Logic functions from three-terminal quantum resistor networks for electron wave computing

C. H. Wu and Diwakar Ramamurthy
Department of Electrical and Computer Engineering, University of Missouri-Rolla, Missouri 65409-0040

(Received 30 July 2001; published 30 January 2002)

Electron transmission characteristics through a generalized three-terminal clean Aharonov-Bohm ring is investigated with an arbitrary terminal configuration. This three-terminal ring is shown to be the most basic quantum resistor network that is suitable for electron wave computing, as we demonstrate in this work. There are four basic classes of three-terminal rings. The scaling relation in each class is deduced. Thus the transmission characteristics in each class are valid from an atomic-scale-sized ring to a mesoscopic-scale-sized one, limited only by the electron phase-breaking length. The Buttiker symmetry rule is essential when searching for basic logic functions. Logic functions such as IF-THEN, AND, OR, XOR, and INVERT are shown here as the basic building blocks for a possible massive parallel electron wave computing machine. The node equation method, linking the wave function of one terminal node with its neighboring terminal nodes, is used. The rules governing each terminal node are summarized. This method is equivalent to the Kirchhoff current conservation law in classical circuit theory.

DOI: 10.1103/PhysRevB.65.075313 PACS number(s): 05.60.Gg, 03.67.Lx, 85.35.Ds, 85.35.Be

I. INTRODUCTION

As our technology for fabricating nanostructure conducting wires continues to advance, in the future it will be possible to produce manmade conducting structures on a near-atomic scale. This will open the possibility for us to synthesize interconnected wired networks. In such a network, the coherent wave nature of electrons will be realized over the entire network at very low temperature, since all inelastic scattering are negligible. Such an interconnected electron waveguide network is a quantum resistor network (QRN) in the sense that the value of the transmission probability from one location to another in the network is provided by a theory based on the Landauer-Buttiker formulation.[1–5] A classic example of such a QRN is a two-terminal Aharonov-Bohm (AB) ring. In such a ring, the transmission probability from an input terminal to an output terminal can be tuned by an applied electric field or magnetic flux. This was demonstrated experimentally and theoretically by many investigators.[4–7] An electron wave inside a clean AB ring was shown in recent experiments to be able to encircle the ring up to six times, and the damping of the AB oscillation amplitude is proportional to the length of the interference paths.[7] This demonstrates that there is a valid size and temperature for which interconnected AB rings can be used for networking applications. It is also clear that only a generalization from a two-terminal ring to a multiterminal ring will allow us to construct a useful network. An interconnected two-terminal ring remains a one-input–one-output network. A sharply varying transmission probability exists only in a small magnetic flux range, for example. This resembles a resonant-tunneling device,[8] and its usefulness is very limited. The minimum requirement to form a QRN is a three-terminal device. This can be achieved by adding a third terminal to an existing two-terminal AB ring. This generalization will prove to be useful for electron wave computing, which we describe in this work. A QRN can be considered as a set of nodes with interconnected quasi-one-dimensional bond paths between the nodes. The computing principle for a QRN is entirely different from the traditional electron computing. In electron wave computing, the switching principle of a transistor is replaced by the interference principle of coherent electron waves inside a QRN. In sequential electronic computing, a high switching speed for a transistor is very desirable. On the other hand, electron wave computing is based on routing and rerouting massive channels of waves in a network to their desirable output locations. Thus the computing time is determined by the propagation time just like optical wave computing. Therefore, a dc response of a QRN is sufficient for our understanding. We show here that it is possible to construct a massive parallel-processing machine just as in optical computing, another branch of wave computing. Many important logic functions needed for an electron wave-computing scheme are shown to be possible in this work. For example, with a three-terminal generalized AB ring, branching or routing an electron wave to one of the two possible paths is now possible. This allows one to construct a logic IF-THEN gate. This was first shown by one of us and collaborators.[9] In many situations, reflections from a QRN are not part of a computation scheme. In this case, any reflection from such a particular node has to be removed from further computation, so that a forward-moving electron wave will not interfere with it. This requires an insertion between the two nodes of a quantum circulator, a three-terminal AB ring, to dump the unwanted computation. For logic function applications, this involves multiple coherent inputs. In this case, it is important to realize that the Buttiker symmetry rule[10] must be taken into consideration in order to gather useful functions.

A three-terminal ring was first investigated in Ref. 9 in a special case when the three terminals are equally spaced. In this work, we generalize the discussion to three arbitrarily spaced terminals. In this general situation, we are able to classify all three-terminal AB rings into four classes, and to deduce the scaling relation in each class. Under those scaling relations, the transmission characteristics of a QRN are valid from an atomic scale to a possible nanoscale as long as inelastic scattering do not play an important role. It is impor-

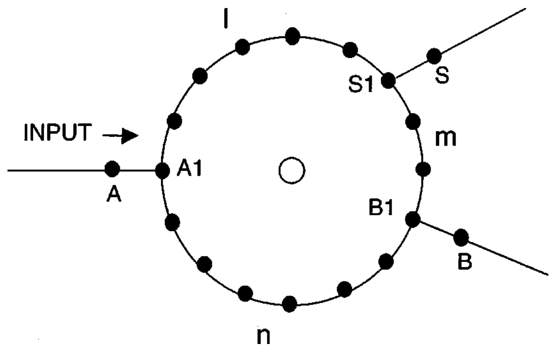


FIG. 1. Generalized three-terminal AB ring with (l, m, n) as the configuration of the three terminals.

tant to realize that generalized three-terminal AB rings are divided into several classes. Certain logic function exists only in a particular class of rings. Classes of rings with even- and odd-numbered lattice spacings between two adjacent terminals offer drastically different transmission characteristics. This is demonstrated in this work. Even in a two-terminal AB ring, it was already shown[11] that even and odd-numbered rings provide entirely different transmission probabilities. When the total number of atoms or sites in a two-terminal AB ring is an even number, the upper and lower paths can be equal, and the reflection probability is a periodic function of the applied magnetic flux with a single periodicity. However, when the total number of atoms in the ring is an odd number, the upper and lower paths must differ by at least one atomic spacing. In this case, the periodicity of the reflection probability is doubled. This universal double periodicity is in contrast with the ensemble-averaged double periodicity from a thick ring, as shown in Ref. 12.

Most discussions of the AB effect employ an *S*-matrix approach.[13] Here we use a more convenient node-equation approach developed by one of us earlier.[11] A network is considered to consist of nodes and bonds connecting the nodes. At each node, a Kirchhoff law must be satisfied, just as in classical circuit theory. This Kirchhoff law is a result of the conservation of electron current at the node. This approach greatly simplifies our analysis for obtaining electron wave functions at all node points in a multiterminal QRN. This is presented in Sec. II. The classification of all three-terminal rings and the scaling relations are shown in Sec. III. Logic functions for wave computing are summarized in Sec. IV, and a conclusion appears in Sec. V.[14,15]

II. NODE EQUATIONS FOR A THREE-TERMINAL GENERALIZED AB RING WITH ONE INPUT

A generalized three-terminal one-dimensional clean AB ring is illustrated in Fig. 1. The total number of atoms or sites (node points) in the ring is $M=l+m+n$, where l, m, and n are the dimensionless separation distances between terminals in units of the atomic spacing d. We will denote (l, m, n) as a set of numbers that characterizes the configuration of a three-terminal AB ring with a threaded magnetic flux in units of $\phi_0=hc/e$, the elementary flux. The three terminals in Fig. 1, labeled A, B, and S, are connected to the ring at the three node points, labeled $A1$, $B1$, and $S1$, respectively. If an input of amplitude a is coming from terminal A into the $A1$ node and a reflection of amplitude b is coming from node point $A1$ to terminal A, then a node equation for node point $A1$ can be written as[11]

$$[\cot kl+\cot kn-iD]\psi(A1)-e^{-in\theta}\csc kn\,\psi(B1)-e^{il\theta}\csc kl\,\psi(S1)=0. \tag{1}$$

Here $D=(1-R)/(1+R)$, and $R=b/a$ is the reflection coefficient. Note that $\psi(A1)$, $\psi(B1)$, and $\psi(S1)$ are the wave functions at node points $A1$, $B1$, and $S1$ respectively. The energy of an incoming electron wave E is related to k by $\hbar^2k^2/2\mu$.

The threaded magnetic flux will provide an additional phase shift to the wave function at $B1$, so that $\psi(B1)$ is changed to $e^{-in\theta}\psi(B1)$, where $\theta=(d/r_0)(\phi/\phi_0)=(2\pi/M)(\phi/\phi_0)$, with r_0 being the radius of the ring. The value of k for such a ring must satisfy $\cos kd=\cos(2\pi/M)[s\mp(\phi/\phi_0)]$, where $s=1,2,3,...$, and, at the Fermi energy, a proper value of s must be picked.[11] Similarly the wave function at the $S1$ node point is changed from $\psi(S1)$ to $e^{+il\theta}\psi(S1)$. Thus a node equation relates the wave function at the center node to its three nearest neighbors.

At node points $B1$ and $S1$, there are no inputs from terminal B and S. However, the input from terminal A results in outgoing waves through terminals B and S. The corresponding node equation can be written by setting $D=-1$ (or $a=0$) in Eq. (1) to obtain

$$[\cot km+\cot kn+i]\psi(B1)-e^{-im\theta}\csc km\,\psi(S1)-e^{in\theta}\psi(A1)=0 \tag{2}$$

and

$$[\cot kl+\cot km+i]\psi(S1)-e^{-il\theta}\csc kl\,\psi(A1)-e^{im\theta}\csc km\,\psi(B1)=0. \tag{3}$$

Equations (1), (2), and (3) are valid for an arbitrary three-terminal AB ring of (l, m, n) configuration. The node equation method is very similar to the Kirchhoff current law in elementary circuit theory. For each node point, there is associated a linear equation connecting the wave function of that node point to all its nearest neighbors through the current conservation law. The rules governing each node point can be summarized as follows.[11]

At a given node point, labeled a node i, connecting to s neighboring node points, labeled j, there is a linear equation relating wave function at node i, $\psi(i)$, to the wave functions of all its neighbors, $\psi(j)$. The coefficients for each wave function are (1) $\Sigma_{j=1}^{s}\cot(kl_{ij})-iD$ for node i. Here the summation is over all j neighbors, and $D=(1-R)/(1+R)$ if the node point receives an external input, otherwise, $D=-1$. The coefficient for all the neighboring nodes is $-\csc(kl_{ij})$.

For the two neighboring ring nodes affected by the threaded magnetic flux, the wave functions are further modified by multiplying a phase factor $e^{\pm il_{ij}\theta}$ by the wave func-

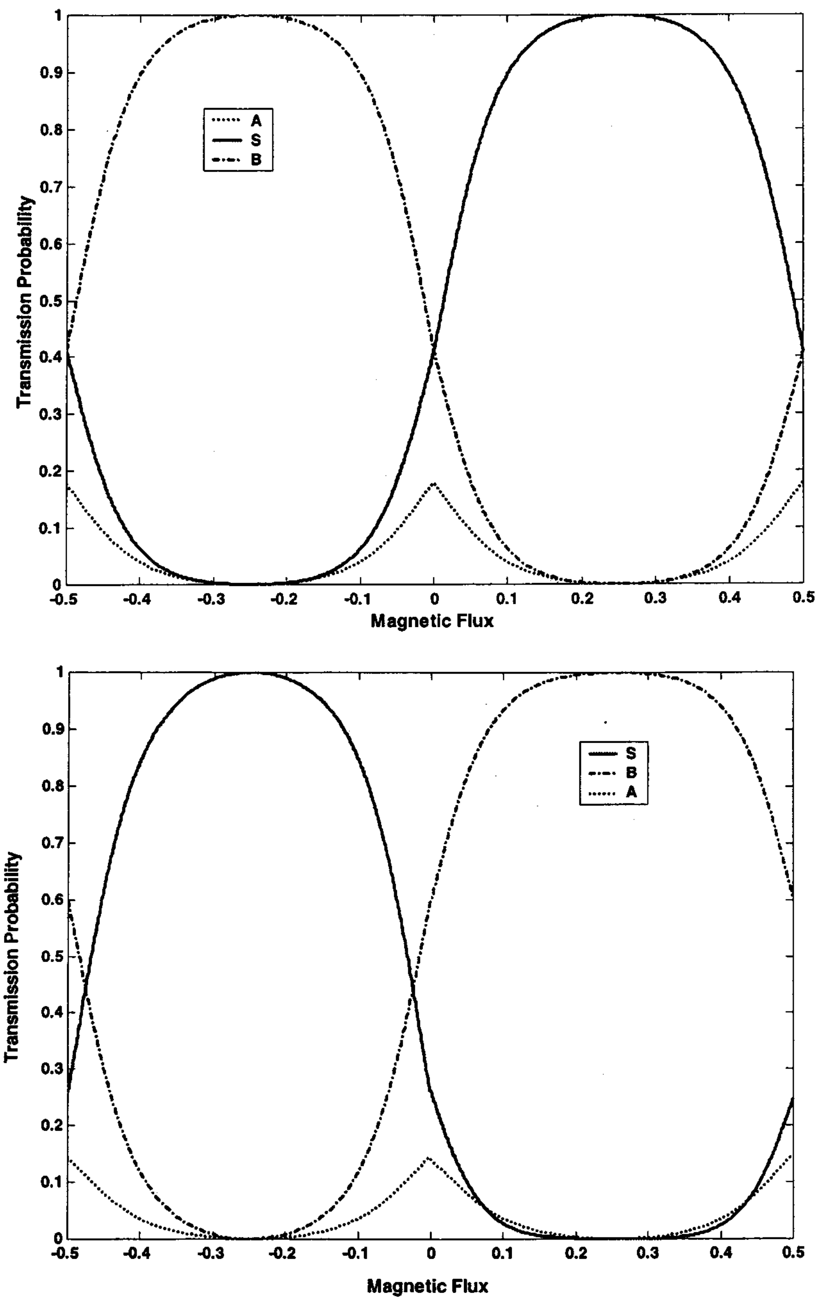


FIG. 2. Transmission and reflection probability for configurations (1,1,1) and (1,3,5) for class (1). Note that at $\phi/\phi_0 = \pm 0.25$ one output terminal has the transmission probability of unity.

tions. Here the positive sign is for the clockwise neighbor and the minus sign for the counter clockwise neighbor, and θ is as given earlier.

The above rules are for free-electron network only. If there is a potential associated with each node point, then a form factor will replace the cos(*kd*) function. This is shown in Ref. 11.

Equations (1), (2), and (3) can be solved to obtain reflection coefficient R by setting the determinant equal to zero. The D function is found to be given by

$$D = \frac{(3r - s - 2t + 2u \cos M\theta) + i(2 - 2\nu)}{(r+s) + i(2-\nu)}, \tag{4}$$

with

$$r = \cot kl + \cot km + \cot kn,$$

$$s = \cot km,$$

$$t = \cot kl \cot km \cot kn,$$

$$u = \csc kl \csc km \operatorname{cso} kn,$$

$$\nu = \cot kl \cot km + \cot km \cot kn + \cot kl \cot kn.$$

The wave functions at node points $A1$, $B1$, and $S1$ can then be solved. The transmission probability from terminal A to terminal B, denoted by $|t_{ab}|^2$, is then given by $|\psi(B1)|^2$. Similar, $|t_{as}|^2 = |\psi(S1)|^2$. It is easy to observe that the Hermittian matrix from Eqs. (1), (2), and (3) insures that the conservation of probability

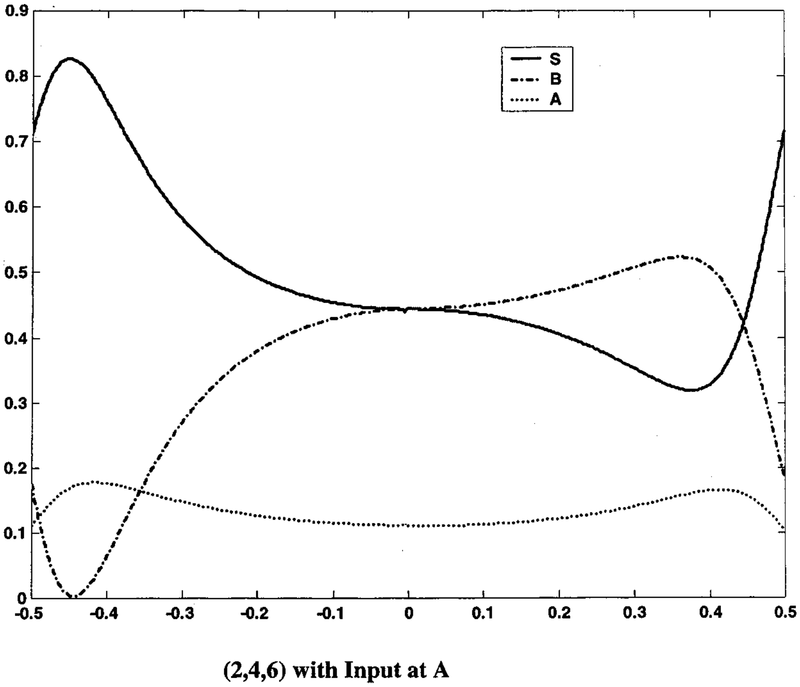


FIG. 3. Transmission and reflection probability for configuration (2,4,6) with input at terminal A for class (2).

$$|R|^2+|t_{ab}|^2+|t_{as}|^2=1 \tag{5}$$

is satisfied.

III. CLASSIFICATION AND SCALING RELATION OF THREE-TERMINAL AB RINGS

Just as there are two basic classes of two terminal AB rings, there are four basic classes of three-terminal rings. They are classified according to whether the set of (l, m, n) numbers are even or odd numbers.

The transmission probabilities are all periodic function of the threaded magnetic flux with a single periodicity ϕ_0. The flux periodicity for the reflection probability, on the other hand, depends on whether the total number of sites in a ring is an even or odd number. An even-numbered ring has a single periodicity and an odd-numbered ring has a double periodicity, just as in the situation in two-terminal rings.[11] Our classifications can be described as follows:

Class (1): When the set of numbers (l, m, n) is comprised of all odd numbers, an input from terminal A can be totally transmitted to either terminal B or terminal S at a specified flux of $\phi/\phi_0=\pm 0.25$. This is shown in Fig. 2 for the configurations of (1,1,1) and (1,3,5). Note that $|t_{as}(\phi/\phi_0)|$

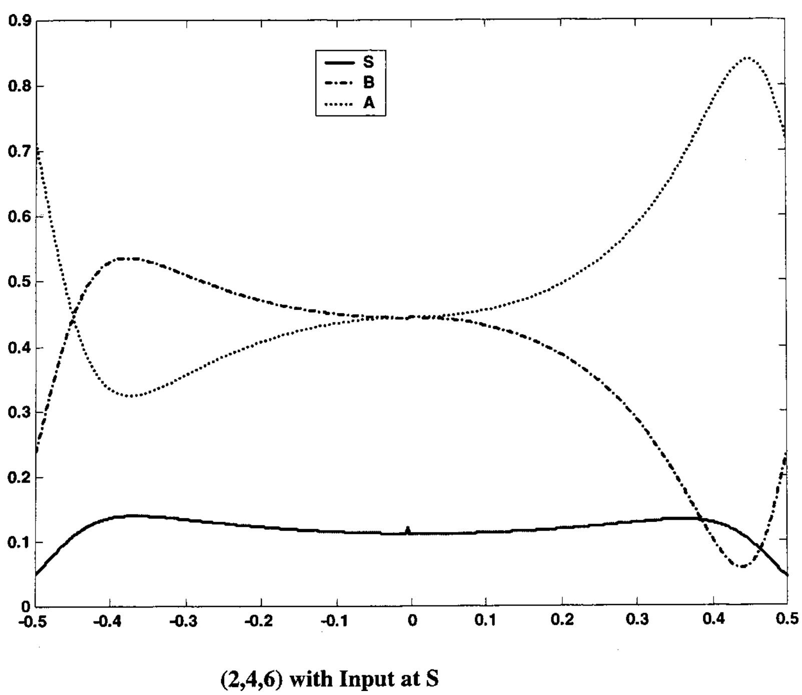
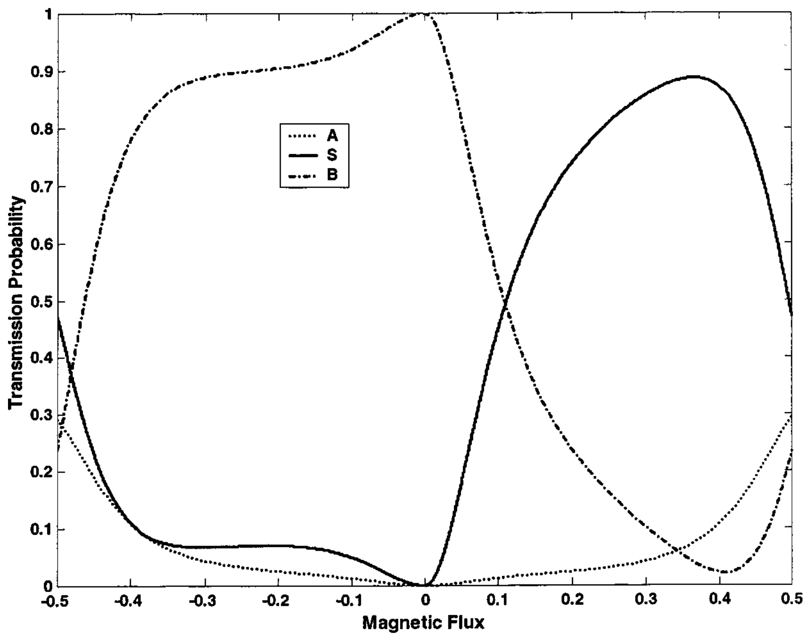


FIG. 4. Transmission and reflection probability for configuration (2,4,6) with input at terminal S for class (2).



(1,1,2) with Input at A

FIG. 5. Transmission and reflection probability for configuration (1,1,2) with input at terminal *A*. This is a class (3) ring with a distributed probability at positive flux value.

$=|t_{ab}(-\phi/\phi_0)|$ symmetry is satisfied in the case when all of (l, m, n) are equal and odd, as shown in Fig. 2 in configuration (1,1,1). This characteristic makes it possible for this device to be used as a two-way master-slave quantum circulator.[8] When this device is inserted between two quantum processors at this specified flux, the reflection is deflected to the dumped terminal and will not interfere with the incoming wave. Thus the problem of handling unwanted reflections in a quantum computing network can be solved by attaching three-terminal quantum circulators at all appropriate places. The same quantum circulator can also be used as a logic IF-THEN gate function when the flux is switched from $\phi/\phi_0=+0.25$ for TRUTH to $\phi/\phi_0=-0.25$ for FALSE. More logic functions are discussed below.

The scaling relation of class (1) rings can generally be described as follows. All the transmission characteristics from an input at terminal *A* at configuration (l, m, n), with a flux value of ϕ/ϕ_0, are exactly the same as those at configuration $(l+2,m+2,n+2)$ with a flux value of $-\phi/\phi_0$. Therefore we will denote such a scaling law as

$$(l,m,n)_{\phi/\phi_0}=(l+2,m+2,n+2)_{-\phi/\phi_0}, \tag{6}$$

so that alternate odd-numbered class (1) rings have the same transmission behavior. This means that a (1,3,5) ring and a (3,5,7) ring transmit inputs exactly the same way, except that their *B* and *S* terminals are interchanged if the input is from terminal *A*. This scaling relation is valid from an atomic-

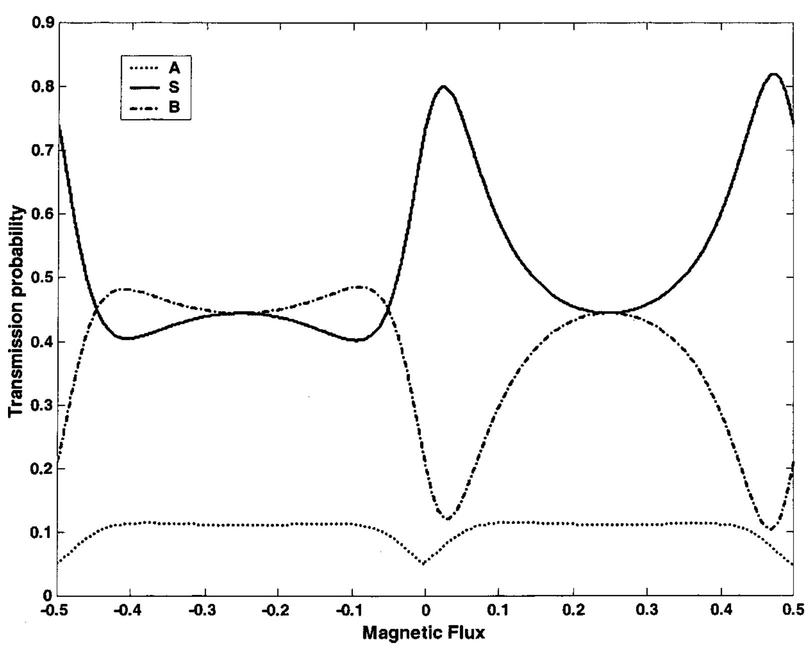


FIG. 6. Transmission and reflection probability for configuration (2,3,4) with input at terminal A. Note that a horizontal line at a transmission probability value near 0.45 will exhibit the symmetry of a probability between terminals *S* and *B* for any flux value. This is a class (4) ring.

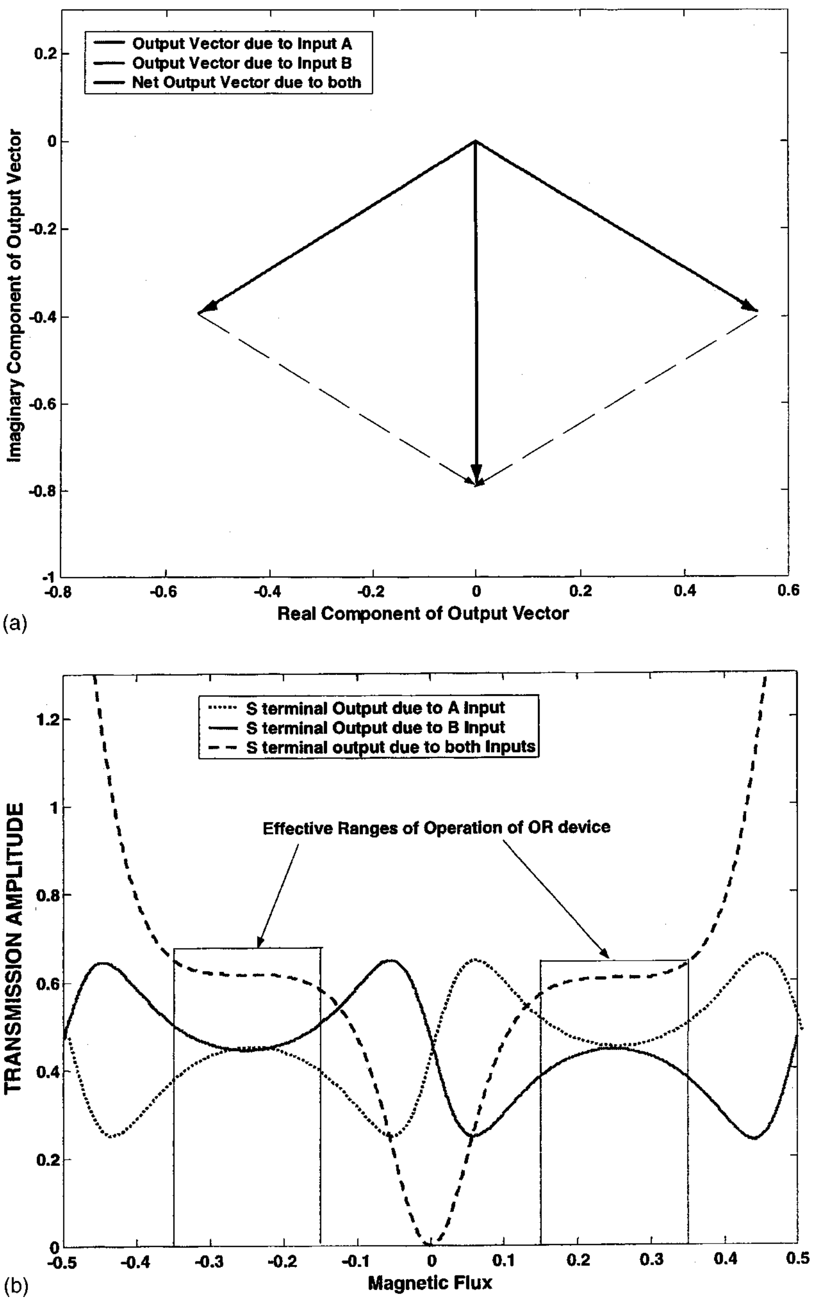


FIG. 7. (a) Vector plot for an OR gate ring (2,2,1) with a phase of B input π at $\phi/\phi_0 = -0.25$. (b) Transmission plot for an OR gate ring (2,2,1) with a phase of B input π.

scale ring to a larger mesoscopic-scale one, as long as the electron phase-breaking length is even larger.

Class (2): When the set of numbers (l, m, n) is comprised of all even numbers, the outputs are always finite all over three terminals, and an input cannot be routed only to a particular terminal by tuning the flux and the reflection probability is finite at any flux. This is shown in Fig. 3 for the (2,4,6) configuration. Note that at near zero flux range, $|t_{as}| = |t_{ab}|$. To show that the Buttiker symmetry rule is satisfied, we provide Fig. 4 with an input from terminal S. Note that $|t_{as}(\phi/\phi_0)| = |t_{sa}(-\phi/\phi_0)|$ is satisfied by comparing the two figures.

The scaling relation of class (2) rings can be summarized as

$$(l,m,n)_{\phi/\phi_0} = (l+4s-2, m+4s-2, n+4s-2)_{(\phi/\phi_0)\pm 0.5}, \tag{7}$$

where $s = 1,2,3,\ldots$. For example, the transmission characteristics for the (2,2,2) configuration at $\phi/\phi_0 = 0$ is the same as those for the (4,4,4) configuration at $\phi/\phi_0 = +0.5$. In other words, the origin of periodicity is shifted by $\phi/\phi_0 = \pm 0.5$.

Class (3): When the set of numbers, (l, m, n) is comprised of two odd numbers and one even number, the transmission probabilities will be distributed all over in a single flux period. However, it is now possible to route an input only to a particular terminal while blocking the rest. This is shown in Fig. 5 at $\phi/\phi_0 = 0$. By observing the flux range from $\phi/\phi_0 = 0$ to -0.35, the transmission probability from terminal A to terminal B is close to 0.9. The rapidly changing behavior of all the transmission curves at positive flux range in Fig. 5 indicates that it is difficult to use this class of rings for logic functions.

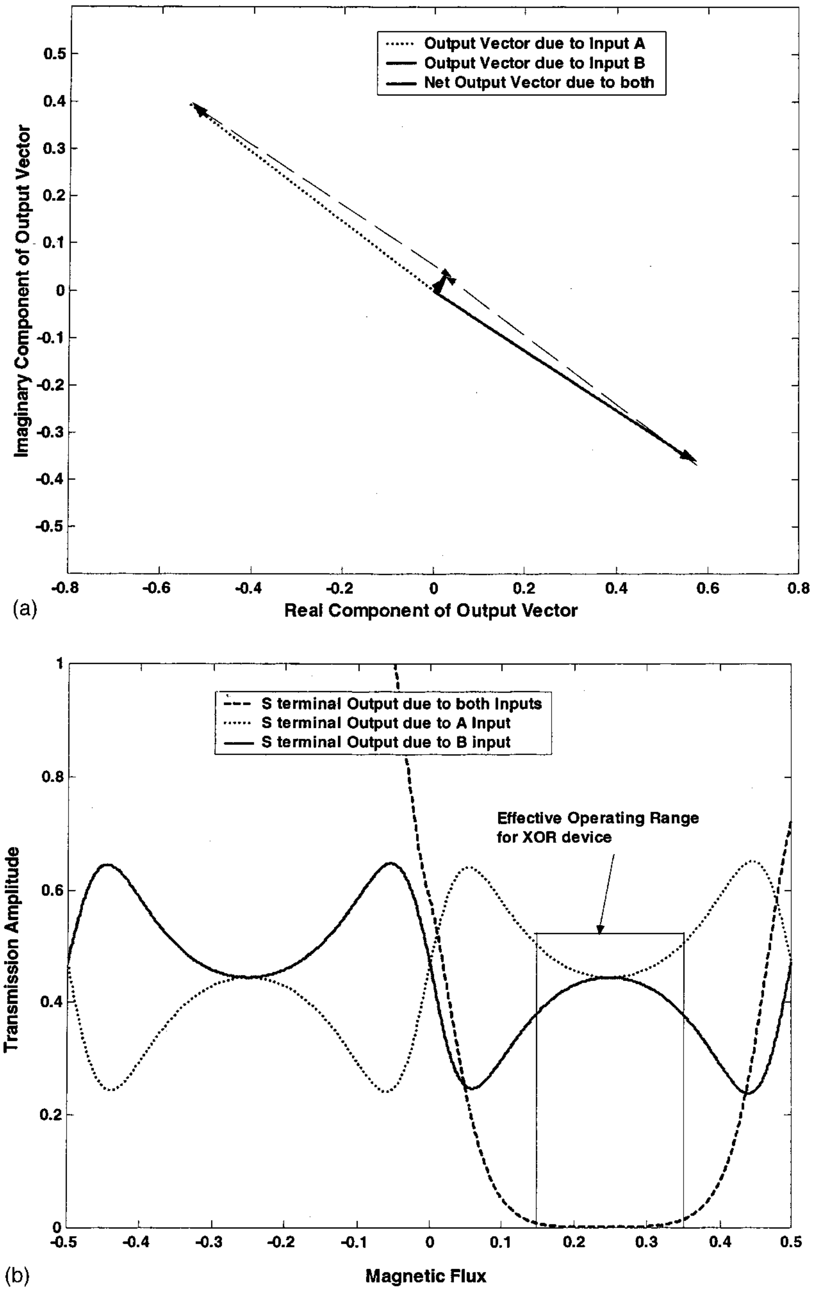


FIG. 8. (a) Vector plot for an XOR gate ring (2,2,1) with phase of $5\pi/8$ at $\phi/\phi_0=0.25$. (b) Transmission plot for an XOR gate ring (2,2,1) with a phase of $5\pi/8$.

There are two scaling relations in class (3) rings. The first scaling relation can be written as

$$(l,m,n)_{\phi/\phi_0}=(3sl,3sm,3sn)_{\phi/\phi_0}, \tag{8}$$

where $s=1,2,3\ldots$.

Magnification of the ring three times gives the same transmission characteristics.

The second scaling relation is

$$(l,m,n)_{\phi/\phi_0}=(l+4s-2,m+4s-2,n+4s-2)_{\phi/\phi_0\pm 0.5}. \tag{9}$$

For example, the (1,2,3) configuration has the same transmission characteristics as in the (3,4,5) configurations with a shift of flux periodicity origin by a half period as in Eq. (7).

Class (4): When the set of numbers (*l, m, n*) is comprised of two even numbers and one odd number, the transmission probabilities show a plateau near 0.45 at $\phi/\phi_0=\pm 0.25$. This is shown in Fig. 6, with a probability symmetry between the *S* and *B* terminals along a value near 0.45 at any magnetic flux value. Note that in a flux range from -0.05 to -0.45, the reflection probability is low and transmission probabilities to both terminals are nearly equal with a value of about 0.45, which is close to the maximum allowed value of 0.5 by the Buttiker symmetry rule. Thus this class of rings is useful for logic functions, as we will show later in Sec. IV.

The scaling relation for class (4) rings can be written as

$$(l,m,n)_{\phi/\phi_0}=(3l,3m,3n)_{\phi/\phi_0}. \tag{10}$$

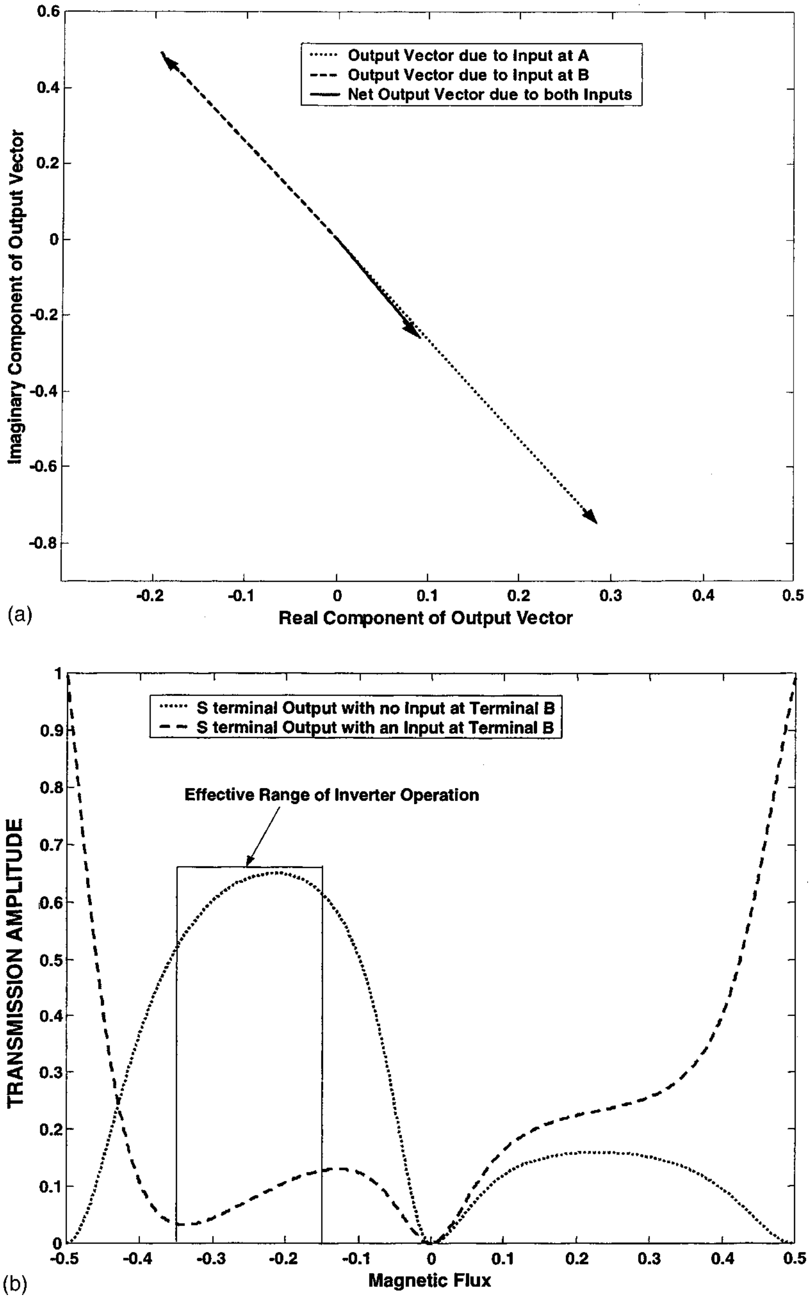


FIG. 9. (a) Vector plot for inverter ring (1,2,3) at $\phi/\phi_0 = -0.25$. (b) Transmission plot for inverter ring (1,2,3).

For example, the (2,3,4) configuration at flux, $\phi/\phi_0 = +0.25$, has the same transmission characteristics as the (6,9,12) configuration at flux, $\phi/\phi_0 = -0.25$, and has an identical behavior as in the (18,27,36) configuration at the same flux value.

Finally, there is a special class for equally spaced rings that needs to be discussed. When $l = m = n$, whether it is an even or odd number, the magnitude of the total amplitudes always equals to unity at any flux:

$$|R + t_{as} + t_{ab}| = 1. \tag{11}$$

This is in addition to the conservation law of Eq. (5).

IV. LOGIC FUNCTIONS FROM THREE-TERMINAL QRN

In Sec. IV, we described the transmission behavior with one input (from terminal *A*) only. For logic function applications, this often requires more than just one input. When two coherent inputs are present at terminals *A* and *B*, the transmission probability at terminal *S* is the square of the vector sum, $|t_{as} + t_{bs}|$. In digital applications, the evaluation of a given logic function usually depends on one of the two measurement methods. One method is to distinguish high and low transmission probabilities by setting a threshold value. The other method is to measure the transmission ratio between two output terminals, or to measure the so called "on-off ratio." We note that for a robust logic operation, the valid range of the threshold flux must be reasonably wide, and the output transmission probability must remain constant.

In order to probe for possible logic functions using interference principle of electron waves in a three-terminal QRN, it is important to utilize the Buttiker symmetry rule first. In the symmetry rule, the transmission probability at terminal i, due to an input at terminal j at a threaded flux of $+\phi/\phi_0$, is the same as the transmission probability at terminal j due to an input at terminal i at threaded flux of $-\phi/\phi_0$. Thus this symmetry rule imposes a condition on the maximum allowed transmission probability at a given terminal to 0.5 when a logic function requires two equal inputs. This also points out that refreshing is needed before later iteration is performed.

For example, in an XOR operation, when the two inputs are $A=1$ and $B=0$, the transmission that goes to terminal S must be HIGH. Similarly, when two inputs are $A=0$ and $B=1$, the transmission at terminal S is also HIGH at the same flux value of ϕ/ϕ_0. This means that when an input is placed at terminal S at a flux value of $-\phi/\phi_0$, the outputs must go equally to terminals A and B. Therefore the best situation of using terminals A and B as two equal inputs is such that terminal S will have maximum transmission probability of 0.5 if the reflection probability is zero. The useful logic functions can be summarized as follows.

(1) *Logic AND-gate function*: This can be achieved using the principle of two vector sum at terminal S when inputs are from terminals A and B. When $A=1$ and $B=0$ or when $A=0$ and $B=1$., the transmission probability at terminal S, $|t_{as}|^2$ or $|t_{bs}|^2$, is low. But when $A=1$ and $B=1$., the two vectors can line up so that $|t_{as}+t_{bs}|^2$ is four times larger than $|t_{as}|^2$ or $|t_{as}|^2$. This is illustrated using a class (1) ring at $\phi/\phi_0=0$, and is discussed in Ref. 11.

(2) *Logic OR-gate function*: The principle for an OR-gate function can be achieved using three equal-sided triangle among the three vectors $|t_{as}|$, $|t_{bs}|$, and $|t_{as}+t_{bs}|$. Therefore, in the three situations of (1) $A=1$ and $B=0$, (2) $A=0$ and $B=1$ and (3) $A=B=1$, the corresponding three transmission probabilities at terminal S are all equally high. Since the vector $|t_{as}|$ or $|t_{bs}|$ can be rotated by providing an extra phase from the input, this triangle relation can be easily satisfied. This is shown in Fig. 7(a) in a class (4) ring with $B=e^{i\pi}$ input. Note that $|t_{as}|^2=|t_{bs}|^2=0.45$ at flux $\phi/\phi_0=-0.25$, which is close to the maximum value of 0.5 imposed by the Buttiker symmery rule. This is shown in Fig. 7(b).

(3) *Logic XOR-gate function*: When two output vectors are nearly equal in magnitude but are in opposite directions, it is possible to construct an XOR gate. The rotation of one of the outputs is provided by the same phase given in the input. This is shown in Fig. 8 in the same class (4) ring with $A=1$ and $B=e^{-i5\pi/8}$ inputs. Therefore, an OR gate and an XOR gate can be both realized by adjusting the phase of one of the inputs with respect to the other.

(4) *Inverter*: In a special case, an XOR gate can also be used as an inverter if we consider one of the inputs simply as a supply line while the other is the input to be inverted. For example, if terminal A is the supply line, then $A=1$ always. In that case we have $B=0$, the output at terminal S is high. But when $B=1$, the output at S is low from the XOR gate, as shown in Fig. 9 at flux $\phi/\phi_0=-0.25$.

This points out that a QRN is neither a passive nor active network. It is simply a network that routes and reroutes two-dimensional vectors along the appropriate paths by the use of threaded magnetic fluxes. Since the QRN is a highly reflective network along the nodes of propagation, the forwarding electron waves need to be refreshed after certain steps. This is in addition to a refreshing requirement imposed by the Buttiker symmetry rule. However, this requirement can be easily achieved using the supply line concept, as we illustrated in this example.

V. CONCLUSION

It is possible that massive parallel channels of thin-wired electron waveguides can become a powerful electron wave computing machine for the future. For that purpose, we have investigated a general three-terminal clean AB ring to be used as components of a QRN. The transmission characteristics can be evaluated using the node equation approach developed. The rings are shown to be divided into four basic classes. Each class has its own distinct transmission characteristics as well as its scaling relation. For logic applications, it is necessary to consider the output as a vector sum due to two inputs as well as the restriction imposed by Buttiker symmetry rule. The concept of quantum circulator, IF-THEN, XOR, OR, AND as well as INVERTER rules, are shown to be possible. Thus higher-order functions, such as half-adder or full-adder functions can be constructed from the logic devices presented here. Those higher order QRNs can be used as a massive routing-rerouting network for very large inputs through the tuning magnetic fluxes. We note that it is desirable that magnetic flux be applied globally. In this scheme, similar classes of rings are grouped together in one location so that the same flux can be applied to all those rings. A QRN is neither a passive nor active network, because the supply line concept for refreshing can be achieved. The unwanted reflection between two processors can be taken care of with a quantum circulator. Therefore, it is possible to build a computing scheme based on the QRN concept for the next generation computer.

[1] R. Landauer, Philos. Mag. **21**, 863 (1970).
[2] R. Landauer and M. Buttiker, Phys. Rev. Lett. **54**, 2049 (1985).
[3] M. Buttiker, Y. Imry, and R. Landauer, Phys. Lett. **96A**, 365 (1983).
[4] M. Buttiker, Y. Imry, and M. Azbel, Phys. Rev. A **30**, 1982 (1984).
[5] M. Buttiker, Y. Imry, R. Landauer, and S. Pinhas, Phys. Rev. B **31**, 6207 (1985).
[6] R. Webb, S. Washburn, C. Umbach, and R. Laibowitz, Phys. Rev. Lett. **54**, 2696 (1985).
[7] A. E. Hensen, A. Kristensen, S. Pederson, C. B. Sorenson, and P. E. Lindelof, Phys. Rev. B **64**, 045327 (2001).
[8] F. Cappaso, S. Sen, F. Beltram, L. M. Lunardi, A. S. Vengurlekar,

P. R. Smith, N. J. Shah, R. J. Malik, and A. Y. Cho, IEEE Trans. Electron Devices **36**, 2065 (1989).
[9] C. H. Wu, W. J. Javurek, and B. Bookout, *New Devices from Three-Terminal Quantum Resistor Networks*, Proceedings of the 1991 International Semiconductor Device Research Symposium, Charlottesville, Virginia, December 1991 (Engineering Academic Outreach, University of Virginia, Charlottesville, 1991), p. 305.
[10] M. Buttiker, Phys. Rev. Lett. **57**, 1761 (1986).
[11] C. H. Wu and G. Mahler, Phys. Rev. B **43**, 5012 (1991).
[12] B. Al'tshuler, A. Aronov, and B. Spivak, Pis'ma Zh. Eksp. Teor. Fiz. **33**, 101 (1981) [JETP Lett. **33**, 94 (1981)].
[13] P. W. Anderson, D. J. Thouless, E. Abrahams, and D. S. Fischer, Phys. Rev. B **22**, 3519 (1980).
[14] C. H. Wu and Diwakar Ramamurthy (unpublished).
[15] R. Landauer, Phys. Today, **42**, 119 (1989).