Comment on "Exact solutions of the Lawrence-Doniach model for layered superconductors"

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In two recent publications [Phys. Rev. B **60**, 7496 (1999); **63**, 054508 (2001)] Kuplevakhsky has questioned the existence and stability of isolated Josephson vortices in layered superconductors. He argued that "vortex planes" rather than isolated vortices correspond to "unconditional minimum" of Gibbs free energy and ruled out "any possibility of single Josephson vortex penetration." In this comment, I disprove those statements and demonstrate that isolated Josephson vortices penetrate layered superconductors and have considerably lower energy than vortex planes.

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Josephson vortices (fluxons) in layered superconductors and stacked Josephson junctions (SJJ's) have been extensively studied both theoretically[3–7] and experimentally.[8,9] However, recently[1,2] Kuplevakhsky "revised previous calculations" and concluded that "the infinite Lawrence-Doniach model does not admit solutions in the form of isolated Josephson vortices" and that vortices exist only in the form of "vortex planes." His three main arguments against fluxons are (i) vortex planes have lower free energy, (ii) the fluxon solution is not unique for given boundary conditions, and (iii) a single fluxon cannot penetrate the stack at any magnetic field H. In this comment I disprove those statements and show by direct numerical simulations that isolated fluxons penetrate layered superconductors and have considerably lower energy than vortex planes.

I'll consider a stack of N identical junctions with interlayer spacing s, Josephson penetration depth λ_J, and London penetration depths λ_{ab} and λ_c. Properties of SJJ's are described by the coupled sine-Gordon equation[5] (CSGE) with magnetic coupling constant S. I'll follow notations of Ref. 7.

Free energy. Free energy of SJJ's is composed of kinetic, magnetic, and Josephson energies. Using the first integral of CSGE,[6] it can be shown that free energy of any isolated solution is twice the Josephson energy [see Eqs. (10) and (29) in Ref. 6]. Figure 1 shows calculated energies of the single fluxon, E_{sing} (data from Ref. 7), and the vortex-plane (in-phase) solution per vortex, $E_{in\text{-}phase}$, vs the number of SJJ's. Parameters of SJJ's are typical for Bi2212 high-T_c superconductor (HTSC). Energies are normalized to the fluxon energy in a single junction E_0. From Fig. 1 it is seen that E_{sing} only slightly increases with N and for $N > \lambda_{ab}/s$ saturates at

$$E_{sing}(N \gg 1) \simeq 3.6E_0. \tag{1}$$

This energy is consistent with $\sim 3.3E_0$ estimated from Clem-Coffey solution.[4] Note that fluxon energies both for stacked and single Josephson junctions have the same order of magnitude. This is due to the fact that the energy is predominantly stored in the fluxon core, which has the same length scale λ_J.[7]

On the other hand, the length scale of the in-phase solution is given by the largest characteristic length λ_N (Ref. 7) and $E_{in\text{-}phase}$ per vortex is $\simeq(\lambda_N/\lambda_J)E_0$. A large difference in length scales of fluxon and Meissner (in-phase) solutions is seen from Fig. 2(a). From Fig. 1 it is seen that unlike E_{sing}, $E_{in\text{-}phase}$ increases rapidly with N together with λ_N. For $N \gg 1$, $\lambda_N \to \lambda_J/\sqrt{1-2|S|} = \lambda_c$ as shown by the dotted line in Fig. 1. And the in-phase solution becomes identical to Eq. (37) from Ref. 2 (note, λ_J in Ref. 2 is our λ_c). The free energy per fluxon for the in-phase solution saturates at the value

$$E_{in-phase}(N \gg 1) = \frac{\lambda_c}{\lambda_J} E_0. \tag{2}$$

From Fig. 1 it is seen that $E_{in\text{-}phase}$ is always larger than E_{sing}, implying that the in-phase solution is unstable inside the stack, in contrast to Kuplevakhsky's erroneous conclusion that the vortex-plane solution corresponds to an "unconditional minimum" of free energy. A similar instability of a laminar solution is known for type-II superconductors.[10] In reality, the in-phase solution maximizes both magnetic and Josephson energies because vortices are placed at the shortest distance s from each other, the flux quantum is squeezed in one junction and the Josephson core has the largest possible size λ_c (note, that the total energy is twice the Josephson energy).

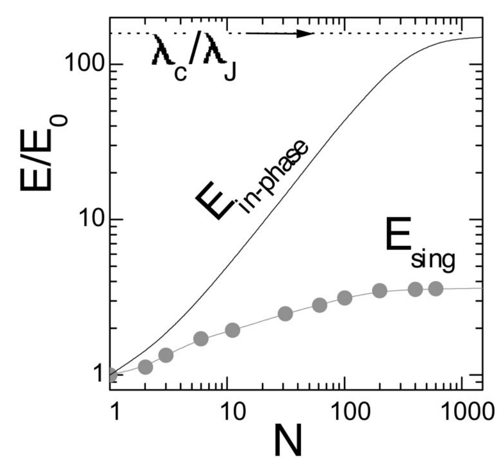

FIG. 1. Calculated energies of a single fluxon and the in-phase (vortex-plane) solution per fluxon for SJJ's with different number of junctions. It is seen that $E_{in\text{-}phase}$ is always larger than E_{single} and that both saturate at $N \to \infty$.

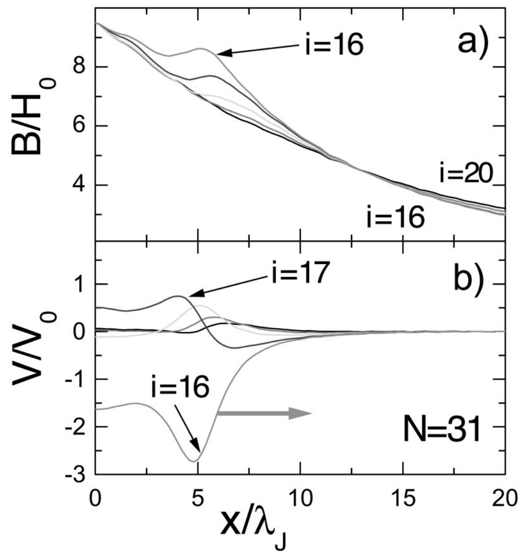

FIG. 2. Numerical simulation of the dynamics of fluxon penetration in SJJ's: snapshots of (a) magnetic induction and (b) voltage in junctions 16–20. From (b) it is clear that a single fluxon entered junction 16, and from (a) it is seen that there is no violation of boundary conditions at $x=0$. Also note a large difference in length scales of fluxon and Meissner (in-phase) solutions in Fig. 2(a).

Uniqueness of fluxon solutions. Kuplevakhsky has claimed that solution of CSGE for given boundary conditions is unique and discarded fluxon solutions as violating this requirement. However, such requirement is incorrect. This can be clearly seen from the limiting case $N=1$, for which nonunique analytic solution is known.[11] For SJJ's the situation is much more complicated: M fluxons can be arranged in N junctions in a number (up to N^M) of quasiequilibrium configurations (modes).[6,9] Existence of multiple quasiequilibrium fluxon modes in SJJ's has been demonstrated both analytically, numerically and experimentally.[6,9]

Fluxon penetration. Boundary conditions require that magnetic induction $B_i(0)=H$, i.e., uniform. Kuplevakhsky has argued that since B_i of the fluxon is nonuniform, it cannot penetrate SJJ's at any H. However, he did not take into account that penetration is essentially a dynamic process. Figure 2 shows a result of numerical solution of the full dynamic CSGE for $N=31$. External magnetic field was slowly increased until fluxons start to penetrate in the stack. Figure 2 represents a snapshot at the beginning of penetration. From Fig. 2(b) it is clear that a single fluxon has penetrated the middle junction $i=16$ and is moving inside the stack as shown by an arrow. The boundary condition, $B_i(0)=H$, is not violated at any instance, as seen from Fig. 2(a). Both for single and stacked Josephson junctions, fluxon penetration can be considered as decomposition of a bound fluxon-antifluxon pair, a "breather,"[12] centered at the edge of the junctions. This does not violate boundary conditions because fluxon and antifluxon fields cancel each other at the center of the breather.

Nevertheless, Kuplevakhsky has correctly concluded that the Meissner state in SJJ's can exist up to a superheating field $H_S=\Phi_0/\pi\lambda_c s$, which is essentially the field in the center of the vortex plane. For HTSC, H_S can be almost two orders of magnitude larger than $H_{c1}=4\pi E_{sing}/\Phi_0 \leqslant 1$ Oe. However, numerical simulations, Fig. 2, have shown that single fluxons (not vortex planes) penetrate SJJ's at $H \sim H_S$. At a finite temperature, fluctuations reduce the penetration field as in conventional type-II superconductors. The strong surface pinning of fluxons is probably responsible for the in-plane penetration field in HTSC being larger than expected H_{c1}.[13]

Finally, I would like to comment on speculations about finite vs infinite systems. In the latest paper[2] Kuplevakhsky argued that fluxons don't exist only in infinite systems, while in finite SJJ's unstable fluxon solution may exist. From Fig. 1 it is clear that for $N \gg \lambda_{ab}/s$ the fluxon solution becomes independent of N and agrees well with Bulaevskii[3] and Clem-Coffey[4] solutions. From this we can say that fluxons do exist in an infinite system if infinity is considered as a limit of large N.

In conclusion, I have shown that single fluxons can penetrate layered superconductors and have free energy much less than the vortex plane (in-phase) state. This is confirmed by direct numerical simulations in the dynamic case and is in strong disagreement with Refs. 1, 2.

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