

Magnetopolarizability of mesoscopic systems

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In order to understand how screening is modified by electronic interferences in a mesoscopic isolated system, we have computed both analytically and numerically the average thermodynamic and time-dependent polarizabilities of two-dimensional mesoscopic samples in the presence of an Aharonov-Bohm flux. Two geometries have been considered: rings and squares. Mesoscopic correction to screening are taken into account in a self consistent way, using the response function formalism. The role of the statistical ensemble (canonical and grand canonical), disorder and frequency have been investigated. We have also computed first order corrections to the polarizability due to electron-electron interactions. Our main results concern the diffusive regime. In the canonical ensemble, there is no flux dependence in the polarizability when the frequency is much smaller than the level spacing. On the other hand, in the grand canonical ensemble for frequencies larger than the mean broadening of the energy levels (but still small compared to the level spacing), the polarizability oscillates with flux, with the periodicity $h/2e$. The order of magnitude of the effect is given by $\delta\alpha/\alpha \propto (\lambda_s/Wg)$, where λ_s is the Thomas-Fermi screening length, W the width of the rings or the size of the squares, and g their average dimensionless conductance. This magnetopolarizability of Aharonov-Bohm rings has been recently measured experimentally [R. Deblock, Y. Noat, H. Bouchiat, B. Reulet and D. Mailly, Phys. Rev. Lett. **84**, 3579 (2000)] and is in good agreement with our grand canonical result.

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I. INTRODUCTION

A. Motivation of the paper

Transport and thermodynamic properties of mesoscopic metallic samples have been known for a long time to be quite sensitive to the quantum phase coherence of the electronic wave functions at low temperature.¹ In particular, it was shown that a mesoscopic ring threaded by a magnetic field exhibits a persistent current periodic with the flux and with a period of one flux quantum $\phi_0 = h/e$. This current was measured in several experiments,²⁻⁴ corresponding to different situations and materials. More recently, the response of a mesoscopic system to a time-dependent magnetic field was studied both theoretically and experimentally.^{5,6}

The purpose of the present paper is to investigate how screening is influenced by electronic interferences in a phase coherent sample. In this spirit, we discuss how the electrical polarizability α , which is the response of a metallic sample to a small electrostatic field and is directly related to screening, is also sensitive to mesoscopic effects. α can be experimentally measured by coupling the mesoscopic samples to a capacitor. For small voltages, corresponding to the linear regime, the change in the capacitance is directly proportional to the average polarizability $\langle\alpha\rangle$ of the particles.

When the typical size L of the particle is much larger than the Thomas-Fermi screening length λ_s , α is mostly determined by the geometrical shape of the particle. For a spherical particle, it is proportional to the volume V of the particle, with a small correction of the order of λ_s/L .

Now, what happens in the case of a quantum coherent sample, i.e., when L is the order of the phase coherence length? Is there a mesoscopic correction to the polarizability, due to contribution from electronic interferences?

In order to answer this question, we have calculated the flux dependent correction to the average polarizability, which we call “mesoscopic correction,” for two-dimensional mesoscopic samples in the presence of an Aharonov-Bohm flux. Two different geometries have been considered, namely, a two-dimensional ring or square in an in-plane electric field. The role of statistical ensemble, disorder, and finite frequency have been investigated. In the case of a time-dependent electric field, we distinguish two different limits, whether the frequency is smaller or larger than the inverse relaxation time, or mean level broadening γ . The case $\omega \ll \gamma$ corresponds to the thermodynamic limit whereas $\omega \gg \gamma$ corresponds to the finite frequency limit. A preliminary account of this work is already published.⁷

B. Scope of the paper

This paper is organized as follows. The definition of the electrical polarizability and the classical result for a macroscopic sample are given in Sec. II.

In Sec. III, we derive the expression of the electrical polarizability of a system of non-interacting electrons, in the spirit of the early work of Gorkov and Eliashberg (GE).⁸ A giant unphysical polarizability is obtained. However, it is possible to discuss in this very simple model the flux dependence of two quantities which contribute to the polarizability, namely, the matrix element of the position (or screened potential) operator and the contribution of energy levels. Then, we examine in Sec. IV how screening can be taken into account. In particular, we show how to incorporate contributions of electronic interferences. Using those results, a general expression for the polarizability and its mesoscopic correction for a time varying electric field are derived in Sec. V.

We then study separately the case of the grand canonical and canonical statistical ensemble. The grand canonical average polarizability is found to depend only on the flux-dependent matrix elements of the screened potential whereas the canonical average depends also on the energy levels. As a result when the frequency is smaller than the level spacing the magnetopolarizability is found much smaller in the canonical ensemble compared to the grand canonical case. Finally, we present results on the Hartree-Fock correction to the zero frequency polarizability. Section VI is devoted to a comparison of these results with recent experiments.

II. BASIC CONCEPTS

A. Screening and polarizability

When an external field $\mathbf{E}_{\text{ext}} = -\text{grad}(\phi_{\text{ext}})$ is applied on a conductor, the charge density adjusts in such a way that the effective field inside the conductor cancels. This phenomenon is called screening. In principle the computation of the potential inside the sample requires to solve a complex N -body Hamiltonian taking into account the long-range Coulomb interaction between electrons. However, it is possible in general to treat the interactions in a mean field approximation. The screened potential inside the conductor is then given by

$$\phi(\mathbf{r}_1) = \phi_{\text{ext}}(\mathbf{r}_1) + \int \delta\rho(\mathbf{r}_2)U(\mathbf{r}_1, \mathbf{r}_2)dr_2, \quad (1)$$

where $\delta\rho(\mathbf{r}_2)$ is the charge density induced at the point \mathbf{r}_2 and $U(\mathbf{r}_1, \mathbf{r}_2)$ is the Coulomb interaction potential. In the linear response limit, $\delta\rho(\mathbf{r}_2)$ can be described by the electric response function $\chi(\mathbf{r}_1, \mathbf{r}_2)$, which relates the variation of charge in the system at \mathbf{r}_2 to a local perturbation at \mathbf{r}_1 , $\delta(\mathbf{r}_1)$:

$$\delta\rho(\mathbf{r}_2) = \int \chi(\mathbf{r}_1, \mathbf{r}_2)\phi(\mathbf{r}_1)dr_1. \quad (2)$$

Relation (1) can then be written more conveniently in a matrix form

$$\phi = (1 - U\chi)^{-1}\phi_{\text{ext}}. \quad (3)$$

In a clean infinite system, the induced charge density is simply related to the screened potential ϕ in Fourier space $\delta\rho(q) = \chi(q)\phi(q)$, where $\phi(q) = \int e^{i\mathbf{q}\cdot\mathbf{r}}\phi(\mathbf{r})d\mathbf{r}$. The problem is more complex in a disordered system, which does not have the translational invariance. In any case for a finite system, it is possible to compute the response function from the eigenstates $|\alpha\rangle$ of the system in the absence of the external field, associated to the eigenvalues ϵ_α :

$$\chi(\mathbf{r}_1, \mathbf{r}_2) = \sum_{\alpha=1}^N \sum_{\beta < \alpha} \frac{\psi_\alpha^*(\mathbf{r}_1)\psi_\beta(\mathbf{r}_2)\psi_\alpha(\mathbf{r}_1)\psi_\beta^*(\mathbf{r}_2)}{\epsilon_\beta - \epsilon_\alpha} + \text{c.c.} \quad (4)$$

In most cases we will identify these eigenstates as the solution of the hamiltonian of the system without electron interactions which will be taken into account only by considering the screening on the applied field. The effect of interactions

on the eigenstates will be however discussed in Sec. VII within the Hartree-Fock approximation.

The application of an electric field on a finite metallic system, results in an induced dipolar moment \mathbf{P} which, is proportional to the applied field when it is sufficiently small. The range of validity of linear response is discussed in Appendix B:

$$\mathbf{P} = \int \rho(\mathbf{r})\mathbf{r}d\mathbf{r} = \alpha\mathbf{E}, \quad (5)$$

where α is by definition the polarizability of the conductor, which can be expressed as a function of the response function

$$\alpha = \frac{1}{E^2} \int \chi(\mathbf{r}_1, \mathbf{r}_2)\mathbf{E}\cdot\mathbf{r}_2\phi(\mathbf{r}_1)dr_1dr_2. \quad (6)$$

B. Classical polarizability

1. Plate with perpendicular electric field

The polarizability of a macroscopic piece of metal is described by classical electrostatics. In this approximation, the dipolar moment can be calculated by solving Poisson equation $\Delta V + \rho/\epsilon_0 = 0$, in all space and imposing as well that the conductor is at a constant potential. The problem is solvable analytically for a sphere of volume V : $\alpha = \epsilon_0(4\pi/3)V$. It is also possible to estimate the polarizability of a metallic plate of thickness a in a constant electric field perpendicular to the plate. The induced charge density is such that the electric field cancels inside the plate. Neglecting border effects, we have

$$\rho(\mathbf{r}) = \frac{E\epsilon_0}{2} \left[\delta\left(x - \frac{a}{2}\right) - \delta\left(x + \frac{a}{2}\right) \right]. \quad (7)$$

Integrating Eq. (5), one finds the classical polarizability

$$\alpha = \epsilon_0 a^3 = \epsilon_0 V. \quad (8)$$

2. Two-dimensional (2D) and 1D systems in longitudinal electric fields

We now examine the case of 2D conductors, and more specifically a strip of length L and width W , with $L \gg W$, in an in-plane electric field perpendicular to its long axis. In order to determine the induced charge density, the effective electric field inside the strip is imposed to be zero. In the limit of an infinite strip, this condition leads to the following equation:

$$\int \frac{1}{2\pi\epsilon_0} \frac{\rho(x')}{x-x'} dx' - E = 0. \quad (9)$$

We have calculated numerically the solution of Eq. (9) in the case of an infinite strip (Fig. 1). In contrast with the three-dimensional case, the charge distribution spreads over the full width of the strip in order to cancel the external field. The numerical solution is very close to a logarithm. Assuming that the density is of the form

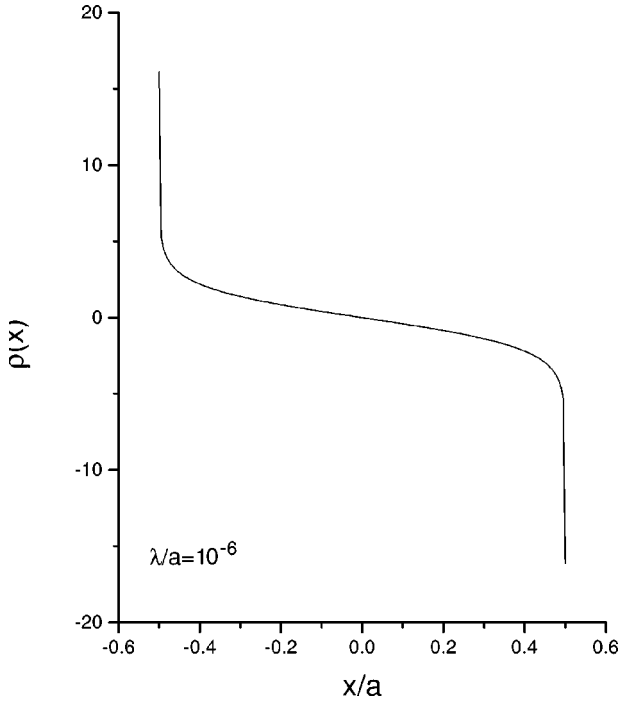


FIG. 1. Induced charge density of a conducting strip in an in-plane uniform electric field: the function diverges logarithmically as a function of the cutoff λ/a .

$\rho(x) \approx EW \ln(W/2-x)/\ln(W/2+x)$, the integral (5) of the previous function gives $\alpha \approx \epsilon_0 W^2 L$.

This result extrapolates to a two-dimensional sample of typical size W , for which the in-plane polarizability scales as $\alpha \sim \epsilon_0 W^3$. For instance the polarizability of a disk of radius R is⁹ $\alpha = \epsilon_0 16R^3/3$.

In the case of a one-dimensional wire, whose length L is much longer than its diameter D , in an electric field along its axis, the polarizability has been shown⁹ to be $\epsilon_0 L^3 \ln(D/L)$. Similarly as in the 2D case, the charge density has to spread over the entire wire in order to screen the external potential.

The case of a 1D ring is also exactly solvable. In order to take screening into account, we first assume that the induced charge density in the ring is of the form $\rho(\theta) = \lambda \cos(\theta) \delta(z) \delta(\rho - R)$, from which we deduce the induced potential in the ring

$$\phi_{\text{ind}}(\theta) = \int_0^{2\pi} \frac{1}{8\pi\epsilon_0} \frac{\rho(\theta')}{\left| \sin\left(\frac{\theta - \theta'}{2}\right) \right|} d\theta'. \quad (10)$$

The integral is logarithmically diverging and it is necessary to introduce a cutoff $\theta_c = W/R$, related to the finite width W of the ring $\phi_{\text{ind}}(\theta) \approx -\lambda \cos(\theta) \ln(\theta_c)/4\pi\epsilon_0$.

The screened potential reads $\phi(\theta) = \cos(\theta)(E + \lambda J)$, with $J = \ln(1/\theta_c)/4\pi\epsilon_0$. The polarizability is obtained either from the potential calculated above, either from the expression of the charge. This self-consistent relation sets the value of the parameter λ , from which we deduce the polarizability of the ring:

$$\alpha = \frac{\alpha_0}{1 + \frac{2J\alpha_0}{R^3}}, \quad (11)$$

where $\alpha_0 = \epsilon_0 \pi^2 R^3$ which reads in the limit $J \gg 1$:

$$\alpha \approx \frac{\epsilon_0 \pi^2 R^3}{\ln\left(\frac{R}{W}\right)}. \quad (12)$$

As a result both longitudinal polarizabilities of a ring and a disk scale like the cube of their radius, just as in the polarizability of a sphere.

C. A mesoscopic correction to the polarizability?

In the following, we define the quantity we wish to calculate. As we have just seen, the classical polarizability of a metallic sample is determined by geometrical factors. However, due to the Pauli principle, charges cannot strictly accumulate on the border of the sample, but the charge density distribution rather extends on the screening length λ_s . This effect reduces the polarizability compared to its classical value by a quantity $\delta\alpha_{TF}$ of the order of λ_s/L , where L is the typical size of the conductor along the electric field. In a coherent sample, electronic interferences might also give rise to a contribution $\delta\alpha_Q$ to the polarizability. The total polarizability can thus be written $\alpha = \alpha_{cl} + \delta\alpha_{TF} + \delta\alpha_Q$. In order to show evidence of this correction, a magnetic flux Φ can be applied through the sample in order to modify the phase difference of electronic trajectories. We will call ‘‘mesoscopic correction’’ the flux-dependent part of the polarizability $\delta\alpha(\Phi)$. We will focus on the ensemble average of this quantity $\overline{\delta\alpha}(\Phi)$, which is $\Phi_0/2$ periodic. We will characterize the flux dependence of $\overline{\delta\alpha}$ by $\delta_\Phi(\alpha) = \overline{\delta\alpha}(\Phi_0/4) - \overline{\delta\alpha}(0)$.

D. Model used for the numerical simulations

Part of the analysis we have done rely on computer simulations. The two-dimensional rings or squares are modeled by the Anderson model. Each atomic site is coupled to its nearest neighbor by a hopping term t . Disorder is introduced by on-site energies randomly distributed in the interval $[-w, w]$. The Anderson Hamiltonian can be written in second quantization

$$H = \sum_k \epsilon_k c_k^\dagger c_k + t e^{i\phi_k} c_k^\dagger c_{k+1} + t e^{-i\phi_k} c_{k+1}^\dagger c_k, \quad (13)$$

where c_k^\dagger is the creation operator associated with site k . The magnetic flux is introduced in this Hamiltonian through the phase factor of the hopping element with $\phi_k = 2\pi\Phi/\Phi_0(x_k - x_{k+1})/L$ for a ring of size L . In the case of squares we have taken into account the penetration of the magnetic field in the sample: the phase factor appearing in the hopping matrix elements is computed from the integral of the potential vector between concerned neighbors. In two dimensions, an estimation of the elastic mean free path is given by⁵

$$l_e \approx 30 \left(\frac{t}{w} \right)^2 \quad (14)$$

allowing us to choose values of t/w so that the system is in the diffusive regime. Averages are always performed over the number of electrons in the system between one quarter and three quarter filling. It has been checked that expression (14) remains valid in this energy range. On the other hand, we have not done disorder averaging which needs much longer computation time. For this reason we have restricted our analysis to a moderate range of disorder, corresponding to the diffusive ergodic regime where disorder and energy averages give identical results.

III. QUANTUM POLARIZABILITY OF A SYSTEM OF NONINTERACTING ELECTRONS

We now examine the electrical response of a quantum system of noninteracting electrons, which was first calculated by GE (Ref. 8) for a metallic grain both in the diffusive and ballistic limit. Their conclusion is quite surprising: in the diffusive regime, α is bigger than the classical polarizability by a factor $(a/a_0)^2$, where a is the typical size of the grain and $a_0 = \hbar^2/(me^2)$ is the Bohr radius. Later on, Rice¹⁰ stressed the lack of screening in the calculation of GE and showed that when screening is taken into account, one recovers the classical polarizability. We will see in the following that it is useful to consider the flux dependence of the polarizability within this crude approximation, since many results remain qualitatively true in the presence of screening.

A. Position of the problem

The polarizability of a system of a number N of noninteracting electrons can be understood as the sensitivity of the energy spectrum to an external electric field, very similar to persistent currents measure the sensitivity of the spectrum to an Aharonov-Bohm flux. As a matter of fact, the eigenstates $|\alpha\rangle$ of the system and the eigenvalues ϵ_α , are modified by an external electric field E :

$$\begin{aligned} \epsilon'_\alpha &= \epsilon_\alpha - \langle \alpha | e \mathbf{E} \cdot \mathbf{r} | \alpha \rangle + \sum_{\beta \neq \alpha} \frac{|\langle \alpha | e \mathbf{E} \cdot \mathbf{r} | \beta \rangle|^2}{\epsilon_\beta - \epsilon_\alpha} + \dots, \\ |\alpha'\rangle &\approx |\alpha\rangle + \sum_{\beta \neq \alpha} \frac{\langle \beta | e \mathbf{E} \cdot \mathbf{r} | \alpha \rangle}{\epsilon_\beta - \epsilon_\alpha} |\beta\rangle + \dots \end{aligned} \quad (15)$$

Consequently, it exists an induced charge density $\delta\rho(\mathbf{r}) = e \sum_{\alpha=1}^N |\psi'_\alpha(\mathbf{r})|^2 - |\psi_\alpha(\mathbf{r})|^2$, associated with the asymmetry of each wave function generated by E . Using the definition (5) of α , the expression of the polarizability in the absence of screening at zero temperature can be found:

$$\alpha = \frac{2e^2}{E^2} \sum_{\alpha=1}^N \sum_{\beta \neq \alpha} \frac{|\langle \alpha | \mathbf{E} \cdot \mathbf{r} | \beta \rangle|^2}{\epsilon_\beta - \epsilon_\alpha}. \quad (16)$$

This expression depends on the eigenenergies of the unperturbed system and its eigen-functions through the matrix elements of the position operator. Due to the energy denomi-

nator, the polarizability will in particular be very sensitive to the electric field induced coupling between the last occupied levels and the first non occupied ones. Note also that this expression of the polarizability is also from Eq. (15) identical to the second derivative of the total energy of the system with respect to electric field:

$$\alpha = - \sum_{\alpha=1}^N \frac{\partial^2 \epsilon_\alpha}{\partial E^2}.$$

These results present strong similarities with the paramagnetic contribution of orbital susceptibility of a mesoscopic ring pierced by a flux line which is identical to expression (16) where the $\mathbf{E} \cdot \mathbf{r}$ is replaced by $e \mathbf{A} \cdot \mathbf{p}$, where \mathbf{A} is the potential vector and \mathbf{P} is the kinetic momentum operator. The total magnetic susceptibility contains also a constant diamagnetic term $-Ne^2/m$ which does not exist in the electric response.^{5,11}

B. One-dimensional Aharonov-Bohm ring

Using expression (16) it is possible to calculate exactly the induced dipolar moment in a 1D nondisordered ring in an in plane electric field. Wave functions only depend on the angle θ , indicating the position in the ring, and satisfy the 1D Shrödinger equation $-(\hbar^2/2mL^2)(\partial^2/\partial\theta^2)\psi(\theta) = E\psi(\theta)$. Furthermore, the magnetic flux, associated with the periodicity in the ring, imposes the boundary condition $\psi(\theta + 2\pi) = e^{i2\pi(\phi/\phi_0)}\psi(\theta)$. This equation can be solved. Using the parity and periodicity of the spectrum, we then order the eigenvalues and the corresponding wavefunctions in ascending order in the interval $[-\phi_0/2, \phi_0/2]$:

$$\begin{aligned} \epsilon_{2p} &= \frac{\hbar^2}{2mL^2} \left(p + \frac{\phi}{\phi_0} \right)^2 \psi_{2p}(\theta) = \frac{1}{\sqrt{L}} e^{i\theta(p + \frac{\phi}{\phi_0})}, \\ \epsilon_{2p+1} &= \frac{\hbar^2}{2mL^2} \left(p - \frac{\phi}{\phi_0} \right)^2 \psi_{2p+1}(\theta) = \frac{1}{\sqrt{L}} e^{i\theta(-p + \frac{\phi}{\phi_0})}. \end{aligned} \quad (17)$$

As a consequence of the particular geometry of the system, the electric field only couples certain states. In particular, the matrix element between adjacent states cancels unless the quantum numbers p et q are such that $p - q = \pm 2$, in this case $\langle p | X | q \rangle = R/4\pi$. The polarizability depends on the parity of the number of electrons N and reads

$$\begin{aligned} \alpha_N(\Phi) &= \frac{e^2 R^2}{8\pi^2} \left(\frac{1}{\epsilon_{N+2}(\Phi) - \epsilon_N(\Phi)} \right. \\ &\quad \left. - \frac{1}{\epsilon_{N+1}(\Phi) - \epsilon_{N-1}(\Phi)} \right). \end{aligned} \quad (18)$$

At zero flux and for $N \gg 1$: $\alpha_N = L^4/8\pi^4 a_0 N$. It is possible to evaluate $\delta_\Phi \alpha$ by computing $(\alpha_{N+1} - \alpha_N)/2$. As a result,

$$\frac{\delta_\Phi \alpha}{\alpha} \approx - \frac{1}{N^2}. \quad (19)$$

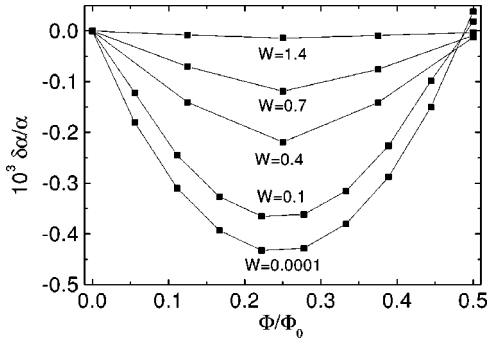


FIG. 2. Magnetopolarizability of a 1D ring, of length $L=100$, calculated with the Anderson model, for several values of the disorder: $w = 10^{-4}$, 0.1, 0.4, 0.7, 1.4.

It is a very small effect, since it decreases rapidly with the number of electrons in the ring. This result remains true for a multichannel ring. We attribute this effect to selection rules inherent to the square lattice which are responsible for the cancellation of the matrix elements of operator X between eigenstates close to a level crossing. In particular it has been shown¹² that these selection rules do not exist in the hexagonal lattice where giant magnetopolarizability is expected for particular values of flux at the same level of approximation.

In order to investigate the effect of disorder we have performed numerical simulations using the Anderson model. On Fig. 2 is plotted the magnetopolarizability for a ring of length $L=100$ and several values of the disorder w . In absence of disorder, the results are in qualitative agreement with formula (19). The effect then decreases when the disorder is increased, with a law close to $1/w^2$. One is tempted to attribute this result to occurrence of localization in the 1D ring. However we will see in the following that this cancellation of the flux-dependent polarizability is also observed in a multichannel ring or disk in the diffusive regime.

C. Diffusive system

In the following we discuss a diffusive system in dimension d , characterized by a diffusion coefficient $D = (1/d)v_F l_e$ where l_e is the elastic mean free path which is assumed to be shorter than the system size a along the electric field. It has been shown^{8,13,14} using semiclassical arguments that the average square matrix element $|\langle \alpha | X | \beta \rangle|^2$ depends mainly on the energy difference $\epsilon = |\epsilon_\beta - \epsilon_\alpha|$ and the Thouless energy $E_c = hD/L^2$. For $\epsilon < E_c$ it is of the order of a/g where $g = E_c/\Delta$ is the ratio between the Thouless energy and the mean level spacing Δ . At higher energy it decreases as $1/\epsilon^2$. This behavior is illustrated in Fig. 3 showing numerical results on a sample for different values of disorder where these two regimes can be clearly distinguished.

It is then easy to deduce the order of magnitude of the polarizability noting that the summation in expression (16) can be restricted to $|\alpha - \beta| < g$ as a result:

$$\alpha \sim e^2 a^2 / \Delta. \quad (20)$$

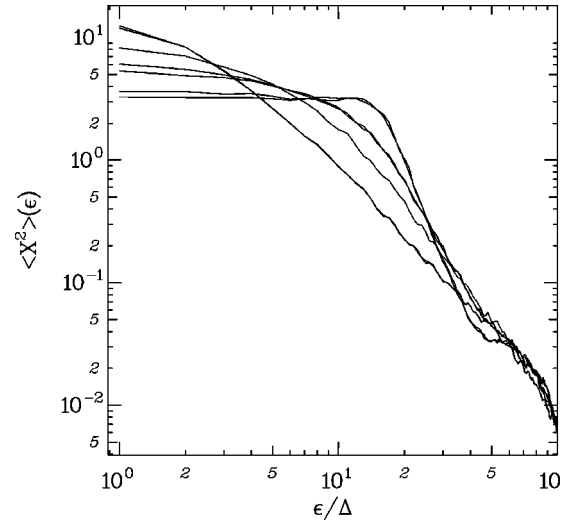


FIG. 3. Energy dependence of the nondiagonal matrix elements in a ring of length $L=60$ with $M=4$ channels, and several values of the disorder: $w = 0.9, 1.2, 1.4, 1.7$. Note the additional step observed for $\epsilon \approx 5\Delta$, which corresponds to the Thouless energy in the transverse direction of the ring hD/W^2 , where W is the width of the ring.

The GE result is then recovered assuming that the Fermi wave length is of the order of the Bohr radius. We will see in the next section that this result is modified by screening. It is, however, worth continuing this analysis in this oversimplified picture of noninteracting electrons in the case of an Aharonov-Bohm ring. From expression (16) the polarizability is expected to exhibit flux dependence both from the matrix elements and energy denominators. We discuss in the following separately these two contributions which is justified in the context of random matrix theory where eigenfunctions and eigenenergies constitute two sets of independent random variables.¹⁵

1. Energy denominator

We focus on the flux dependence of the quantity $K = \langle \sum_{\alpha < \beta} [1/(\epsilon_\beta - \epsilon_\alpha)] \rangle$. This quantity depends only on the energy differences $(\epsilon_\alpha - \epsilon_\beta)$, and can be expressed as a function of $R(\epsilon)$, the two levels correlation function :

$$K = \int_0^{E_{\max}} d\epsilon \int_0^\epsilon du \frac{R(u)}{u}. \quad (21)$$

In the diffusive regime, $R(s)$ is well described by random matrix theory¹⁵ and the average probability to find two degenerate adjacent levels is zero. This property is characteristic of level repulsion in the spectrum of a random matrix, which is stronger in a system where reversal symmetry is broken compared to a system where it is not. As a consequence, K decreases as a function of the flux at low magnetic flux and increases back in the vicinity of $\phi_0/2$ where time reversal symmetry is recovered.

We have calculated numerically K in a disordered ring of length $L=80$ and width $a=8$ for several value of the on-site disorder w corresponding to the diffusive regime. Figure 4

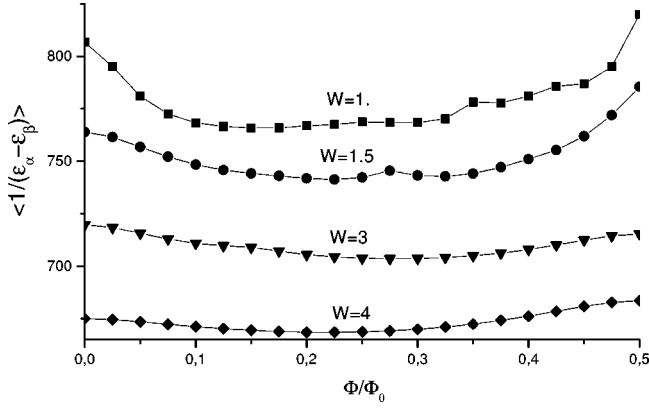


FIG. 4. K calculated numerically in a ring 80×8 , for different values of the disorder $w = 1, 1.5, 3, 4$.

illustrate the effect described above. In the absence of other contributions, this term would give rise to a negative magnetopolarizability.

2. Matrix element

On Fig. 5 are plotted the flux dependence of the average square of the diagonal and nondiagonal matrix elements for a diffusive ring. It can be noted that they have opposite flux dependences, resulting from the fact that $\text{Tr}(X^2) = \sum_\alpha X_{\alpha\alpha}^2 + \sum_{\alpha \neq \beta} X_{\alpha\beta}^2$ is flux independent. The diagonal elements decrease as a function of the flux, whereas the nondiagonal elements increases.

The time reversal property of the operator \mathbf{X} implies that its diagonal matrix elements are real even function of flux and can be developed in successive powers of $\cos(2\pi\phi/\phi_0)$.

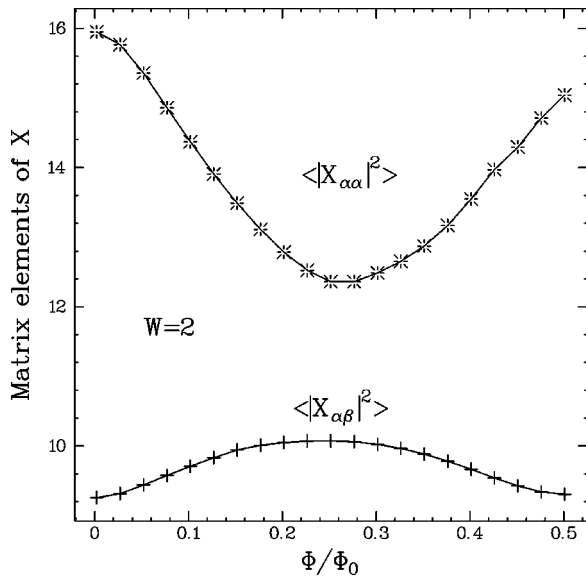


FIG. 5. Flux dependence of the diagonal and nondiagonal matrix elements in a ring of length $L = 60$ and $M = 4$ channels, for $w = 2$. The factor 2 at zero flux between the diagonal and nondiagonal matrix elements is due to the fact that only the states such that $\epsilon_\alpha < \epsilon_\beta$ are taken into account in the calculation of the non diagonal matrix elements.

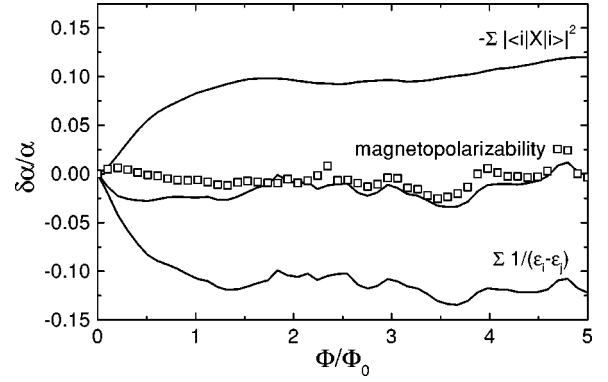


FIG. 6. Flux dependence of the matrix element and energy denominators contributions to the average unscreened polarizability of a disordered square 30×30 , calculated with the Anderson model for $w = 3$. These two contributions nearly compensate yielding to the absence of magnetopolarizability. Full line: sum of the flux dependences of energy denominator and average square matrix elements. Squares: exact calculation.

$\langle X_{\alpha,\alpha}^2 \rangle$ is maximum for multiple values of $\phi_0/2$. It is possible to evaluate analytically this flux dependence of the diagonal matrix elements from random matrix theory, using the relation between $X_{\alpha\alpha}$ and the sensitivity of the energies to an electric field $X_{\alpha\alpha} = (\partial\epsilon_\alpha/\partial E)_{E=0}$. Since electric field preserves time-reversal symmetry, the typical value of the derivative of the energy levels with respect to the electric field $\langle |\partial\epsilon_\alpha/\partial E_0|^2 \rangle$ is proportional to $1/\beta$. Consequently, $|X_{\alpha,\alpha}|^2(\phi = \phi_0/4) = \frac{1}{2}|X_{\alpha,\alpha}|^2(\phi = 0)$. Figure 5 shows that this last result is qualitatively true. Note also that this flux dependence of the variance of the diagonal elements of the dipolar operator was also found from numerical simulations in the Anderson model by Uski *et al.*¹⁶ Therefore, in absence of the energy denominators $1/(\epsilon_\beta - \epsilon_\alpha)$, the flux dependence of the matrix elements would give rise to a positive magnetopolarizability. As a result this flux dependence is opposite to the contribution of the energy denominators. Moreover we can see on Fig. 6 that these contributions almost exactly cancel and there is no magnetopolarizability in this model. It is also possible on Fig. 6 to check the validity of random matrix theory by comparing the exact calculation of the average magnetopolarizability from expression (16) with the result obtained assuming independence between matrix elements and energy denominators. Our results confirm that RMT provide a reasonably good description of a mesoscopic system in the diffusive regime.

At this stage it is interesting to emphasize that the flux dependence of the average square of the X operator matrix element is opposite to the same quantity related to the current operator which changes sign by time reversal symmetry. So when computing the magnetic susceptibility the contribution of the matrix elements and energy denominators are of the same sign. This effect is related to the existence of a finite average current in the canonical ensemble for a diffusive ring.

Note that both persistent current and zero frequency polarizability can be expressed as a function of the free energy

of the system $I = -\partial F(E, \phi)/\partial \phi$ and $\alpha = -\partial^2 F(E, \phi)/\partial E^2$. So,

$$\frac{\partial \alpha}{\partial \phi} = \frac{\partial^2 I_{\text{per}}}{\partial E^2}. \quad (22)$$

Let us emphasize that the absence of thermodynamic magnetopolarizability is thus related to the insensitivity of persistent currents to an electrostatic field and can be qualitatively understood using a semiclassical argument. The effect of the electrostatic potential slowly varying at the scale of the Fermi wavelength can be included into the phase of the electronic wave functions. Persistent currents can be computed from the integral of the classical action on a circular orbit. The contribution of the static electrostatic potential on such an orbit $(e/\hbar v_f) \int V(\mathbf{r}) ds$, where $V(\mathbf{r}) = Er \cos(\theta)$ and $ds = rd\theta$ obviously cancels out. This argument is helpful to understand why the polarizability is independent of the flux in this very low frequency regime. We will see, however, in Sec. V that a nonzero magnetopolarizability is found at finite frequency in the grand canonical ensemble or at finite temperature in the canonical ensemble where the energy denominators contribution disappears.

IV. SCREENING

A. The Thomas-Fermi approximation

As mentioned above the mean field approximation reduces the complicated many body problem of interacting electrons to a much simpler one, in which electrons move in an effective potential $\phi(\mathbf{r})$, resulting from the screening of the applied potential by other electrons. Therefore, it goes beyond a simple electrostatic calculation by taking into account the kinetic energy of electrons as well as their fermionic character.¹⁷

In the linear regime, i.e for a small external potential, the induced charge density is simply proportional to the effective potential:^{18,19}

$$\delta\rho(\mathbf{r}) = -e^2 n(E_F) \phi(\mathbf{r}) = -\epsilon_0 k_s^2 \phi(\mathbf{r})$$

at 3D with $k_s = \sqrt{e^2 n(E_F)/\epsilon_0}$ and

$$\delta\rho(\mathbf{r}) = -e^2 n(E_F) \phi(\mathbf{r}) = -\epsilon_0 k_s \phi(\mathbf{r}) \delta(z)$$

for a conducting plane at $z=0$ with $k_s = e^2 n(E_F)/\epsilon_0$.

The resolution of the self-consistent Eq. (1) within this Thomas-Fermi approximation for a 3D sample in the presence of a uniform external applied field, gives rise to a screened potential whose value is of the order of E/k_s which is confined to the border of the sample within $\lambda_s = 1/k_s$. This result is in principle not true any more in a 2D system where classical screening already involves charge displacement in the whole system. However, since the charge distribution is always singular on the edge of the sample it is possible to approximate the screened potential for a disk of radius R by $\Phi(r, \theta) = (E/k_s) F(R-r) \cos(\theta)$ where F is a peaked function centered on zero of width $1/k_s$. When screening is taken into account the expression of the quantum polarizability at $T=0$ reads

$$\alpha = \frac{2e^2}{E^2} \sum_{\alpha \neq \beta} \frac{\langle \beta | \phi_{TF}(\mathbf{r}) | \alpha \rangle \langle \beta | \mathbf{E} \cdot \mathbf{r} | \alpha \rangle}{\epsilon_\alpha - \epsilon_\beta}. \quad (23)$$

Within this approximation the GE result for the polarizability is modified by a factor $1/(ak_s)^2$ and becomes identical to the classical result.

B. Beyond the Thomas-Fermi approximation

As we have seen, the Thomas-Fermi approximation does not take into account contribution of the screening due to electronic interferences. They indeed give rise to a quantum correction for the response function χ , and consequently to the effective potential ϕ and the polarizability α :

$$\chi = \chi_{TF} + \delta\chi,$$

$$\phi = \phi_{TF} + \delta\phi,$$

$$\alpha = \alpha_{TF} + \delta\alpha,$$

where χ is the response to the local field. Assuming that those corrections are small compared to the Thomas-Fermi value, it is possible to show, in agreement with Efetov²⁰ (see Appendix A for the detailed calculation), that $\delta\alpha$ can be simply expressed as a function of the mesoscopic correction to the one electron response function $\delta\chi$ and the Thomas-Fermi potential

$$\delta\alpha \approx \frac{1}{E^2} \text{Tr}(\phi_{TF} \delta\chi \phi_{TF}). \quad (24)$$

In the following we discuss the response to a time-dependent electric field. We will see that dynamical polarizability can be very different from the static one.

V. EXPRESSION OF THE POLARIZABILITY

A. Response to time-dependent potential

The application of a time-dependent external potential $V(t) = e\mathbf{E} \cdot \mathbf{r} \exp(i\omega t)$ raises the problem of the relaxation of the system towards equilibrium. This process is made by inelastic processes characterized by a typical time scale, related to inelastic collisions.

In the limit of a weak coupling, this process can be described by a master equation on the density matrix:¹¹

$$i\hbar \frac{\partial \rho}{\partial t} = [H_0 + V(t), \rho] - i\gamma(\rho - \rho_{\text{eq}}). \quad (25)$$

The parameter γ represents the typical broadening of the energy levels and characterizes dissipation processes, allowing the relaxation of the system towards equilibrium. The density matrix ρ_{eq} satisfies the condition $[H, \rho_{\text{eq}}] = 0$, $H = H_0 + e\mathbf{E} \cdot \mathbf{r} \exp(i\omega t) + \phi_{\text{ind}}(\mathbf{r}, \gamma, t)$ being the Hamiltonian of the system in the mean field approximation, in an ac electric field at frequency ω . Two limits are to be considered. When the frequency of the time-dependent potential is small compared to γ ; the system follows the potential and stays at

every moment at equilibrium. On the contrary, at high frequency compared to γ , the system is always out of equilibrium.

In addition, we assume that the effective potential is always in phase with the external one. This is justified if the dissipative part of the polarizability is very small compared to the nondissipative part ($\alpha'' \ll \alpha'$).²¹

The expression of the density matrix is then obtained by solving the master equation. From it, we deduce the response function and later on the polarizability and its quantum correction $\delta_\phi \alpha$, which can be expressed as a function of the eigenstates and eigenvalues of the Hamiltonian H_0 .

$$\alpha = -\frac{2e^2}{E^2} \text{Re} \left(\sum_{\alpha \neq \beta} \frac{f_\alpha - f_\beta}{\epsilon_\alpha - \epsilon_\beta} \frac{\epsilon_\alpha - \epsilon_\beta - i\gamma}{\epsilon_\alpha - \epsilon_\beta + \omega - i\gamma} \langle \beta | \phi_{TF} | \alpha \rangle \right) \times \langle \beta | \mathbf{E} \cdot \mathbf{r} | \alpha \rangle + \frac{\gamma}{\gamma + i\omega} \sum_{\alpha} \frac{\partial f_{\alpha}}{\partial \epsilon_{\alpha}} \langle \alpha | \phi_{TF} | \alpha \rangle \langle \alpha | \mathbf{E} \cdot \mathbf{r} | \alpha \rangle, \quad (26)$$

$$\delta_\phi \alpha = -\frac{2e^2}{E^2} \delta_\phi \times \text{Re} \left(\sum_{\alpha \neq \beta} \frac{f_\alpha - f_\beta}{\epsilon_\alpha - \epsilon_\beta} \frac{\epsilon_\alpha - \epsilon_\beta - i\gamma}{\epsilon_\alpha - \epsilon_\beta + \omega - i\gamma} |\langle \beta | \phi_{TF} | \alpha \rangle|^2 \right) + \frac{\gamma}{\gamma + i\omega} \sum_{\alpha} \frac{\partial f_{\alpha}}{\partial \epsilon_{\alpha}} |\langle \alpha | \phi_{TF} | \alpha \rangle|^2. \quad (27)$$

B. Quantum correction to the polarizability in the diffusive regime for the different statistical ensembles

In this section, we examine the differences between the canonical and grand canonical ensembles. The canonical ensemble CE corresponds to the situation for which the number of electrons in the system is fixed. It is in particular the case for electrically isolated systems. The chemical potential is determined self-consistently by the condition $N = \sum_i f_0(\epsilon_i - \mu)$, and therefore depends on the flux through the energy levels. On the other hand, in the grand canonical ensemble GCE, the system can exchange electrons with the thermodynamic reservoir, which imposes the value of the chemical potential. A physical realization consists in connecting the ring to a large metallic pad. At finite temperature, the occupation of the energy levels is spread over an energy interval of the order of the temperature. As a result, the sensitivity to the flux dependence of the chemical potential is suppressed when $T \gg \Delta$, where Δ is the mean level spacing. Therefore the differences between CE and GCE are expected to disappear with increasing temperature and frequency. We first discuss the polarizability in the GCE.

1. Grand canonical ensemble

Due to the ergodicity property in the diffusive regime, the disorder average is equivalent to the average over the number of electrons. This property simplifies the calculation. By averaging over the whole spectrum, one can show that

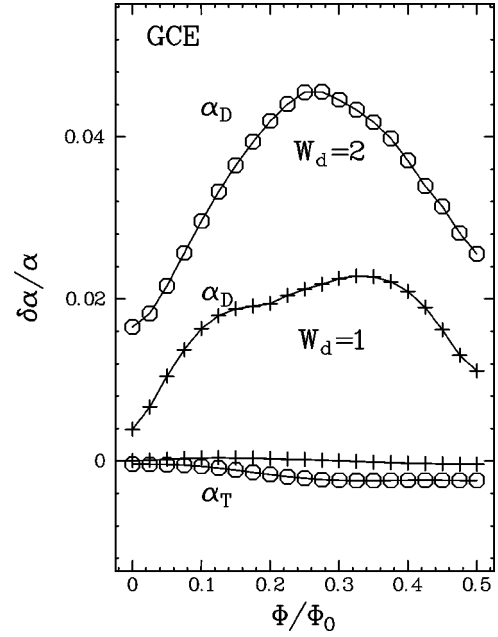


FIG. 7. Dynamical ($\omega \gg \gamma$) and thermodynamical ($\omega \ll \gamma$) polarizability in the grand canonical ensemble of a disordered ring 80×8 , calculated with the Anderson model for $w=1$ and $w=2$.

$\langle (f_\alpha - f_\beta) / (\epsilon_\alpha - \epsilon_\beta) \rangle = -1 / \delta\mu$, where $\langle \rangle \equiv (1 / \Delta\mu) \int_{\mu - \delta\mu/2}^{\mu + \delta\mu/2}$. Considering only regimes where $\omega \ll \Delta$ and $\gamma \ll \Delta$ formula (27) greatly simplifies:

$$\delta_\phi \alpha = \frac{2e^2}{E_F} \delta_\phi \left(\sum_{\alpha \neq \beta} |\langle \beta | \phi_{TF} | \alpha \rangle|^2 + \frac{\gamma}{\gamma + i\omega} \sum_{\alpha} \delta(\epsilon_\alpha - \mu) |\langle \alpha | \phi_{TF} | \alpha \rangle|^2 \right). \quad (28)$$

Two frequency regimes can be distinguished, depending whether the frequency is smaller or larger than the relaxation energy γ . In the limit $\omega \ll \gamma$, which corresponds to a static electric field, the polarizability is simply equal to the flux dependence of the trace of the screened potential

$$\delta_\phi \alpha_s^{\text{GCE}} = \frac{2e^2}{E_F} \delta_\phi \text{Tr}(\phi_{TF}^2). \quad (29)$$

Since the trace is independent of the basis which is considered, this quantity is independent of the magnetic flux. Therefore, at low frequency, there is no mesoscopic correction to the polarizability in the GCE as shown on Fig. 7.

We now turn to the dynamical limit, i.e., for $\omega \gg \gamma$. As for the canonical ensemble, we consider a diffusive ring of radius R and width W . The relaxation term cancels and only remains the term

$$\delta_\phi \alpha_D^{\text{GCE}} = \frac{2e^2}{E_F} \delta_\phi \left(\sum_{\alpha > \beta} |\langle \beta | \phi_{TF} | \alpha \rangle|^2 \right). \quad (30)$$

Therefore, one needs to evaluate the matrix element $|\phi_{\alpha,\beta}^{TF}|^2$. It is then possible to use the semiclassical calculation of McMillan²² which yields to a general expression of the av-

erage squares the matrix elements of the operator $e^{iq \cdot r}$ in a diffusive system which generalizes semi-classical expressions of the matrix elements of \mathbf{X} relevant to the unscreened case discussed above:

$$|\langle \alpha | e^{iq \cdot r} | \beta \rangle|^2 = \frac{\Delta}{\pi \hbar} \frac{Dq^2}{D^2 q^4 + (\epsilon_\alpha - \epsilon_\beta)/\hbar^2}. \quad (31)$$

In order to determine the relevant spacial frequencies of the screened potential, this latter is then decomposed in Fourier series $\phi_{TF}(\mathbf{r}) = \sum_{n=-M/2}^{M/2} A_n e^{iq_n^+ \cdot r} + e^{iq_n^- \cdot r}$, where $\mathbf{q}_n^{\pm} = (n\pi/W)\mathbf{u}_r \pm (2\pi/L)\mathbf{u}_\theta$. At low energy, the main contribution to the matrix elements of the screened potential is dominated by the terms with the smallest wavevector corresponding to $n=0$, $q_{\min} = 2\pi/L$ for which

$$\phi_{\alpha,\beta}^2 \approx \frac{1}{Dq_{\min}^2} = \frac{\Delta}{\pi E_c} \left(\frac{8R\lambda_s}{3\pi^2 W} \right)^2, \quad (32)$$

where λ_s is the screening length, $E_c = \hbar D/2\pi R$ is the Thouless energy, and Δ the mean level spacing between energy levels.

We can then estimate the flux dependence of the matrix elements $|\phi_{\alpha\beta}|^2$ as was done in Sec. III for the unscreened potential

$$\begin{aligned} \delta_\phi \sum_{\beta \neq \alpha} \langle |\phi_{\alpha\beta}| \rangle^2 &= -\delta_\phi \langle |\phi_{\alpha\alpha}| \rangle^2 \\ &= \langle |\phi_{\alpha\alpha}|^2 \rangle_{\text{GUE}} - \langle |\phi_{\alpha\alpha}|^2 \rangle_{\text{GOE}} \\ &= \frac{1}{2} \langle |\phi_{\alpha\beta}|^2 \rangle_{\text{GOE}}. \end{aligned} \quad (33)$$

Thus, the mesoscopic correction to the polarizability reads

$$\frac{\delta\alpha_D^{\text{GCE}}}{\alpha_0} = \left(\frac{8}{3\pi^3} \right) \frac{\lambda_s}{W} \frac{1}{g}, \quad (34)$$

where α_0 is the classical value of the polarizability. We took the 2D limit corresponding to a disk for which $\alpha_0 = \epsilon_0 \frac{16}{3} R^3$. For a quasi-1D ring, $\alpha_0 = [\epsilon_0 \pi^2 R^3 / \ln(R/W)]$ (see Sec. II B). Only the numerical factor in formula (34) is modified if one considers this last value. This result is in good agreement with the calculation of Blanter and Mirlin²³ using supersymmetry techniques.

The magnetopolarizability increases when a magnetic flux is applied, corresponding to a *positive* magnetopolarizability. A noticeable result is that this effect is inversely proportional to the conductance and thus increases with the disorder in the diffusive regime. It is expected, however, to decrease again with disorder in the localized regime when wave functions and eigenenergies become insensitive to the Aharonov-Bohm flux.

This disorder dependence of $\delta\alpha/\alpha$ is illustrated on Fig. 8. On the other hand, the magnetopolarizability decreases with the electron density n_e , since the conductance increases and the screening length decreases with n_e .

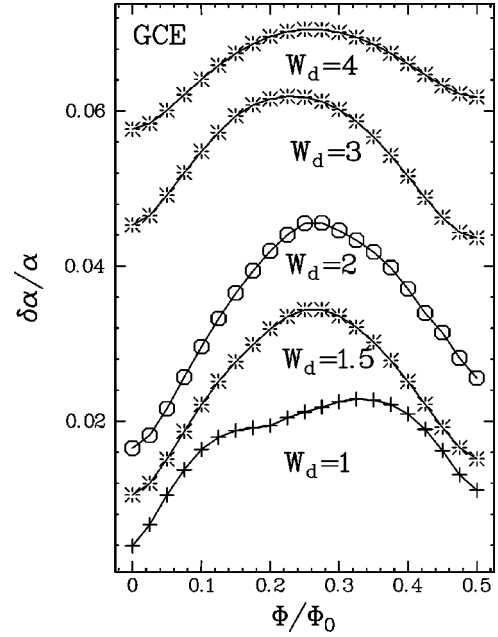


FIG. 8. Dynamical polarizability ($\omega \gg \gamma$) in the grand canonical ensemble in a disordered ring 80×8 , calculated with the Anderson model for several values of the disorder. $w = 1, 1.5, 2, 3, 4$.

The ring geometry is particularly favorable to observe this effect. The extrapolation of Eq. (34) leads to $\delta\alpha/\alpha \sim (\Delta/2E_c)(\lambda_s/a)$ for a two-dimensional sample such as a disk or a square of typical size a and $\delta\alpha/\alpha = (\Delta/2E_c)(\lambda_s/R)^2$ for a sphere. In a ring etched in a semiconductor heterojunction GaAs/GaAlAs with the following parameters: $L = 8 \mu\text{m}$, $\lambda_s = 400 \text{ \AA}$, $M = 10$, $E_c = 7\Delta$, we obtain $\delta\alpha/\alpha \sim 3 \times 10^{-3}$.

2. Canonical ensemble

From relation (27), we deduce that at zero temperature and zero frequency the quantum correction to the polarizability reads in the canonical ensemble

$$\delta_\phi \alpha = \frac{2e^2}{E^2} \delta_\phi \left(\sum_{\alpha=1}^N \sum_{\beta=N+1}^{N_t} \frac{|\langle \alpha | \phi_{TF}(\mathbf{r}) | \beta \rangle|^2}{\epsilon_\beta - \epsilon_\alpha} \right), \quad (35)$$

where N_t is the total number of states. Note that this expression is very similar to the Gorkov Eliashberg one where matrix elements of the X operator are replaced by the matrix elements of the screened potential. Just as previously discussed in the absence of screening, the flux dependence of the energy denominators compensates exactly the one of the matrix element. It is important to note that this compensation is now also found by Blanter and Mirlin in supersymmetry calculations at zero frequency²⁴ contrary to what was stated previously²³ when the contribution of three levels correlation function was not correctly taken into account. We believe that the symmetry argument given in the preceding section still hold for a screened potential. However, it is also pointed out in this recent Ref. 24 that a finite magnetopolarizability can be expected in the canonical ensemble as soon as the frequency is not negligible compared to the level spacing due

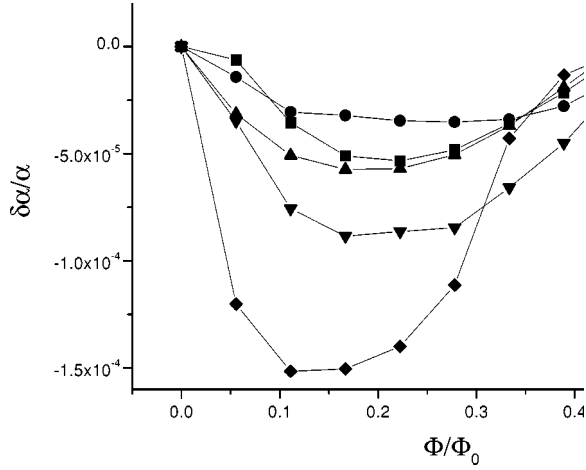


FIG. 9. Correction to the canonical polarizability by interactions in a ring of length $L=60$ and $M=4$ channel, for several values of the disorder $w=0.9, 1.2, 1.4, 1.7$.

to the decrease of the contribution coming from the energy denominators. A similar effect is obtained when increasing the temperature as shown precisely in Ref. 5. The same authors²⁴ also emphasize that the magnetopolarizability oscillates with frequency on the average level spacing scale, such as the level spacing distribution.

C. Effect of interactions in the canonical ensemble

We have shown that in the canonical ensemble in the diffusive regime, there is no mesoscopic correction to the polarizability. However, we have neglected in this calculation electron-electron interactions.

In order to investigate a possible effect of electron-electron interaction a first and simple approach consists in taking the interaction potential as a perturbation. Rather than a Coulomb potential, we use an on-site interaction $U\delta(\mathbf{r}-\mathbf{r}')$. This approximation is justified for high electronic density, since the interaction between two electrons is then strongly screened by other electrons. In first order in U , the variation in total energy can be expressed in terms of the local electronic density²⁵ $n(\mathbf{r}) = \sum_{\alpha=1}^N |\langle \alpha | \mathbf{r} \rangle|^2$:

$$\delta E_{\text{tot}} = U \int n(\mathbf{r})^2 dr. \quad (36)$$

The correction to the canonical polarizability is obtained then by the formula

$$\delta\alpha_{\text{int}} = - \frac{\partial^2(\delta E_{\text{tot}})}{\partial^2 E}. \quad (37)$$

On Fig. 9 numerical calculations of this quantity $\delta\alpha_{\text{int}}/\alpha$ are shown for a ring for various values of disorder.

Interactions give rise to a negative magnetopolarizability. The effect is more important in the ballistic regime than in the diffusive regime, in which it does not seem to depend on disorder. It gives rise to a magnetopolarizability which is quite enhanced compared to its value in the absence of inter-

actions. Nevertheless, the order of magnitude of this effect remains small compared to the dynamical polarizability.

VI. CONCLUSION: COMPARISON WITH RECENT EXPERIMENTS

We have calculated the average polarizability of mesoscopic rings and squares. In the grand canonical ensemble, we found that there exists a positive magnetopolarizability for frequencies larger than the typical broadening of the energy levels γ . The relative effect $\delta\alpha/\alpha$ scales as $1/g$, where g is the dimensionless conductance. In the canonical ensemble at zero frequency and zero temperature, the magnetopolarizability cancels in the diffusive regime, whereas in the ballistic regime a small negative effect is found. Differences between canonical and grand canonical ensemble disappear at frequencies or temperatures larger than the level spacing. In the following table, are summarized the results obtained for $\delta\alpha/\alpha$ in the ballistic and diffusive regime, for the canonical and grand canonical ensemble.

	Ballistic regime	
	$\omega \ll \gamma$	$\Delta \gg \omega \gg \gamma$
C	$-\frac{1}{4N^2}$	$-\frac{1}{4N^2}$
GC	0	0
	Diffusive regime	
	$\omega \ll \gamma$	$\Delta \gg \omega \gg \gamma$
C	0	0
GC	0	$\left(\frac{8}{3\pi^3}\right) \frac{\lambda_F}{W} \frac{1}{g}$

These results are in good agreement with recent experiments where the magnetopolarizability of two-dimensional rings has been recently measured by Deblock and co-workers.^{26,27} The sample investigated was an array of rings fabricated by electronic lithography in an heterojunction GaAs/GaAlAs. A resonant technique was used in which the rings are coupled to the capacitive part of a high frequency superconducting resonator.

The polarizability exhibits oscillations as a function of the flux, with a periodicity corresponding to $\phi_0/2$ through a ring. The order of magnitude as well as the sign of the effect is in good agreement with our grand canonical results. In addition, it decreases as a function of electronic density, according to the $1/g$ dependence of formula (34).

Taking into account the fact that these experiments were done at a frequency which is of the order of $1/3$ of the level spacing, where differences between canonical and grand canonical results are strongly reduced, it can be shown that this result is in good agreement with theoretical predictions.^{24,27} A complete check of the theory would imply to perform new experiments at other values of frequency.

APPENDIX A: DETAILS ON THE CALCULATION OF THE MESOSCOPIC CORRECTION TO THE POLARIZABILITY

In this appendix, we derive in more details the expression of the mesoscopic correction to the polarizability. In the fol-

lowing, we will note $\Xi = \Xi_{TF} + \delta\Xi$ and $\xi = \xi_{TF} + \delta\xi$, the response to the external and internal field respectively, defined in the matrix form as

$$\delta\rho = \chi\phi = \Xi\phi_{\text{ext}}. \quad (\text{A1})$$

The relation (1) which relates the induced charge density to the response function can be written in the matrix form

$$\delta\rho = \chi(1 - U\chi)^{-1}\phi_{\text{ext}}. \quad (\text{A2})$$

The effective potential is also related to the response function by 3. The polarizability in the Thomas-Fermi approximation reads

$$\begin{aligned} \alpha_{TF} &= \frac{1}{E}\text{Tr}(x\chi_{TF}\phi) = \frac{1}{E}\text{Tr}[\chi_{TF}(1 - U\chi_{TF})^{-1}\phi_{\text{ext}}] \\ &= \frac{1}{E}\text{Tr}(x\Xi_{cl}\phi_{\text{ext}}). \end{aligned} \quad (\text{A3})$$

with $\Xi_{cl} = \chi_{TF}(1 - U\chi_{TF})^{-1}$. The next step is to find the quantum correction $\delta\Xi$ to the screened response function. We set by definition

$$\Xi = \chi(1 - U\chi)^{-1} = \Xi_{cl} + \delta\Xi. \quad (\text{A4})$$

Assuming that quantum corrections are small compared to the Thomas-Fermi value, second order terms can be neglected:

$$(1 - U\chi_{TF})\Xi - U\delta\chi\Xi_{cl} = \chi_{TF} + \delta\chi,$$

$$\delta\Xi = \delta\chi(1 - U\chi)^{-1}(1 + U\Xi_{cl}).$$

Using the fact that $1 + U\Xi_{cl} = \chi_{TF}^{-1}\Xi_{cl}$, we show that the quantum correction to the response function can be expressed:

$$\delta\Xi = \Xi_{cl}\chi_{TF}^{-1}\delta\chi\chi_{TF}^{-1}\Xi_{cl}. \quad (\text{A5})$$

The quantum correction to the polarizability $\delta\alpha = \text{Tr}(x\delta\Xi\phi_{\text{ext}})$ can then be written

$$\delta\alpha = \text{Tr}(\phi_{TF}\delta\chi\phi_{TF}),$$

where we have used that $\chi_{TF}^{-1}\Xi_{cl}\phi_{\text{ext}} = \phi_{TF}$.

APPENDIX B: DISCUSSION OF THE LINEAR RESPONSE APPROXIMATION

In order to estimate the validity of the linear response, we have calculated the total energy of a ring as a function of the

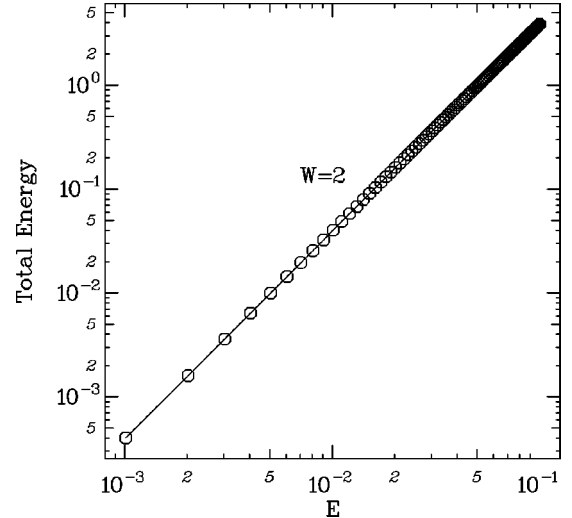


FIG. 10. Total energy versus electric field in units of t/eR calculated in the Anderson model in a ring 40×5 , and for a disorder $w=2$.

electric field (screening is not here considered). It can be expanded as

$$U(E) = U_0 - d_0E - \alpha E^2 + \dots$$

The coefficient d_0 correspond to the spontaneous dipolar moment of the ring, resulting from fluctuations of the charge in the presence of disorder. The ensemble average of d_0 is zero. The energy average varies like E^2 at small field. As it is shown on Fig. 10, a deviation from the linear behavior is hardly observed before a critical field E_{max} such that

$$eE_{\text{max}}R \sim E_F, \quad (\text{B1})$$

where R is the radius of the ring.

This critical value is less restrictive than the criteria⁸ and justifies the use of the linear response up to very high fields. For instance, in a 2D electron gas, obtained in a semiconductor heterojunction GaAs/GaAlAs for which $E_F \sim 30$ K, our criteria allows to use linear response up to fields such that $ER \sim 2$ mV.

¹Y. Imry, *Introduction to Mesoscopic Physics* (Oxford University Press, Oxford, 1997).

²L.P. Lévy, G. Dolan, J. Dunsmuir, and H. Bouchiat, *Phys. Rev. Lett.* **64**, 2074 (1990).

³V. Chandrasekhar *et al.*, *Phys. Rev. Lett.* **67**, 3578 (1991).

⁴D. Mailly, C. Chapelier, and A. Benoit, *Phys. Rev. Lett.* **70**, 2020 (1993).

⁵B. Reulet and H. Bouchiat, *Phys. Rev. B* **50**, 2259 (1994).

⁶A. Kamenev, B. Reulet, H. Bouchiat, and Y. Gefen, *Europhys. Lett.* **28**, 391 (1994).

⁷Y. Noat, B. Reulet, and H. Bouchiat, *Europhys. Lett.* **36**, 701 (1996).

⁸L.P. Gor'kov and G.M. Eliashberg, *Sov. Phys. JETP* **21**, 940 (1965).

- ⁹L. Landau and E. Lifshitz, *Electrodynamics of Continuous Media* (Mir, Moscow, 1969).
- ¹⁰S. Strässler, M.J. Rice, and P. Wyder, Phys. Rev. B **6**, 2575 (1972); M.J. Rice, W.R. Schneider, and S. Strässler, *ibid.* **8**, 474 (1973).
- ¹¹N. Trivedi and D.A. Browne, Phys. Rev. B **38**, 9581 (1988).
- ¹²P. Fulde and A.A. Ovchinnikov, Eur. Phys. J. B **17**, 623 (2000).
- ¹³O. Entin-Wohlman and B. Mühlischlegel, Z. Phys. B: Condens. Matter **80**, 385 (1990).
- ¹⁴K. Frahm, B. Mühlischlegel, and R. Németh, Z. Phys. B: Condens. Matter **78**, 91 (1990).
- ¹⁵L. M. Metha, *Random Matrices* (Academic, New York, 1991).
- ¹⁶V. Uski, B. Mehlig, and R.A. Römer, Ann. Phys. (Leipzig) **7**, 437 (1998).
- ¹⁷N. W. Ashcroft and N. D. Mermin, *Solid State Physics* (Saunders, Philadelphia, 1976).
- ¹⁸R. Berkovits and B.L. Altshuler, Phys. Rev. B **46**, 12 526 (1992).
- ¹⁹R. Berkovits and B. Altshuler, Europhys. Lett. **19**, 115 (1992).
- ²⁰K.B. Efetov, Phys. Rev. Lett. **76**, 1908 (1996).
- ²¹Y. Noat, B. Reulet, H. Bouchiat, and D. Mailly, Superlattices Microstruct. **23**, 621 (1998).
- ²²W.L. McMillan, Phys. Rev. B **24**, 2739 (1981).
- ²³Y. Blanter and A. Mirlin, Phys. Rev. B **57**, 4566 (1998).
- ²⁴Y. Blanter and A. Mirlin, Phys. Rev. B **63**, 113315 (2001).
- ²⁵M. Ramin, B. Reulet, and H. Bouchiat, Phys. Rev. B **51**, 5582 (1995).
- ²⁶R. Deblock, Y. Noat, H. Bouchiat, B. Reulet, and D. Mailly, Phys. Rev. Lett. **84**, 5379 (2000).
- ²⁷R. Deblock, Y. Noat, H. Bouchiat, B. Reulet, and D. Mailly, cond-mat/0106474 (unpublished).