# Superconducting fluctuations in granular metals with a large coupling between the grains

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We study the fluctuation conductivity of superconducting granular metals at low temperatures and strong magnetic field destroying the Cooper pairs. Explicit calculations are performed for larger values of the coupling between the grains than those considered in previous works. We show that in a broad region of the coupling constants the superconducting fluctuations still significantly reduce the conductivity leading to a negative magnetoresistance.

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## I. INTRODUCTION

During the last decade study of electric properties of nonhomogeneous metals attracted a lot of attention. In particular, granular metals have been investigated recently in a number of experimental works.[1,2] Theory of superconducting fluctuations in the granulated superconductors was suggested recently in Refs. 3,4.

In these works a three-dimensional (3D) array of grains placed in a strong magnetic field $H>H_c$, where $H_c$ is the field destroying the superconducting gap in the grains, and at low temperatures $T\ll T_c$, where $T_c$ is the superconducting transition temperature, was considered. It was demonstrated that the correction to the fluctuation conductivity is *negative* and this effect should exist even at zero temperature. The resistivity increases and only at extremely strong magnetic fields, $H\gg H_c$, reaches its classical value. Therefore the system exhibits a *negative magnetoresistance*. This theory may explain existing experiments.[1]

It was important for the calculations presented in Ref. 3 that the dimensionless conductance satisfied the condition $g\ll\Delta_0/\delta$, here $\Delta_0$ is the BCS gap at zero magnetic field and $\delta$ is the mean level spacing. The same effect of the negative magnetoresistance was obtained in a recent paper[5] for two-dimensional (2D) homogeneous superconducting samples at low temperatures $T\ll T_c$ and strong magnetic field $H>H_c$. The limit of the homogeneous metal is opposite to the one considered in the works[3,4] because the dimensionless conductance $g$ of a homogeneous sample is proportional to $k_F l$, where $k_F$ and $l$ are the Fermi momentum and the mean free path, respectively, and can be very large. The negative magnetoresistance can also be seen under certain circumstances in high-$T_c$ superconductors[6–8] for high temperatures $T>T_c$ and low magnetic fields $H<H_c$.

In the present paper we generalize the results of Refs. 3,4 to larger values of the tunneling dimensionless conductance $g$. In particular, the assumption that the dimensionless conductance, $g$, is restricted from above by $\Delta_0/\delta$ is now dropped. This means that the structure of the granular metal becomes more similar to that of a bulk metal. The main question we are dealing with in this paper is if the superconducting fluctuations may cause a negative magnetoresistance in the granulated systems with the larger coupling between the grains. At the same time, our region of parameters is different from the limit of a homogeneous metal. We assume that

$$1\ll g\ll E_T/\delta, \tag{1.1}$$

where $E_T=D/R^2$ is the Thouless energy of a single grain, $D$ is the diffusion coefficient, and $R$ is the radius of the grain. The last inequality in Eq. (1.1), although being more general than that used previously,[3,4] means that the granular structure is still important for our consideration.

The granular material we consider now consists of a 3D array of metallic grains with a typical diameter of the grains of 120±20 Å. The electrons can tunnel from one grain to another. It is this tunneling that determines the properties of the entire system. Inside the grains there can be impurities and the shape of each grain is not perfect, so that the electrons are scattered randomly by the boundaries. Since the hopping amplitude is not very large, the macroscopic charge transfer is determined by the ratio of the hopping amplitude $t$ to the mean level spacing $\delta$ or, in other words, by the dimensionless conductance $g=(\pi^2/4)(t/\delta)^2$. In the limit $t\gg\delta$ the discreteness of the energy spectrum in a single grain is not resolved and therefore the electron motion is diffusive through many grains. This limit corresponds to a macroscopically weak disorder and results in a large dimensionless conductance $g\gg 1$. Below, we restrict our consideration by this limit.

Let us discuss what happens with a granular metal at low temperatures. Below the critical temperature $T_c$, the electron-phonon interaction leads to the formation of a superconducting gap in each grain and Cooper pairs appear. Applying a strong magnetic field one destroys the superconducting gap in each grain and comes to the picture of a normal metal with superconducting fluctuations. Our calculations are performed in this regime.

We assume that the energy parameters are ordered as follows:

$$\delta\ll t,\Delta_0\ll E_T. \tag{1.2}$$

The last inequality in Eq. (1.2) means that the size of a single grain $R$ is much smaller than the coherence length $\xi_0$. In this limit the superconducting fluctuations in a single grain are zero dimensional. We want to emphasize that all energies are smaller than the Thouless energy $E_T$, and as a consequence the behavior of the system does not depend on grain boundaries or on individual scattering processes. Due to the large values of the conductance $g \gg 1$ we may neglect weak localization and charging effects. Therefore all the effects considered below are entirely due to the superconducting fluctuations.

The superconducting pairing inside the grains can be destroyed by both the orbital mechanism and the Zeeman splitting. The critical magnetic field $H_c^{\rm or}$ destroying the superconductivity in a single grain in this case can be estimated as $H_c^{\rm or} R \xi \approx \phi_0$, where $\phi_0 = hc/e$ is a flux quantum, $R$ is the radius of a single grain, and $\xi = \sqrt{\xi_0 l}$ is the superconducting coherence length. The Zeeman critical magnetic field $H_c^z$ can be written as $g \mu_B H_c^z = \Delta_0$, where $\mu_B$ is Bohr's magneton and $g$ is the Landé factor. The ratio of these two fields can be written in the form $H_c^{\rm or}/H_c^z \approx R_c/R$, where $R_c = \xi (p_0 l)^{-1}$. For $R > R_c$ the orbital critical magnetic field is smaller than the Zeeman critical magnetic field $H_c^{\rm or} < H_c^z$ and the superconductivity is suppressed by the orbital motion of electrons. Although the Zeeman mechanism can be easily included in the present consideration, we consider now only the orbital mechanism of the destruction of the superconductivity. This limit is opposite to the one considered in Ref. 9, where the Zeeman splitting was assumed to be the main mechanism of destruction of the Cooper pairs. A broader region of the conductance $g$ used in the present paper makes the calculation somewhat more difficult than previously because one has to consider additional diagrams and calculate them using more complicated expressions for integrands. The remainder of the paper is organized as follows. In Sec. II we formulate the model. In Sec. III we discuss the fluctuation conductivity of granular metals. In Sec. IV we discuss the weak localization correction to conductivity of granular metals. Our results are summarized in the conclusion.

## II. THE MODEL

We assume that the grains are packed in a 3D lattice surrounded by an isulator. The grains are coupled with each other and therefore the electrons can hop from one grain to another. Inside the grains as well on the surface there can be impurities and the electrons can interact with phonons. The Hamiltonian of the system can be written in the form

$$\hat{H} = \hat{H}_0 + \hat{H}_T, \tag{2.1}$$

here $\hat{H}_0$ describes a single grain with electron-phonon interaction in the presence of a strong magnetic field and is given by

$$\hat{H}_0 = \sum_{i,k} E_{i,k} a^\dagger_{i,k} a_{i,k} - |\lambda| \sum_{i,k,k'} a^\dagger_{i,k} a^\dagger_{i,-k} a_{i,-k'} a_{i,k'} + \hat{H}_{\rm imp}, \tag{2.2}$$

where $i$ stands for the number of the grain, $k \equiv (\mathbf{k},\uparrow), -k \equiv (-\mathbf{k},\downarrow)$. The quantity $\lambda$ is an interaction constant and $\hat{H}_{\rm imp}$ describes the elastic interaction of the electrons with impurities. The interaction in Eq. (2.2) contains diagonal matrix elements only. This form of the interaction can be used provided the superconducting gap $\Delta_0$ satisfies the last inequality in Eq. (1.2). The second term in Eq. (2.1) describes tunneling of electrons from grain to grain and is given by

$$\hat{H}_T = \sum_{i,j,p,q} t_{i,j,p,q} a^\dagger_{ip} a_{jq} \exp\left( i \frac{e}{c} \mathbf{A} \cdot \mathbf{d}_{ij} \right) + \text{c.c.}, \tag{2.3}$$

where $\mathbf{A}$ is the vector potential and $\mathbf{d}_{ij}$ is a vector connecting the center of grain $i$ with the center of a neighboring grain $j$ $(|\mathbf{d}_{ij}| = 2R)$. The operator $a^\dagger_{ip}$ is the creation operator of an electron in grain $i$ in the state $p$ and $a_{ip}$ is the annihilation operator of an electron in grain $i$ in the state $p$.

## III. FLUCTUATION CONDUCTIVITY

In this section we consider the conductivity of granular metals in detail. The dc conductivity $\sigma$ is related to the operator of the electromagnetic response as[10]

$$\sigma = \lim_{\omega \to 0} \frac{Q^R(\omega)}{-i\omega}, \tag{3.1}$$

where $Q^R(\omega)$ is the analytical continuation of $Q(i\omega_\nu)$ into the upper complex half plane and is called the *retarded* operator of the electromagnetic response and $\omega$ is the frequency of the external electromagnetic field. In order to calculate $Q(i\omega_\nu)$ we use Matsubara's diagram technique.[10] After calculation of $Q(i\omega_\nu)$ for imaginary frequencies we have to carry out the analytical continuation of $Q(i\omega_\nu)$ into the region of real frequencies: $i\omega_\nu \to \omega + i0^+$. All diagrams which contribute to the conductivity of granular metals are shown in Fig. 2. The same class of diagrams describe the conductivity of bulk metals.[5,11] Scattering of the electrons inside the grains by impurities is included in the Born approximation, giving rise to a scattering mean free time $\tau$ and resulting in a renormalization of the single electron normal state Green's function to $G^0(i\varepsilon_n, \mathbf{p}) = [i\varepsilon_n - \xi(\mathbf{p}) + i/2\tau \,\text{sgn}(\varepsilon_n)]^{-1}$, here $\varepsilon_n = (2n+1)\pi T$ is the fermion frequency and $\xi(\mathbf{p}) = \varepsilon(\mathbf{p}) - \varepsilon_F$ is the electron energy counted from the Fermi level. For $l \ll L_c$, where $l$ is the mean free path and $L_c$ is the cyclotron radius, we can treat the Green's function in the quasiclassical approximation. In this approximation the magnetic field results in the appearance of an additional phase

$$G(i\varepsilon_n, \mathbf{r}-\mathbf{r}') = G^0(i\varepsilon_n, \mathbf{r}-\mathbf{r}') \exp\left( \frac{ie}{c} \int_{\mathbf{r}}^{\mathbf{r}'} \mathbf{A} d\mathbf{s} \right). \tag{3.2}$$

Each wavy line in the diagrams represents the propagator of the superconducting fluctuations $K(i\Omega_k, \mathbf{q})$:

$$K(i\Omega_k, \mathbf{q}) = -\frac{1}{\nu_0} \left[ \ln\left( \frac{\mathcal{E}_0(H) + |\Omega_k|}{\Delta_0} \right) + \eta(\mathbf{q}) \right]^{-1}, \tag{3.3}$$

where $\Omega_k = 2k\pi T$ is the boson frequency, $\nu_0$ is the density of states on the Fermi surface, $\eta(\mathbf{q}) = 8/3\pi (g\delta/\Delta_0) \Sigma_{i=1}^3 [1 - \cos(q_i d)]$ describes the tunneling of electrons from grain to

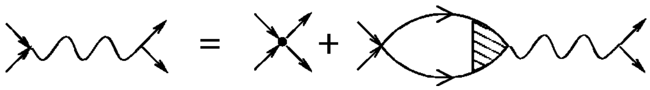

FIG. 1. The Dyson equation in the ladder approximation for the propagator of the superconducting fluctuations. The black point represents the coupling constant and the shaded three-point vertex stands for the renormalized impurity vertex.

grain; $\mathcal{E}_0(H)=(2/5)(\phi/\phi_0)^2 E_T$, where $\phi$ is the magnetic flux through the grain. The propagator of superconducting fluctuations, Eq. (3.3), is presented by the sum of all diagrams with two incoming and two outgoing lines in Fig. 1.

The impurity vertex entering these diagrams is represented as a shaded half circle and has the form[3,4]

$$\lambda(i\varepsilon_n, i\Omega_k - i\varepsilon_n, \mathbf{q}) = \frac{1}{\tau}\frac{\theta[-\varepsilon_n(\Omega_k-\varepsilon_n)]}{|2\varepsilon_n-\Omega_k|+\mathcal{E}_0(H)+\frac{16}{\pi}g\delta\sum_{i=1}^{3}[1-\cos(q_i d)]}, \tag{3.4}$$

where $\theta(x)$ is the Heaviside function and the third term in the denominator describes tunneling processes from grain to grain. Each external vertex is given by $-2et\mathbf{d}\sin(\mathbf{p}\cdot\mathbf{d})$, where $e$ is the charge of an electron, $t$ is the tunneling amplitude, and $\mathbf{d}$ is a vector connecting the centers of two neighboring grains. In the following subsections we consider the contributions to the fluctuation conductivity that arise from these diagrams.

### A. Correction to the conductivity due to suppression of DOS

In this section we consider the correction to conductivity due to suppression of the density of states (see diagrams 5–10). This (DOS) contribution arises from corrections to the density of states due to the superconducting fluctuations. Even at strong magnetic fields ($H>H_c$) there are still some electrons that form fluctuational Cooper pairs. These electrons are bound and cannot simultaneously take part in one-electron charge transfer. This results in a reduction of the number of carriers for the one-electron charge transfer and the conductivity decreases. The analytical expression for the operator of the electromagnetic response taking into account summation over the spin indices has the form

$$Q(i\omega_\nu)=\frac{8}{3}\sum_{i=1}^{3} T\sum_{\Omega_k}\int\frac{d^3\mathbf{q}}{(2\pi)^3}K(i\Omega_k,\mathbf{q})T \times\sum_{\varepsilon_n} C^2(i\varepsilon_n, i\Omega_k-i\varepsilon_n,\mathbf{q})I(i\Omega_k,i\varepsilon_{n+\nu}), \tag{3.5}$$

where a factor of 2 originates from a similar diagram shown in Fig. 2 which gives the same contribution to conductivity. The analytical expression for $I(i\Omega_k,i\varepsilon_{n+\nu})$ in Eq. (3.5) is given by

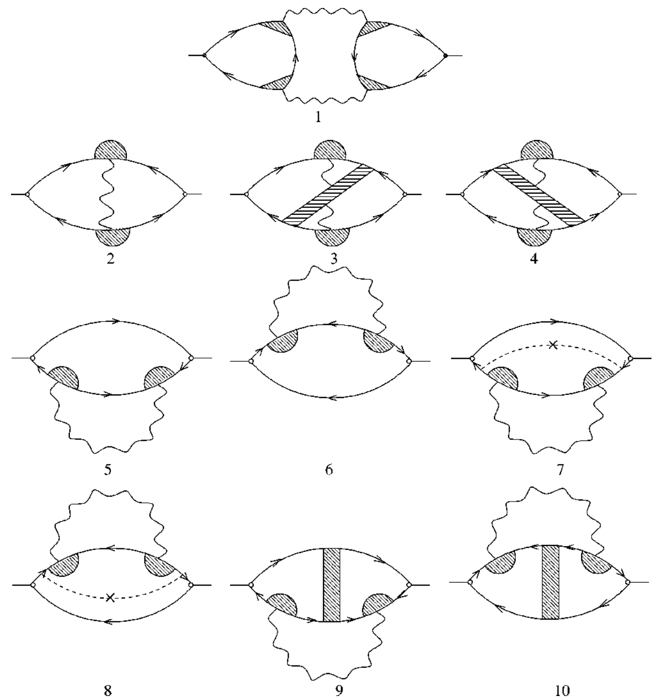


FIG. 2. Diagrams for the leading order contribution to the fluctuation conductivity of granular metals. Wavy lines symbolize the propagator of the superconducting fluctuations, thin solid lines with arrows are the normal state Green's functions averaged over impurity positions and shaded semicircles are vertex corrections arising from impurities. Dashed lines with central crosses are additional impurity renormalizations and shaded blocks are impurity ladders. Diagram 1 is the Aslamazov-Larkin (AL) contribution, diagram 2 is the Maki-Thompson (MT), 5, 6, 7, and 8 are the density of states (DOS) diagrams. Diagrams 3,4 and 9,10 arise when one averages the DOS and MT diagrams over impurities.

$$I(i\Omega_k,i\varepsilon_{n+\nu})=\int\frac{d^3\mathbf{q}'}{(2\pi)^3}\sin^2(q_i' d)\frac{4e^2t^2d^2V^2}{(2\pi\nu_0\tau)^2}T \times\sum_{\varepsilon_n}\int\frac{d^3\mathbf{p}_1 d^3\mathbf{p}_2}{(2\pi)^6}G(i\varepsilon_{n+\nu},\mathbf{p}_1) \times G^2(i\varepsilon_n,\mathbf{p}_2)G(i\Omega_k-i\varepsilon_n,\mathbf{p}_2), \tag{3.6}$$

where $\mathbf{p}_1$ and $\mathbf{p}_2$ denote the momenta of electrons inside the grains, $\mathbf{q}'$ is the quasimomentum, and $\varepsilon_{n+\nu}=\varepsilon_n+\omega_\nu$. Inside the Green's functions we may put $\Omega_k$ equal to zero because the characteristic frequency of the superconducting fluctuation propagator $K(i\Omega_k,\mathbf{q})$ is of the order $\Omega_k\sim\Delta_0$ which is much smaller than the Thouless energy $E_T$. The integral over $\mathbf{p}_2$ in Eq. (3.6) is only nonzero when the poles of the Green's functions corresponding to one grain lie on different sides of the axis of the real numbers. Therefore $\varepsilon_n$ and $\Omega_k-\varepsilon_n$ must have different signs. In all other cases the result equals to zero. For $\Omega_k,\varepsilon_n,\omega_\nu\ll 1/\tau$ we obtain for $I(i\Omega_k,i\varepsilon_{n+\nu})$

$$I(i\Omega_k,i\varepsilon_{n+\nu})=\frac{4ge^2}{\nu_0^2\pi^2 d}\theta(-\varepsilon_n\varepsilon_{n+\nu})\theta[-\varepsilon_n(\Omega_k-\varepsilon_n)]. \tag{3.7}$$

Now using Eq. (3.7) we calculate the sum over $\varepsilon_n$ in Eq. (3.6)

$$D(i\Omega_k,i\omega_\nu,\mathbf{q})=T\sum_{\varepsilon_n} C^2(i\Omega_k,i\Omega_k-i\varepsilon_n,\mathbf{q})I(i\Omega_k,i\varepsilon_{n+\nu})$$
$$=\frac{16ge^2}{d}T\sum_{n=-\nu}^{-1}\frac{\theta[-\varepsilon_n(\Omega_k-\varepsilon_n)]}{(|2\varepsilon_n-\Omega_k|+4\pi T\alpha_{\mathbf{q}})^2}. \tag{3.8}$$

Carrying out the summation over the $\varepsilon_n$ in Eq. (3.8) we obtain for the function $D(i\Omega_k,i\omega_\nu,\mathbf{q})$ the following result:

$$D(i\Omega_k,i\omega_\nu,\mathbf{q})=-\frac{ge^2}{\pi^2 dT}\theta(\Omega_k+\omega_\nu)\times\left[\psi'\left(\frac{1}{2}+\frac{2\omega_\nu+\Omega_k}{4\pi T}+\alpha_{\mathbf{q}}\right)-\psi'\left(\frac{1}{2}+\frac{|\Omega_k|}{4\pi T}+\alpha_{\mathbf{q}}\right)\right], \tag{3.9}$$

where $\psi(x)$ is the logarithmic derivative of the Gamma-function and $\alpha_{\mathbf{q}}$ is given by

$$\alpha_{\mathbf{q}}=\frac{1}{4\pi T}\left(\mathcal{E}_0(H)+\frac{16}{\pi}g\delta\sum_{i=1}^{3}[1-\cos(q_i d)]\right). \tag{3.10}$$

The second term in Eq. (3.10) arises due to the renormalization of the Cooperons when tunneling processes from grain to grain are taken into account. We insert Eq. (3.9) into Eq. (3.5) and present the operator of the electromagnetic response in the following form:

$$Q(i\omega_\nu)=\frac{8}{3}\sum_{i=1}^{3}T\sum_{\Omega_k}\int\frac{d^3\mathbf{q}}{(2\pi)^3}D(i\Omega_k,i\omega_\nu,\mathbf{q})K(i\Omega_k,\mathbf{q}). \tag{3.11}$$

Now the function $Q(i\omega_\nu)$ must be continued analytically into the upper complex half plane of the frequency. The analytical continuation in Eq. (3.11) is carried out in the Appendix. As a result, we obtain for the operator of the electromagnetic response $Q(i\omega_\nu)$ after analytical continuation the following expression:

$$Q^R(\omega)=\frac{-i\omega 2ge^2}{\pi^4 dT^2\Delta_0\nu_0}\psi''\left(\frac{1}{2}+\frac{\Delta_0}{4\pi T}\right)\sum_{i=1}^{3}\int\frac{d^3\mathbf{q}}{(2\pi)^3}\times\left[\int_0^{\Omega_{\max}}\coth\frac{\Omega}{2T}\frac{\Omega d\Omega}{\frac{\Omega^2}{\Delta_0^2}+\tilde{\eta}^2(\mathbf{q})}+\int_0^\infty\frac{1}{\sinh^2\frac{\Omega}{2T}}\frac{\Omega^2}{\frac{\Omega^2}{\Delta_0^2}+\tilde{\eta}^2(\mathbf{q})}\frac{d\Omega}{2T}\right], \tag{3.12}$$

where $\Omega_{\max}\sim\Delta_0$ is an upper cutoff, $\tilde{\eta}(\mathbf{q})=\eta(\mathbf{q})+2h$ and $h=(H-H_c)/H_c$ is the reduced magnetic field. In Eq. (3.12) we may put $\mathbf{q}=\mathbf{0}$ inside $\psi(x)$ because the main contribution to the integral over $\mathbf{q}$ comes from small momentum and in this case $\alpha_{\mathbf{q}}$ is a slowly varying function of $\mathbf{q}$. The remaining integrals over $\Omega$ in Eq. (3.12) can be easily calculated and the final result for the electromagnetic response is

$$Q^R(\omega)=-i\omega\frac{2ge^2\Delta_0}{\pi^4 dT^2\nu_0}\psi''\left(\frac{1}{2}+\frac{\Delta_0}{4\pi T}\right)\sum_{i=1}^{3}\int\frac{d^3\mathbf{q}}{(2\pi)^3}\times\left[\ln\left(\frac{\xi}{\tilde{\eta}}\right)-\frac{1}{2\xi}-\psi(\xi)+\xi\psi'(\xi)-1\right], \tag{3.13}$$

where we introduced the dimensionless parameter $\xi=\Delta_0\tilde{\eta}/4\pi T$. For very low temperature $T\ll\Delta_0\tilde{\eta},(\xi\gg 1)$ we may use the asymptotic expansion for $\psi(\xi)$ and finally obtain the correction to the conductivity due to the suppression of the density of states

$$\frac{\sigma^{\mathrm{DOS}}}{\sigma_0}=-\frac{2}{\pi}\frac{\delta}{\Delta_0}\left\langle\ln\left(\frac{1}{\tilde{\eta}(\mathbf{q})}\right)\right\rangle, \tag{3.14}$$

where $\langle\cdots\rangle=V\int_0^{2\pi/d}\cdots[d^3\mathbf{q}/(2\pi)^3]$, and $\sigma_0=8ge^2/\pi R$ is the classical conductivity of a granular metal. One can see that the DOS diagram gives a negative contribution to the conductivity. The absolute value of $\sigma^{\mathrm{DOS}}$ is a decreasing function of the magnetic field $H$ and reaches its maximum value at the critical magnetic field $H=H_c$. The absolute value of this maximum can be estimated as[3,4]

$$\left|\frac{\sigma^{\mathrm{DOS}}_{\max}}{\sigma_0}\right|\sim\frac{\delta}{\Delta_0}\ln\left(\frac{\Delta_0}{g\delta}\right). \tag{3.15}$$

This maximum value is smaller than unity and this fact ensures our expansion. The conductivity $\sigma^{\mathrm{DOS}}$ is independent of the temperature and therefore it remains finite in the limit $T\to 0$. This fact indicates that there are still virtual Cooper pairs even at zero temperature and strong magnetic fields. The quantity $\sigma^{\mathrm{DOS}}$ becomes comparable with $\sigma_0$ when $g$ is of the order of unity. Such values of $g$ mean that we would not be far from the metal-insulator transition. In this case we have to take into account all localization effects and Eq. (3.14) can be used only for rough estimates.

We would like to note that Eq. (3.14) does not differ from those written in Refs. 3,4. However, other contributions to the conductivity considered in the next subsections may change, so the extension of the calculations to the entire region specified by Eq. (1.1) is not as simple.

As it was shown in Refs. 3,4 we can neglect the higher order corrections to the DOS, an example is shown in Fig. 3. At low temperature $T\ll\Delta_0\tilde{\eta}$ this diagram contains an additional small factor of $(\delta/\Delta_0)\ln\tilde{\eta}$.

In the following sections we discuss the Aslamazov-Larkin (AL) and Maki-Thompson (MT) contributions to the fluctuation conductivity. It turns out that the AL contribution is proportional to $T^2$ whereas the MT contribution can be

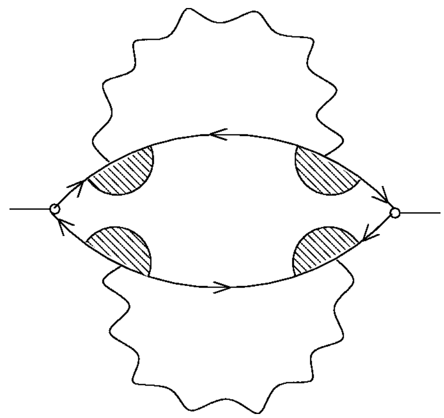

FIG. 3. Higher order correction to the DOS.

divided into two parts. One is proportional to $T^2$ at low temperatures and the other one remains finite when $T \to 0$.

### B. Aslamazov-Larkin contribution to conductivity

This contribution originates from the ability of virtual Cooper pairs to carry an electrical current.[12] The diagram for the operator of the electromagnetic response is represented by the first diagram in Fig. 2 and its analytical expression has the following form:

$$Q^{\mathrm{AL}}(i\omega_\nu)=\frac{4}{3}\sum_{i=1}^{3} T\sum_{\Omega_k}\int\frac{d^3\mathbf{q}}{(2\pi)^3}K(i\Omega_k,\mathbf{q})\times K(i\Omega_k-i\omega_\nu,\mathbf{q})B^2(i\Omega_k,i\omega_\nu,\mathbf{q}), \tag{3.16}$$

$B(i\Omega_k,i\omega_\nu,\mathbf{q})$ describes the block of the Green's functions

$$\begin{aligned}B(i\Omega_k,i\omega_\nu,\mathbf{q})&=-i\int\frac{d^3\mathbf{q}'}{(2\pi)^3}\sin(q_i'd)\cos[(q_i-q_i')d]\frac{et^2dV^2}{(\pi\nu_0\tau)^2}T\\&\times\sum_{\varepsilon_n}\int\frac{d^3\mathbf{p}_1d^3\mathbf{p}_2}{(2\pi)^6}G(i\varepsilon_{n+\nu},\mathbf{p}_1)\\&\times G(i\Omega_k-i\varepsilon_{n+\nu},\mathbf{p}_1)G(i\varepsilon_n,\mathbf{p}_2)\\&\times G(i\Omega_k-i\varepsilon_{n+\nu},\mathbf{p}_2)\\&\times C(i\varepsilon_{n+\nu},i\Omega_k-i\varepsilon_{n+\nu},\mathbf{q})C(i\varepsilon_n,i\Omega_k-i\omega_\nu,\mathbf{q}),\end{aligned} \tag{3.17}$$

where $\sin(q_i'd)$ denotes the current vertex and $\cos[(q_i-q_i')d]$ describes the tunneling vertex. In this integral we may put $\Omega_k=0$ and $\omega_\nu=0$ because in the vicinity of the critical magnetic field $H_c$ the leading contribution to the response $Q^{\mathrm{AL}}$ arises from the fluctuation propagators rather than from the frequency dependence of the function $B$, Eq. (3.17). Therefore we neglect the dependence of function $B$ on frequencies $\Omega_k$ and $\omega_\nu$. Integrating over the momenta $\mathbf{p}_1,\mathbf{p}_2$, and quasimomentum $\mathbf{q}'$, then summing over the internal frequency $\varepsilon_n$ we obtain in the limit of low temperature for function $B(\mathbf{q})$ the result

$$B(\mathbf{q})=-i\frac{32ged}{\pi\Delta_0}\int\sin(q_i'd)\cos[(q_i-q_i')d]\frac{d^3\mathbf{q}'}{(2\pi)^3}. \tag{3.18}$$

As in Sec. III A, in order to calculate the response $Q^{\mathrm{AL}}$, we have to make the analytical continuation from the Matsubara frequency $\omega_\nu$ to real values of $\omega$. This can be achieved by transforming the sum over $\Omega_k$ in Eq. (3.16) into a contour integral. This procedure allows one to carry out the analytical continuation of $Q(i\omega_\nu)$ into the upper complex half plane, $i\omega_\nu\to\omega+i0^+$. As a result for the sum over the $\Omega_k$ in Eq. (3.16) we obtain

$$T\sum_{\Omega_k}K(i\Omega_k,\mathbf{q})K(i\Omega_k-i\omega_\nu,\mathbf{q})\to-i\omega\frac{2\pi T^2}{3\nu_0^2\Delta_0^2}\frac{1}{\tilde{\eta}^4}. \tag{3.19}$$

Using Eqs. (3.18) and (3.19) we finally obtain for the AL contribution to the fluctuation conductivity

$$\frac{\sigma^{\mathrm{AL}}}{\sigma_0}=\frac{64}{27}g\frac{\delta^2T^2}{\Delta_0^4}\sum_{i=1}^{3}\left\langle\frac{\sin^2(q_id)}{\tilde{\eta}^4(\mathbf{q})}\right\rangle, \tag{3.20}$$

which agrees with the results obtained in Refs. 3,4. One can see that $\sigma^{\mathrm{AL}}$ gives a positive contribution to the fluctuation conductivity and is proportional to $T^2$, therefore it vanishes in the limit $T\to0$.

Let us estimate the right hand side in Eq. (3.20) for the case of low temperatures, $T\ll\Delta_0\tilde{\eta}$, and near the critical magnetic field, $h\ll g\delta/\Delta_0$. The main contribution to the integral in Eq. (3.20) comes from small momenta. Therefore we can make an expansion in $\mathbf{q}$ of $\sin(q_id)$ and $\cos(q_id)$. Retaining only the first nonvanishing terms and extending the range of integration from $2\pi/d$ to infinity we obtain

$$\frac{\sigma^{\mathrm{AL}}}{\sigma_0}\sim g^{-3/2}\frac{T^2}{\Delta_0^{3/2}\delta^{1/2}}\left(\frac{H_c}{H-H_c}\right)^{3/2}. \tag{3.21}$$

The AL contribution grows when approaching the critical magnetic field $H_c$ thus leading to a decrease of resistivity. In order to determine which contribution to the conductivity will dominate we compare $\sigma^{\mathrm{AL}}$ with the maximum value of $\sigma^{\mathrm{DOS}}$

$$\left|\frac{\sigma^{\mathrm{AL}}}{\sigma^{\mathrm{DOS}}_{\max}}\right|\sim\frac{g^{-3/2}T^2}{\Delta_0^{1/2}\delta^{3/2}}\ln^{-1}\left(\frac{\Delta_0}{g\delta}\right)\left(\frac{H_c}{H-H_c}\right)^{3/2}. \tag{3.22}$$

For $h\ll g\delta/\Delta_0$ and sufficiently low temperatures, $T\ll\Delta_0\tilde{\eta}$, one can see from Eq. (3.22) that $|\sigma^{\mathrm{AL}}/\sigma^{\mathrm{DOS}}|\ll1$. This means that the AL contribution cannot change the monotonous increase of the resistivity of granular metals when decreasing the magnetic field.

### C. Maki-Thompson contribution to conductivity

This contribution to the conductivity comes from coherent electron scattering forming a Cooper pair on impurities.[13,14] The MT contribution is represented by the second diagram in Fig. 2. The analytical expression for the operator of the electromagnetic response for the MT contribution to fluctuation conductivity is given by

$$Q^{\mathrm{MT}}(i\omega_\nu)=\frac{2}{3}\sum_{i=1}^{3}T\sum_{\Omega_k}\int\frac{d^3\mathbf{q}}{(2\pi)^3}K(i\Omega_k,\mathbf{q})B(i\Omega_k,i\omega_\nu,\mathbf{q}),\tag{3.23}$$

where $K(i\Omega_k,\mathbf{q})$ is the propagator of the superconducting fluctuations and $B(i\Omega_k,i\omega_\nu,\mathbf{q})$ is a function describing the contribution of the loop. This function has the form

$$\begin{aligned}B(i\Omega_k,i\omega_\nu,\mathbf{q})&=\int\sin(q_i'd)\sin[(q_i-q_i')d]\frac{d^3\mathbf{q}'}{(2\pi)^3}\frac{e^2t^2d^2V^2}{(\pi\nu_0\tau)^2}T\\&\quad\times\sum_{\varepsilon_n}\int\frac{d^3\mathbf{p}_1d^3\mathbf{p}_2}{(2\pi)^6}G(i\varepsilon_n,\mathbf{p}_1)\\&\quad\times G(i\Omega_k-i\varepsilon_n,\mathbf{p}_1)G(i\varepsilon_{n-\nu},\mathbf{p}_2)\\&\quad\times G(i\Omega_k-i\varepsilon_{n-\nu},\mathbf{p}_2)\\&\quad\times C(i\varepsilon_n,i\Omega_k-i\varepsilon_n,\mathbf{q})C(i\varepsilon_{n-\nu},i\Omega_k-i\varepsilon_{n-\nu},\mathbf{q}).\end{aligned}\tag{3.24}$$

In evaluating the sum over the Matsubara frequency $\varepsilon_n$ it is useful to break up the sum into two parts. In the first part $\varepsilon_n$ is in the domains $]-\infty,-\omega_\nu[$ and $[0,\infty[$. This gives rise to the *regular part* of the MT diagram. The second (*anomalous*) part of the MT diagram arises from the summation over the $\varepsilon_n$ in the domain $[-\omega_\nu,0[$. Using this we can perform the sum over the $\varepsilon_n$ and after integration over the momenta we can write the function $B$ as a sum of an *anomalous* $B^{\mathrm{an}}$ and a *regular* $B^{\mathrm{reg}}$ contribution to the MT diagram

$$B(i\Omega_k,i\omega_\nu,\mathbf{q})=-32\frac{ge^2}{d}\cos(q_id)(B^{\mathrm{an}}+B^{\mathrm{reg}}),\tag{3.25}$$

where

$$\begin{aligned}B^{\mathrm{an}}=&-\frac{1}{4\pi}\frac{\theta(\Omega_k)\theta(\omega_\nu-\Omega_{k+1})}{\omega_\nu+4\pi T\alpha_{\mathbf{q}}}\left[\psi\left(\frac{1}{2}+\frac{2\omega_\nu-\Omega_k}{4\pi T}+\alpha_{\mathbf{q}}\right)\right.\\&\left.-\psi\left(\frac{1}{2}+\frac{\Omega_k}{4\pi T}+\alpha_{\mathbf{q}}\right)\right],\end{aligned}\tag{3.26}$$

$$B^{\mathrm{reg}}=\frac{1}{8\pi\omega_\nu}\left[\psi\left(\frac{1}{2}+\frac{2\omega_\nu+\Omega_k}{4\pi T}+\alpha_{\mathbf{q}}\right)-\psi\left(\frac{1}{2}+\frac{\Omega_k}{4\pi T}+\alpha_{\mathbf{q}}\right)\right].\tag{3.27}$$

Note that the anomalous and regular part have different signs and the anomalous part has an additional diffusion pole in comparison with the regular one.

#### *1. The anomalous MT contribution to conductivity*

The term $B^{\mathrm{an}}$ corresponds to the *anomalous* MT contribution, it appears in the case of a special pole arrangement in the integration over momenta in Eq. (3.24), when the poles of the Green's functions corresponding to different grains lie on different sides of the axis of the real numbers. In order to calculate the fluctuation conductivity we have to insert $B^{\mathrm{an}}$ into Eq. (3.23). Then, one should perform the analytical continuation of $Q^{\mathrm{an}}(i\omega_\nu)$ to real values of the external frequency $\omega$. Finally, using Eq. (3.1) we get the conductivity. To this end we represent $Q^{\mathrm{an}}$ in the following form:

$$Q^{\mathrm{an}}(i\omega_\nu)=-\frac{16ge^2}{3\pi d\nu_0}\sum_{i=1}^{3}\int\frac{d^3\mathbf{q}}{(2\pi)^3}\frac{\cos(q_id)F(\omega_\nu,\mathbf{q})}{\omega_\nu+4\pi T\alpha_{\mathbf{q}}}.\tag{3.28}$$

The function $F(i\omega_\nu,\mathbf{q})$ is given by[15]

$$F(i\omega_\nu,\mathbf{q})=T\sum_{\Omega_k=0}^{\omega_{\nu-1}\;\prime}f(\Omega_k,\omega_\nu,\mathbf{q}),\tag{3.29}$$

where

$$f(i\Omega_k,i\omega_\nu,\mathbf{q})=\frac{\psi\left(\frac{1}{2}+\frac{2\omega_\nu-\Omega_k}{4\pi T}+\alpha_{\mathbf{q}}\right)-\psi\left(\frac{1}{2}+\frac{\Omega_k}{4\pi T}+\alpha_{\mathbf{q}}\right)}{2h+\frac{|\Omega_k|}{\Delta_0}+\eta(\mathbf{q})}.$$

Because of the presence of the Heaviside function $\theta$ in Eq. (3.26), $B^{\mathrm{an}}$ is only nonzero in the domain $[\Omega_1,\omega_{\nu-1}]$. The upper limit of the sum over $\Omega_k$ in Eq. (3.29) depends on the external frequency $\omega_\nu$. Therefore it is not correct simply to make the substitution $i\omega_\nu\to\omega$. Note that we have calculated Eq. (3.26) only for $\Omega_k\geqslant 0$ but one can easily obtain the corresponding expression for $\Omega_k<0$ replacing $\Omega_k$ by $|\Omega_k|$. Therefore, instead of summing over all values of $\Omega_k$ we extract the term $\Omega_k=0$ and multiply the sum over $\Omega_k>0$ by a factor of 2. The prime on the sum in Eq. (3.29) indicates that the term with $\Omega_k=0$ should be multiplied by 1/2. The analytical continuation is achieved by transformation of the sum into a contour integral. The final result has the following form:[15]

$$\begin{aligned}F^R(\omega,\mathbf{q})=&-\frac{i\omega}{4\pi T}\int_0^\infty\frac{d\Omega}{\sinh^2\frac{\Omega}{2T}}\\&\times\frac{\psi\left(\frac{1}{2}+\frac{i\Omega}{4\pi T}+\alpha_{\mathbf{q}}\right)-\psi\left(\frac{1}{2}-\frac{i\Omega}{4\pi T}+\alpha_{\mathbf{q}}\right)}{2h-\frac{i\Omega}{\Delta_0}+\eta(\mathbf{q})}.\end{aligned}\tag{3.30}$$

For $\Omega\ll 1$ we expand the numerator in a Taylor series around $\Omega=0$ and confine ourselves to the first nonvanishing term. Then we obtain

$$F^R(\omega,\mathbf{q}) = \frac{2i\omega}{(4\pi T)^2\Delta_0}\psi'\left(\frac{1}{2}+\frac{\Delta_0}{4\pi T}\right)\int_0^\infty \frac{d\Omega}{\sinh^2\frac{\Omega}{2T}}\frac{\Omega^2}{\frac{\Omega^2}{\Delta_0^2}+\tilde{\eta}^2}. \tag{3.31}$$

The last integral can be calculated in a similar way as the second integral in Eq. (3.12) in Sec. III A and we obtain for the anomalous MT contribution to conductivity the following result:

$$\frac{\sigma_{\text{an}}^{\text{MT}}}{\sigma_0} = \frac{4\pi\delta T^2}{9\Delta_0^2}\sum_{i=1}^{3}\left\langle\frac{\cos(q_i d)}{4\pi T\alpha_{\mathbf{q}}\tilde{\eta}^2(\mathbf{q})}\right\rangle. \tag{3.32}$$

Let us estimate the anomalous MT contribution to the conductivity. The main contribution to the integral in Eq. (3.32) comes from small momenta $\mathbf{q}$, therefore for $h \ll g\delta/\Delta_0$ we expand the numerator and denominator in powers of $\mathbf{q}$. Confining ourselves to the first nonvanishing order in $\mathbf{q}$, we obtain the following result:

$$\frac{\sigma_{\text{an}}^{\text{MT}}}{\sigma_0} \sim g^{-3/2}\frac{T^2}{\Delta_0^{3/2}\delta^{1/2}}\left(\frac{H_c}{H-H_c}\right)^{1/2}. \tag{3.33}$$

From Eq. (3.33) we see that $\sigma_{\text{an}}^{\text{MT}}$ gives a positive contribution to the fluctuation conductivity and grows when approaching the critical magnetic field $H \to H_c$. The anomalous MT contribution is proportional to $T^2$ as temperature goes to zero. At zero temperature the anomalous MT contribution vanishes.

It is interesting to compare $\sigma_{\text{an}}^{\text{MT}}$ with the contribution to conductivity that arises from the suppression of the density of states $\sigma^{\text{DOS}}$. The ratio of these two quantities is given by

$$\left|\frac{\sigma_{\text{an}}^{\text{MT}}}{\sigma_{\text{max}}^{\text{DOS}}}\right| \sim \frac{g^{-3/2}T^2}{\Delta_0^{1/2}\delta^{3/2}}\ln^{-1}\left(\frac{\Delta_0}{g\delta}\right)\left(\frac{H_c}{H-H_c}\right)^{1/2}. \tag{3.34}$$

One can see that at $T \ll \Delta_0\tilde{\eta}$ the anomalous MT contribution is small compared to $\sigma_{\text{max}}^{\text{DOS}}$. Thus, we conclude that the anomalous MT contribution cannot change the monotonous increase of the resistivity of a granular superconductor when decreasing the magnetic field.

#### *2. The regular MT contribution to conductivity*

Let us now investigate the contribution to the conductivity arising from the *regular* part of the MT diagram. The operator of the electromagnetic response can be written as

$$Q^{\text{reg}}(i\omega_\nu) = \frac{8ge^2}{3\pi d\omega_\nu\nu_0}\sum_{i=1}^{3} T\sum_{\Omega_k}\int\frac{d^3\mathbf{q}}{(2\pi)^3}\cos(q_i d) \times\frac{\psi\left(\frac{1}{2}+\frac{2\omega_\nu+\Omega_k}{4\pi T}+\alpha_{\mathbf{q}}\right)-\psi\left(\frac{1}{2}+\frac{\Omega_k}{4\pi T}+\alpha_{\mathbf{q}}\right)}{2h-\frac{i\Omega}{\Delta_0}+\eta(\mathbf{q})}. \tag{3.35}$$

As before, we can write this sum as a contour integral. Then, we have to perform the analytical continuation to real values of the external frequency $i\omega_\nu$: $i\omega_\nu \to \omega + i0^+$. We expand the resulting expression in a Taylor series up to the second order in $\omega$. The static term is canceled by a similar term in $Q(0)$. As a result we obtain for the operator of the electromagnetic response

$$Q_{\text{reg}}^{\text{MT}} = \frac{-2i\omega ge^2}{9\pi^4 dT^2\nu_0\Delta_0}\psi''\left(\frac{1}{2}+\frac{\Delta_0}{4\pi T}\right)\int_0^{\Omega_{\max}}\coth\frac{\Omega}{2T}\frac{\Omega\, d\Omega}{\frac{\Omega^2}{\Delta_0^2}+\tilde{\eta}^2}. \tag{3.36}$$

Performing the integration in Eq. (3.36) we obtain for the regular MT contribution to conductivity in the low temperature limit

$$\frac{\sigma_{\text{reg}}^{\text{MT}}}{\sigma_0} = -\frac{2}{9\pi}\frac{\delta}{\Delta_0}\sum_{i=1}^{3}\left\langle\cos(q_i d)\ln\left(\frac{1}{\tilde{\eta}(\mathbf{q})}\right)\right\rangle. \tag{3.37}$$

Let us estimate the integral in the right hand side of Eq. (3.37). At very low temperatures, $T \ll \Delta_0\tilde{\eta}$, and near the critical magnetic field $H_c$, $h \ll g\delta/\Delta_0$, it turns out that

$$\frac{\sigma_{\text{reg}}^{\text{MT}}}{\sigma_0} \sim -\frac{\delta}{\Delta_0}. \tag{3.38}$$

The regular MT part gives a negative contribution to the fluctuation conductivity and it is independent of temperature, so that even at zero temperature $\sigma_{\text{reg}}^{\text{MT}}$ remains finite and reduces the conductivity. We have shown before that at very low temperature neither $\sigma^{\text{AL}}$ nor $\sigma_{\text{an}}^{\text{MT}}$ can change the monotonous increase of resistivity caused by the suppression of the density of states and the regular MT contribution.

### D. Diagrams No. 3, 4, 7, 8 and 9,10

These diagrams arise when the DOS and MT diagrams are averaged over the impurity positions. This averaging process results in the appearance of an additional Cooperon connecting two different grains with each other. Let us consider the diagram 9 in Fig. 2, the diagram 10 can be calculated in the same way. The analytical expression of the operator of the electromagnetic response reads as follows:

$$Q(i\omega_\nu) = \frac{4}{3}\sum_{i=1}^{3} T\sum_{\Omega_k}\int\frac{d^3\mathbf{q}}{(2\pi)^3}K(i\Omega_k,\mathbf{q})B(i\Omega_k,i\omega_\nu,\mathbf{q}), \tag{3.39}$$

where $K(i\Omega_k,\mathbf{q})$ is the propagator of the superconducting fluctuations and $B(i\Omega_k,i\omega_\nu,\mathbf{q})$ corresponds to the contribution of the loop. A factor of 2 in Eq. (3.39) comes from the summation over spin indices and another factor of 2 originates from a similar diagram shown in Fig. 2. The analytical expression for the loop can be written as

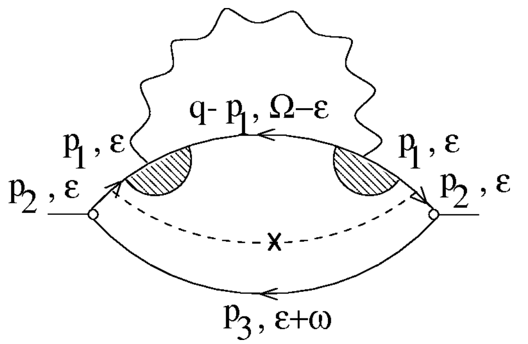


FIG. 4. DOS-type diagram with an additional impurity renormalization.

$$
\begin{aligned}
B(i\Omega_k,i\omega_\nu,\mathbf{q})=4\int \sin(q_i'd)\frac{d^3\mathbf{q}'}{(2\pi)^3}\int \sin(q_i''d)\\
\times\frac{d^3\mathbf{q}''}{(2\pi)^3}\frac{e^2t^2d^2V^2}{(2\pi\nu_0\tau)^3}T\sum_{\varepsilon_n}\int\frac{d^3\mathbf{p}_1d^3\mathbf{p}_2}{(2\pi)^6}\\
\times G(i\varepsilon_{n+\nu},\mathbf{p}_1)G^2(i\varepsilon_n,\mathbf{p}_2)\\
\times G(i\Omega_k-i\varepsilon_n,\mathbf{p}_2)C^2(i\varepsilon_n,i\Omega_k-i\varepsilon_n,\mathbf{q})\\
\times C(i\varepsilon_{n+\nu},i\Omega_k-i\varepsilon_n,\mathbf{q}),
\end{aligned}
\tag{3.40}
$$

where $\mathbf{p}_1$ and $\mathbf{p}_2$ denote the momenta in the different granules and $\mathbf{q}',\mathbf{q}''$ are quasimomenta. As before we can set $\Omega_k$ inside the Green's functions equal to zero. We can immediately see that after integration over $\mathbf{q}',\mathbf{q}''$ the function $B(i\Omega_k,i\omega_\nu,\mathbf{q})$ is equal to zero and the contribution to the fluctuation conductivity from this diagram vanishes. The same reason holds for the diagram 3 and 4 in Fig. 2 which come from the averaging of the MT diagram over impurity positions. Also the diagrams 7 and 8 of Fig. 2 do not contribute to the fluctuation conductivity. For simplicity we present the diagram 7 once again in Fig. 4.

One can easily see that the corresponding analytical expression of such a diagram contains a term of the form

$$
\int\frac{d^3\mathbf{p}_2}{(2\pi)^3}G^A(\varepsilon,\mathbf{p}_2)G^A(\varepsilon,\mathbf{p}_2)=0.
$$

Both Green's functions have poles on the same side of the complex plane and therefore the integral is zero. Thus, we may neglect diagrams 7 and 8. As we have shown, only the diagrams 1 (AL), 2 (MT), 5, and 6 (DOS) make a contribution to the fluctuation conductivity of granular metals.

### E. Final Formulas

The calculations presented in the previous subsections show that in granular superconducting metals fluctuations make a considerable contribution to the conductivity in the normal phase at low temperatures and strong magnetic field. The fluctuation conductivity of granular metals is given by the DOS, Aslamazov-Larkin and Maki-Thompson contribution.

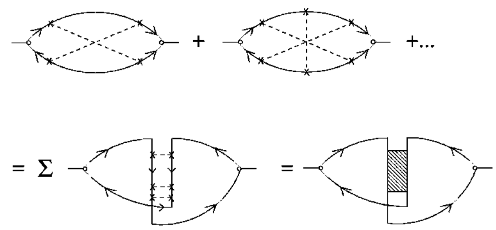


FIG. 5. Weak localization correction to conductivity. The shaded block denotes the renormalized Cooperon.

$$
\sigma^{\mathrm{fl}}=\sigma^{\mathrm{DOS}}+\sigma^{\mathrm{AL}}+\sigma^{\mathrm{MT}}.
\tag{3.41}
$$

Other contributions from the diagrams in Fig. 2 are equal to zero. At low temperatures, $T\ll\Delta_0\tilde{\eta}$, the final result for the total fluctuation conductivity of granular metals can be written as

$$
\begin{aligned}
\frac{\sigma^{\mathrm{fl}}}{\sigma_0}=-\frac{\delta}{\Delta_0}\Bigg[\frac{1}{2\pi}\left\langle\ln\left(\frac{1}{\tilde{\eta}(\mathbf{q})}\right)\right\rangle\\
-\sum_{i=1}^{3}\Bigg(\frac{64}{27}g\frac{\delta^2T^2}{\Delta_0^4}\left\langle\frac{\sin^2(q_id)}{\tilde{\eta}^4(\mathbf{q})}\right\rangle\\
+\frac{4\pi}{9}\frac{T^2}{\Delta_0^2}\left\langle\frac{\cos(q_id)}{4\pi T\alpha_{\mathbf{q}}\tilde{\eta}^2(\mathbf{q})}\right\rangle\\
-\frac{2}{9\pi}\left\langle\cos(q_id)\ln\left(\frac{1}{\tilde{\eta}(\mathbf{q})}\right)\right\rangle\Bigg)\Bigg].
\end{aligned}
\tag{3.42}
$$

Equation (3.42) is the main result of our paper. For low temperatures, $T\ll\Delta_0\tilde{\eta}$ and strong magnetic field, $H>H_c$ the main contribution comes from the first and from the last terms in the brackets in Eq. (3.42). As a consequence, the superconducting fluctuation contribution to the conductivity of granular metals is *negative*. The quantitative result of Eq. (3.42) is written for the region $1\ll g\ll(E_T/\delta)$. In the next section we show that the correction to the conductivity due to weak localization effects can be neglected.

## IV. WEAK LOCALIZATION CORRECTION TO CONDUCTIVITY OF GRANULAR METALS

The weakly localized regime (WLR) is the regime where interference effects between different plane waves, being treated independently, start to play a role. The interference of plane waves leads to an increase of the probability to find an electron at a certain place and this effect results in a reduction of the conductivity. The diagram corresponding to the WL correction to conductivity[16,17] is shown in Fig. 5.

For the WL correction of granular metals[3,4] we obtain

$$
\delta\sigma^{\mathrm{WL}}=-\frac{16}{3}ge^2\delta d^2\sum_{i=1}^{3}\int\frac{C(0,\mathbf{q})}{2\pi\nu_0}\cos(q_id)\frac{d^3\mathbf{q}}{(2\pi)^3},
\tag{4.1}
$$

where $C(0,\mathbf{q})$ is the Cooperon taken at the frequency $\omega=0$ and quasimomentum $\mathbf{q}$. Using Eq. (3.4) and making the integration over quasimomentum **q** in Eq. (4.1) we obtain the final result for the WL correction to conductivity of granular metals

$$\frac{\delta\sigma^{\mathrm{WL}}}{\sigma_0} \sim -g^{-3/2}\left(\frac{\Delta_0}{\delta}\right)^{1/2}. \tag{4.2}$$

Similar to the case of a homogeneous sample, the WL correction to conductivity has the negative sign.

Now let us compare the WL correction with $\sigma^{\mathrm{DOS}}$. The ratio of these two quantities is given by

$$\left|\frac{\delta\sigma^{\mathrm{WL}}}{\sigma^{\mathrm{DOS}}_{\mathrm{max}}}\right| = g^{-3/2}\left(\frac{\Delta_0}{\delta}\right)^{3/2}\ln^{-1}\left(\frac{\Delta_0}{g\delta}\right). \tag{4.3}$$

For large $g \gg 1$, this ratio is small and therefore the WL correction is always smaller than the correction which arises from the suppression of the density of states.

## V. CONCLUSION

We have obtained the fluctuation conductivity of granular metals at low temperatures and strong magnetic field. Our main result is given by Eq. (3.42). To obtain this result we assumed that the dimensionless conductance $g$ satisfies the inequality (1.1). Therefore, the granular structure of the metal is essential for our consideration. We have generalized the previous studies[3,4] to the entire region $1 \ll g \ll E_T/\delta$. This case is still different from the case of 2D homogeneous superconductor[5] where the dimensionless conductance was assumed to be very large (of order $k_F l$).

One can see that $\sigma^{\mathrm{DOS}}$ and $\sigma^{\mathrm{MT}}_{\mathrm{reg}}$ give a *negative contribution* to the fluctuation conductivity, whereas the AL and the anomalous MT contributions are positive. This leads to a competition between the positive and the negative contributions. But as we have shown, at low temperatures, $T \ll T_c$, and strong magnetic field, $H \gg H_c$, this negative contribution cannot be compensated by the positive contributions and therefore the entire fluctuation conductivity is negative. The situation holds even at $T=0$ where the AL and anomalous MT contributions are equal to zero.

Qualitatively results are depicted in Fig. 6, where the typical curve for the dependence of the fluctuation resistivity on the reduced magnetic field $h$ at low temperature is presented. The curve reaches the value of the classical resistivity $R_0$ asymptotically only in extremely strong magnetic fields. It was shown in Ref. 3 that for the granular metals the real transition into the superconducting state occurs not at $H_c$ but at a lower field $H_{c_2}$, which is due to the electron motion over many grains. Thus, in order to take into account the macroscopic orbital electron motion we have to replace $H_c$ by $H_{c_2}$ in our formulas.

## ACKNOWLEDGMENTS

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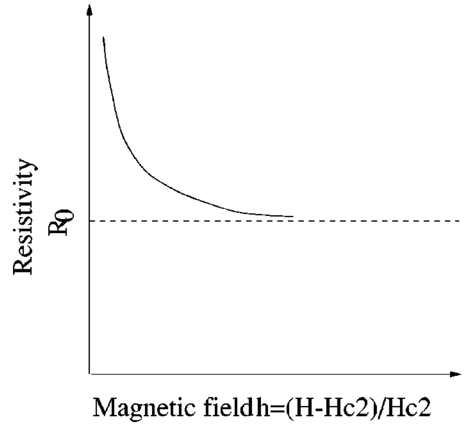


FIG. 6. Schematic picture of the resistivity of a granulated superconductor at fixed temperature as a function of the magnetic field.

## APPENDIX

In this appendix we carry out the analytical continuation in Eq. (3.11) and obtain the result for the operator of electromagnetic response $Q^R(\omega)$, Eq. (3.12). The lower limit in the sum over $\Omega_k$ in Eq. (3.11) depends on the external frequency $\omega_\nu$. Therefore the analytical continuation of this expression in the region of real frequencies is not correct by simply making the replacement $i\omega_\nu \to \omega + i0^+$, it can be achieved by a transformation of the sum into a contour integral

$$T\sum_{\Omega_k=-\omega_\nu}^{\infty} D(i\Omega_k, i\omega_\nu, \mathbf{q})K(i\Omega_k, \mathbf{q}) \to \frac{1}{4\pi i}\int_{C_1+C_2} \coth\frac{z}{2T} D(z, i\omega_\nu, \mathbf{q})K(z,\mathbf{q})dz, \tag{A1}$$

where the contours of integration $C_1$ and $C_2$ are shown in Fig. 7. We should interpret the propagator of the superconducting fluctuations as the *retarded* propagator $K^R$ when $\mathrm{Im}(z) > 0$ and as the *advanced* propagator $K^A$ when $\mathrm{Im}(z) < 0$. In the vicinity of the critical magnetic field, $(H - H_c)/H_c \ll 1$, we can expand the logarithm in the denominator of $K$ and the retarded or advanced form of this quantity has the following form:[3]

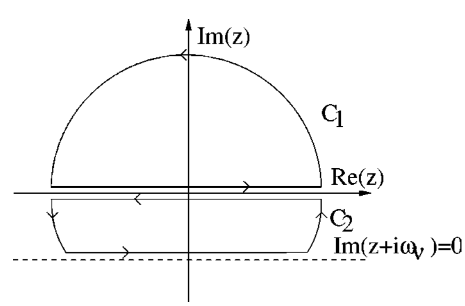


FIG. 7. Contours of integration $C_1$ and $C_2$, the arrows indicate the way going along the paths.

$$K^{R,A}(-i\Omega,\mathbf{q}) = -\frac{1}{\nu_0}\left(2h \mp \frac{i\Omega}{\Delta_0} + \eta(\mathbf{q})\right)^{-1}, \quad \text{(A2)}$$

where $h=(H-H_c)/H_c$.

The contribution to the integral from the large circle vanishes if we extend the contour to $i\infty$, so that only the paths along the horizontal lines with $\mathrm{Im}(z)=0$ and $\mathrm{Im}(z+i\omega_\nu)=0$ remain. Thus for the right hand side in Eq. (A1) we obtain

$$\int_{-\infty}^{+\infty}\coth\frac{z}{2T}D^R(-iz,i\omega_\nu,\mathbf{q})K^R(-iz,\mathbf{q})\frac{dz}{4\pi i} + \int_{-\infty}^{\infty}\coth\frac{z}{2T}D^A(-iz,i\omega_\nu,\mathbf{q})K^A(-iz,\mathbf{q})\frac{dz}{4\pi i} + \int_{-\infty-i\omega_\nu}^{+\infty-i\omega_\nu}\coth\frac{z}{2T}D^A(-iz,i\omega_\nu,\mathbf{q})K^A(-iz,\mathbf{q})\frac{dz}{4\pi i}, \quad \text{(A3)}$$

where $D^R,D^A$ are the retarded, advanced forms of the function $D(i\Omega_k,i\omega_\nu,\mathbf{q})$. They are given by

$$D^{R,A}(\Omega,\omega,\mathbf{q}) = -\frac{ge^2}{\pi^2 dT}\left[\psi'\left(\frac{1}{2}-\frac{2i\omega+i\Omega}{4\pi T}+\alpha_{\mathbf{q}}\right) - \psi'\left(\frac{1}{2}\mp\frac{i\Omega}{4\pi T}+\alpha_{\mathbf{q}}\right)\right]. \quad \text{(A4)}$$

In the third integral in Eq. (A3) we make the substitution of variables $z+i\omega_\nu \to z$ and use the fact that $\coth[(z+i\omega_\nu)/2T]=\coth(z/2T)$. Now we can simply make the analytical continuation: $i\omega_\nu \to \omega$. Finally we obtain for the sum over $\Omega_k$ in Eq. (A1) the following result:

$$\int_{-\infty}^{+\infty}\frac{d\Omega}{4\pi i}\coth\frac{\Omega}{2T}D^R(-i\Omega,\omega,\mathbf{q})K^R(-i\Omega,\mathbf{q}) + \int_{-\infty}^{+\infty}\frac{d\Omega}{4\pi i}\left(\coth\frac{\Omega-\omega}{2T}-\coth\frac{\Omega}{2T}\right) \times D^A(-i\Omega,\omega,\mathbf{q})K^A(-i\Omega,\mathbf{q}). \quad \text{(A5)}$$

Since we are interested in the dc conductivity it is sufficient to retain the linear term in $\omega$ in Eq. (A5) then we obtain

$$T\sum_{\Omega_k=-\omega_\nu}^{\infty} \to i\omega\frac{ge^2}{4\pi^4 dT^3}\int_{-\infty}^{+\infty}\coth\frac{\Omega}{2T}\psi''\left(\frac{1}{2}-\frac{i\Omega}{4\pi T}+\alpha_{\mathbf{q}}\right) \times K^R(-i\Omega,\mathbf{q})\frac{d\Omega}{4\pi i} - \frac{\omega}{2T}\int_{-\infty}^{+\infty}\frac{D^A(-i\Omega,0,\mathbf{q})K^A(-i\Omega,\mathbf{q})}{\sinh^2\frac{\Omega}{2T}}\frac{d\Omega}{4\pi i}. \quad \text{(A6)}$$

The main contribution to the integral over $\Omega$ in Eq. (A6) comes from small values of the frequency therefore we may put $\Omega=0$ inside $\psi(x)$ in the first integral and extend the integrand by $i\Omega/\Delta_0+\tilde{\eta}$. One can easily see that this integral is logarithmically divergent and it has to be cut off at $\Omega_{\max}\sim\Delta_0$. In the second integral we make an expansion in $\Omega$, retaining only the first nonvanishing term and then we extend the resulting expression by $-i\Omega/\Delta_0+\tilde{\eta}$. As a result for the operator of electromagnetic response we obtain Eq. (3.12).


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