Detecting gapless excitations above ferromagnetic domain walls

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In a two- or three-dimensional quantum ferromagnetic *XXZ* model, a low-energy excitation mode above a magnetic domain wall is gapless, whereas all of the usual spin wave excitations moving around the whole crystal are gapful. Although this surprising fact was already proved in a mathematically rigorous manner, the gapless excitations have not yet been detected experimentally. For this issue, we show theoretically that the evidence of the gapless excitations appears as the dynamical fluctuations of the experimental observable, magnetoresistance, in a ferromagnetic wire. We also discuss other methods (e.g., ferromagnetic resonance and neutron scattering) to detect the gapless excitations experimentally.

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I. INTRODUCTION

In ferromagnetic spin systems, the low-energy excitations above the ground states can be described by the conventional spin wave theory under the assumption that no magnetic domain structure affects the low-energy spectrum. However, this assumption is not necessarily valid. In fact, it was recently proved[1,2] in a mathematically rigorous manner that, in a two- or higher-dimensional quantum ferromagnetic *XXZ* model with arbitrary spin S, an excitation mode above a magnetic domain wall is gapless, whereas all the usual spin wave excitations moving around the whole crystal are gapful. Namely, all the gapless excitations are confined in the narrow region along the domain walls.

This locality makes it very difficult to detect the gapless excitations in experiments. Thus the gapless excitations have not been found experimentally so far. However, quite recent technology has made it possible to create only one single domain wall in nanoscale ferromagnets. For example, the resistance contributions due to a single ferromagnetic domain wall were actually measured in a magnetoresistance experiment for a metallic wire.[3] An experimentally important issue is whether the gapless excitations can be actually detected or not. We can list at least two experimental methods which have a possibility to detect directly the spectrum of the gapless excitations. These are ferromagnetic resonance and neutron scattering. As is well known, by these methods, one can get the Fourier transform of the spin-spin correlation which is directly related to the low energy excitations above the ground states of the system at a very low temperature. Clearly the dominant part of the Fourier spectrum consists of the contributions from the usual spin wave excitations. In order to obtain only the spectrum of the gapless mode, the small contributions must be separated from the dominant contributions due to the usual spin wave excitations. Since we can expect that this detection will succeed by overcoming the technical problem in the near future, we will briefly discuss this issue in Sec. V below.

The main aim of this paper is to propose another experimental method to find the evidence of the gapless excitations above a ferromagnetic domain wall. The method is to detect the dynamical fluctuations due to the gapless excitations in the magnetoresistance. Although the method is indirect in comparison to the above direct methods, the detection of the dynamical fluctuations due to the gapless excitations is itself very interesting and challenging problem from both theoretical and experimental points of view. In order to get the dynamical fluctuations of the resistance, we calculate the transmission coefficient for a single conduction electron through the ferromagnetic domain wall with gapless excitations derived by deforming the spin configuration of the domain wall. It turns out that the transmission coefficient depends on the detailed structures of gapless excitations. In particular, when a kink excitation moves along the domain wall owing to quantum or thermal fluctuations, the transmission coefficient varies depending on the position of the kink above the domain wall. Thus the dynamical fluctuations due to the gapless mode appear through the transmission coefficient. According to the Landauer formula, the conductance is proportional to the transmission coefficient through the wire. As a result, the dynamical fluctuations due to the gapless mode appear as the resistance fluctuations.

This paper is organized as follows: In Sec. II, we introduce a model in which a single electron interacts with a ferromagnetic domain wall which is realized in the quantum spin-1/2 *XXZ* model with boundary fields. Section III is devoted to a brief review about the domain wall ground states and low energy excitations above the ground states in the *XXZ* model. In Sec. IV, we calculate the transmission coefficient of the electron through the domain wall with a low energy kink excitation. We also estimate the realistic values of the parameters in the model for the detection of the dynamical fluctuation of the domain wall in experiments. In Sec. V, we discuss the detection of the gapless mode above the domain walls in neutron scattering and ferromagnetic resonance experiments. Section VI is devoted to summary.

II. A SINGLE ELECTRON COUPLED TO A FERROMAGNETIC DOMAIN WALL

In order to treat the scattering problem of conduction electrons in a ferromagnetic wire, we introduce a ferromag-

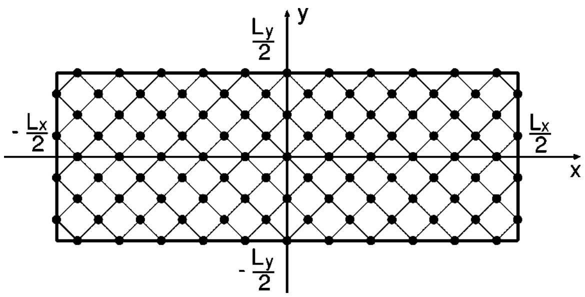

FIG. 1. Diagonal lattice Λ in the $L_x \times L_y$ box Ω.

netic XXZ-Kondo model in which a single conduction electron interacts with a single ferromagnetic domain wall of localized spins. For simplicity, we define the model on a two-dimensional box

$$\Omega := [-L_x/2, L_x/2] \times [-L_y/2, L_y/2], \tag{1}$$

although we can treat the same system in higher dimensions. The total Hamiltonian of the model consists of three terms as

$$H = H_{\mathrm{el}} + H_{\mathrm{dw}} + H'. \tag{2}$$

As usual the kinetic term H_{el} for the conduction electron with the mass μ is given by

$$H_{\mathrm{el}} = -\frac{\hbar^2}{2\mu}\triangle = -\frac{\hbar^2}{2\mu}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right). \tag{3}$$

The XXZ Hamiltonian H_{dw} for a ferromagnetic domain wall of localized spins[4] is

$$H_{\mathrm{dw}} = -J\sum_{\langle \mathbf{a},\mathbf{b}\rangle}[S_{\mathbf{a}}^{(1)}S_{\mathbf{b}}^{(1)} + S_{\mathbf{a}}^{(2)}S_{\mathbf{b}}^{(2)} + \Delta S_{\mathbf{a}}^{(3)}S_{\mathbf{b}}^{(3)}] - J\sqrt{\Delta^2-1}\sum_{\mathbf{a}\in B_+} S_{\mathbf{a}}^{(3)} + J\sqrt{\Delta^2-1}\sum_{\mathbf{b}\in B_-} S_{\mathbf{b}}^{(3)} \tag{4}$$

with the nearest neighbor spin-spin interactions with the positive exchange integral J and the Ising anisotropy $\Delta > 1$. Here $\mathbf{S}_{\mathbf{a}} = (S_{\mathbf{a}}^{(1)}, S_{\mathbf{a}}^{(2)}, S_{\mathbf{a}}^{(3)})$ is the spin-1/2 operator of a localized spin at the site $\mathbf{a} = (a_x, a_y)$ in the two-dimensional diagonal lattice

$$\Lambda := \{(ma, na) \in \Omega \mid \text{ integers } m,n \text{ satisfy } m+n = \text{even}\} \tag{5}$$

with a lattice constant a, see Fig. 1. Here, in order to create a domain wall along the y axis, we have taken the lattice to be diagonal, and taken the magnitude of the localized spins spin 1/2 although the model can be easily extended to that with arbitrary spin S. Further, for simplicity, we have taken $4Ma = L_x$, $2(2N+1)a = L_y$ with positive integers M,N. In order to make a single domain wall ground state, we have applied the boundary fields $\pm J\sqrt{\Delta^2-1}$ on the set of the boundary sites

$$B_\pm := \{(a_x, a_y) \in \Lambda \mid a_x = \pm L_x/2\}. \tag{6}$$

The interaction between the electron spin $\mathbf{s}$ and the localized spins $\mathbf{S}_{\mathbf{a}}$ is given by

$$H' = \sum_{\mathbf{a}\in\Lambda} u(\mathbf{r}-\mathbf{a})[s^{(1)}S_{\mathbf{a}}^{(1)} + s^{(2)}S_{\mathbf{a}}^{(2)} + \Delta' s^{(3)}S_{\mathbf{a}}^{(3)}] \tag{7}$$

with the anisotropy Δ', where the function $u(\mathbf{r})$ of $\mathbf{r} = (x,y)$ satisfies a short range condition

$$u(\mathbf{r}) = 0 \quad \text{for} \quad |\mathbf{r}| > r_0 > 0 \tag{8}$$

with a constant $r_0 \approx a$.

III. DOMAIN WALL GROUND STATES AND LOW ENERGY EXCITATIONS

Before studying the scattering of the electron by a low energy excitation above a domain wall, let us briefly review the domain wall ground states and the low energy excitations above the ground states. The exact domain wall ground state[4–7] of the Hamiltonian H_{dw} of Eq. (4) is given by

$$\Phi(z) = \bigotimes_{\mathbf{a}=(ma,na)\in\Lambda} \frac{|\uparrow\rangle_{\mathbf{a}} + zq^m|\downarrow\rangle_{\mathbf{a}}}{\sqrt{1+|z|^2q^{2m}}}, \tag{9}$$

where $z = e^{l/2+i\phi}$ with two real numbers $l, \phi; q \in (0,1)$ is defined by $\Delta = (q+q^{-1})/2$, and $|\uparrow\rangle_{\mathbf{a}}$ and $|\downarrow\rangle_{\mathbf{a}}$ are the spin up and down states at the lattice site $\mathbf{a}$, respectively. The position of the domain wall is specified with l in the x axis and the angle ϕ is a quantum mechanical phase corresponding to the degree of freedom of the rotation about the third axis of the spin. In the following, we choose $l=0$, i.e., the center of the domain wall is at $x=0$ in the x axis, and the wall is itself along the y axis.

Recently the excitations above of the domain wall ground states in the ferromagnetic XXZ model with the anisotropy $\Delta > 1$ and with the spin $S \geqslant 1/2$ have been intensively investigated.[1,2,8–12] Among many results, the most surprising result about the quantum domain walls is that, in two or higher dimensions, gapless excitations appear above the domain wall ground states,[1,2] whereas, in one dimension, all the excitations have a finite energy gap above all the ground states.[1,8,12] Here we should note that all the excitations above the translationally invariant ferromagnetic ground states always have a finite energy gap in any dimension and for any spin S because of the Ising anisotropy. In other words, the usual spin wave excitations always exhibit a finite energy gap. Thus the existence of the domain wall makes low energy excitations gapless in two or higher dimensions.

A reader might think that the appearance of the gapless mode is a trivial consequence of the translational symmetry breaking, and the mode is nothing but Nambu-Goldstone mode. But the lattice translations are clearly discrete. In addition, no gapless mode appears in one dimension although the system shows the same symmetry breaking. The reason of the gap is that all the domain walls in one dimension are a zero-dimensional pointlike object. As a result, a local deformation relying on the translational symmetry always gives either another domain wall ground state or a high-energy excitation.[7,8,12] On the other hand, in two or higher dimensions, all the domain walls are an infinitely extended object.

In this case, a local deformation for a domain wall ground state can give a low-energy excitation because creating another domain wall ground state needs a nonlocal deformation. Thus the gapless mode cannot be constructed by using only the translational symmetry, and cannot be simply interpreted as Nambu-Goldstone mode.

This surprising fact was found by Koma and Nachtergaele,[1,8] and they first proved the existence of a finite energy gap in one dimension and then proved the existence of a gapless excitation in two dimensions. The latter result was extended to three or higher dimensions by Matsui.[2] In their mathematical proof, the gapless excitations were constructed by deforming the ferromagnetic domain wall locally.[1,2,9,10] In realistic situations, we can expect that the corresponding low energy excitations appear as kinks of a domain wall. Namely, creating a kink is the simplest deformation for the domain wall.

IV. THE TRANSMISSION COEFFICIENT OF THE ELECTRON THROUGH A DOMAIN WALL

In this section, we will calculate the transmission coefficient of the conduction electron through the domain wall with a kink excitation. For this purpose, let us see first the properties of the low-energy excitations above the domain wall ground states which were briefly discussed in the previous section.

Although we need at least two kinks to construct a low energy excitation with a local support above a domain wall ground state, we will consider only a single kink above the domain wall. Namely we study the effect of a single kink in the electric transport through the domain wall. In other words, it is enough to consider only a single kink to see the effect of the gapless mode on the transmission coefficient of the electron through the domain wall. In order to construct a single kink above the domain wall ground state (9), we twist the quantum mechanical phase along the y direction, i.e., along the domain wall. The explicit form of the kink state is

$$\Phi(z;\varphi)=\bigotimes_{\mathbf{a}=(ma,na)\in\Lambda}\frac{|\uparrow\rangle_{\mathbf{a}}+zq^{m}e^{i\varphi(na)}|\downarrow\rangle_{\mathbf{a}}}{\sqrt{1+|z|^2q^{2m}}} \tag{10}$$

with the phase

$$\varphi(na)=\begin{cases}-\delta, & na<y_0,\\ \delta, & na>y_0,\\ 0, & na=y_0,\end{cases} \tag{11}$$

where δ is a real number, and y_0 is the position of the kink. In passing, we can treat a general deformation

$$z\to z\exp[\gamma(na)+i\varphi(na)] \tag{12}$$

with real functions $\gamma(na)$, $\varphi(na)$ in the same way.

In a real material, we can expect that the motion of the kink is much slower than that of the conduction electron. Under this assumption, the effective Hamiltonian for the electron is given by taking the expectation of the total Hamiltonian H of Eq. (2) about the kink state (10) as[13]

$$\begin{aligned}\tilde{H}_{\rm eff}&=\langle\Phi(z;\varphi),H\Phi(z;\varphi)\rangle-\text{const}\\ &=-\frac{\hbar^2}{2\mu}\triangle-\frac{J'\Delta'}{4}\tan h\left(\frac{x}{\lambda}\right)\sigma^{(3)}\\ &\quad-\frac{J'}{4}\text{sech}\left(\frac{x}{\lambda}\right)\exp[i\{\varphi(y)+\phi\}\sigma^{(3)}]\sigma^{(1)},\end{aligned} \tag{13}$$

where $(\sigma^{(1)},\sigma^{(2)},\sigma^{(3)})$ is the Pauli matrix, the domain wall width is given by $\lambda=a/|\log q|$, and we have replaced the discrete variables (ma,na) with the continuous ones (x,y) and chosen $u(\mathbf{r})=-J'$ with the positive exchange integral J' in the interaction range because the qualitative scattering behavior of the electron by the domain wall potential is expected to be independent of the detailed lattice structure. The effective Hamiltonian (13) is an extension of the effective Hamiltonian phenomenologically obtained by Cabrera and Falicov[14] in the case without kinks. For a double exchange model, see Ref. 15.

Let us consider a wire with the width L_y in the y direction and with the infinitely long length $L_x=+\infty$ in the x direction. We impose the Dirichlet boundary conditions $\psi(x,\pm L_y/2)=0$ in the y direction for the wave function $\psi(\mathbf{r})$ of the electron. Since the scattering potential of the domain wall depends on the position y_0 of the kink, we can expect that the transmission coefficient $\mathcal{T}$ through the domain wall potential varies depending on the position y_0. In order to see the dependence explicitly, we shall introduce an approximation. For this purpose, consider first the case with no domain wall, i.e., no effective domain wall potential in the Hamiltonian (13). Then the wave function $\psi(\mathbf{r})$ of the electron has a product form $\psi(\mathbf{r})=\psi_x(x)\psi_y(y)$ because the Hamiltonian $\tilde{H}_{\rm eff}$ of Eq. (13) is exactly of the free electron form. Now our approximation is as follows: We restrict the wave function $\psi_y(y)$ to the sector of the ground state

$$\psi_y^{(0)}(y)=\sqrt{\frac{2}{L_y}}\cos\left(\frac{\pi y}{L_y}\right). \tag{14}$$

Namely, we ignore all the excitations from the lowest subband to the higher subbands. Although this approximation is not necessarily justified in a realistic situation, we believe that a similar position dependence inevitably appears in the transmission coefficient $\mathcal{T}$. With the approximation, the effective Hamiltonian $H_{\rm eff}$ is given by

$$\begin{aligned}H_{\rm eff}=\langle\psi_y^{(0)},\tilde{H}_{\rm eff}\psi_y^{(0)}\rangle&=-\frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial x^2}-\frac{J'\Delta'}{4}\tanh\left(\frac{x}{\lambda}\right)\sigma^{(3)}\\ &\quad-\frac{J'}{4}\text{sech}\left(\frac{x}{\lambda}\right)A(y_0,L_y)\sigma^{(1)}+\text{const},\end{aligned} \tag{15}$$

where

$$A(y_0,L_y)=\sqrt{\cos^2\delta+f(y_0,L_y)\sin^2\delta} \tag{16}$$

with

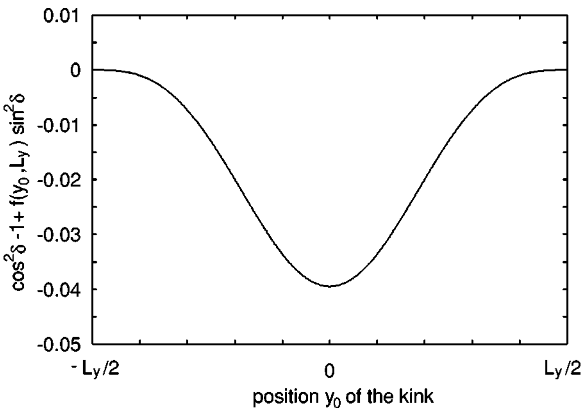

FIG. 2. The quantity $\cos^2\delta - 1 + f(y_0,L_y)\sin^2\delta$ in Eq. (19) as a function of the position y_0 of the kink for $\delta = 0.2$. The effect due to the kink in the scattering of the electron is maximized if the kink is at the center in the y axis.

$$f(y_0,L_y) = \left[\frac{2y_0}{L_y} + \left(\frac{1}{\pi}\right) \sin\left(\frac{2\pi y_0}{L_y}\right) \right]^2, \tag{17}$$

and we have chosen

$$\phi = \tan^{-1}[f(y_0,L_y)\tan\delta]. \tag{18}$$

For $\delta = 0$, we recover the well-known effective Hamiltonian[13,14] for the electron with the single domain wall with no kink. Clearly the transmission coefficient $\mathcal{T}$ is a function of $A(y_0,L_y)$. Let us consider the case with a small δ, i.e., with a very low energy excitation above the domain wall. We expand $\mathcal{T} = \mathcal{T}[A(y_0,L_y)]$ as

$$\mathcal{T} = \mathcal{T}_0 + \mathcal{C}[\cos^2\delta - 1 + f(y_0,L_y)\sin^2\delta] + \mathcal{O}(\delta^4), \tag{19}$$

where $\mathcal{T}_0$ is the transmission coefficient in the case with no kink, and $\mathcal{C}$ is a constant. The second term varies as the position y_0 of the kink varies from $-L_y/2$ to $L_y/2$, see Fig. 2.

Thus the conductance which is proportional to the transmission coefficient $\mathcal{T}$ varies depending on the kink position y_0 when the kink moves along the domain wall, owing to the thermal fluctuations, or to the driving force of a magnetic field. In other words, when the kink position y_0 dynamically fluctuates, the resistance also fluctuates reflecting the motion of the kink above the domain wall.

In a realistic experiment, the widths and the temperatures of the two-dimensional wire must be controlled so as to realize a single kink above a single domain wall. The realization of the single kink can be expected for the scale of wire widths comparable to the domain wall width and for low temperatures compared to the usual spin wave gap $\gamma = 2dSJ(\Delta - 1)$ with $d = 3$ because creating many kinks within a short length of the single domain wall clearly costs much energy. However, a very narrow wire would not show lower excitation energy than the usual spin wave gap. Such proper ranges of wire widths and of temperatures to realize the single kink would be attainable in present experiments. For example, for iron, the typical values for the wire widths L_y and the temperatures T are estimated as $L_y \approx \lambda \approx 40$ nm and $T \approx \gamma/k_B \approx 0.03$ K with Boltzmann constant k_B. Here the value of the gap $\gamma = 6SJ(\Delta - 1)$ is estimated by using $\Delta = (q + q^{-1})/2$ and $\lambda = a/|\log q|$ with the realistic values $S = 1$, $\lambda = 40$ nm and $a = 3 \times 10^{-10}$ m in Ref. 16 and with $J/k_B \approx 160$ K estimated also in the book.

V. THE GAPLESS MODE IN NEUTRON SCATTERING AND MAGNETIC RESONANCE EXPERIMENTS

Let us discuss a possibility of detecting the gapless mode in the other experimental methods, i.e., neutron scattering or ferromagnetic resonance experiments. The dominant part of their signals consists of the contributions from the usual ferromagnetic spin wave excitations which have a finite energy gap. The spectrum of the gapless mode appears below those of the spin wave excitations. The signal from the gapless mode is expected to be very weak in comparison with those from the spin wave modes because the spin wave excitations move around the whole crystal, whereas the gapless mode is confined in the narrow regions along the domain walls. A theory to describe the domain walls and the gapless mode was given in Refs. 9,12.

Using neutron scattering, the Fourier transforms of the spin-spin correlation functions can be obtained experimentally. The results include the information of the energy-momentum relations for the usual spin wave and the gapless modes. However, some technical problems arise in the experiment as follows. As is well known, a neutron scattering experiment is useless for a very small sample, and so a small sample to realize a single kink may be unsuitable for the experiment. Even if the experiment could be performed for such a small sample, the intensity of the signal from the gapless mode is expected to be very weak as mentioned above. But, seeing the recent progress of the nanoscale technology, we can expect that the detection of the gapless mode in the neutron scattering experiments will succeed by overcoming these technical problem in the near future. The dynamical spin-spin correlation for a domain wall state was studied theoretically, and the contribution from the gapless mode was obtained within a random phase approximation.[11]

A ferromagnetic resonance experiment is also expected to be useful for detecting the gapless mode. In addition the experiment generally has the advantage that the observed signal profile strongly depends on the geometry of a sample and an external magnetic field. These properties may be exploited for detecting the gapless mode. For example, consider a three-dimensional anisotropic $L_x \times L_y \times L_z$ system satisfying $L_z \ll L_y \ll L_x$, i.e., a long wire with the anisotropic widths L_y, L_z. For small widths L_y, L_z, the usual spin waves in the y and z directions become standing waves with a discrete spectrum because of the finite size effect.[17] Further, for a sufficiently small L_z, the energies of the standing waves in the z direction become much higher than the rest of the spectrum. In this situation, we can ignore the modes in the z direction. Assume that the face of a single domain wall entered the wire is perpendicular to the longitudinal x direction. Then we can expect that the signal from the excitation modes in the y direction is different from that without domain walls because there appear peculiar excitations, such as a kink excitation, due to the presence of the domain wall.

Namely we can expect that the spectrum of the gapless mode is observed below that of the usual spin waves, by controlling the wire width L_y. For example, for iron, that realistic size L_y is estimated as the order of 100 nm which is the order of the domain wall width. Technically this order of the size is already attainable in a ferromagnetic resonance experiment.[18]

In a ferromagnetic resonance experiment, the frequency of the applied microwaves must be very low because the energy of the gapless mode is expected to be very low. In order to estimate the magnitude of the frequency, consider the sample in Ref. 3 as an example. Then the external magnetic field $\mu_0 H$ to create a domain wall into the sample is of the order of 10 mT. The corresponding microwave frequency is estimated as $1 \sim 100$ MHz or less. Thus the natural ferromagnetic resonance in the absence of the static magnetic field or using the terrestial magnetic field seems to be suitable for detecting the gapless mode above domain walls. The oscillating magnetic field of the applied microwave is set to be either perpendicular or parallel to the internal or terrestial magnetic field. This argument about the microwaves is based on discussions with Kou Furukawa and Takeji Takui.[18]

The ferromagnetic resonance for a sample including pointlike magnetic objects, such as localized magnetic impurities, has been often investigated so far. However, as far as we know, a system including extended magnetic objects with an internal degree of freedom, such as domain walls with kinks, has not yet been investigated.

VI. SUMMARY

We have studied the possibility of experimentally detecting the gapless mode above the ferromagnetic domain walls. The existence of the mode was already proved in a mathematically rigorous manner, but the evidence of the mode has not yet been observed in any experiment. For this issue, we have theoretically showed the possibility that the evidence of the gapless mode appears experimentally as the dynamical fluctuations of the magnetoresistance in ferromagnetic nanoscale wires. We have also discussed the possibility of detecting the gapless mode in neutron scattering and magnetic resonance experiments. The difficulty of the experimental detection comes from the fact that the signal from the gapless mode is very weak in comparison with those from the usual spin wave modes because the spin wave excitations move around the whole crystal, whereas the gapless mode is confined in very narrow regions along the domain walls. But this difficulty is merely a technical problem and we believe that progress of nanoscale technology would make it possible to overcome this difficulty and to succeed in detecting the gapless mode in the near future.

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