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Short-range interactions and a Bose metal phase in two dimensions

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We show here based on a one-loop scaling analysis that short-range interactions are strongly irrelevant perturbations near the insulator-superconductor (IST) quantum critical point. The lack of any proof that short-range interactions mediate physics which is present only in strong coupling leads us to conclude that short-range interactions are strictly irrelevant near the IST quantum critical point. Hence, we argue that no physics, such as the formation of a uniform Bose metal phase can arise from an interplay between on-site and nearest-neighbor interactions.

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The standard model used to study[1–13] the insulator-superconductor transition in thin films is the commensurate Bose-Hubbard model or equivalently the charging model for an array of Josephson junctions. The Hamiltonian for this model

$$\hat{H}=\frac{1}{2}\sum_{ij}\hat{n}_i V_{ij}\hat{n}_j - J\sum_{\langle ij\rangle}\cos(\phi_i-\phi_j) \tag{1}$$

consists of a charging term V_{ij} and nearest-neighbor Josephson coupling between grains possessing a superconducting phase ϕ_i. The operator, $\hat{n}_i$ is the boson number operator for site i. In the on-site version of this model, $V_{ij}=\delta_{ij}V_0$, where V_0 is the capacitance charging energy for each junction. The corresponding free-energy functional for the on-site charging model lies in the $O(3)$ universality class which possesses a single quantum critical point[1] signalling the loss of phase coherence once V_0/J exceeds a critical value.

The recent experiments of Kapitulnik and co-workers[14] in which a metallic phase has been observed to intervene between the superconductor and the eventual insulating phase suggests that perhaps two phase transitions accompany the loss of phase coherence in a 2D superconductor: (1) superconductor to Bose metal and (2) Bose metal to insulator. These experimental results as well as earlier observations[15–17] of a similar metallic phase have stimulated a re-examination[18–21] of the physics of phase-only models. In this context, Das and Donaich[21] have appended to the standard on-site charging model a nearest-neighbor charging term with amplitude V_1. Concluding that V_1 is a relevant perturbation, they find that short-range interactions mediate a critical point in which a Bose metal phase obtains once the size of each grain is increased beyond a critical value such that $V_1>V_0$. The Bose metal phase of Das and Doniach[21] is a uniform phase lacking both phase and charge order. Hence, this phase is translationally and rotationally invariant. This result is surprising because the critical point in a Josephson junction array (JJA) is controlled by the standard ϕ^4 Wilson-Fisher critical point. It is well-known that short-range interactions are irrelevant near the Wilson-Fisher critical point.[22] Moreover, no critical point is generated regardless of the magnitude of V_1. Hence, the work of Das and Doniach[21] stands in stark contrast to the standard view. In addition, Fazio and Schön[23] have analyzed the nearest-neighbor charging model as well and have shown that at $T=0$, no phase exists lacking both charge and phase order.

Motivated by the disagreement between the standard picture[22,23] and the Das-Doniach result,[21] we take a closer look at the nearest-neighbor charging model. We show that as long as on-site Coulomb interactions are present, that there is no signature that screened interactions of any type are relevant through one-loop order. Two conclusions are possible. Either the physics controlled by short-range interactions is strictly a strong coupling problem with no weak-coupling signature, or short-range interactions are irrelevant at each order in perturbation theory in agreement with the standard view. The lack of any proof that short-range interactions flow to strong coupling leads us to conclude that it is unlikely that short-range interactions can mediate homogeneous phases near the quantum critical point associated with loss of phase coherence.

To establish this result, we write the partition function for the phase-only model in the standard way as a path integral

$$Z=\int \mathcal{D}\phi e^{-S}, \tag{2}$$

where the statistical weight for each path,

$$S=\frac{1}{2}\int d\tau\sum_k \frac{\dot{\phi}(\mathbf{k})\dot{\phi}(-\mathbf{k})}{V(\mathbf{k})}-\int d\tau\sum_{\langle ij\rangle} J_{ij}\cos(\phi_i-\phi_j), \tag{3}$$

defines the effective action for our problem. Here, $V(\mathbf{k})$ is the Fourier transform of V_{ij}. At the outset, we place no restriction on the range of V_{ij}. To simplify this action, we first decouple the charging term by introducing[4] an auxilliary real gauge field, $A_0(\mathbf{k})$, through the identity

$$\begin{aligned}\exp\left[-\frac{1}{2}\int d\tau\sum_{\mathbf{k}}\dot{\phi}(\mathbf{k})\frac{1}{V(\mathbf{k})}\dot{\phi}(-\mathbf{k})\right]\\ =\int \mathcal{D}A_0\exp\Bigg\{-\frac{1}{2}\int_{\mathbf{k},\omega}\frac{A_0(\mathbf{k},\omega)A_0(-\mathbf{k},-\omega)}{e^{-2}V(\mathbf{k})-1}\\ -\frac{1}{2e^2}\int d\tau\sum_i(\dot{\phi}_i-eA_0(i))^2\Bigg\}.\end{aligned} \tag{4}$$

The coupling constant e is a free parameter which will be determined later. The second step is to decouple the $\exp(\phi_i)$ terms in the standard way[1] by introducing the complex field $\psi_i(\tau)$, which will play the role of the order parameter in an effective Landau-Ginzburg theory. The final expression for the partition function

$$Z=\int \mathcal{D}\psi\mathcal{D}A_0 e^{-S} \tag{5}$$

is obtained by integrating over the auxilliary fields. The effective action now takes the form,

$$S=\frac{1}{2}\int d^dx d\tau\left[|(\partial_\tau-ieA_0)\psi|^2+|\nabla\psi|^2+r|\psi|^2+\frac{u}{2}|\psi|^4\right]$$
$$+\frac{1}{2}\int_{\mathbf{k},\omega}\frac{A_0(\mathbf{k},\omega)A_0(-\mathbf{k},-\omega)}{e^{-2}V(\mathbf{k})-1}=S_0+S_1. \tag{6}$$

Consider first the case of long-range Coulomb interactions. In this case, the constant, e, plays the role of the electric charge, $e^*=2e$, and $V(k)=(e^*)^2/k^\sigma$ where $\sigma=2$ for D=3 and $\sigma=1$ for 2D. Consequently, the pure Coulomb part of the action reduces to

$$S_1=\frac{1}{2}\int_{\mathbf{k},\omega}k^\sigma A_0(\mathbf{k},\omega)A_0(-\mathbf{k},-\omega), \tag{7}$$

which is the Fisher and Grinstein[4] result.

What about short-range interactions? We simplify to the case considered by Das and Doniach[21] and truncate $V(\mathbf{k})$ at the nearest-neighbor level:

$$V(\mathbf{k})=V_0+2V_1(\cos k_x+\cos k_y). \tag{8}$$

It is crucial in our derivation that $V_0\neq0$. As has been considered previously, when $V_0=0$ but $V_1\neq0$, the nature of the $T=0$ transition changes fundamentally when compared with the $V_0\neq0$ case. In the former case, that is, $V_0=0$ but $V_1\neq0$, the $T=0$ transition is of the Berezinskii-Kosterlitz-Thouless kind.[23] In the long wavelength limit, $V(\mathbf{k})=V_0+4V_1-V_1k^2$, which is convenient to write in the form, $e^2-V_1k^2$ where we have fixed the free parameter, $e^2=V_0+4V_1$. Consequently, the pure gauge part of the action simplifies to

$$S_1=-\frac{e^2}{2V_1}\int_{\mathbf{k},\omega}\frac{A_0(\mathbf{k},\omega)A_0(-\mathbf{k},-\omega)}{k^2}. \tag{9}$$

Upon rescaling the gauge field, $A_0(\mathbf{k},\omega)\to i\sqrt{V_1/e^2}A_0(\mathbf{k},\omega)$, we arrive at the working form for the action,

$$S=\frac{1}{2}\int d^dx d\tau\left[|(\partial_\tau+gA_0)\psi|^2+|\nabla\psi|^2+r|\psi|^2+\frac{u}{2}|\psi|^4\right]$$
$$+\frac{1}{2}\int_{\mathbf{k},\omega}A_0(\mathbf{k},\omega)\frac{1}{k^2}A_0(\mathbf{k},\omega), \tag{10}$$

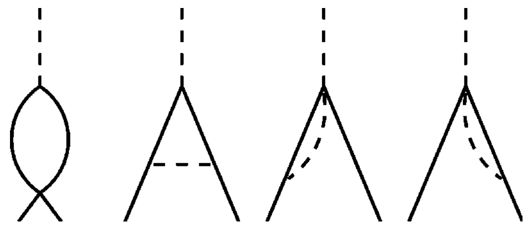

FIG. 1. Diagrams that contribute to the renormalization of g through one-loop order. The dashed line represents the propagator for the $A_0(\mathbf{k},\omega)$ field. The solid lines are given by the Gaussian propagator $(\omega^2+q^2+r)^{-1}$.

where the constant $g=\sqrt{(V_1(V_0+4V_1))}=e\sqrt{V_1}$. Clearly, when $V_1=0$, $g=0$ and the rescaled fields, $A_0(\mathbf{k},\omega)$, vanish leading to the standard on-site charging model. Hence, the relevance of short-range interactions can be deduced entirely from the scaling properties of the coupling constant g.

Performing the standard tree-level rescaling with the rescaling parameter $b>1$, we find that the momentum and frequency scale as $q'=qb$ and $\omega'=\omega b^z$, with z the dynamical exponent. At the tree level, $z=1$ and the anomalous dimension exponent vanishes, $\eta=0$. Hence, the ψ and A_0 fields scale as

$$A_0=b^\mu A_0', \quad \mu=\frac{d+z-2}{2},$$

$$\psi=b^\lambda\psi', \quad \lambda=\frac{d+2+z}{2}. \tag{11}$$

Combining these scaling relations with the rescaling of the momentum and the frequency arising from the integrations in the action, we arrive at our key result

$$g'=g\frac{b^{\mu+\lambda}}{b^{(3d+3z)/2}}=gb^{-(d+z)/2}, \tag{12}$$

namely that g has a negative eigenvalue. Hence, upon successive renormalization transformations, the physics controlled by g can have no effect on the underlying quantum phase transition.

Does the irrelevance of g still persist beyond the tree level? To answer this question, we derive the scaling equations for g through one-loop order. The relevant diagrams that contribute are shown in Fig. 1.[24] We evaluate these diagrams using the standard frequency-momentum shell RG approach in which we integrate out the fields $A_0(\omega,\mathbf{k})$ and $\psi(\omega,\mathbf{k})$ for momenta and frequencies satisfying the constraint $\Lambda/b<\omega<\Lambda$, and $\Lambda/b<k<\Lambda$ with the upper momentum and frequency cutoffs $\Lambda_\omega=\Lambda_k=\Lambda=1$. Setting $b=e^l$, we obtain

$$\frac{dg_l}{dl}=-\left(\frac{d+1}{2}\right)g_l-2A_lg_lu_l-B_lg_l^3 \tag{13}$$

as the differential form for the scaling equation for g. The coefficients, A_l and B_l are given by

$$A_l = \frac{2K_d}{(2\pi)^{d+1}} \left[\int_0^1 dq \frac{q^{d-1}}{(q^2+1+r_l)^2} + \int_0^1 \frac{d\omega}{(1+\omega^2+r_l)^2} \right] \tag{14}$$

and

$$B_l = \frac{2K_d}{(2\pi)^{d+1}} \left[\int_0^1 dq q^{d+1} \frac{1+2q^2+2r_l}{(1+q^2+r_l)^2} + \int_0^1 d\omega \frac{2+\omega^2+2r_l}{(1+\omega^2+r_l)^2} \right], \tag{15}$$

where K_d is the area of a d-dimensional unit sphere. These coefficients are positive and depend on the scaling length l through the parameter r_l. Hence, from the structure of the scaling equation, for g_l, Eq. (13), we find that the $g=0$ fixed point is stable through one-loop order. That is, there is no signature in weak coupling that finite g can drive a different critical point. This conclusion is consistent with the standard view that as long as the broken symmetry state is rotationally and translationally invariant, the critical point is of the Wilson-Fisher type where it is well known that short-range interactions cannot lead to a different critical point.

Because $g \propto \sqrt{V_1}$, the one-loop scaling equation for g necessarily implies that nearest-neighbor interactions are irrelevant and as a consequence cannot change the phase diagram of the on-site charging model, contrary to the claims of Das and Doniach.[21] Simply put, nearest neighbor interactions constitute an irrelevant perturbation in the phase-disordering transition in complete agreement with the standard scaling arguments.[22] Because the critical point in the JJA model is of the Wilson-Fisher type, V_1, regardless of its magnitude, cannot mediate a critical point separating phases that are translationally and rotationally invariant. This automatically rules out a V_1-mediated uniform Bose metal phase.

However, short-range interactions can mediate an inhomogenous charge-ordered phase such as a supersolid.[12,13] In such instances, the effective field theory reduces[12] to two coupled $O(3)$ vector models. Nonetheless, if the broken symmetry state is rotationally and translationally invariant, the critical point is of the Wilson-Fisher type where short-range interactions are strictly irrelevant. In light of this conclusion, the only candidate for a Bose metal phase that remains is our recent proposal[18] that in the standard quantum disordered regime, a cancellation arises between the exponentially long quasiparticle scattering time and the exponentially small quasiparticle population, leading ultimately to a finite dc conductivity.

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