Magnon delocalization in ferromagnetic chains with long-range correlated disorder

Rodrigo P. A. Lima and Marcelo L. Lyra
Departamento de Física, Universidade Federal de Alagoas, 57072-970 Maceió, AL, Brazil

Elton M. Nascimento and Antônio D. de Jesus
Departamento de Física, Universidade Estadual de Feira de Santana, 44031-460 Feira de Santana, BA, Brazil

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We study one-magnon excitations in a random ferromagnetic Heisenberg chain with long-range correlations in the coupling constant distribution. By employing an exact diagonalization procedure, we compute the localization length of all one-magnon states within the band of allowed energies E. The random distribution of coupling constants was assumed to have a power spectrum decaying as $S(k)\propto 1/k^{\alpha}$. We found that for $\alpha<1$, one-magnon excitations remain exponentially localized with the localization length ξ diverging as $1/E$. For $\alpha=1$ a faster divergence of ξ is obtained. For any $\alpha>1$, a phase of delocalized magnons emerges at the bottom of the band. We characterize the scaling behavior of the localization length on all regimes and relate it with the scaling properties of the long-range correlated exchange coupling distribution.

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I. INTRODUCTION

The properties of quasiparticle excitations in low-dimensional disordered systems has been the subject of recent intensive investigations due to the possible applications of random structures and superlattices in new devices. In many particular cases of interest, one-dimensional Hamiltonians can incorporate the main aspects of the disorder in what concerns the spatial distribution of quasiparticle excitations, such as optical, acoustic, electronic, and spin waves. Scaling theory establishes that in one-dimensional systems with uncorrelated disorder, the characteristic length of excitations is finite for any amount of disorder as a consequence of exponentially localized states.[1] Correlations in the disorder distribution can stabilize extended states. Resonant extended states emerge, for example, in one-dimensional electronic systems with randomly distributed impurity segments[2–4] and interpenetrating Anderson chains.[5] Recently, it has been demonstrated that long-range correlations in the disorder distribution can be responsible for the emergence of a phase of extended states within the band of allowed energies, with mobility edges separating localized and extended states.[6,7] A recent optical experiment has demonstrated this phenomenon[8] which has been proposed for use in the development of window filters in electronic, acoustic, or photonic nonperiodic structures.[9]

Spin-wave excitations in ferromagnetic chains with randomly distributed exchange couplings J have similar features as those of electronic excitations in chains with a particular distribution of pair-correlated off-diagonal disorder.[10–14] The localization length of the low-energy one-magnon excitations diverges as $\xi(E)\propto E^{-\phi}$, where ϕ depends on the particular form of the disorder distribution in the vicinity of $J=0$.[11,12] A diverging length of low-energy excitations is also observed in disordered harmonic chains.[15,16] In both cases, this behavior is associated with the presence of an uniformly ordered ground state.

Long-range correlations in electronic systems with off-diagonal disorder can stabilize extended states in a more effective way than in systems with diagonal disorder.[17] As the magnon equations of motion in ferromagnetic spin chains can be mapped onto those of electronic chains with off-diagonal disorder, one would expect the one-magnon excitations in disordered ferromagnetic chains to be also quite sensitive to the presence of long-range correlations in the random exchange distribution. In this paper we study, via direct diagonalization and finite-size scaling, the nature of the one-magnon eigenstates in a disordered chain with long-range correlated disorder with power spectrum decaying as $S(k)\propto k^{-\alpha}$. Our results indicate that a phase of extended magnons emerges at the bottom of the band for $\alpha>1$. We characterize the scaling aspects of the localization length and relate it with those of the disorder distribution.

II. MODEL, FORMALISM, AND NUMERICAL PROCEDURE

The Hamiltonian describing a spin-1/2 quantum Heisenberg ferromagnetic chain with random exchange couplings has the form

$$H=-2\sum_{n=1}^{L} J_n\hat{\mathbf{S}}_n\cdot\hat{\mathbf{S}}_{n+1}, \tag{1}$$

the ground state of the chain being the ordered state with all spins aligned. The one-magnon excitations have the general form $|\Psi\rangle=\Sigma_n b_n|n\rangle$, where $|n\rangle$ represents a state with the spin on the nth site flipped with respect to the ground-state orientation. The coefficients b_n can be shown to satisfy the difference equation

$$(J_{n-1}+J_n)b_n-J_{n-1}b_{n-1}-J_n b_{n+1}=Eb_n, \tag{2}$$

where E is the excitation energy and we used units of $\hbar=1$. For an uncorrelated distribution of the exchange couplings, the one-magnon excitations are exponentially localized in the thermodynamic limite for any $E>0$. The uniform $E=0$ mode is quite particular since this mode is not sensitive to the presence of disorder. This feature is responsible for the emergence of effectively extended low-energy excitations in finite chains.[11,12] The typical localization length ξ diverges as $E\rightarrow 0$ with a power law whose exponent depends on the specific form of the distribution $P(J)$ at the vicinity of $J=0$. For distributions with a finite average $\langle 1/J\rangle$, the inverse localization length vanishes linearly with E. However, for distributions with a diverging $\langle 1/J\rangle$, the excitations with low energy become more localized and only a slower divergence $\xi\propto E^{-\phi}$ takes place with an exponent $\phi<1$ which depends of the specific behavior of $P(J)$ at small values of J. This transition at $E=0$, which exists even for uncorrelated disorder, is due to the fact that the ground state is not affected by the disorder. In contrast, this feature is not present in the usual electronic problem with uncorrelated disorder.

In this work we will investigate the role played by long-range correlations in the exchange coupling distributions. In the analog electronic system, it has been recently demonstrated that long-range correlations can drastically modify the localized character of the excitations and even induce the emergence of a phase of delocalized states near the center of the band.[17]

In order to introduce long-range correlations in the disorder distribution, the exchange couplings J_n will be considered to be in such a sequence to describe the trace of a fractional Brownian motion with a specified spectral density $S(k)\propto 1/k^{\alpha}$, where k is related to the wavelength λ of the modulations on the random exchange landscape by $k=1/\lambda$. For $\alpha=0$, one recovers the traditional disordered ferromagnetic chain model with δ-correlated disorder, $\langle J_n J_{n'}\rangle=\langle J^2\rangle\delta_{n,n'}$. The exponent α describes the self-similar character of the random distribution and the persistent character of its increments. Following an approach based on the use of discrete Fourier transforms,[18–20] a power-law spectral density is imposed by construction whenever the exchange couplings are given by

$$J_n=\sum_{k=1}^{L/2}\left[k^{-\alpha}\left(\frac{2\pi}{L}\right)^{(1-\alpha)}\right]^{1/2}\cos\left(\frac{2\pi nk}{L}+\phi_k\right), \qquad (3)$$

where L is the number of sites and ϕ_k are $L/2$ random phases uniformly distributed in the interval $[0,2\pi]$. The equation above is a general decomposition of a power-law correlated potential where randomness is present only in the relative phases of the Fourier components. We normalize the width of the disorder distribution to keep it finite and size independent with $\langle J^2\rangle-\langle J\rangle^2=1$. Such procedure implies that the couplings generated by Eq. (3) have to be rescaled by $L^{-\alpha/4}$. In what follows, we shift the couplings to have $\langle J_n\rangle=4$ to enforce all generated couplings to be strictly positive. Those rare sequences not satisfying this bound were not considered on the subsequent analysis. The typical dependence of the coupling constant landscape with the spectral density exponent α is similar to those reported in Ref. 6. The most relevant aspect is that the landscape becomes progressively less rough as α is increased, favoring therefore delocalization.

Our numerical procedure consisted in obtaining all eigenenergies and one-magnon eigenstates for finite chains of size L by direct diagonalization and to infer the thermodynamic behavior through a finite size scaling analysis. We performed a configurational average over distinct disorder realizations such that, for each chain size, we computed 16×10^4 eigenstates. From each normalized eigenstate ($\Sigma_n b_n^2=1$), we computed the inverse participation ratio $Y=\Sigma_n b_n^4$ and used it to characterize the localized and delocalized nature of these one-magnon states. The typical localization length will be estimated by $\xi=1/Y$. In general, the localization length is finite for exponentially localized states and diverges linearly with L for truly extended states. In the next section, we report our results for its scaling behavior for distinct long-range correlated coupling constants characterized by the power spectrum exponent α.

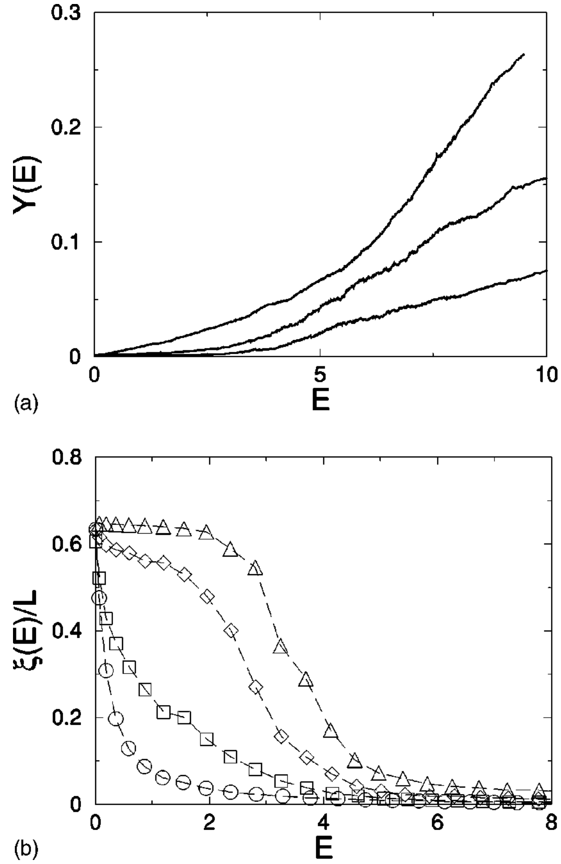

FIG. 1. (a) The inverse participation ratio as a function of the one-magnon energy E for a chain with $L=1600$ spins. From top to bottom $\alpha=0.0,1.0,1.5$. The linear dependence $Y\propto E$ for low-energy excitations observed for $\alpha=0$ is typical of uncorrelated potentials with finite $\langle 1/J\rangle$. For $\alpha>1$ a phase of low-energy delocalized excitations emerges. (b) The scaled localization length ξ/L vs E for chains with $L=1600$. From bottom to top $\alpha=0.0,1.0,1.5,2.0$. The phase of delocalized states appears as a plateau for $\alpha>1$.

III. RESULTS AND DISCUSSIONS

The behavior of the inverse participation ratio Y and the typical localization length ξ as a function of the one-magnon energy are shown in Figs. 1(a) and 1(b), respectively, as obtained from chains with $L=1600$ sites and several power spectrum exponents α. For $\alpha=0$, which mimics an uncorrelated disorder distribution, we found that $Y\propto E$ at low-energy

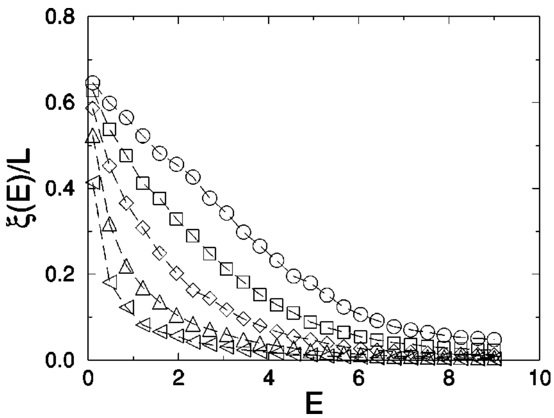

FIG. 2. The scaled localization length ξ/L vs E for $\alpha = 0.5$. From top to bottom $L = 100,200,400,800,1600$. For excitations with finite energies, $\xi(L)/L \rightarrow 0$ as L increases, characterizing localized states. The uniform mode at $E = 0$ remains delocalized.

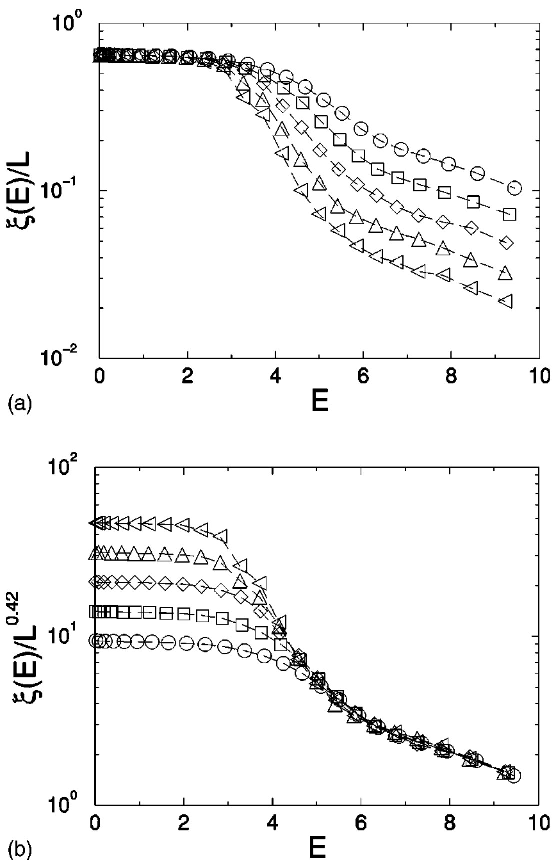

FIG. 3. (a) The scaled localization length ξ/L vs E for $\alpha = 2.0$. From top to bottom $L = 100,200,400,800,1600$. The phase of delocalized states appears as a size-independent plateau. The critical energy separating localized and delocalized states is $E_c = 3.0 \pm 0.2$. (b) The scaled localization length $\xi/L^{0.42}$ vs E for $\alpha = 2.0$ showing a data collapse for $E > E_c$. From bottom to top $L = 100,200,400,800,1600$. The size dependence of the characteristic length of such exponentially localized states reflects their sensitivity to the rescaling of the local disorder.

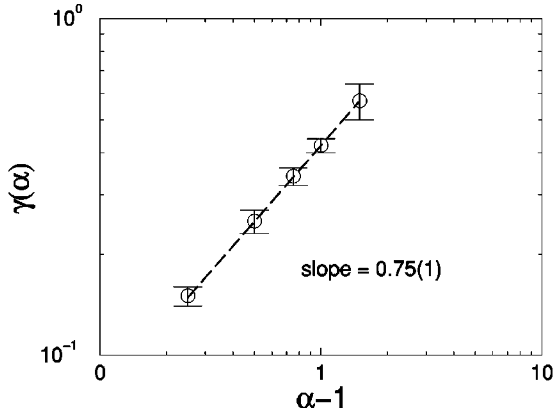

FIG. 4. Estimated values of the exponent γ related to the anomalous scaling of the localization length above E_c for several values of the correlation exponent α. These data suggest $\gamma(\alpha) \propto (\alpha - 1)^{0.75}$.

excitations, as expected. This trend remains in the range of $\alpha < 1$. In this regime, we also found the localization length to be size independent over the entire band of allowed energy values (except for finite-size scaling corrections near $E = 0$), even though the local disorder scales down as $L^{-\alpha/4}$. Our results indicate that the localization length of the excitations is not sensitive to such rescaling in this regime. In Fig. 2, we plot the normalized localization length $\xi(E)/L$ for $\alpha = 0.5$

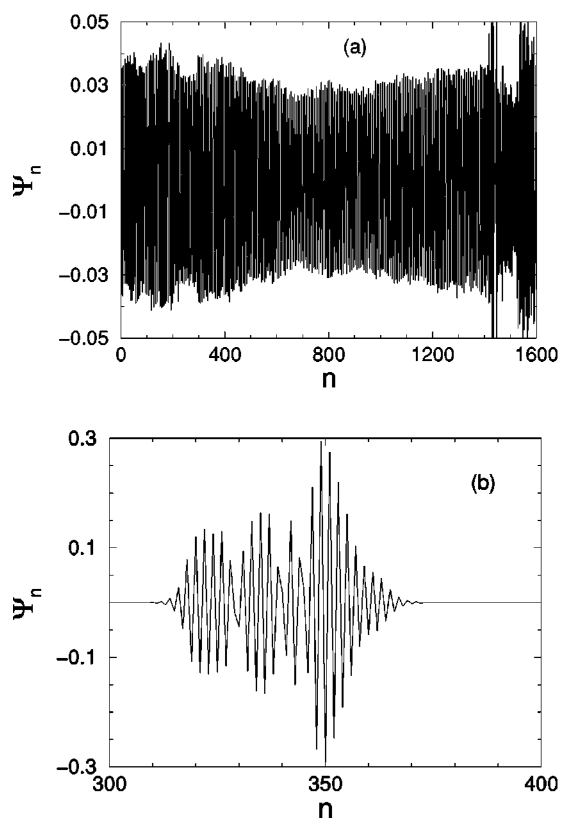

FIG. 5. Typical wave functions representing delocalized and localized one-magnon states of a long-range correlated ferromagnetic chain with $\alpha = 1.75$. (a) Delocalized state near $E = 2.0$. (b) Localized state near $E = 8.0$.

and several chain sizes to illustrate the comments above. $\xi(E=0)/L$ remains finite in the thermodynamic limit once the uniform mode remains delocalized. For any finite energy, the one-magnon excitations are exponentially localized with $\xi(E)/L \rightarrow 0$ as L increases.

For $\alpha=1$, a new scaling behavior was identified where at low energies $Y \propto E^{\phi}$, with $\phi>1$, indicating that in this case the excitations have a stronger tendency to become delocalized. It is important to mention that a similar feature ($\phi>1$) was reported to be present in the analog electronic chain model with long-range correlated hopping amplitudes.[17] For any $\alpha>1$, the data suggest that a phase of delocalized low-energy magnon states emerges, which is seen quite clear in Fig. 1(b) where ξ/L achieves a plateau. Therefore, long-range correlations can stabilize extended one-magnon states whenever the power spectrum exponent $\alpha>1$.

To further characterize the scaling behavior of the localization length in the regime of strong correlations that allows for the existence of delocalized states, we employed a finite-size scaling study of data from chains with $L=1600,800,400,200$, and 100 sites. The behavior of $\xi(E)/L$ is shown in Figure 3a for the typical case of $\alpha=2$. For $E<E_c=3.0(2)$ the data collapse characterizes a phase with delocalized states. E_c depends on the correlation exponent α and on the disorder width. The one-magnon eigenstates remain exponentially localized for $E>E_c$. However, the localization length in this regime becomes sensitive to the size-dependent rescaling of the coupling constants. We found that the localized states for $\alpha=2$ have $\xi \propto L^{\gamma}$ with $\gamma=0.42$ as illustrated by the collapse of data above E_c in Fig. 3(b). This size dependence is weaker than the expected for a similar rescaling of the local couplings in chains with uncorrelated disorder. The additional factor is due to the correlated nature of the disordered potential and its consequently enforced finite width. The new scaling exponent γ depends on the correlation exponent α in a nontrivial way. In Fig. 4, we report our estimated values for γ. Our results suggest a nonlinear relation $\gamma(\alpha) \propto (\alpha-1)^{0.75}$, although we could not envisage a simple scaling argument to support this fact. In Fig. 5, we show typical wave functions representing the localized and delocalized nature of one-magnon excitations within each phase.

IV. SUMMARY AND CONCLUSIONS

In this work, we investigated the nature of one-magnon excitations in ferromagnetic quantum chains with long-range correlated disorder. We found that a phase of delocalized magnons emerges whenever the power spectrum exponent characterizing the strength of correlations is $\alpha>1$, in close relationship to the behavior of the analog electronic model with off-diagonal disorder. Chains with distinct sizes and long-range correlations in the coupling exchange distribution need to have the local disorder scaled down as the size L increases in order to keep the disorder and bandwidth finite. For $\alpha<1$, the localized spin excitations are insensitive to such local disorder rescaling. On the other side, for $\alpha>1$, the localized states acquire a size-dependent localization length reflecting its sensitivity to the local disorder rescaling. However, such states should not be confused with critical multifractal states whose participation ratios also have a power-law size dependence.[21–24] The present localized states have exponential tails and their anomalous scaling is linked to the scaling property of the underlying long-range correlated potential. It would be valuable to investigate the spin-wave dynamics in these new regimes and the possible relation between superdiffusion and correlation exponents. This information would be valuable for possible future spin-wave dynamics experiments on correlated ferromagnetic chains and/or nonperiodic ferromagnetic superlattices.

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