Spin injection into ballistic layers and resistance modulation in spin field-effect transistors

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(Received 23 August 2001; published 8 February 2002)

The problem of electron transport through low-dimensional ballistic electron layers contacted to ferromagnetic leads is studied theoretically. Using the drift-diffusion approach to describe the potential distribution in the leads, we derive the general expression for the device resistance in terms of both the spin polarization in the leads and partial spin-dependent transmission probabilities. It is found that, due to the dimensionality mismatch between the leads and low-dimensional layer, the resistance depends rather on the total transmission probability T than on the spin polarization in the leads. We calculate T for both two-dimensional (2D) and 1D layers in order to investigate the efficiency of the current modulation in the field-effect and magnetoresistance measurements. The results are compared with recent experimental data.

DOI: 10.1103/PhysRevB.65.085319 PACS number(s): 72.25.−b, 85.75.Hh

I. INTRODUCTION

Spin-polarized electron transport in microstructures has been the focus of attention since the proposal[1] of an electronic device where the spin precession in two-dimensional (2D) or 1D electron gas is controlled by an external electric field via spin-orbit coupling. This field modifies the device resistance provided the degree of spin polarization of the electron gas in the conduction channel of the device is nonzero. The possibility to vary the spin-orbit coupling constant in low-dimensional semiconductor layers by the external electric field has been demonstrated in a number of experiments.[2–4] It appeared to be difficult,[5,6] however, to achieve an efficient injection of spin-polarized electrons from ferromagnetic metallic leads to semiconductor layers. Theoretical studies[7,8] applied to transport across the interfaces between 3D materials (3D-3D contacts), have uncovered a fundamental obstacle for spin injection: the conductivity mismatch between the two materials. Indeed, when this obstacle is bypassed by choosing semimagnetic semiconductors instead of ferromagnetic metals as spin aligners, a robust, more than 80%, spin injection across 3D-3D interfaces is observed.[9–11] Another way to achieve spin injection is concerned with quantum-mechanical spin-filtering properties of the interfaces and has been recently discussed for 3D-3D (Refs. 8 and 12), 3D-2D (Refs. 13 and 14), and 3D-quasi-1D (Ref. 15) contacts.

Despite all efforts, a practical implementation of the idea of spin field-effect transistors[1] still remains an elusive goal. Although devices containing very short (nearly ballistic) 2D electron layers contacted to ferromagnetic leads have been fabricated, no electric field-effect modulation has been reported for them so far. A weak resistance modulation in such devices, about 0.2% at temperatures below 4.2 K, was observed in the magnetoresistance measurements, when the injector-detector magnetization configuration was switched from parallel to antiparallel.[16,17] The weakness of the resistance modulation has been attributed mostly to the deviation of the electron transport from the ballistic regime and to the spin-flip scattering. Though this conclusion is supported by the observation of a decrease of the resistance modulation with increasing temperature, the weakness of the effect at low temperatures still requires other explanations. In particular, it was mentioned[18] that limitations on the spin injection, similar to those outlined in Ref. 7, should also be expected for devices with 3D-2D contacts. However, a comprehensive quantitative theory of spin-polarized transport in such devices is still missing.

In this paper we study the transport of spin-polarized electrons in conditions when the dimensionalities of the spin-aligning leads and of the semiconductor layer between them are different. This study covers 3D-2D-3D and 3D-1D-3D structures, and also can be applied to 2D-1D-2D structures (Fig. 1). Electron transport in the low-dimensional layer is assumed to be ballistic, while transport in the leads is diffusive. The paper is organized as follows. In Sec. II we derive general expressions for the degree of spin polarization of the current and for the resistance of the device. These quantities depend on both the spin polarization of electrons in the leads and the partial transmission probabilities for spin-up and spin-down electrons, including spin-flip transmission. In Sec. III we present a quantum-mechanical calculation of the transmission probabilities through 2D and 1D layers and analyze the limits of the device resistance modulation. In Sec. IV the results are briefly discussed and compared to recent experimental data.

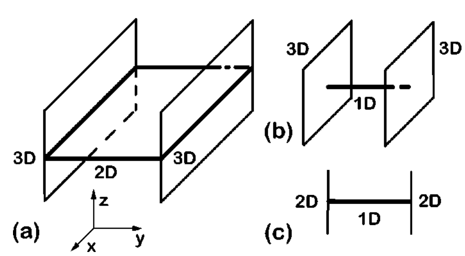

FIG. 1. Schematic representation of the structures under consideration. The low-dimensional layer is placed between the leads of higher dimensionality.

II. SPIN POLARIZATION AND BALLISTIC RESISTANCE

To describe the electric response of the structures under consideration, we use the drift-diffusion approach developed in Ref. 19 and applied in Refs. 7 and 8 to 3D-3D-3D structures. Consider two leads, the left $(-)$ and the right $(+)$ ones, characterized by the different conductivities $\sigma_{\uparrow}^{\pm}$ and $\sigma_{\downarrow}^{\pm}$ for spin-up $(\uparrow)$ and spin-down $(\downarrow)$ electron states. The distribution of electrochemical potentials in the leads is determined by the following equations:

$$\Delta[\sigma_{\uparrow}^{\pm}\mu_{\uparrow}^{\pm}(\mathbf{r})+\sigma_{\downarrow}^{\pm}\mu_{\downarrow}^{\pm}(\mathbf{r})]=0, \tag{1}$$

$$\Delta[\mu_{\uparrow}^{\pm}(\mathbf{r})-\mu_{\downarrow}^{\pm}(\mathbf{r})]-l_{s\pm}^{-2}[\mu_{\uparrow}^{\pm}(\mathbf{r})-\mu_{\downarrow}^{\pm}(\mathbf{r})]=0, \tag{2}$$

which follow from the continuity equations for the current densities $\mathbf{j}_{\uparrow,\downarrow}^{\pm}(\mathbf{r})=-\sigma_{\uparrow,\downarrow}^{\pm}\nabla\mu_{\uparrow,\downarrow}^{\pm}(\mathbf{r})$, provided that spin relaxation is taken into account. The latter is characterized by the effective lengths $l_{s\pm}$, which are assumed to be much greater than the mean-free-path lengths in the leads. Equations (1) and (2) should be supplemented by the boundary conditions, whose form is determined by the geometry of the leads and of the layer between them, as specified in Fig. 1. A detailed consideration of electron transport is given below for 3D-2D-3D structures. A rather straightforward generalization of the obtained results to 3D-1D-3D and 2D-1D-2D structures is given thereafter.

In 3D-2D-3D structures, the translational symmetry of the problem along the x direction renders the potentials $\mu_{\uparrow,\downarrow}^{\pm}$ independent of x. The first pair of boundary conditions follows from current conservation:

$$-\sigma_{\uparrow,\downarrow}^{-}\nabla_y\mu_{\uparrow,\downarrow}^{-}(y,z)|_{y=-0}=\delta(z)J_{\uparrow,\downarrow}^{-}, \tag{3}$$

$$-\sigma_{\uparrow,\downarrow}^{+}\nabla_y\mu_{\uparrow,\downarrow}^{+}(y,z)|_{y=L+0}=\delta(z)J_{\uparrow,\downarrow}^{+}, \tag{4}$$

where $J_{\uparrow,\downarrow}^{-}$ and $J_{\uparrow,\downarrow}^{+}$ are the currents entering the 2D layer at $y=0$ and exiting it at $y=L$, respectively. These four currents are not independent of each other, since $J_{\uparrow}^{-}+J_{\downarrow}^{-}=J_{\uparrow}^{+}+J_{\downarrow}^{+}=J$, where J is the total current. We assume that the layer thickness is smaller than the mean-free-path length. Therefore, from the point of view of the drift-diffusion approximation, this thickness should be set to zero, which justifies the use of the δ functions in Eqs. (3) and (4). Another set of boundary conditions describes application of the external potentials V_- and V_+ to the leads:

$$\mu_{\uparrow}^{-}(y,z)=\mu_{\downarrow}^{-}(y,z)=V_-, \quad \sqrt{y^2+z^2}=r_-, \tag{5}$$

$$\mu_{\uparrow}^{+}(y,z)=\mu_{\downarrow}^{+}(y,z)=V_+, \quad \sqrt{(y-L)^2+z^2}=r_+, \tag{6}$$

where $r_{\pm}$ are large in comparison to $l_{s\pm}$. An exact definition of the equipotential surface is not essential and is chosen here for convenience only. Changing it, one would modify only the resistances of the bulk lead regions which enter the theory as additional series resistances and can be excluded from consideration (see below).

Using Eqs. (3)–(6) one can write separate boundary conditions for the variables $g^{\pm}(\mathbf{r})=\sigma_{\uparrow}^{\pm}\mu_{\uparrow}^{\pm}+\sigma_{\downarrow}^{\pm}\mu_{\downarrow}^{\pm}$ and $u^{\pm}(\mathbf{r})=\mu_{\uparrow}^{\pm}-\mu_{\downarrow}^{\pm}$. Thus, Eqs. (1) and (2) for these variables become independent of each other, and their solutions can be written as follows:

$$g^{-}(y,z)=\sigma^{-}V_-+J\ln(\sqrt{y^2+z^2}/r_-)/\pi,$$

$$g^{+}(y,z)=\sigma^{+}V_+-J\ln[\sqrt{(y-L)^2+z^2}/r_+]/\pi \tag{7}$$

and

$$u^{-}(y,z)=-A^{-}\int\frac{dq}{2\pi}\frac{\cos(qz)}{\sqrt{q^2+l_{s-}^{-2}}}e^{\sqrt{q^2+l_{s-}^{-2}}y},$$

$$u^{+}(y,z)=A^{+}\int\frac{dq}{2\pi}\frac{\cos(qz)}{\sqrt{q^2+l_{s+}^{-2}}}e^{-\sqrt{q^2+l_{s+}^{-2}}(y-L)}, \tag{8}$$

where $\sigma^{\pm}=\sigma_{\uparrow}^{\pm}+\sigma_{\downarrow}^{\pm}$ are the total conductivities of the lead regions and $A^{\pm}=J_{\uparrow}^{\pm}/\sigma_{\uparrow}^{\pm}-J_{\downarrow}^{\pm}/\sigma_{\downarrow}^{\pm}$. The potentials $\mu_{\uparrow,\downarrow}^{-}(y,z)$ and $\mu_{\uparrow,\downarrow}^{+}(y,z)$ appear to be logarithmically divergent as $(y,z)\to(0,0)$ and $(y,z)\to(L,0)$, respectively. This is a consequence of the crudity of the drift-diffusion approximation (for more accuracy, one has to solve a kinetic equation in the vicinity of the 3D-2D transition regions). Nevertheless, one can introduce a cutoff at the mean-free-path lengths $l_{\pm}$, the smallest parameters of the drift-diffusion theory. This cannot affect the accuracy considerably, as far as the logarithmic scaling is concerned. Accordingly, we introduce $\mu_{\uparrow,\downarrow}^{-}=\mu_{\uparrow,\downarrow}^{-}(-l_-,0)$ and $\mu_{\uparrow,\downarrow}^{+}=\mu_{\uparrow,\downarrow}^{+}(L+l_+,0)$, and obtain

$$\mu_{\uparrow,\downarrow}^{-}=V_--R_-J+\frac{K_0(l_-/l_{s-})}{\pi\sigma^{-}}\left[J_{\downarrow,\uparrow}^{-}-\frac{\sigma_{\downarrow,\uparrow}^{-}}{\sigma_{\uparrow,\downarrow}^{-}}J_{\uparrow,\downarrow}^{-}\right], \tag{9}$$

$$\mu_{\uparrow,\downarrow}^{+}=V_++R_+J-\frac{K_0(l_+/l_{s+})}{\pi\sigma^{+}}\left[J_{\downarrow,\uparrow}^{+}-\frac{\sigma_{\downarrow,\uparrow}^{+}}{\sigma_{\uparrow,\downarrow}^{+}}J_{\uparrow,\downarrow}^{+}\right], \tag{10}$$

where K_0 is the modified Bessel function. According to the initial assumptions, its arguments l_-/l_{s-} and l_+/l_{s+} are small, and the limits of accuracy of this treatment permit us to replace $K_0(l_{\pm}/l_{s\pm})$ by $\ln(l_{s\pm}/l_{\pm})$ in the following. In Eqs. (9) and (10) we have also introduced the resistances of the leads according to $R_{\pm}=(\pi\sigma^{\pm})^{-1}\ln(r_{\pm}/l_{\pm})$.

To find the relation between J and V_--V_+, one has to connect the four currents $J_{\uparrow,\downarrow}^{\pm}$ to four potentials $\mu_{\uparrow,\downarrow}^{\pm}$ given by Eqs. (9) and (10). Considering the electron subsystems with different spins in each lead as four independent reservoirs[20] and assuming that electron motion in the 2D layer is ballistic, we directly apply the Landauer-Büttiker formalism for a four-terminal device[21] to obtain a set of equations

$$J_{\uparrow}^{-}=G[(T_{\uparrow\uparrow}^{-+}+T_{\uparrow\downarrow}^{-+}+R_{\uparrow\downarrow}^{-})\mu_{\uparrow}^{-}-T_{\uparrow\uparrow}^{+-}\mu_{\uparrow}^{+}-T_{\downarrow\uparrow}^{+-}\mu_{\downarrow}^{+}-R_{\downarrow\uparrow}^{-}\mu_{\downarrow}^{-}], \tag{11}$$

$$J_{\downarrow}^{-}=G[(T_{\downarrow\downarrow}^{-+}+T_{\downarrow\uparrow}^{-+}+R_{\downarrow\uparrow}^{-})\mu_{\downarrow}^{-}-T_{\downarrow\downarrow}^{+-}\mu_{\downarrow}^{+}-T_{\uparrow\downarrow}^{+-}\mu_{\uparrow}^{+}-R_{\uparrow\downarrow}^{-}\mu_{\uparrow}^{-}], \tag{12}$$

$$J_\uparrow^+ = G[T_{\uparrow\uparrow}^{-+}\mu_\uparrow^- + T_{\downarrow\uparrow}^{-+}\mu_\downarrow^- + R_{\downarrow\uparrow}^+\mu_\downarrow^+ - (T_{\uparrow\uparrow}^{+-} + T_{\uparrow\downarrow}^{+-} + R_{\uparrow\downarrow}^+)\mu_\uparrow^+], \tag{13}$$

$$J_\downarrow^+ = G[T_{\downarrow\downarrow}^{-+}\mu_\downarrow^- + T_{\uparrow\downarrow}^{-+}\mu_\uparrow^- + R_{\uparrow\downarrow}^+\mu_\uparrow^+ - (T_{\downarrow\downarrow}^{+-} + T_{\downarrow\uparrow}^{+-} + R_{\downarrow\uparrow}^+)\mu_\downarrow^+]. \tag{14}$$

Here $T_{jj'}^{\alpha\beta}$ $(\alpha,\beta=\pm,\ j,j'=\uparrow\downarrow)$ is the probability for an electron in spin state j coming from the lead α to be transmitted into spin state j' of the lead β $(\beta\neq\alpha)$, and $R_{jj'}^{\alpha}$ $(j\neq j')$ is the probability of the electron in spin state j coming from the lead α to be reflected back, into the state with the opposite spin. The coefficients $T_{jj'}^{\alpha\beta}$ and $R_{jj'}^{\alpha}$ correspond to the Fermi energy and are averaged over the angle of incidence (see the details in the next section) in such a way that the conductance G introduced in Eqs. (11)–(14) is equal to the Sharvin conductance of a 2D point contact (per unit length), expressed through the 2D electron density n_{2D} in the ballistic layer as $G=(e^2/h)\sqrt{2n_{2D}/\pi}$. The physical meaning of Eqs. (11)–(14) is the following.[21] The reservoir (αj) injects a current $G[T_{j\uparrow}^{\alpha\beta}+T_{j\downarrow}^{\alpha\beta}+R_{jj'}^{\alpha}](\mu_j^\alpha-\mu_0)$, while the current from three other reservoirs coming into (αj) is equal to $G[T_{\uparrow j}^{\beta\alpha}(\mu_\uparrow^\beta-\mu_0)+T_{\downarrow j}^{\beta\alpha}(\mu_\downarrow^\beta-\mu_0)+R_{j'j}^{\alpha}(\mu_{j'}^\alpha-\mu_0)]$. The difference between these currents gives the current J_j^α. Here μ_0 is the reference potential, which is smaller than or equal to the lowest of the four potentials $\mu_{\uparrow,\downarrow}^{\pm}$. In equilibrium, when all these potentials are equal to each other, all the currents $J_{\uparrow,\downarrow}^{\pm}$ must be equal to zero, so that one can write the following linear relations:

$$T_{\uparrow\uparrow}^{-+}+T_{\uparrow\downarrow}^{-+}+R_{\uparrow\downarrow}^{-}=T_{\uparrow\uparrow}^{+-}+T_{\downarrow\uparrow}^{+-}+R_{\downarrow\uparrow}^{-},$$

$$T_{\downarrow\downarrow}^{-+}+T_{\downarrow\uparrow}^{-+}+R_{\downarrow\uparrow}^{-}=T_{\downarrow\downarrow}^{+-}+T_{\uparrow\downarrow}^{+-}+R_{\uparrow\downarrow}^{-},$$

$$T_{\uparrow\uparrow}^{-+}+T_{\downarrow\uparrow}^{-+}+R_{\downarrow\uparrow}^{+}=T_{\uparrow\uparrow}^{+-}+T_{\uparrow\downarrow}^{+-}+R_{\uparrow\downarrow}^{+},$$

$$T_{\downarrow\downarrow}^{-+}+T_{\uparrow\downarrow}^{-+}+R_{\uparrow\downarrow}^{+}=T_{\downarrow\downarrow}^{+-}+T_{\downarrow\uparrow}^{+-}+R_{\downarrow\uparrow}^{+}.$$

On the other hand, we would like to stress that, in general, $T_{jj'}^{\alpha\beta}\neq T_{j'j}^{\beta\alpha}$ and $R_{jj'}^{\alpha}\neq R_{j'j}^{\alpha}$. This asymmetry can appear even in the absence of a magnetic field, because the spin-orbit interaction (Rashba) term in the Hamiltonian of the ballistic layer (see next section) changes its sign when the momentum is reversed, so that for spin-polarized electrons the forward and backward directions of motion are not equivalent. However, if the magnetization vectors of the leads are in the yz plane [see Fig. 1(a)], i.e., are directed perpendicular to the effective magnetic field created in the ballistic layer by the current (Rashba field), the coefficients $T_{jj'}^{\alpha\beta}$ and $R_{jj'}^{\alpha}$ become symmetric. The asymmetry of the partial spin-dependent probabilities $T_{jj'}^{\alpha\beta}$ and $R_{jj'}^{\alpha}$ is not a violation of the Casimir-Onsager-Büttiker symmetry relation for two-terminal devices because, since the states with different spins are considered as separate reservoirs, one has an *effective* four-terminal device instead of a two-terminal one. It is convenient to introduce the total (summed over the spins) transmission probability

$$T=T^{\alpha\beta}=T^{\beta\alpha}=\sum_{jj'}T_{jj'}^{\alpha\beta}, \tag{15}$$

which describes the total current $J=GT(\mu^- - \mu^+)$ in conditions when the potentials for both spin states in each lead are equal, $\mu_\uparrow^\alpha=\mu_\downarrow^\alpha=\mu^\alpha$. The symmetry of the total transmission probability with respect to the lead indices is a manifestation of the Casimir-Onsager-Büttiker symmetry, because T is related to the *physical* two-terminal device. This symmetry formally follows from the linear relations for $T_{jj'}^{\alpha\beta}$ and $R_{jj'}^{\alpha}$ written above. According to its definition, the total transmission probability T can vary from 0 to 2.

The partial transmission probabilities are essentially related to the properties of the interfaces and of the ballistic layer itself. The ballistic layer acts like a spin filter if $T_{\uparrow\uparrow}^{\alpha\beta}\neq T_{\downarrow\downarrow}^{\alpha\beta}$ or $T_{\uparrow\downarrow}^{\alpha\beta}\neq T_{\downarrow\uparrow}^{\alpha\beta}$. Next, if $T_{\uparrow\downarrow}^{\alpha\beta}$ and $T_{\downarrow\uparrow}^{\alpha\beta}$ are nonzero, the layer acts like a spin mixer. The spin-filtering and spin-mixing properties are not necessarily connected to each other. The spin mixing can be associated with the Rashba term in the Hamiltonian. It is important for the layers with considerable spin-orbit coupling, like InAs or InSb. The spin-filtering property can exist[13] because of the different Fermi velocities for spin-up and spin-down electrons in the leads, without regard to the spin-orbit coupling.

Below we use Eqs. (9)–(14) to calculate the spin polarization of the current $\delta J^{\pm}=J_\uparrow^{\pm}-J_\downarrow^{\pm}$ and the device resistance $R=(V_- - V_+)/J-R_- -R_+$ (per unit length) for the case of *identical* leads, when $\sigma^+=\sigma^-=\sigma$, $l_{s-}=l_{s+}=l_s$, and $l_-=l_+=l$, and the absolute values of magnetization are the same in both leads. If the magnetization configuration of the leads is *parallel* (P), one has $\sigma_{\uparrow,\downarrow}^-=\sigma_{\uparrow,\downarrow}^+\equiv\sigma_{\uparrow,\downarrow}$, $T_{\uparrow\downarrow}^{\alpha\beta}=T_{\downarrow\uparrow}^{\alpha\beta}$, and $R_{jj'}^-=R_{j'j}^+$. Assuming also that the vectors of magnetization of the leads are in the yz plane, we reduce the number of independent coefficients to 4, written as $T_{\uparrow\uparrow}=T_{\uparrow\uparrow}^{\alpha\beta}$, $T_{\downarrow\downarrow}=T_{\downarrow\downarrow}^{\alpha\beta}$, $T_{\uparrow\downarrow}=T_{jj'}^{\alpha\beta}$ $(j\neq j')$, and $R_{\uparrow\downarrow}=R_{jj'}^{\alpha}$. A calculation gives

$$\delta J_P^- = \delta J_P^+ = J\,\frac{(T_{\uparrow\uparrow}-T_{\downarrow\downarrow})+2\beta\gamma Q_P/(1-\beta^2)}{T+2\gamma Q_P/(1-\beta^2)}, \tag{16}$$

$$R_P=G^{-1}\frac{1+\gamma P_P/(1-\beta^2)}{T+2\gamma Q_P/(1-\beta^2)}, \tag{17}$$

where $\beta=(\sigma_\uparrow-\sigma_\downarrow)/\sigma$ characterizes spin polarization of the currents in the leads, $Q_P=2T_{\uparrow\uparrow}T_{\downarrow\downarrow}+T_{\uparrow\downarrow}(T_{\uparrow\uparrow}+T_{\downarrow\downarrow})+TR_{\uparrow\downarrow}$, and $P_P=(1-\beta)^2T_{\uparrow\uparrow}+(1+\beta)^2T_{\downarrow\downarrow}+2\beta^2T_{\uparrow\downarrow}+2R_{\uparrow\downarrow}$. For *antiparallel* (A) configuration, when the magnetization of the right lead is inverted, one has $\sigma_{\uparrow,\downarrow}^-=\sigma_{\downarrow,\uparrow}^+\equiv\sigma_{\uparrow,\downarrow}$, $T_{jj}^{\alpha\beta}=T_{j'j'}^{\beta\alpha}$ $(j\neq j')$, $T_{jj'}^{\alpha\beta}=T_{j'j}^{\beta\alpha}$ $(j\neq j')$, and $R_{jj'}^-=R_{j'j}^+$. If the vectors of magnetization of the leads are in the yz plane, we again have only four independent transmission coefficients defined as $T_{\uparrow\uparrow}=T_{jj}^{\alpha\beta}$ $(j=\uparrow,\downarrow)$, $T_{\uparrow\downarrow}=T_{\uparrow\downarrow}^{-+}=T_{\downarrow\uparrow}^{+-}$, $T_{\downarrow\uparrow}=T_{\downarrow\uparrow}^{-+}=T_{\uparrow\downarrow}^{+-}$, and $R_{\uparrow\downarrow}=R_{jj'}^{\alpha}$. The spin polarization and the resistance are given by the equations

$$\delta J_A^- = -\delta J_A^+ = J\frac{(T_{\uparrow\downarrow}-T_{\downarrow\uparrow})+2\beta\gamma Q_A/(1-\beta^2)}{T+2\gamma Q_A/(1-\beta^2)}, \quad (18)$$

$$R_A = G^{-1}\frac{1+\gamma P_A/(1-\beta^2)}{T+2\gamma Q_A/(1-\beta^2)}, \quad (19)$$

where $Q_A = 2T_{\uparrow\downarrow}T_{\downarrow\uparrow}+T_{\uparrow\uparrow}(T_{\uparrow\downarrow}+T_{\downarrow\uparrow})+TR_{\uparrow\downarrow}$ and $P_A=(1-\beta)^2T_{\uparrow\downarrow}+(1+\beta)^2T_{\downarrow\uparrow}+2\beta^2T_{\uparrow\uparrow}+2R_{\uparrow\downarrow}$. Equations (18) and (19) can be obtained from Eqs. (16) and (17) by the formal substitutions $T_{\uparrow\uparrow}\to T_{\uparrow\downarrow}$, $T_{\downarrow\downarrow}\to T_{\downarrow\uparrow}$, and $T_{\uparrow\downarrow}\to T_{\uparrow\uparrow}$. One should remember, however, that the transmission probabilities $T_{jj'}$ for parallel and antiparallel configurations are different. In Eqs. (16)–(19) we have introduced the characteristic dimensionless parameter

$$\gamma = \frac{2G}{\pi\sigma}\ln\frac{l_s}{l}. \quad (20)$$

The results (16)–(20) are not modified essentially for the 3D-1D-3D and 2D-1D-2D structures shown in Figs. 1(b) and 1(c). In the 2D-1D-2D case, the geometry of the drift-diffusion problem is the same as in the 3D-2D-3D case, and the results are formally valid if we replace G by the fundamental conductance quantum e^2/h and consider σ as the sheet resistance of the 2D leads. In the 3D-1D-3D case, the geometry is different, and the electrochemical potentials depend on three coordinates. The theory, however, is very similar to the one presented above. It is based on the boundary conditions

$$-\sigma^-_{\uparrow,\downarrow}\nabla_y\mu^-_{\uparrow,\downarrow}(x,y,z)|_{y=-0} = \delta(x)\,\delta(z)J^-_{\uparrow,\downarrow}, \quad (21)$$

$$-\sigma^+_{\uparrow,\downarrow}\nabla_y\mu^+_{\uparrow,\downarrow}(x,y,z)|_{y=L+0} = \delta(x)\,\delta(z)J^+_{\uparrow,\downarrow}, \quad (22)$$

while another pair of boundary conditions relating $\mu^\pm_{\uparrow,\downarrow}(x,y,z)$ to the external potentials away from the 3D-1D contacts can be chosen the same as in Eqs. (5) and (6). The results (16)–(19) are valid in this case if we replace G by e^2/h and set γ equal to

$$\gamma = \frac{e^2}{h}\frac{1}{\pi\sigma l}. \quad (23)$$

The appearance of the term l^{-1} results from inverse divergences of the potentials $\mu^\pm_{\uparrow,\downarrow}(x,y,z)$ as functions of the distances to the contact points. The cutoff of these divergences is again taken at the mean-free-path length, and we neglected $1/l_s$ in comparison to $1/l$. The loss of accuracy is greater than in the case of logarithmic divergence, and one can speak only about order-of-magnitude estimates. Nevertheless, it is clear that γ does not depend on the spin-relaxation length and is of the order of $(k_F l)^{-2}$, where k_F is the Fermi wave number in the leads. This means that this parameter is small, as long as metallic conductivity in the leads is assumed. According to Eq. (20), the parameters γ for 3D-2D-3D and 2D-1D-2D structures are estimated as $(p/k_F)(k_F l)^{-1}\ln(l_s/l)$, where $p=\sqrt{2\pi n_{2D}}$ is the Fermi wave number in the 2D layer and $(k_F l)^{-1}\ln(l_s/l)$, respectively. These parameters are also small. It is important to stress that the factors $(k_F l)^{-1}\ln(l_s/l)$ and $(k_F l)^{-2}$ are of the order of relative weak localization corrections to the 2D and 3D conductivities, respectively, and the spin-relaxation length l_s plays the role of the dephasing length. The fundamental reasons for the smallness of the parameter γ is the *dimensionality mismatch* between the leads and the layers, whose dimensionality is lower. This mismatch sets an obstacle for spin injection.

The results (16) and (18) show that spin polarization of the current can occur as a consequence of two effects. The first is related to the spin polarization in the leads (the terms proportional to β), and the second is due to the spin-filtering properties of the contacts [the terms proportional to $T_{\uparrow\uparrow}-T_{\downarrow\downarrow}$ in Eq. (16) and to $T_{\uparrow\downarrow}-T_{\downarrow\uparrow}$ in Eq. (18)]. If there is no spin filtering, $T_{\uparrow\uparrow}=T_{\downarrow\downarrow}=T_P$, $T_{\uparrow\downarrow}=T_{\downarrow\uparrow}=T_A$, and Eqs. (16) and (18) can be written as

$$\frac{|\delta J_i|}{J} = \frac{2|\beta|\gamma(T_i+R_{\uparrow\downarrow})}{1-\beta^2+2\gamma(T_i+R_{\uparrow\downarrow})}, \quad i=P,A, \quad (24)$$

while Eqs. (17) and (19) become

$$R_i = \frac{1}{TG}+\frac{1}{G}\frac{\beta^2\gamma}{1-\beta^2+2\gamma(T_i+R_{\uparrow\downarrow})}, \quad i=P,A. \quad (25)$$

Since the parameter γ is small, the spin injection is expected to be weak in this case, unless $|\beta|$ is very close to 1. On the other hand, if spin filtering exists, the spin injection occurs even if we neglect the terms proportional to $\gamma/(1-\beta^2)$. Nevertheless, the resistance R at small $\gamma/(1-\beta^2)$ is mostly determined by the *total* transmission probability T. The control of the resistance, which is crucial for operation of the spin field-effect transistor,[1] thus depends mostly on the possibility to control T.

The simplest way to evaluate the transmission probabilities in Eqs. (16)–(19) is to assume adiabatic contacting, when the electrochemical potentials for spin-up and spin-down states are continuous across the contacts between the leads and the low-dimensional layer. These are, in fact, the conditions considered by Datta and Das.[1] Following Ref. 1, in the 1D case one can write $R_{\uparrow\downarrow}=0$, $T_{\uparrow\uparrow}=T_{\downarrow\downarrow}=\cos^2\varphi$ and $T_{\uparrow\downarrow}=T_{\downarrow\uparrow}=\sin^2\varphi$, where $\varphi=\alpha m^* L/\hbar^2$ is the field-effect-controlled phase expressed through the electron effective mass m^* and Rashba constant α in the layer. These expressions are valid for both parallel and antiparallel configurations of the magnetization. The ballistic wire acts like a spin mixer, but not like a spin filter. The total transmission probability (15) is equal to 2. The resistances $R=R_i$ for parallel $(i=P)$ and antiparallel $(i=A)$ configurations are

$$R_i \simeq \frac{h}{2e^2}\left[1+\frac{2\beta^2\gamma}{1-\beta^2}-\left(\frac{2\beta\gamma}{1-\beta^2}\right)^2 T_i\right], \quad (26)$$

where $T_P=\cos^2\varphi$ and $T_A=\sin^2\varphi$. To obtain Eq. (26), we expanded the second term of the right-hand side of Eq. (25) in powers of a small parameter $\gamma/(1-\beta^2)$. The control of R through φ is as weak as γ^2. The magnetoresistance, i.e., the difference R_P-R_A, has the same weakness. In other words, the spin field-effect transistor cannot work efficiently in the form it was initially proposed,[1] because of the fundamental reasons which render the parameter γ small and thereby suppress the spin injection.

Fortunately, the realistic contacts are far from adiabatic and can provide spin filtering as well. As was pointed out by Grundler,[13] the abrupt contacts between ferromagnetic materials and 2D layers act like spin filters in a natural way, since the Fermi velocities for spin-up and spin-down electrons in the ferromagnets are different and the quantum-mechanical transmission probabilities depend on these velocities. Using this Fermi velocity mismatch, Grundler has estimated[14] the magnetoresistance of a 2D device with two ferromagnetic leads. The theory of Ref. 14, however, has to be improved, since the model of one-dimensional transport used there has not been justified, the transmission has been considered only for x-polarized magnetization [for the geometry shown in Fig. 1(a)], and the motion of electrons in the x direction has been neglected. The latter approximation apparently led to an overestimation of the relative modulation of the resistance. It is important to note that the Rashba term in the 2D Hamiltonian mixes the motion of an electron in x and y directions, so that the symmetry of the spinor wave function depends on the direction of the wave vector in the xy plane [see, for example, Eq. (34) of the next section]. As a result, the transmission probability acquires a nontrivial dependence of the 2D wave vector. This effect has been completely neglected in Refs. 13 and 14. The theory[15] of spin injection through a single interface betwen 3D and quasi-1D materials has the same drawback, since the mixing (induced by the Rashba term) between 1D subband states[22] has been neglected. In the next section we overcome these limitations and calculate the transmission probabilities through both 2D and 1D layers for different magnetization directions in the leads.

III. TRANSMISSION PROBABILITY

To calculate the transmission probabilities, Grundler[13,14] used a simple model, which reduced the problem of electron transfer between the regions of different dimensionality to a one-dimensional quantum-mechanical problem. It has been already pointed out[23] that such a one-dimensional transport model gives a reasonable order-of-magnitude estimate of the exact numerical results for the transmission. However, as far as we know, nobody noticed that this model is a very good approximation in the case when the Fermi wave number in 3D material is much greater than the inverse length of size quantization in the quantum well where the 2D layer is created. To demonstrate it, let us consider the transmission problem in more detail. Following Kriman and Ruden,[23] we consider a single abrupt contact between a 2D conductor from the left ($<$) and a 3D conductor from the right ($>$), at $y=0$. The wave function of the Fermi electron coming from the side of the 2D conductor is written as

$$\Psi_<(x,y,z)=[e^{i\sqrt{k_<^2-k_0^2}y}+r_0e^{-i\sqrt{k_<^2-k_0^2}y}]\chi_0(z)e^{ik_xx}+\sum_{n=1}^{\infty}r_ne^{\sqrt{k_n^2-k_<^2}y}\chi_n(z)e^{ik_xx}, \tag{27}$$

where k_n are discrete wave numbers of the quantized subbands. Only the lowest quantized subband $(n=0)$ is assumed to be occupied by electrons. The higher subbands with $n=1,2,\ldots$ contribute to evanescent reflected modes. The wave function on the side of the 3D conductor is given in the integral representation

$$\Psi_>(x,y,z)=\int\frac{dk}{2\pi}t_ke^{i\sqrt{k_>^2-k^2}y}e^{ikz}e^{ik_xx}, \tag{28}$$

where $k=k_z$, and the factor $i\sqrt{k_>^2-k^2}$ in the exponent should be replaced by $-\sqrt{k^2-k_>^2}$, when $|k|>k_>$. In Eqs. (27) and (28) we introduced the wave numbers $k_<=\sqrt{2m_<(\varepsilon_F-U)/\hbar^2-k_x^2}$ and $k_>=\sqrt{2m_>\varepsilon_F/\hbar^2-k_x^2}$, where $m_<$ and $m_>$ are the effective masses, ε_F is the Fermi energy counted from the conduction-band bottom in the 3D material, and U is the potential energy offset between the contacting materials. The boundary conditions expressing continuity of Ψ and $m^{-1}d\Psi/dy$ give us

$$(1+r_0)\chi_0(z)+\sum_{n=1}^{\infty}r_n\chi_n(z)=\int\frac{dk}{2\pi}t_ke^{ikz} \tag{29}$$

and

$$i\sqrt{k_<^2-k_0^2}(1-r_0)\chi_0(z)+\sum_{n=1}^{\infty}r_n\sqrt{k_n^2-k_<^2}\chi_n(z)=i\frac{m_<}{m_>}\int\frac{dk}{2\pi}t_k\sqrt{k_>^2-k^2}e^{ikz}. \tag{30}$$

Multiplying these equations by e^{-ikz}, integrating them over z, and introducing the overlap integrals $I_{kn}=\int dze^{-ikz}\chi_n(z)$, one can exclude t_k and obtain a single equation

$$iI_{k0}[1-\lambda_k]-iI_{k0}r_0[1+\lambda_k]+\sum_{n=1}^{\infty}I_{kn}r_n\left[\sqrt{\frac{k_n^2-k_<^2}{k_<^2-k_0^2}}-i\lambda_k\right]=0,$$

$$\lambda_k=\frac{m_<}{m_>}\sqrt{\frac{k_>^2-k^2}{k_<^2-k_0^2}}, \tag{31}$$

which is equivalent to an infinite system of linear equations for r_n, $n=0,1,\ldots$. The coefficients I_{kn} become small when $|k|$ exceeds the inverse length of size quantization in the quantum well where the 2D layer is created. If this inverse length (estimated as k_0) is much smaller than $k_>$, one can neglect the dependence of λ_k on k, replacing $\sqrt{k_>^2-k^2}$ by $k_>$. Then, multiplying Eq. (31) by I_{km}^* and integrating it over k with use of the orthogonality property $\int dkI_{km}^*I_{kn}=2\pi\delta_{nm}$, we obtain $r_0=(v_<-v_>)/(v_<+v_>)$ and $r_n=0$ for $n\neq0$. Here $v_>=\hbar k_>/m_>$ and $v_<=\hbar\sqrt{k_<^2-k_0^2}/m_<$ are Fermi velocities of 3D and 2D electrons in the y direction. Therefore, the contribution from the higher subbands is negligible, and the transmission probability $1-|r_0|^2$ is given by a well-known expression corresponding to the one-dimensional transmission-reflection problem[24] (the

x-dependent part of the wave function is conserved across the interface and its inclusion is trivial). From the qualitative point of view, this correspondence follows from the fact that electron transitions between 3D and 2D layers at $k_> \gg k_0$ involve only those electrons in 3D material whose $|k_z|$ is small in comparison to $k_>$, so that they effectively move along y direction. The contacts between 3D metals and semiconductor quantum wells satisfy this requirement, because for them $k_> \sim 10^8$ cm^{-1} and $k_0 \sim 10^6$ cm^{-1}, and can be considered within the one-dimensional transport model. This greatly simplifies the subsequent calculation.

Below we calculate the probability of transmission from a metallic lead to another through a 2D semiconductor layer between them. The metal is a ferromagnet, and the semiconductor layer is characterized by a considerable spin-orbit coupling which adds the Rashba term $\alpha[\hat{\boldsymbol{\sigma}} \times \hat{\mathbf{p}}]\mathbf{n}_z$ in the 2D Hamiltonian.[25] Here $\hat{\boldsymbol{\sigma}}$ is the vector of the Pauli matrices, $\hbar \hat{\mathbf{p}}$ is the operator of the 2D momentum, and $\mathbf{n}_z$ is the unit vector in the z direction. The Rashba constant α can be modified by application of an electric field in the z direction, since this field modifies the symmetry of the confining potential. Below we need to use another form of the Rashba term, generalized for the case of a coordinate-dependent α. The Rashba term is written as an anticommutator $(\alpha[\hat{\boldsymbol{\sigma}} \times \hat{\mathbf{p}}] + [\hat{\boldsymbol{\sigma}} \times \hat{\mathbf{p}}]\alpha)\mathbf{n}_z/2$. This form can be justified, for example, by derivation of the Rashba term from the multiband Kane Hamiltonian.

The wave function in such a ferromagnet-semiconductor-ferromagnet system is presented as $\Psi(x,y,z) = e^{ip_x x}\chi(z)\boldsymbol{\psi}(y)$, and the one-dimensional Schrödinger equation $\hat{H}(y)\boldsymbol{\psi}(y) = \varepsilon \boldsymbol{\psi}(y)$ is written for the two-component spinor $\boldsymbol{\psi} = (\psi_\uparrow, \psi_\downarrow)$. The 2×2 matrix Hamiltonian $\hat{H}$ is given by

$$\hat{H}(y) = -\frac{d}{dy}\frac{\hbar^2}{2m(y)}\frac{d}{dy} + \frac{\hbar^2 p_x^2}{2m(y)} + \hat{U}(y) - \frac{i}{2}\hat{\sigma}_x\left[\alpha(y)\frac{d}{dy} + \frac{d}{dy}\alpha(y)\right] - \hat{\sigma}_y \alpha(y) p_x, \tag{32}$$

where $m(y)$ is the coordinate-dependent effective mass, which is assumed to be a scalar, and $\hat{U}(y) = U_0(y) + \Sigma_i \hat{\sigma}_i U_i(y)$ is the potential energy matrix, $i = x,y,z$. The component U_0 changes from U_0^M in the leads to U_0^* in the semiconductor layer. The components U_i are nonzero only in the ferromagnetic leads, and the vector $\mathbf{U} = (U_x, U_y, U_z)$ gives the direction of the magnetization $\mathbf{M}$ there. On the other hand, $\alpha(y)$ is assumed to be nonzero (equal to α) only in the semiconductor layer, $0 < y < L$. The Hamiltonian (32) is similar to that used by Hu and Matsuyama[15] in their study of spin injection across a single ferromagnet-semiconductor interface. The only difference is that these authors considered magnetization perpendicular to the interface, introduced a lateral confinement of 2D electron gas, and added a δ-function scattering potential at the interface. The Hamiltonian (32) allows us to derive the boundary conditions expressing the continuity of both $\boldsymbol{\psi}(y)$ and

$$\left[\frac{\hbar^2}{m(y)}\frac{d}{dy} + i\hat{\sigma}_x \alpha(y)\right]\boldsymbol{\psi}(y) \tag{33}$$

across the interfaces. The wave function in the semiconductor layer is written as

$$\boldsymbol{\psi} = \sum_{q=\pm} a_q \begin{pmatrix} p_1 \\ q p_{1y} - i p_x \end{pmatrix} e^{iqp_{1y}y} + \sum_{q=\pm} b_q \begin{pmatrix} p_2 \\ -q p_{2y} + i p_x \end{pmatrix} e^{iqp_{2y}y}, \tag{34}$$

where $p_{1,2y} = \sqrt{p_{1,2}^2 - p_x^2}$, $p_1 = p - m^*\alpha/\hbar^2$, $p_2 = p + m^*\alpha/\hbar^2$, and $\hbar p = \sqrt{2m^*(\varepsilon - U_0^*) + (m^*\alpha/\hbar)^2}$. If ε is the Fermi energy, p_1 and p_2 are the Fermi wave numbers for the two branches of the spin-split electron spectrum, while p is the averaged (effective) Fermi wave number of 2D electron gas, which is expressed through its density according to $p = \sqrt{2\pi n_{2D}}$.

The explicit form of the wave function in the metallic leads is written below for the simplest case $\mathbf{M}||\hat{z}$, when the potential energy matrix is diagonal. For a parallel magnetization configuration,

$$\boldsymbol{\psi}|_{y<0} = \begin{pmatrix} c_1 e^{ik_1 y} + r_1 e^{-ik_1 y} \\ c_2 e^{ik_2 y} + r_2 e^{-ik_2 y} \end{pmatrix}, \quad \boldsymbol{\psi}|_{y>L} = \begin{pmatrix} t_1 e^{ik_1 y} \\ t_2 e^{ik_2 y} \end{pmatrix}, \tag{35}$$

where $k_1 = \sqrt{2m_M(\varepsilon - U_0^M - U_z)/\hbar^2}$, $k_2 = \sqrt{2m_M(\varepsilon - U_0^M + U_z)/\hbar^2}$, and m_M is the effective mass in the ferromagnet. If ε is the Fermi energy, k_1 and k_2 are the Fermi wave numbers for spin-up and spin-down states in the ferromagnet. The cases $c_1 = 1$, $c_2 = 0$ and $c_1 = 0$, $c_2 = 1$ correspond to incoming electrons in the states $\uparrow$ and $\downarrow$, respectively. If the right-lead magnetization is inverted, one should permute k_1 and k_2 in $\boldsymbol{\psi}|_{y>L}$. The eight coefficients $a_\pm$, $b_\pm$, $r_{1,2}$, and $t_{1,2}$ are expressed through c_1 and c_2 after a solution of the system of eight linear equations generated through application of the boundary conditions at $y=0$ and $y=L$ to the wave functions (34) and (35). The p_x-dependent transmission probabilities $T_{jj'}^{-+}(p_x)$ obtained in this way should be averaged over p_x as

$$T_{jj'}^{-+} = \frac{1}{2p}\int_{-\infty}^{\infty} T_{jj'}^{-+}(p_x)\, dp_x. \tag{36}$$

This gives us the transmission probabilities $T_{jj'}^{\alpha\beta}$ entering the equations of Sec. II.

The problem of transmission through a 1D semiconductor layer is simpler, since free motion in the x direction is absent. We have $\Psi(x,y,z) = \chi(x,z)\boldsymbol{\psi}(y)$ and put $p_x = 0$ in the Hamiltonian (32). One should remember, however, that this substitution is not straightforward because of the spin-orbit-induced mixing between 1D subband states,[22] which leads to some renormalization[26] of both the effective mass m^* and the Rashba constant α. One can neglect this renormalization if $\alpha/\hbar$ is small in comparison to the Fermi velocity $\hbar p/m^*$ in the 1D layer. The wave function (34) in the 1D case is

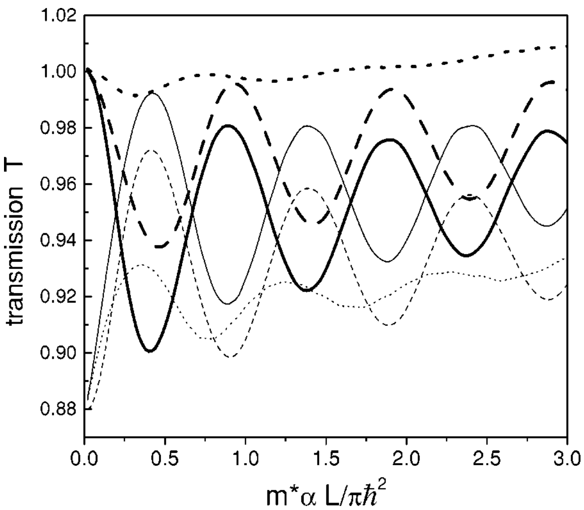

FIG. 2. Probabilities of transmission through a 1-μm-long 2D layer for $\mathbf{M}\|\hat{z}$ (solid line), $\mathbf{M}\|\hat{y}$ (dashed line), and $\mathbf{M}\|\hat{x}$ (dotted line) as a function of $m^*\alpha L/\pi\hbar^2$. Thick and thin lines correspond to parallel and antiparallel magnetization configurations. The leads are Fe electrodes, and the 2D electron gas density n_{2D} is 10^{11} cm^{-2}.

considerably simplified, since $p_{1,2y}=p_{1,2}$, and there is no need of an averaging procedure like Eq. (36) once the transmission probabilities $T_{jj'}^{-+}$ are calculated in the quantum-mechanical problem.

The transmission probabilities for both 1D and 2D cases contain two oscillating contributions, with phases $(p_1+p_2)L=2pL$ and $(p_2-p_1)L=2m^*\alpha L/\hbar^2=2\varphi$. We will neglect the first kind of oscillations by averaging the transmission probability over the period $\delta p=\pi/L$. From the physical point of view, it corresponds to thermal averaging of the transmission probabilities[27] in conditions when the temperature is large in comparison to $\hbar^2 p/k_B m^* L$. The oscillations associated with the Rashba phase 2φ are not influenced by such averaging, since φ is energy independent. The results given below are obtained for $L=1$ μm, $m_M=m_0$, and $m^*=0.035m_0$. The latter is a reasonable value of the conduction-band electron mass in InAs quantum wells.

Figure 2 represents the total transmission probability T for a 2D layer with density $n_{2D}=10^{11}$ cm^{-2}, corresponding to Fermi wave number $p=0.79\times10^6$ cm^{-1}. The transmission is plotted as a function of φ/π (for given m^* and L, the values of α lie in the reasonable range $\alpha\sim10$ meV nm) and shows characteristic oscillations. The leads are assumed to be Fe $[k_1=1.05\times10^8$ cm^{-1} and $k_2=0.44\times10^8$ $\mathrm{cm}^{-1}]$.[13] The field-effect modulation of the transmission for $\mathbf{M}\|\hat{z}$ reaches about 10% at small α and decreases with an increase of α. For $\mathbf{M}\|\hat{y}$ the modulation at small α is about 6%, but its decrease is weaker. At $\mathbf{M}\|\hat{x}$, when the spins of the injected electrons are nearly aligned with the Rashba field, the field-effect modulation is weakest, but the magnetoresistance is highest, about 8% on average. The behavior of the transmission probability for $Ni_{40}Fe_{60}$ leads $[k_1=1.05\times10^8$ cm^{-1} and $k_2=0.65\times10^8$ $\mathrm{cm}^{-1}]$ and $Ni_{80}Fe_{20}$ leads $[k_1=1.05\times10^8$ cm^{-1} and $k_2=0.88\times10^8$ $\mathrm{cm}^{-1}]$ is very similar, but the modulation is considerably weaker because of a smaller Fermi velocity mismatch for spin-up and spin-down electrons. We obtained a degree of modulation of less than 4% for $Ni_{40}Fe_{60}$ and about 0.4% for $Ni_{80}Fe_{20}$ for all orientations of $\mathbf{M}$. With an increase of the electron density, the modulation weakens. Figure 3 shows the behavior of transmission coefficients at $n_{2D}=5\times10^{11}$ cm^{-2}.

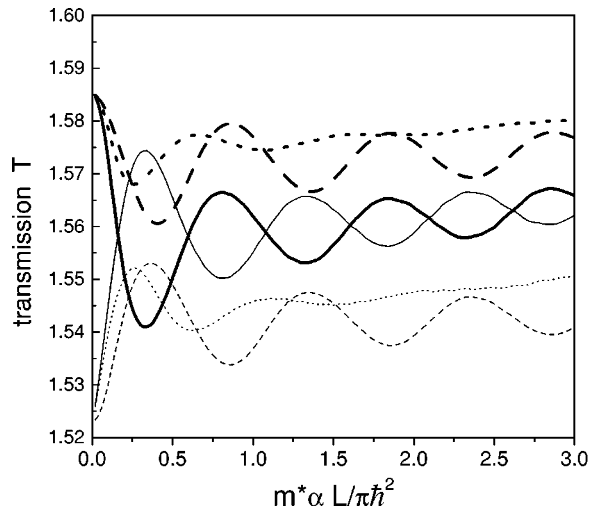

FIG. 3. The same as in Fig. 2 for $n_{2D}=5\times10^{11}$ cm^{-2}.

In Fig. 4 we plot the transmission probability through a 1D layer. As in the case shown in Fig. 2, we assume Fe leads and Fermi wave number $p=0.79\times10^6$ cm^{-1}. The transmission does not depend on the angle of $\mathbf{M}$ in the yz plane, because a rotation of the magnetization vector in this plane does not change the component M_x parallel to the Rashba field. The transmission oscillates as the Rashba phase is changed; the oscillations are nearly harmonic and do not decay with an increase of α, in contrast to the 2D case. The amplitude of the oscillations is maximal for $\mathbf{M}\perp\hat{x}$. The trans-

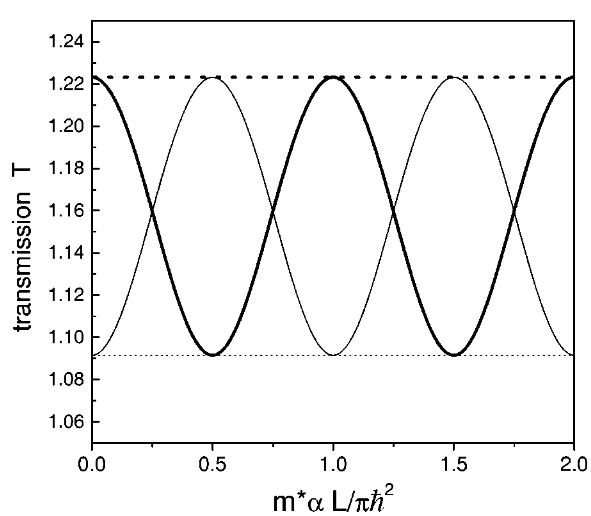

FIG. 4. Probabilities of transmission through a 1-μm-long 1D wire for $\mathbf{M}\perp\hat{x}$ (solid line) and $\mathbf{M}\|\hat{x}$ (dotted line) as a function of $m^*\alpha L/\pi\hbar^2$. The leads are Fe electrodes, and the Fermi wave number is $p=0.79\times10^6$ cm^{-1}. Thick and thin lines correspond to parallel and antiparallel magnetization configurations.

mission for $\mathbf{M}||\hat{x}$ does not depend on the Rashba phase, because the spin-up and spin-down states do not mix. For this orientation, the difference between the transmission probabilities T_{par} and T_{ant} for parallel and antiparallel polarization configurations is equal to the amplitude of the field-effect modulation of transmission at $\mathbf{M}\perp\hat{x}$. One can obtain analytical expressions for these coefficients,[14] which gives

$$\frac{T_{par}-T_{ant}}{T_{par}}=\frac{(k_1-k_2)^2[k_1k_2/m_M^2-(p/m^*)^2]}{(k_1+k_2)^2[k_1k_2/m_M^2+(p/m^*)^2]}. \quad (37)$$

The degree of modulation defined by this equation is maximal at $p\rightarrow 0$ when it reaches 16.7% for the case of Fe leads. With the increase of electron density, the modulation weakens and goes through its minimum (zero) at $p_{min}\simeq 2.4\times 10^6$ cm^{-1} (in the case of Fe leads). A similar behavior takes place for 2D layers as well, though analytical results for that case cannot be obtained. We stress that the given value of $p_{\rm min}$ corresponds to a Fermi energy of about 60 meV and cannot be reached in the present artificial quantum wires. On the other hand, in InAs-based quantum wells p_{min} corresponds to the densities $n_{2D}\sim 10^{12}$ cm^{-2}, which are typical for those systems.

We notice that the transmission shown in Figs. 2–4 does not depend on the direction of **M** if α is zero. This is a consequence of the isotropic model of the ferromagnets we use. Next, one can check that if **M** is reversed in both leads simultaneously, the transmission is not changed even at nonzero α. This is a manifestation of the Casimir-Onsager-Büttiker symmetry, which recently has been emphasized[28,29] for the conductance of a single interface between a ferromagnet and a Rashba semiconductor. It is interesting that in the case of a 1D Rashba semiconductor, this conductance is not sensitive to the direction of **M** at all.[29] Applying our formalism to this particular problem, we obtain the following expression for the conductance:

$$G=\frac{e^2}{h}\sum_{i=1,2}\frac{4\xi_i}{(1+\xi_i)^2}, \quad \xi_i=\frac{k_i}{p}\frac{m^*}{m_M}, \quad (38)$$

which coincides with the result of Ref. 29 under the assumption $m^*=m_M$. Therefore, in order to control the conductance either by electric-field modulation of α or by magnetization axis rotation, it is essential to have a device with two ferromagnetic leads.

IV. DISCUSSION

Among the issues raised in the new field of spin electronics, the problem of electrical spin injection from spin-aligning (for example, ferromagnetic) leads into semiconductor layers appears to be particularly challenging. The large degree of spin polarization of the current has been obtained only in the case of injection from a semimagnetic semiconductor to a bulk (3D) semiconductor layer. However, to implement the idea of a spin transistor as proposed by Datta and Das, one needs to use low-dimensional, ballistic layers. In the first part of this paper we have shown that there is a fundamental obstacle, caused by the dimensionality mismatch, for the injection of spin-polarized currents to layers whose dimensionality is lower than the dimensionality of the leads. As a result, the contribution of the spin polarization in the leads to the spin polarization in the low-dimensional layer appears to be small. If the dimensionality mismatch is 1 or 2, the relative magnitude of such contribution, given above by the dimensionless parameter γ, is of the order of weak localization correction to 2D or 3D conductivities, respectively. Moreover, the parameter γ for 3D-2D-3D structures contains an extra factor, the ratio of the Fermi wave numbers in 2D and 3D layers, which is also small in the case of metallic leads. Due to the smallness of γ, the resistance of the device depends rather on the total transmission probability T than on the spin polarization in the leads.

Fortunately, even if we neglect the small terms proportional to γ, the spin-filtering effect in the contacts between the leads and the semiconductor layer still offers a possibility to inject spin-polarized electrons and to control the resistance. In principle, using the spin-filtering property of abrupt contacts discussed in Refs. 13 and 14 it is possible to get up to 15% of the resistance modulation in 1D layers (quantum wires) using Fe leads. Somewhat smaller values are expected for 2D layers with the same Fermi wave number of the electrons. The use of FeNi alloys reduces the resistance modulation. The increase of the electron density in the layers has the same effect. In particular, the resistance modulation is expected to be small for electron densities n_{2D} around

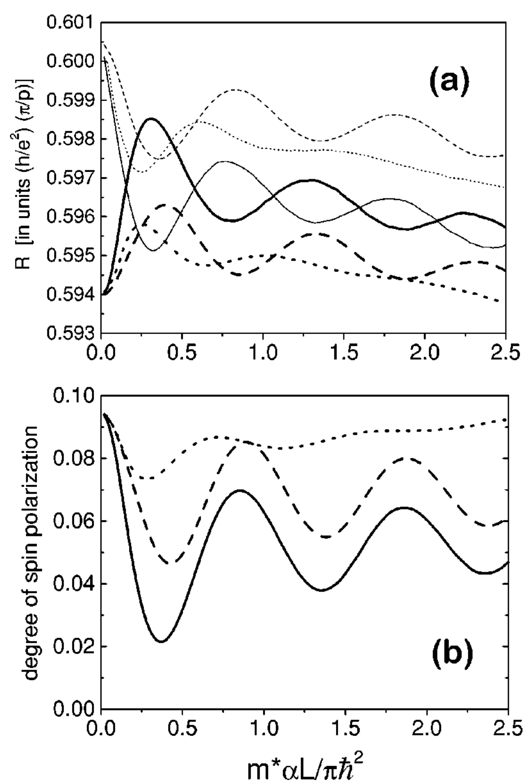

FIG. 5. The resistance R (a) and the degree of spin polarization for parallel magnetization configurations (b) calculated using the experimental parameters of Ref. 17 [$Ni_{40}Fe_{60}$ leads, $m^*=0.05m_0$, $n_{2D}=1.7\times 10^{12}$ cm^{-2}, and $L=0.45$ μm] for $\mathbf{M}||\hat{z}$ (solid line), $\mathbf{M}||\hat{y}$ (dashed line), and $\mathbf{M}||\hat{x}$ (dotted line).

10^{12} cm^{-2} (for InAs layers with Fe leads). In this sense, the samples with $n_{2D} = 1.7 \times 10^{12}$ cm^{-2} used in the recent experiment[17] are, probably, not the best choice. It is important to compare the results of our calculations to the experimental data. Using the experimental values $L = 0.45\mu\mathrm{m}$, $m^* = 0.05m_0$, and parameters k_1 and k_2 of $\mathrm{Ni}_{40}\mathrm{Fe}_{60}$ leads, we calculated the resistance according to $R = (h/e^2)\sqrt{\pi/2n_{2D}}\ T^{-1}$, neglecting the contributions proportional to γ. Within the same approximation, the degree of spin polarization of the injected current for a parallel magnetization configuration is estimated as $(T^{-+}_{\uparrow\uparrow} - T^{-+}_{\downarrow\downarrow})/T$. The results are plotted in Fig. 5 as functions of the Rashba phase for different polarizations of **M** in the leads. Actually, only the case $\mathbf{M} || \hat{x}$ has been investigated experimentally. For this polarization we obtain a magnetoresistance of about 0.5% on average. This value is not far from the value of 0.2% obtained experimentally. The degree of spin polarization of the injected current for parallel magnetization configurations is found to be about 9%, which is twice larger than the experimental value. Nearly the same values were obtained for samples with channel length $L = 1.8$ $\mu\mathrm{m}$, also investigated in the experiment. We stress that the theory[15] of spin injection through a single ferromagnet/semicondictor interface also gives a degree of spin polarization of about 10% when realistic parameters of the device are used in the calculation.

To explain the larger values of the resistance modulation and spin polarization obtained theoretically, we would like to emphasize that realistic contacts between the metal and 2D semiconductor layer are not completely abrupt, as was assumed in the calculations. The presence of a finite transition region between the metal and 2D layer should increase the adiabaticity of the ballistic electron motion and, therefore, decrease both the spin-filtering effect and the possibility to control R (the completely adiabatic case is considered in the end of Sec. II). The discrepancy between experimental and theoretical values can also be attributed to a more complex experimental geometry, involving several interdigitally placed ferromagnetic layers.

In conclusion, our theory suggests that the modulation of resistance of the ballistic spin field-effect transistor with ferromagnetic leads contacted to 2D or 1D semiconductor layers in the geometry shown in Fig. 1 cannot be made efficient. A modulation of several percent can, in principle, be achieved in 2D devices through the use of Fe leads and low enough ($\sim 2 \times 10^{11}$ cm^{-2}) 2D electron density. Creation of 1D devices is desirable, but the modulation there is not expected to increase considerably in comparison to 2D devices. A possible way to create an efficient transistor device is to make the spin-aligning leads from materials with a total spin polarization of carriers, such as semimagnetic semiconductors[9–11] (in magnetic fields at low temperatures) or half-metallic magnets.[30]

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