Bloch walls in strongly driven easy-plane ferromagnets

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We show that the action of strong rapidly oscillating transverse magnetic field in easy-plane ferromagnet gives rise to the appearance of effective *biaxial magnetic anisotropy* and new soliton solutions in the form of Bloch and Néel domain walls. *Exact analytical expressions* describing the conversion of a Néel wall into the stationary Bloch wall under dissipation are derived on the ground of the adiabatic approximation which are in *good* agreement with the numerical simulations of the basic strongly perturbed Landau-Lifshitz equation.

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I. INTRODUCTION

Strong and rapid perturbations can be a source of drastic change in the nonlinear system behavior and can lead to dynamical stabilization via the appearance of new soliton solutions. This phenomenon seems to be a general character in different fields of condensed matter physics. As an example Josephson junctions under external ac force can support stable propagation of localized fluxons synchronized with a rotating background.[1] In a nonlinear optical medium with quasi-phase-matched quadratic nonlinearity, averaging over rapid periodic modulations of nonlinear susceptibility induces Kerr effects, such as self- and cross-phase modulations.[2] In view of the above-mentioned phenomenon magnetic systems leave much room for study.

Ferromagnets strongly driven by a transverse magnetic field were the subject of special interest for a long time after the discovery of spin-wave instabilities by Suhl.[3] Since then many papers dealing with truncated spin-wave mode models have been published (for a review see Ref. 4 and the recent paper in Ref. 5). The regime far above the spin-wave instabilities remains an open question.

The dynamics of nonlinear excitations as well as their equilibrium and thermodynamical properties in quasi-one-dimensional easy-plane ferromagnets (EPFM's) has been widely investigated over the last few decades.[6–9] The existence of free π kinks in EPFM's under a rapidly oscillating mean-zero magnetic field has been shown in Ref. 10 in the frame of the sine-Gordon equation (sGE).[11] The equation governing a nonlinear spin dynamics is known to be the Landau-Lifshitz equation (LLE) which can be mapped onto the sGE only in the limit of low magnetic fields and small off-easy-plane spin excursions.[12]

It is well known that a magnetic field applied within the easy plane breaks the integrability of the system. Nevertheless, in the case of strong and rapidly oscillating magnetic fields one expects the averaged nontrivial dynamics to reveal integrability.

In this paper we study strongly driven EPFM's. Multiscale expansion is employed to derive the averaged Landau-Lifshitz equation describing the dynamics of a *biaxial* ferromagnet possessing a stationary Walker solution in the presense of damping.[13] We also study the dissipative time evolution of a Néel wall into the stationary state which depends on the amplitude of the dc magnetic field and the ratio of the ac magnetic field amplitude to the driving frequency.

The outline of the paper is as follows. In Sec. II, the model for EPFM's is introduced. In Sec. III the multiple-time-scale approach is used to derive the averaged Hamiltonian. Section IV is devoted to the properties of new solutions in the form of Bloch walls. In Sec. V numerical and analytical results concerning the transformation of the Néel wall into a stationary Bloch wall under damping and additional dc magnetic field are presented. The conclusions are summarized in Sec. VI.

II. MODEL

The Hamiltonian for a ferromagnetic chain has the form[14]

$$\mathcal{H}=-J\sum_i \mathbf{S}_i\cdot\mathbf{S}_{i+1}+A\sum_i (S_i^z)^2-\sum_i \mathbf{S}_i\cdot\mathbf{H}, \tag{1}$$

where $\mathbf{S}_i$ stands for the classical spin vector measured in units of the Bohr magneton μ_B at the ith site along the chain, i.e., along the z axis. The first term describes the nearest-neighbor ferromagnetic exchange interaction of strength $J>0$. The parameters A and $\mathbf{H}$ are the easy-plane anisotropy constant and the magnetic field along the x direction, respectively. For the quasi-one-dimensional ferromagnet $CsNiF_3$, the parameters take the values $J=23.6$ K and $A=4.5$ K, which corresponds to a magnetic field of 18 kG.

The spin dynamics is governed by the LLE with the phenomenological Gilbert damping term:

$$\dot{\mathbf{S}}_i=-\gamma\mathbf{S}_i\times\frac{\delta\mathcal{H}}{\delta\mathbf{S}_i}-\frac{\alpha}{S}\mathbf{S}_i\times\dot{\mathbf{S}}_i, \tag{2}$$

where $\mathbf{S}=S(\cos\phi\sin\theta,\sin\phi\sin\theta,\cos\theta)$, $-\delta\mathcal{H}/\delta\mathbf{S}_i$ is the effective magnetic field at site i, $\gamma=g\mu_B$ is the gyromagnetic ratio and α is the dissipation strength.

A continuum model of EPFM's can be derived in the usual way; see Ref. 14. Let us put for simplicity $S=1$ and rescale the variables as $\mathbf{s}\to\mathbf{S}$, $t\to 2A\gamma t$, $z^2\to z^2 2A/Ja^2$, and $H\to H/2A$.

In the new variables LLE with driving magnetic field $H(t/\epsilon)=H+h\sin(t/\epsilon)$ takes the form

$$\mathbf{s}_t=\mathbf{s}\times[\mathbf{s}_{zz}-s^z\mathbf{e}^z+H(t/\epsilon)\mathbf{e}^x]-\alpha\mathbf{s}\times\mathbf{s}_t, \tag{3}$$

where subscripts (t and z) denote derivatives (with respect to time t and space z coordinates), $\mathbf{e}^z$ and $\mathbf{e}^x$ are unit vectors along the z and x axes, respectively, superscripts denote the indicated components of spin vector $\mathbf{s}$ and $\epsilon=1/\omega \ll 1$, and ω is the dimensionless driving frequency.

III. MULTIPLE-TIME-SCALE TECHNIQUE

In order to find an averaged equation we employ the multiple-time-scale expansion technique.[15] We introduce the multiple time scales (a fast time variable $\tau=\omega t=t/\varepsilon$ and slow time variables $t_n=\varepsilon^n t$, with $n=0,1,2,\ldots$) as follows:

$$\frac{\partial}{\partial t}=\frac{1}{\varepsilon}\frac{\partial}{\partial\tau}+\frac{\partial}{\partial t_0}+\varepsilon\frac{\partial}{\partial t_1}+\varepsilon^2\frac{\partial}{\partial t_2}+\cdots,$$

$$\mathbf{s}=\mathbf{S}+\varepsilon\mathbf{s}_1+\varepsilon^2\mathbf{s}_2+\cdots, \tag{4}$$

$$\mathbf{s}_t=\mathbf{S}_{t_0}+\mathbf{s}_{1\tau}+\varepsilon(\mathbf{S}_{t_1}+\mathbf{s}_{1t_0}+\mathbf{s}_{2\tau})+\varepsilon^2(\mathbf{S}_{t_2}+\mathbf{s}_{1t_1}+\mathbf{s}_{2t_0}+\mathbf{s}_{3\tau})+\cdots, \tag{5}$$

where we introduce the slowly varying envelope function $\mathbf{S}(t_n)$, which depends on the slow times t_n. The fast oscillating components $\mathbf{s}_n(\mathbf{S},\tau)$ $(n=1,2,\ldots)$ depend on $\mathbf{S}$ and fast time τ.

To separate the fast oscillations from the slowly varying field, we substitute expressions (4) and (5) into Eq. (3) and expand in powers of ε by the proper avoiding secular terms.

For the leading order of (ε^0), we have

$$\mathbf{S}_{t_0}=\mathbf{S}\times(\mathbf{S}_{zz}-S^z\mathbf{e}^z+H\mathbf{e}^x)-\alpha\mathbf{S}\times\mathbf{S}_{t_0}, \tag{6}$$

$$\mathbf{s}_{1\tau}=h\sin\tau\mathbf{S}\times\mathbf{e}^x-\alpha\mathbf{S}\times\mathbf{s}_{1\tau}. \tag{7}$$

From Eq. (7) we find

$$\mathbf{s}_1=-h\cos\tau(\mathbf{S}\times\mathbf{e}^x-\alpha\mathbf{S}\times[\mathbf{S}\times\mathbf{e}^x]); \tag{8}$$

here, we assume that $\alpha\ll 1$, since dissipation is small in ferromagnets of high purity. Therefore we can neglect all terms proportional to α^2 and $\alpha^2h^2\epsilon^2$ since they are small in comparison with the α ones.

In the next order ε^1, we find $\mathbf{S}_{t_1}=0$ and

$$s_2^x=h\sin\tau[(S^z)^2-(S^y)^2],$$

$$s_2^y=h\sin\tau S^xS^y-\frac{h^2}{2}S^y\cos 2\tau,$$

$$s_2^z=-h\sin\tau S^xS^z-\frac{h^2}{2}S^z\cos 2\tau. \tag{9}$$

Let us write the formula for $\mathbf{s}^2$, expressed in components of slowly varying spin $\mathbf{S}$ up to terms of order $O(\Delta^2)$, and average over the oscillation time period of the driving magnetic field. Thus we get $(S^x)^2+(1+\Delta)[(S^y)^2+(S^z)^2]=1$. From this we find it necessary to rescale $\mathbf{S}$ components provided the conservation of the spin length holds:

$$S^x\to S^x,\quad S^y\to\sqrt{1+\Delta}S^y,\quad S^z\to\sqrt{1+\Delta}S^z. \tag{10}$$

By using the rescaled slowly varying spin components (10) we obtain averaged LLE's up to terms of the order $\mathcal{O}(\Delta^2)$ which can be written as follows:

$$S_t^x=(S^yS_{zz}^z-S^zS_{zz}^y)-(1-2\Delta)S^yS^z,$$

$$S_t^y=S^zS_{zz}^x-S^xS_{zz}^z+(1-\Delta)S^zS^x+S^zH,$$

$$S_t^z=S^xS_{zz}^y-S^yS_{zz}^x-\Delta S^xS^y-S^yH, \tag{11}$$

where $\Delta=(\varepsilon h)^2/2$. We do not write the dissipation term on the right-hand side (RHS) of these equations [this term is simply reduced to $-\alpha(1-2\Delta)\mathbf{S}\times\mathbf{S}_t$].

The Hamiltonian of the averaged system from Eqs. (11) expressed in the original variables t, z, and H is

$$\mathcal{E}_{av}=\int_{-\infty}^{\infty}\frac{dz}{a}\left[\frac{Ja^2}{2}\mathbf{S}_z^2+A(S^z)^2-S^xH\right]+\int_{-\infty}^{\infty}\frac{dz}{a}\times\{A\Delta[(S^y)^2-(S^z)^2]\}+\mathcal{O}(\Delta^2). \tag{12}$$

This is exactly the total energy of a biaxial ferromagnet. Let us choose a new system of coordinates such that the old system (x,y,z) goes into the new system (z,x,y). The biaxial ferromagnet in the new parametrization has two easy axes x and z with easy-axis anisotropy constants equal to $A(1-2\Delta)$ and $A(1-\Delta)$, respectively.

IV. BLOCH WALLS

Now after averaging one integrable system, the uniaxial ferromagnet under rapid perturbation, we treat another integrable system, the biaxial ferromagnet which is rather well studied.[16,17] This system is known to be completely integrable in the absence of damping and an external magnetic field.[18]

There are two degenerate spin configurations which minimize the energy (12): uniform configurations with all spins pointing either along the positive or along the negative x direction. Due to the effective easy-axis anisotropy in Eq. (12), the transition region will have a finite width and form a Bloch wall (or soliton). A static Bloch wall connects the anisotropy minima $\phi=0$ and $\phi=\pi$ within the easy plane.

Now let us consider moving Bloch wall configurations. We use dimensionless variables for convenience. In the absence of damping and a static magnetic field the exact Walker solution for averaged system can be expressed as[19]

$$S^x=-\tanh u,$$

$$S^y=\cos\varphi\,\mathrm{sech}\,u,$$

$$S^z=-\sin\varphi\,\mathrm{sech}\,u, \tag{13}$$

where $\varphi=\varphi_0=\text{const}$ and $u=q\mathcal{E}(\varphi)(z-vt)/2$. The parameter q is the topological charge of soliton and is defined as $q=\frac{1}{2}[S_x(z=\infty)-S_x(z=-\infty)]$. In this paper we consider further only the case with $q=1$. The velocity and energy of the Bloch wall are given by

$$v=(1-\Delta)\sin 2\varphi/\mathcal{E}(\varphi), \tag{14}$$

$$\mathcal{E}(\varphi)=2\sqrt{\sin^2\varphi+\Delta\cos^2\varphi}. \tag{15}$$

The static Bloch and Néel walls correspond to $\varphi_0=0$ and $\pi/2$, respectively.

The maximum velocity of the Bloch wall is $v_{max}=1-\sqrt{\Delta}$ which is finite and the width of the soliton shrinks with increasing velocity. There are two branches in the $\mathcal{E}(v)$ curves. We refer to solitons with $\mathcal{E}<\mathcal{E}(v_{max})$ and $\mathcal{E}>\mathcal{E}(v_{max})$, where $\mathcal{E}(v_{max})=2\Delta^{1/4}$, as belonging to the lower and upper branches, respectively. Solitons in the upper branch have smaller width than in the lower one at the same velocity as is evident from Eq. (15). The bloch wall in the lower branch resembles outwardly the π kink of the sGE. The essential distinction between sGE theory and the LLE one is that π kinks of the sGE (Ref. 10) do not exist in the upper branch.

We have performed numerical simulations of the LLE by making use of the implicit finite-difference method for integration.

In order to obtain initial conditions let us substitute Bloch wall solution (13) into Eqs. (8), (9), and (4). After straightforward calculations we get an equation which connects spin vector **s** with its slowly varying component **S**. In order to recover spin length S conservation we should divide the expressions for s^y and s^z by a factor of $\sqrt{1+\Delta^2}$. It is valid because we work with an accuracy up to terms of order $\mathcal{O}(\Delta^2)$. After some simple algebra we arrive at

$$s^x(0,z)=-\tanh u(0,z),$$

$$s^y(0,z)=\cos(\varphi-\epsilon h)/\cosh u(0,z),$$

$$s^z(0,z)=-\sin(\varphi-\epsilon h)/\cosh u(0,z). \tag{16}$$

We determine the Bloch wall velocity from the zero crossing of the S_x spin component. We can use this numerical definition for the soliton velocity as long as strong asymmetric shape distortions are absent. Figure 1 shows the mean energy versus the mean velocity $\mathcal{E}(v)$. The coincidence between the analytical and numerical data is surprisingly good for smaller Δ, when corrections to Eq. (11) of the order $O(\Delta^2)$ are negligible.

We also compare the propagation of two different solitons in the lower and upper branches of the energy curve for the same velocity $v=0.5$ and $\Delta=0.125$. Both solitons move equally and begin to substantially deviate from the initial solution (13) after a time of the order of $3/\Delta$ in dimensionless variables. Note that the soliton in the upper branch begins to be destroyed under strong radiation slightly faster than the one in the lower branch. It is clear from this fact that the upper branch is more unstable to all appearances. Similar branches of $\mathcal{E}(v)$ curves arise also in the soliton dynamics in EPFM's in static magnetic fields without driving[12,20] which is a distinctive feature of the LLE.

V. DISSIPATIVE CONVERSION OF A NÉEL WALL INTO A STATIONARY BLOCH WALL

The most interesting case concerns stationary moving Bloch walls[13] generated by strong and rapidly oscillating non-mean-zero magnetic fields under damping.

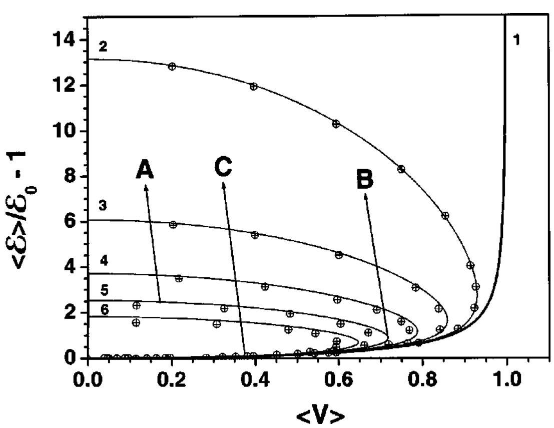

FIG. 1. Mean Bloch wall energy against mean velocity with $H=0$, $\alpha=0$ for different Δ. Scatter graph with crossed circles is obtained by numerical solution of the LLE. Solid curves correspond to analytical expression for $\mathcal{E}(v)$: Eqs. (14) and (15). (1) sine-Gordon limit; Δ = 0.005 (2), 0.02 (3), 0.045 (4), 0.08 (5), and 0.125 (6). The figure is symmetric with respect to transformation $\langle v\rangle\rightarrow-\langle v\rangle$. Labels *A*, *B*, and *C* correspond to initial state on the upper branch, the state with maximum velocity, and final stationary state on the lower branch (see Figs. 2 and 3).

A steady state can exist only in the lower branch of the $\mathcal{E}(v)$ curve. The parameters of the stationary Walker solution up to order φ^2 have the form

$$\varphi_0=H/[(1-3\Delta)\alpha],$$

$$\mathcal{E}=2\sqrt{\Delta+\varphi_0^2},\quad v=2H/[\alpha(1-2\Delta)\mathcal{E}]. \tag{17}$$

From Eq. (17) we conclude that a steady state exists provided the stationary velocity is not too close to the maximum velocity and the static magnetic field H is smaller than $(1-\Delta)\alpha$. From this we get the condition $\Delta>\varphi_0^2$. At the same time Δ should not be so large, since we neglect terms of order Δ^2.

The results of numerical simulations are presented in Fig. 2. We compare parameters obtained from the simulations for different values of Δ at fixed H with the theoretical values calculated from Eqs. (17).

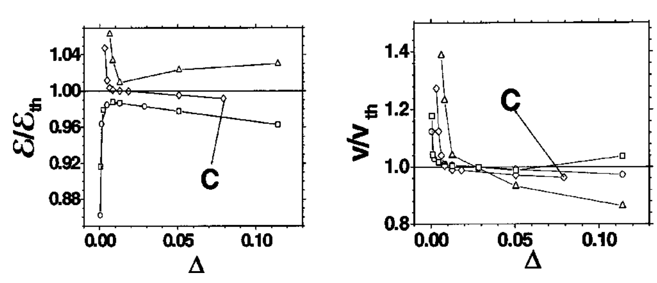

FIG. 2. The ratios of $\mathcal{E}/\mathcal{E}_{th}$ and v/v_{th} vs Δ, where $\mathcal{E}_{th}$, v_{th} are calculated by Eqs. (17). $H=10^{-4}$ (□), 10^{-3} (○), 10^{-2} (◇), and 2×10^{-2} (△). The damping $\alpha=0.1$.

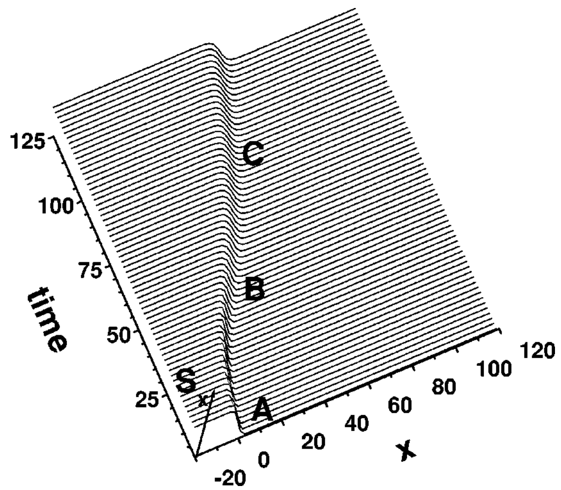

FIG. 3. Profile of the spin component S_x as a function of space and time describing switching between Néel and Bloch domain walls. Initial velocity of the soliton is 0.18 in dimensionless units. Soliton passes from the upper (A) to the lower branch (C) going through the state with maximum velocity (B) and approaches its stationary state with velocity 0.375. See also Figs. 1 and 2. Static magnetic field $H=10^{-2}$, $\Delta=0.08$, and $\alpha=0.1$.

By solving the LLE with initial conditions (16) in the upper branch we observed the transformation of the Néel wall into a stationary Bloch wall. The damping plays a fundamental role in this process. The energy and velocity of the soliton gradually approach their stationary values due to dissipation.

A typical picture of conversion is depicted in Fig. 3. We represented the time evolution of the space distribution for the slowly varying S_x spin component. This component coincides with s_x with an accuracy up to $\mathcal{O}(\Delta^2)$.

The soliton with an initial velocity of 0.18 passes from the upper to the lower branch and broadens its width until it approaches the stationary state with a velocity of about 0.32. The analytical expression (17) gives $v\approx 0.33$, but the true value is 0.32 in accordance with Fig. 2. We can calculate the time evolution of the soliton energy for the case with mean-zero magnetic field $\langle H\rangle=0$ assuming that switching occurs adiabatically. The time dependence of the energy for the switching process has the form

$$\mathcal{E}(t)=\Delta^{-1/4}\sqrt{\cosh\nu(t)/\cosh\eta(t)}, \tag{18}$$

$$v(t)=0.5(1-\Delta)\Delta^{-1/4}/\sqrt{\cosh\nu(t)\cosh\eta(t)}, \tag{19}$$

where $\eta(t)=\ln(\tan\phi_0)-\alpha(1-3\Delta)t$, $\nu(t)=0.5\ln(\Delta)-\eta$, and $\phi_0=\phi(0)$. Analytical and numerical results are in good agreement despite the strong shape distortions of the soliton during switching (see Fig. 4).

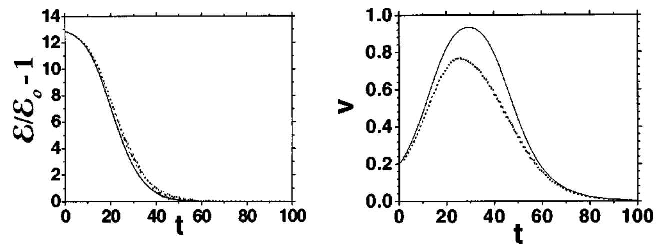

FIG. 4. Wall energy and velocity vs time for the switching in mean-zero magnetic field. Solid and dotted lines correspond to $\mathcal{E}(t)$, $v(t)$ calculated from Eqs. (18), (19) and from numerical simulations of LLE, respectively. $\Delta=0.005$, $\alpha=0.1$, and initial soliton velocity $v_0=0.2$.

Besides, one-soliton solutions, two- and many-soliton solutions are also possible in this averaged integrable system.

As far as applications of the obtained results are concerned, one acquires a posssibilty to manage the effective easy-axis anisotropy constant by making use of strong driving magnetic fields applied to easy-plane ferromagnets.

We give some estimates for realistic physical parameters relevant to the experimental situation with domain walls. The driving frequency and amplitude are $\omega\sim 10^{10}$ Hz and $h\sim 10$ G. The parameter $\Delta\sim 0.01$–0.1. The static magnetic field and domain wall width are $H\sim 1$ G and $\zeta\sim 0.1$–10 μm, respectively.

VI. CONCLUSIONS

In this paper, based on multiscale expansion of the Landau-Lifshitz equation with rapid perturbation we have shown that strong rapidly ocillating transverse magnetic fields in easy-plane ferromagnets induce biaxial magnetic symmetry and lead to the appearance of new solutions in the form of Bloch and Néel walls. The parameters of averaged biaxial systems were calculated. We have found a new switching effect which may occur when the Néel wall converts into a stationary Walker solution in the presence of damping. This is a consequence of the almost integrable averaged system. We have described this phenomenon analytically based on the adiabatic approximation. The time evolution of the energy and the velocity of the soliton is derived. Analytical results have been confirmed by numerical simulations of the fully perturbed Landau-Lifshitz equation with damping and external magnetic fields. A stability analysis of the derived solutions requires a separate investigation.

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