\{z,\overline{z}\}$ and $\{w=1/z^*,\overline{w}=1/\overline{z^*}\}$, which is guaranteed to happen if the degenerate chirality states are unimodular, $|z_\pm|=1$. In the case of a symmetrically pinned wall, one has in addition that $z_+(x)=-z_+^*(-x)$; this, together with the ferromagnetic boundary condition at the chain ends, imply that the instanton fields satisfy, $z(x,\tau)=-z^*(-x,\tau)$ (same for $\overline{z}$). From the form of the action, see Eqs. (4) and (5), it follows that the second functional derivatives, $\delta^2S/\delta z^2$ etc., at the point (x,τ) are the complex conjugate of the corresponding terms in $(-x,\tau)$. The matrix of second derivatives separates in two blocks according to the sign of x, and the full determinant, which is the product of the blocks determinant, is real. This result was used to obtain Eq. (17).

[1] C. Sangregorio, T. Ohm, C. Paulsen, R. Sessoli, and D. Gatteschi, Phys. Rev. Lett. **78**, 4645 (1997); J.R. Friedman, M.P. Sarachik, J. Tejada, and R. Ziolo, *ibid.* **76**, 3830 (1996); L. Thomas, F. Lionti, R. Ballou, D. Gatteschi, R. Sessoli, and B. Barbara, Nature (London) **383**, 145 (1996).

[2] See P.C.E. Stamp, E.M. Chudnovsky, and B. Barbara, Int. J. Mod. Phys. B **6**, 1355 (1992); E.M. Chudnovsky and J. Tejada, *Macroscopic Quantum Tunneling of the Magnetic Moment* (Cambridge University Press, Cambridge, 1998), and references therein.

[3] D. Loss, D.P. DiVincenzo, and G. Grinstein, Phys. Rev. Lett. **69**, 3232 (1992).

[4] W. Wernsdorfer and R. Sessoli, Science **284**, 133 (1999).

[5] A. Garg, Europhys. Lett. **22**, 205 (1993).

[6] P. Perera and M.J. O'Shea, Phys. Rev. B **53**, 3381 (1996).
[7] S. Mangin, G. Marchal, W. Wernsdorfer, A. Sulpice, K. Hasselbach, D. Mailly, and B. Barbara, Europhys. Lett. **39**, 675 (1997).
[8] W. Wernsdorfer, K. Hasselbach, A. Benoit, B. Barbara, B. Doudin, J. Meier, J.-Ph. Ansermet, and D. Mailly, Phys. Rev. B **55**, 11 552 (1997); H. Yamazaki, G. Tatara, K. Katsumata, K. Ishibashi, and Y. Aoyagi, J. Magn. Magn. Mater. **156**, 135 (1996); K.M. Hong and N. Giordano, J. Phys. C **8**, L301 (1996).
[9] R. Rajaraman, *Solitons and Instantons* (North-Holland, Amsterdam, 1982).
[10] H.B. Braun and D. Loss, Phys. Rev. B **53**, 3237 (1996).
[11] S. Takagi and G. Tatara, Phys. Rev. B **54**, 9920 (1996).
[12] E. Fradkin, *Field Theories of Condensed Matter Systems* (Addison Wesley, New York, 1991).
[13] J.R. Klauder, Phys. Rev. D **19**, 2349 (1979); J.R. Klauder, in *Path Integrals*, Proceedings of the NATO Advanced Summer Institute, edited by G.J. Papadopoulos and J.T. Devreese (Plenum, New York, 1978).
[14] M. Stone, K.-S. Park, and A. Garg, J. Math. Phys. **41**, 8025 (2000).
[15] S. Coleman, *Aspects of Symmetry* (Cambridge University Press, Cambridge, 1995).
[16] J.A. Freire, D.P. Arovas, and H. Levine, Phys. Rev. Lett. **79**, 5054 (1997).
[17] W.H. Press, S.A. Teukolsky, W.T. Vetterling, and B.P. Flannery, *Numerical Recipes* (Cambridge University Press, Cambridge, 1986).
[18] M.J.D. Powell, in *Numerical Methods for Nonlinear Algebraic Equations*, edited by P. Rabinowitz (Gordon and Breach, London, 1970).
[19] L.H. Ryder, *Quantum Field Theory* (Cambridge University Press, Cambridge, 1985).
[20] D.D. Awschalom, J.F. Smyth, G. Grinstein, D.P. DiVincenzo, and D. Loss, Phys. Rev. Lett. **68**, 3092 (1992).
[21] B.A. Ivanov, A.K. Kolezhuk, and V.E. Kireev, Phys. Rev. B **58**, 11 514 (1998).