Counting statistics for entangled electrons

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The counting statistics (CS) for charges passing through a coherent conductor comprise the most general quantity that characterizes electronic transport. The CS depend not only on the transport properties of the conductor, but also on the correlations among particles which compose the incident beam. In this paper we present general results for the CS of entangled electron pairs traversing a beam splitter, and we show that the probability that Q charges have passed is not binomial, as in the uncorrelated case, but is symmetric with respect to the average transferred charge. We furthermore consider the joint probability for transmitted charges of a given spin, and we show that the signature of entanglement distinctly appears in a correlation which is not present for a nonentangled case.

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I. INTRODUCTION

Probably one of the most striking features of quantum mechanics is entanglement,[1] which refers to the nonlocal correlations existing, even in the absence of interaction, between two (spatially separated) parts of a given quantum system. Besides the fundamental interest in the generation and detection of entanglement, a great deal of interest has been brought forth by its role in quantum information, which has attracted vast effort due to the very important impact of its potential applications, ranging from quantum computation to quantum teleportation.[2] Entanglement is the main ingredient in all known examples of quantum speed-up in quantum computation and communication.

Most of the work on entanglement was performed in optical systems with photons,[3] cavity QED systems,[4] and ion traps.[5] Only recently have people begun to study how to generate and manipulate entangled pairs in a solid-state environment. The prototype setup was discussed in Ref. 6, where it was shown that the presence of spatially separated pairs of entangled electrons can be revealed by using a beam splitter, as in Fig. 1, and by measuring the correlations of the current fluctuation (noise) at the exiting terminals (labeled by 3 and 4 in the figure). Provided that the electrons injected into leads 1 and 2 are in an entangled state,

$$|\psi\rangle=\frac{1}{\sqrt{2}}(\hat{a}^\dagger_{2\downarrow}\hat{a}^\dagger_{1\uparrow}\pm\hat{a}^\dagger_{2\uparrow}\hat{a}^\dagger_{1\downarrow})|0\rangle, \quad (1)$$

bunching and antibunching behaviors are found depending on whether state $|\psi\rangle$ is a spin singlet (lower sign) or a spin triplet (upper sign). More precisely, current noise is enhanced by a factor of 2 with respect to nonentangled states in the former case, and suppressed to zero in the latter. Note that while this allows one to detect a singlet entangled state, it does not discriminate between entangled and nonentangled triplets (unless spin-dependent detectors are employed[6]). Given the general setup, in order to find the signatures of entanglement in the noise spectrum one needs a physical realization of both the *entangler* (that enables the pair production) and the beam splitter. As the entangler one can resort to the phenomenon of Andreev reflection in hybrid normal-superconducting systems, as discussed in Refs. 7–9. Besides electrons, it is possible to produce entangled states with Cooper pairs in superconducting nanocircuits[10] or, by coupling a mesoscopic Josephson junctions with superconducting resonators,[11,12] between Cooper pairs and the resonator mode.

In this paper we consider the same approach as in Ref. 6, and take for granted the existence of an *entangler*. We address the question of whether the study of the full statistics of charge transport[13] at exit terminals 3 and 4 of such systems can provide more information (as compared to the noise) on the correlation of the injected particles in terminals 1 and 2. The main result of this paper is that not only the value of the noise characterizes the entangled singlet state with respect to uncorrelated states (as shown in Ref. 6); in addition, the whole probability distribution for the transfer of charges is qualitatively modified. More precisely, we show that the probability distribution relative to incident particles in the entangled singlet state is not binomial, in contrast to the case of uncorrelated injected states; moreover, it is symmetric around its average value. In addition, we show that the use of spin-sensitive electron counters, on the one hand, provides a more stringent tool for detecting entangled states which is

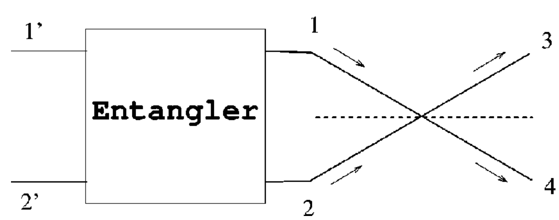

FIG. 1. The prototype setup consists of an *entangler* connected to a beam splitter. The entangler produces pairs of entangled electrons from a source of uncorrelated particles entering from terminals 1' and 2'. In the beam splitter, the entangled electrons injected into terminals 1 and 2 are transmitted and reflected into terminals 3 and 4 by the semitransparent mirror (dashed line). No backscattering into leads 1 and 2 is allowed.

based on general properties of the probability distribution. On the other hand, as already noticed in Ref. 6, it allows one to distinguish between entangled and nonentangled triplet states. The paper is organized as follows. In Sec. II we give a brief review of the scattering approach for counting statistics. Then, in Sec. III, we apply it to the case of a beam splitter with entangled electrons. We first present the results for the statistics of transmitted charges in a single terminal, and then consider the cross correlation. We finally summarize all the results in Sec. IV.

II. SCATTERING APPROACH

In the calculation of counting statistics we adopt the scattering approach of Landauer and Buttiker.[14–16] Within this framework, the transport properties of a metallic phase-coherent structure attached to n reservoirs are determined by the matrix S of scattering amplitudes. Such amplitudes are defined through asymptotic wave functions, known as scattering states, for particles in the leads (which connect the reservoirs to the sample). In one dimension, for example, such scattering states arising from a unitary flux of particles at energy E, originating in the ith reservoir, read

$$\varphi_i(x)=\frac{e^{ik_i(E)x}+r_i(E)e^{-ik_i(E)x}}{\sqrt{hv_i(E)}} \tag{2}$$

for the ith lead, and

$$\varphi_j(x)=\frac{t_{ji}(E)e^{-ik_j(E)x}}{\sqrt{hv_j(E)}} \tag{3}$$

for the jth lead, with $j\neq i$. Here $r_i(E)$ is the reflection amplitude for particles at energy E with wave vector $k_i(E)$ and group velocity $v_i(E)$ in the ith lead, and $t_{ji}(E)$ is the transmission amplitude from lead i to lead j. Note that $|r_i|^2$ is the probability for a particle to reflect back into the ith lead, and $|t_{ji}|^2$ is the probability for the transmission of a particle from lead i to lead j. In the second quantization formalism, the field operator $\hat{\psi}_{j\sigma}(x,t)$ for spin σ particles in lead j is built from scattering states, and is defined as

$$\hat{\psi}_{j\sigma}(x,t)=\int dE\frac{e^{-iEt/\hbar}}{\sqrt{hv_j(E)}}[\hat{a}_{j\sigma}(E)e^{ik_jx}+\hat{\phi}_{j\sigma}(E)e^{-ik_jx}], \tag{4}$$

where $\hat{a}_{j\sigma}(E)$ $[\hat{\phi}_{j\sigma}(E)]$ is the destruction operator for incoming (outgoing) particles at energy E, with spin σ in lead j. Such operators are related through the scattering matrix S of the structure as

$$\begin{pmatrix}\hat{\phi}_{1\uparrow}\\ \hat{\phi}_{1\downarrow}\\ \hat{\phi}_{2\uparrow}\\ \vdots\end{pmatrix}=S\begin{pmatrix}\hat{a}_{1\uparrow}\\ \hat{a}_{1\downarrow}\\ \hat{a}_{2\uparrow}\\ \vdots\end{pmatrix}, \tag{5}$$

and obey anticommutation relations

$$\{\hat{a}^{\dagger}_{i\sigma}(E),\hat{a}_{j\sigma'}(E')\}=\delta_{i,j}\delta_{\sigma,\sigma'}\delta(E-E'). \tag{6}$$

In the case of two- and three-dimensional leads, one can separate longitudinal and transverse particle motions. Since the transverse motion is quantized, the wave function relative to the plane perpendicular to the direction of transport is characterized by a set of quantum numbers which identifies the channels of the lead. Such channels are referred to as "open" when the corresponding longitudinal wave vectors are real, since they correspond to propagating modes. Note that the case of a single open channel corresponds to a one-dimensional lead.

As far as charge transport is concerned, the quantities which are most frequently considered are the conductance and the noise, the latter arising due to the discrete nature of the charge carriers, even at zero temperature. However it is more general to consider the probability distribution for the transfer of charges,[17,18] of which conductance and noise are the first and second moments, respectively. Following Refs. 17 and 18, within the scattering approach the characteristic function of the probability distribution for the transfer of particles in a structure attached to n leads at a given energy E can be written as

$$\chi_E(\vec{\lambda})=\left\langle \prod_{j=1,n} e^{i\lambda_j(\hat{N}_I^{j\uparrow}+\hat{N}_I^{j\downarrow})} \prod_{j=1,n} e^{-i\lambda_j(\hat{N}_O^{j\uparrow}+\hat{N}_O^{j\downarrow})}\right\rangle, \tag{7}$$

where the brackets $\langle\cdots\rangle$ stand for the quantum-statistical average in thermal equilibrium. Assuming a single channel per lead, $\hat{N}^{j\sigma}_{I(O)}$ is the number operator for incoming (outgoing) particles with spin σ in lead j, and $\vec{\lambda}$ is a vector of n real numbers, one for each open channel. Number operators can be written in terms of the above operators as $\hat{N}_I^{j\sigma}=\hat{a}^{\dagger}_{j\sigma}\hat{a}_{j\sigma}$ and $\hat{N}_O^{j\sigma}=\hat{\phi}^{\dagger}_{j\sigma}\hat{\phi}_{j\sigma}$. Note that Eq. (7) is simply a generalization of the spinless, single-channel case, for which it is easy to show that

$$\chi_E(\lambda):=\sum_{m,n=0}^{1} P_E(m,n)e^{i\lambda m}e^{-i\lambda n}=\langle e^{i\lambda\hat{N}_I}e^{-i\lambda\hat{N}_O}\rangle. \tag{8}$$

Here $P_E(m,n)$ is the joint probability for m particles to propagate to the right and n particles to propagate to the left, with energy E.

For long measurement times t,[19] the total characteristic function χ is the product of contributions from different energies, so that

$$\chi(\vec{\lambda})=e^{t/h\int dE\,\log\chi_E(\vec{\lambda})}, \tag{9}$$

and the joint probability distribution for transferring Q_1 electronic charges in lead 1, Q_2 in lead 2, etc., is given by

$$P(Q_1,Q_2,\ldots)=\frac{1}{(2\pi)^n}\int_{-\pi}^{+\pi}d\lambda_1 d\lambda_2\ldots\chi(\vec{\lambda})e^{i\vec{\lambda}\cdot\vec{Q}}. \tag{10}$$

In Refs. 20 and 21 it was first proved that in a quantum conductor with a single open channel the distribution probability is binomial, in contrast to the classical case where the distribution is Poissonian. In Ref. 22 the characteristic function was generalized to many open channels, and an explicit expression for its cumulants was obtained. This allowed one to prove that the probability distribution for a tunnel barrier with very small transmission recovers a Poissonian distribution. Counting statistics was so far studied for several systems including a hybrid normal-metal–superconductor structure,[18,23,24] metallic diffusive wires,[22,25] and chaotic cavities.[26] As far as the experimental measurement is concerned, two possible schemes for measuring the counting statistics were recently proposed in Ref. 27.

In the rest of the paper we specialize to the beam splitter of Fig. 1, for which $n=4$. In analogy with the optical case, we consider the ideal situation where particles injected from branch 1 (2) impinge on a semitransparent mirror, from which they are transmitted into branch 4 (3) and reflected into branch 3 (4).

III. CHARACTERISTIC FUNCTION FOR ENTANGLED ELECTRONS

We concentrate on a calculation of the probability distribution for the transfer of particles in leads 3 and 4, when particles are injected from leads 1 and 2. Since we are not interested in counting the particles passing through the entering leads 1 and 2, we set $\lambda_1=\lambda_2=0$, so that Eq. (7) becomes

$$\chi_E(\lambda_3,\lambda_4)=\langle e^{i\lambda_3(\hat{N}_I^{3\uparrow}+\hat{N}_I^{3\downarrow})}e^{i\lambda_4(\hat{N}_I^{4\uparrow}+\hat{N}_I^{4\downarrow})}\times e^{-i\lambda_3(\hat{N}_O^{3\uparrow}+\hat{N}_O^{3\downarrow})}e^{-i\lambda_4(\hat{N}_O^{4\uparrow}+\hat{N}_O^{4\downarrow})}\rangle. \qquad (11)$$

We assume, as usual, that the incoming particles are independent, and originate from reservoirs. Therefore, we set the chemical potentials of reservoirs connected to leads 3 and 4 to zero, and chemical potentials of reservoirs connected to leads 1 and 2 either to zero or to eV. At zero temperature, the statistical average over the Fermi distribution function in Eq. (11) simplifies to the expectation value onto the state containing two electrons in the energy range $0<E<\mathrm{eV}$, with opposite spin either in lead 1 or 2. The situation we are interested in corresponds to the propagation of entangled incident states from branches 1 and 2, as if originating from an *entangler*. Such a device provides incident electrons, at the same given energy E, described by the state[28]

$$|\psi\rangle=\frac{1}{\sqrt{2}}[\hat{a}_{2\downarrow}^{\dagger}(E)\hat{a}_{1\uparrow}^{\dagger}(E)\pm\hat{a}_{2\uparrow}^{\dagger}(E)\hat{a}_{1\downarrow}^{\dagger}(E)]|0\rangle. \qquad (12)$$

In Eq. (12) the minus sign refers to the spin singlet and the plus sign to the spin triplet. For $0<E<\mathrm{eV}$, Eq. (11) reduces to

$$\chi_E(\lambda_3,\lambda_4)=\langle e^{-i\lambda_3(\hat{N}_O^{3\uparrow}+\hat{N}_O^{3\downarrow})}e^{-i\lambda_4(\hat{N}_O^{4\uparrow}+\hat{N}_O^{4\downarrow})}\rangle, \qquad (13)$$

since the incoming states do not contain incoming particles from leads 3 and 4. By using the identity

$$e^{-i\lambda_j\hat{N}_O^{j\sigma}}=[1+(e^{-i\lambda_j}-1)\hat{N}_O^{j\sigma}] \qquad (14)$$

$[(\hat{N}_O^{j\sigma})^2=\hat{N}_O^{j\sigma}$ are projector operators], the evaluation of $\chi_E(\lambda_3,\lambda_4)$ is reduced to a calculation of expectation values of number operators and their products. The procedure is further simplified by assuming no backscattering into terminals 1 and 2, so that the scattering matrix obeys the relation

$$\begin{pmatrix}\hat{\phi}_{3\sigma}\\ \hat{\phi}_{4\sigma}\end{pmatrix}=\begin{pmatrix}r_{31}^{\sigma} & t_{32}^{\sigma}\\ t_{41}^{\sigma} & r_{42}^{\sigma}\end{pmatrix}\begin{pmatrix}\hat{a}_{1\sigma}\\ \hat{a}_{2\sigma}\end{pmatrix}, \qquad (15)$$

if no spin-mixing processes are present. Here r_{ij}^{σ} (t_{ij}^{σ}) is the reflection (transmission) amplitude for an incoming particle from lead j to be reflected (transmitted) into lead i.

In the case of entangled incident states [Eq. (12)], we find that

$$\chi_E(\lambda_3,\lambda_4)=\left(\frac{1}{2}-A\right)(e^{-2i\lambda_3}+e^{-2i\lambda_4})+2Ae^{-i(\lambda_3+\lambda_4)}, \qquad (16)$$

where

$$A=\tfrac{1}{2}[T^{\uparrow}T^{\downarrow}+R^{\uparrow}R^{\downarrow}\pm(r_{42}^{\uparrow}t_{41}^{\uparrow *}r_{42}^{\downarrow *}t_{41}^{\downarrow}+t_{41}^{\uparrow}r_{42}^{\uparrow *}t_{41}^{\downarrow *}r_{42}^{\downarrow})] \qquad (17)$$

with the upper sign referring to the triplet state and the lower sign referring to the singlet state. $R^{\sigma}=|r_{31}^{\sigma}|^2=|r_{42}^{\sigma}|^2$ and $T^{\sigma}=|t_{32}^{\sigma}|^2=|t_{41}^{\sigma}|^2$ are reflection and transmission probabilities, respectively. Note that the second equalities in the above relationships are completely general in the case of no backscattering. For comparison, in the case of uncorrelated incoming particles the characteristic function is given by

$$\chi_E(\lambda_3,\lambda_4)=\prod_{\sigma=\uparrow,\downarrow}(R^{\sigma}e^{-i\lambda_3}+T^{\sigma}e^{-i\lambda_4}). \qquad (18)$$

As it appears from Eqs. (16) and (18), the characteristic function relative to entangled pairs of incident particles [Eq. (16)] possesses a different structure with respect to the one relative to the ordinary situation of independent particles [Eq. (18)]. In particular, while Eq. (18) depends only on probability coefficients, the characteristic function for entangled electrons depends directly on the scattering amplitudes. Furthermore, unlike Eq. (16), Eq. (18) can be factorized into spin-up and spin-down contributions, reflecting the fact that, in an ordinary situation, electrons with different spins undergo independent scattering processes. In the simplest case of spin-independent transport, such that $r_{ij}^{\uparrow}=r_{ij}^{\downarrow}$ and $t_{ij}^{\uparrow}=t_{ij}^{\downarrow}$, the constant in Eq. (17) takes the value $A=\frac{1}{2}(|t|^2-|r|^2)^2$ for the entangled singlet and $A=1/2$ for the entangled triplet. This implies that pairs of particles in an entangled triplet state show the same characteristic function as for nonentangled triplets (of the form $|\psi\rangle=\hat{a}_{1\sigma}^{\dagger}\hat{a}_{2\sigma}^{\dagger}$), namely,

$$\chi_E(\lambda_3,\lambda_4)=e^{-i(\lambda_3+\lambda_4)}. \qquad (19)$$

Note, moreover, that the result given in Eq. (19) for nonentangled triplets does not depend on transport amplitudes.

It is worthwhile noting that if we allow for spin-polarized transport, for example using ferromagnetic metals for terminals 3 and 4, the characteristic functions for all cases will be distinguished from each other. The constant A in Eq. (16), in fact, will take the value

$$A=\tfrac{1}{2}(t^{\uparrow\star}t^{\downarrow}\pm r^{\uparrow\star}r^{\downarrow})(t^{\uparrow}t^{\downarrow\star}\pm r^{\uparrow}r^{\downarrow\star}) \tag{20}$$

in the case of a symmetric beam splitter (where $r_{31}^{\sigma}=r_{42}^{\sigma}=r$ and $t_{32}^{\sigma}=t_{41}^{\sigma}=t$). This causes the characteristic function of the entangled spin triplet to differ from the one relative to nonentangled triplets, since in the latter case χ_E is again given by Eq. (19), independent of scattering amplitudes.

A. Counting statistics on a single terminal

Let us now turn our attention to the probability distributions for the transfer of particles. As already mentioned in Sec. II, these can be easily computed by a Fourier transform of the total characteristic function [Eq. (9)], so that the probability for transferring a number of Q_α electronic charges, regardless their spin, into lead α is given by

$$P(Q_\alpha)=\frac{1}{2\pi}\int_{-\pi}^{+\pi}d\lambda_\alpha\chi(\lambda_\alpha)e^{i\lambda_\alpha Q_\alpha}. \tag{21}$$

Note that $\chi(\lambda_\alpha)$ is obtained from the complete $\chi(\vec{\lambda})$ by setting to zero every λ_β with $\beta\neq\alpha$. In the limit of small bias voltage V and zero temperature, the total characteristic function [Eq. (9)] can be reduced to $\chi(\vec{\lambda})=[\chi_0(\vec{\lambda})]^M$ with $M=eVt/h$, in such a way that one only needs to calculate the characteristic function at zero energy. For entangled incident particles [see Eq. (12)] we find that

$$P(Q_3)=\sum_{k=|Q_3-M|}^{M}\binom{M}{k}\left(\frac{1}{2}-A\right)^k(2A)^{M-k}\binom{k}{\frac{Q_3-M+k}{2}}, \tag{22}$$

with the sum restricted to values of k such that (Q_3-M+k) is an even number, and $P(Q_3)=0$ for $Q_3>2M$. It is easy to show that distribution (22) is symmetrical with respect to the position of the maximum $(Q_3=M)$, independent of the scattering amplitudes. This result is in contrast with the ordinary situation of independently injected particles where, as expected, the distribution is binomial,

$$P(Q_3)=\binom{2M}{Q_3}R^{Q_3}(1-R)^{2M-Q_3}, \tag{23}$$

and centered around the value $Q_3=2MR$, for spin-independent transport (the factor 2 comes from the spin degeneracy). Note that the width of Eq. (22), for a spin singlet is double with respect to the ordinary case of Eq. (23), and zero for the triplet. In particular, for an entangled spin triplet we have

$$P(Q_3)=\delta_{Q_3,M}, \tag{24}$$

equal to the nonentangled triplet states.

Let us now assume spin-dependent transport. In such a case the distribution $P(Q_3)$, relative to the triplet entangled state, broadens to a finite width, and is distinguished from the two nonentangled triplet states, which remain of the form of Eq. (24). Such a broadening is due to the fact that the constant A in Eq. (16) is no longer equal to 1/2, but instead is given by expression (20). As an example, in Fig. 2 we plot the probability distribution as a function of the number of charges Q_3, relative to the various incident particle states for a beam splitter characterized by $R^{\uparrow}=0.2$, $R^{\downarrow}=0.1$, and $M=50$. The thin (thick) solid line represents the counting statistics relative to the entangled singlet (triplet) state, whereas the dashed line is the counting statistics for the ordinary independent particle state. The curve relative to the entangled triplet has acquired a finite width, and becomes distinguished from nonentangled triplets whose distribution is a Kronecker delta at $Q_3=50$ (not shown in the figure). Note that, since the shot noise is proportional to the variance of $P(Q_3)$ through the relation[13] $s_{33}t/2e^2=\langle\langle Q_3Q_3\rangle\rangle$, where

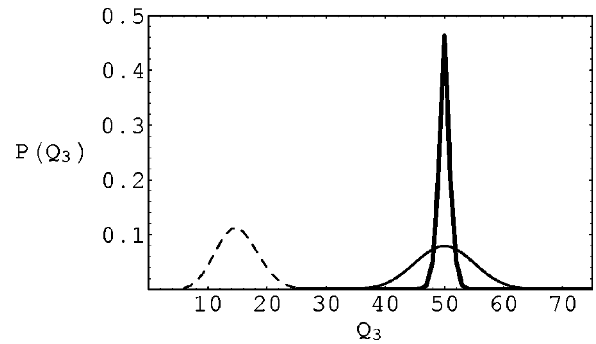

FIG. 2. Single-terminal counting statistics $P(Q_3)$ for a spin-insensitive electron counter. The dashed line is relative to uncorrelated electrons; the thin line and the bold line are relative to entangled singlet and triplet electrons, respectively. The spin-dependent beam splitter is characterized by $R^{\uparrow}=0.2$, $R^{\downarrow}=0.1$, and $M=50$. The mean number of uncorrelated particles depends only on the reflection probabilities, since electrons are injected from lead 1, and $\langle Q_3\rangle=M(R^{\uparrow}+R^{\downarrow})=15$. In the case of entangled particles, the mean number depends on the sum of reflection and transmission probabilities, and it is $\langle Q_3\rangle=M=50$.

$$\langle\langle Q_3Q_3\rangle\rangle=i^2\left.\frac{\partial^2\log\chi(\vec{\lambda})}{\partial\lambda_3^2}\right|_{\vec{\lambda}=0}, \tag{25}$$

we have that

$$s_{33}=\frac{4e^3V}{h}\left(\frac{1}{2}-A\right) \tag{26}$$

for entangled particles, and

$$s_{33}=\frac{2e^3V}{h}[R^{\uparrow}(1-R^{\uparrow})+R^{\downarrow}(1-R^{\downarrow})] \tag{27}$$

for independent particles. For completeness we mention that the dashed curve in Fig. 2, relative to incoming uncorrelated electrons, corresponds to the distribution

$$P(Q_3)=\sum_{k=\max[0,Q_3-M]}^{\min[Q_3,M]}\binom{M}{k}(R^{\uparrow})^k(1-R^{\uparrow})^{M-k}\binom{M}{Q_3-k}\times(R^{\downarrow})^{Q_3-k}(1-R^{\downarrow})^{M-Q_3+k}, \tag{28}$$

which is a convolution of binomial distributions relative to the two different spin species.

To conclude this section, let us now consider a slightly different situation in which we suppose we are able to count the number of electronic charges for a given spin, for example, by placing a spin-up electron counter on terminal 3 and a spin-down electron counter on terminal 4. The appropriate expression for the characteristic function reads

$$\chi_E(\lambda_3,\lambda_4)=\langle e^{-i\lambda_3\hat{N}_O^{3\uparrow}}e^{-i\lambda_4\hat{N}_O^{4\downarrow}}\rangle, \tag{29}$$

giving

$$\chi_E(\lambda_3,\lambda_4)=\left(\frac{1}{2}-A\right)(e^{-i\lambda_3}+e^{-i\lambda_4})+A[1+e^{-i(\lambda_3+\lambda_4)}] \tag{30}$$

in the case of entangled incident particles from lead 1 and 2. It is worthwhile noting that for either $\lambda_3=0$ or $\lambda_4=0$, function (30) is independent of A and, in particular, is equal for singlet and triplet states. This results in the following expression for the probability of separately counting Q_3 spin-up charges in terminal 3,

$$P^{\uparrow}(Q_3)=\binom{M}{Q_3}\frac{1}{2^M}, \tag{31}$$

and Q_4 spin-down charges in terminal 4:

$$P^{\downarrow}(Q_4)=\binom{M}{Q_4}\frac{1}{2^M}. \tag{32}$$

For completeness, we mention that the characteristic function in the ordinary case of independent incident particles reads

$$\chi_E(\lambda_3,\lambda_4)=(T^{\uparrow}+R^{\uparrow}e^{-i\lambda_3})(R^{\downarrow}+T^{\downarrow}e^{-i\lambda_4}), \tag{33}$$

which gives the following binomial probability distribution:

$$P^{\uparrow}(Q_3)=\binom{M}{Q_3}(R^{\uparrow})^{M-Q_3}(1-R^{\uparrow})^{Q_3}. \tag{34}$$

For nonentangled spin triplets we have

$$\chi_E(\lambda_3,\lambda_4)=e^{-i\lambda_3}, \tag{35}$$

which yields

$$P^{\uparrow}(Q_3)=\delta_{Q_3,M}. \tag{36}$$

B. Counting statistics on both terminals: Joint probability

Let us now consider the joint probability for transferring a number of Q_α and Q_β electronic charges into, respectively, lead α and β, given by

$$P(Q_\alpha,Q_\beta)=\int_{-\pi}^{+\pi}\frac{d\lambda_\alpha}{2\pi}\frac{d\lambda_\beta}{2\pi}\chi(\lambda_\alpha,\lambda_\beta)e^{i\lambda_\alpha Q_\alpha+i\lambda_\beta Q_\beta}. \tag{37}$$

We can distinguish between two situations: (i) spin-insensitive counters with χ_E given by Eq. (13); and (ii) spin-sensitive counters with χ_E given by Eq. (29). In case (i) it is easy to show that

$$P(Q_3,Q_4)=P(Q_3)\,\delta_{2M,Q_3+Q_4}=P(Q_4)\,\delta_{2M,Q_3+Q_4} \tag{38}$$

holds, which merely expresses the conservation of particles. $2M$ being the total number of particles injected from leads 1 and 2 over time t, and Q_3 the number of particles exiting lead 3, $Q_4=2M-Q_3$ will be the number of particles recorded by the counter in lead 4. $P(Q_3,Q_4)$, therefore, expresses the correlations due to particles conservation. This makes explicit the fact that a measure of the joint probability distribution on terminals 3 and 4 does not give more information than a measure of the probability distribution on a single terminal. Note, in particular, that this implies that the cross-terminal shot noise is equal in magnitude to the same-terminal shot noise, but with opposite sign: $s_{34}=-s_{33}$. The picture changes completely when the constraint of conservation of particles being counted is lifted, for example, by using spin-selective counters. This can be realized when a spin-up electron counter is placed on terminal 3 and a spin-down electron counter on terminal 4. Note that the number of particles counted is equal to $2M$ only in the case where there are no spin-down particles exiting lead 3 and no spin-up particles exiting lead 4. In the case of pairs of entangled incident particles, the joint probability of counting Q_3 spin-up charges in lead 3 and Q_4 spin-down charges in lead 4 is given by

$$P^{\uparrow\downarrow}(Q_3,Q_4)=\sum_{k=|Q_3-Q_4|}^{\min[Q_3+Q_4,2M-(Q_3+Q_4)]}\binom{M}{k}\left(\frac{1}{2}-A\right)^k A^{M-k}\times\binom{k}{\frac{Q_3-Q_4+k}{2}}\binom{M-k}{\frac{Q_3+Q_4-k}{2}}, \tag{39}$$

with the sum restricted to values of k such that $[Q_3\pm(Q_4-k)]$ is an even number. We see immediately that in the present case Eq. (38) does not hold and, in particular, $P^{\uparrow\downarrow}(Q_3,Q_4)$ cannot be expressed in terms of $P^{\uparrow}(Q_3)$ and $P^{\downarrow}(Q_4)$. This means, in contrast to case (i), that a measure of $P^{\uparrow\downarrow}(Q_3,Q_4)$ provides more information than $P^{\uparrow}(Q_3)$ or $P^{\downarrow}(Q_4)$ alone, and reflects the fact that particles counted in terminals 3 and 4 are correlated in a nontrivial way. Conversely, in the ordinary situation of independent incident particles coming from terminal 1 with χ_E given by Eq. (33), we have that

$$P^{\uparrow\downarrow}(Q_3,Q_4)=\binom{M}{Q_3}(R^{\uparrow})^{M-Q_3}(T^{\uparrow})^{Q_3}\binom{M}{Q_4}\times(R^{\downarrow})^{M-Q_4}(T^{\downarrow})^{Q_4}, \tag{40}$$

which can be written as

$$P^{\uparrow\downarrow}(Q_3,Q_4)=P^{\uparrow}(Q_3)P^{\downarrow}(Q_4). \tag{41}$$

Equation (40) confirms that the transfers of spin-up charges into lead 3 and spin-down charges into lead 4 are independent processes, since the joint probability is equal to the product of probabilities on individual terminals. For completeness, we note that when $A=1/2$ in Eq. (39), i.e., the injected particles are in the entangled triplet states, we have

$$P^{\uparrow\downarrow}(Q_3,Q_4)=\binom{M}{Q_3}\frac{1}{2^M}\delta_{Q_3,Q_4}, \tag{42}$$

and, when the triplets are nonentangled,

$$P^{\uparrow\downarrow}(Q_3,Q_4)=\delta_{Q_3,M}\delta_{Q_4,0}. \tag{43}$$

Remarkably the two expressions above are different even for spin-independent transport.

The net result is that the relationship between joint probability, on the one hand, and single-terminal probabilities, on the other hand, depends on the specific incident particle state. For entangled singlet electrons, in particular, such a relationship does not exist and furthermore $P^{\uparrow\downarrow}(Q_3,Q_4)$ depends on the scattering amplitudes while $P^{\uparrow}(Q_3)$ does not. The relevant consequence is that a measure of such a spin-sensitive counting statistics can provide an unambiguous means of detecting entangled singlet, triplet, or nonentangled states, since it relies on properties of the characteristic function rather than on the value of quantities like shot noise. In practice, one should separately measure $P^{\uparrow}(Q_3),P^{\downarrow}(Q_4)$, and finally $P^{\uparrow\downarrow}(Q_3,Q_4)$, and compute the ratio

$$p_{34}=\frac{P^{\uparrow\downarrow}(Q_3,Q_4)}{P^{\uparrow}(Q_3)P^{\downarrow}(Q_4)}. \tag{44}$$

If $p_{34}=1$ independently of Q_3 and Q_4, we are in the ordinary situation of independent particles injected either from lead 1 or 2. If $p_{34}=1$, but with $P^{\uparrow\downarrow}\neq 0$, only in the point $(M,0)$ of the (Q_3,Q_4) plane and zero everywhere else, then we are in the presence of nonentangled triplets. If $p_{34}\neq 1$, but different from zero only along the direction $Q_3=Q_4$, we are in the presence of triplet entangled states. Finally, if $p_{34}\neq 1$ and is finite independently of Q_3 and Q_4 we are in the presence of a singlet entangled state. For definiteness, note that we do not consider other kinds of correlations. The ratio p_{34} provides a signature of entanglement in the absence of interaction and spin-flip scattering. As an example, in Figs. 3 and 4 we plot distribution (39) and the ratio p_{34}, respectively, for a singlet entangled state injected in a spin-independent beam splitter characterized by $T=0.7$ and $M=50$. Figure 3 shows that $P^{\uparrow\downarrow}$ possesses an elongated shape along the direction $Q_4=M-Q_3$, which becomes sharper as T goes toward 1/2. Figure 4 shows that p_{34} varies very considerably in the (Q_3,Q_4) plane: this allows an easy distinction between different injected particles states. As a final remark we note that the cross-terminal shot noise in the case of independent injected particles is zero, whereas in the entangled case it is

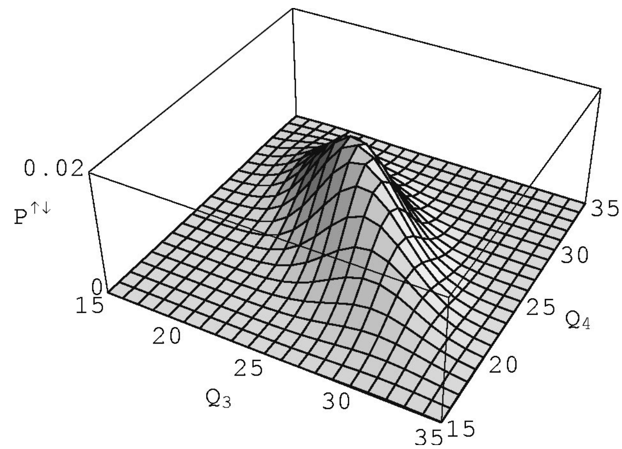

FIG. 3. Joint probability $P^{\uparrow\downarrow}(Q_3,Q_4)$ for a spin-up electron counter placed on lead 3 and a spin-down electron counter placed on lead 4. The 3D plot is relative to entangled singlet electrons injected from leads 1 and 2. The beam splitter is characterized by $T=0.7$ and $M=50$.

$$s_{34}^{\uparrow\downarrow}=\frac{2e^3V}{h}\left(A-\frac{1}{4}\right), \tag{45}$$

nonzero even for triplets. This is in contrast with case (i), where the conservation of counted particles implies that cross-terminal shot noise is always equal in magnitude (with opposite sign) to same-terminal shot noise.

IV. CONCLUSIONS

In this paper we have studied the counting statistics of a beam splitter when pairs of entangled electrons are injected from the entering terminals 1 and 2. First we considered the situation in which spin-insensitive electron counters are placed on terminals 3 and 4. We found, on the one hand, that the single-terminal probability distribution relative to singlet entangled electrons qualitatively differs from the one relative to uncorrelated electrons. In the former case, in fact, the

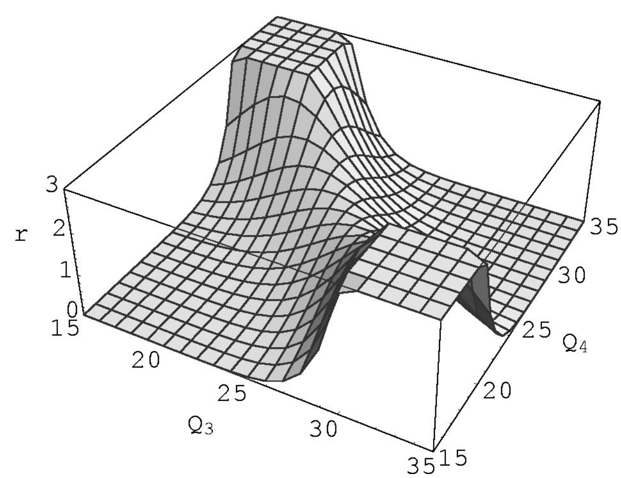

FIG. 4. 3D plot of the ratio $p_{34}(Q_3,Q_4)$ defined in Eq. (44) relative to entangled singlet particles injected into leads 1 and 2. The beam splitter is characterized by $T=0.7$ and $M=50$.

distribution is not binomial, in contrast to the latter case; furthermore it is symmetric with respect to the average number of transmitted charges. On the other hand, we found that distributions relative to the triplet states, both entangled and nonentangled, are equal, and given by unity when the charge transferred is M and zero otherwise. Triplet states can be distinguished, however, when the transport is spin polarized, for example when ferromagnetic terminals are used. If this is the case, the single-terminal counting statistics for the entangled triplet broadens to a finite width, while the nonentangled triplets remain as before. Interestingly we also noted that the joint probability for counting Q_3 electrons arriving in lead 3 and Q_4 electrons arriving in lead 4 does not contain more information than single-terminal probabilities because of the conservation of particles. Such a constraint can be lifted by using spin-sensitive electron counters, for example, by placing a spin-up counter on terminal 3 and a spin-down counter on terminal 4. In this case the joint probability unambiguously characterizes the state of the incident electrons. In particular we found that, unlike in the uncorrelated case, in the presence of entanglement the joint probability cannot be expressed as a product of single-terminal probabilities. In addition, triplet states exhibit distinguished joint probabilities depending on whether they are entangled or not. Note that the single-terminal counting statistics for the entangled states is also binomial as for the uncorrelated case, but with probability of the two outcomes being equal to 1/2, and therefore independent of scattering amplitudes and the total angular momentum of the pair. Operatively, we concluded by showing that the ratio defined in Eq. (44) can serve as a tool for discerning among the differently correlated incident electron states. As shown in Sec. III B, a plot of such a ratio as a function of the number of transferred charges provides an easy and definite way to identify entangled singlet and triplet states from factorizable, uncorrelated incident states. We believe that these results can be used to detect the presence of entanglement in electronic systems, and to provide an additional means for studying and understanding the production and manipulation of entangled electrons.

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[28] In the limit of zero temperature and small bias (the limit we consider in detail), our calculations exactly correspond for the noise to the results obtained in Ref. 6.