

Laughlin-type wave function of two-dimensional electrons in a tilted magnetic field

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We study the fractional quantum Hall states in the tilted magnetic field. A many-particle wave function of the ground state, which is similar to that of Laughlin's, is constructed in the Landau gauge. We show that in the limit of thermodynamics, the concept of the composite fermion is still valid in the presence of the in-plane field.

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Recently, the two-dimensional electron system (2DES) in tilted magnetic field has attracted great interest from both experimentalists and theorists. The magnetotransport experiments on high mobility samples in GaAs/AlGaAs heterostructures revealed different classes of correlated many-electron states.¹ The most prominent findings are the discoveries of the giant anisotropy in the resistivity near half filling of the top most LL.^{2,3} It is revealed that anisotropy occurs when the 2DES is applied by an in-plane magnetic field. The easy direction of transport is perpendicular to the in-plane field. It is generally accepted that the highly anisotropic transport is related to the formation of the unidirectional charge-density-wave (UCDW) state, i.e., the stripe phase.^{4,5} The possibility of existence of the UCDW was predicted by Koulakov, Fogler, and Shklovskii⁶ based on an earlier work on the Hartree-Fock treatment of the high Landau levels. Specifically, the exotic $\nu=5/2$ fractional quantum Hall state, which shows no anisotropy in perpendicular magnetic field, becomes highly anisotropic when the external field is tilted an angle. Contrary to other odd-denominator filling states, which occur as a Jain series, the Hall plateau of the incompressible $\nu=5/2$ state is explained as the appearance of the ground state of spin-singlet pairing of composite fermions (CF's).⁷ However, the spin-polarized p -wave BCS pairing of CF's, or the Moore-Read (MR) Pfaffian wave function,⁸ may be another possibility,⁹ which was recently suggested to be favorable.^{10,11} Studies by Eisenstein *et al.*¹² in the tilted field experiments have shown that the plateau disappears if the tilted angle θ exceeds a critical value. The explanation of the experiments from the point of view of the singlet pairing can be understood as a gain in Zeeman energy.¹³ However, the Lande g factor is about 30% larger than expected. On the other hand, in the picture of p -wave pairing of CF's, how the tilted field violates the spin-polarized paired Hall state is still puzzling. Yu *et al.*¹⁴ proposed a mechanism to solve the above puzzle. They considered that there exists a competition between the instabilities of the CF Fermi surface to the formation of the UCDW and the paired Hall state. When the tilted angle is small, the pairing state dominates. But as the tilted angle increases, the UCDW takes energetically over the paired Hall state as the ground state, which transforms the incompressible state to the compressible state. A recent experiment by Pan *et al.*¹⁵ supports their suggestion.

Here comes the question. One may ask if the concept of the composite fermion is still valid when the magnetic field is tilted at an angle. In the present work, we prove that in the limit of thermodynamics, one can construct a Laughlin-type wave function based on a similar reasoning of the original work by Laughlin. The concept of a composite fermion in a tilted field can be deduced from the analysis of the Laughlin-type wave function in an analogous way taken by J. K. Jain.¹⁶

Consider an electron moving in a torus geometry under the influence of a strong magnetic field which is tilted an angle θ to the x - y plane, with $B_x = B \tan \theta$ and $B_z = B$. The electron is confined in a harmonic potential $V(z) = \frac{1}{2} m_b \Omega^2 z^2$ in the z direction, where m_b is the band mass of the electron. If the characteristic frequency $\Omega \gg \omega_c$, where $\omega_c = \hbar / m_b l_0^2$ is the electron cyclotron frequency in the perpendicular magnetic field B , then the electrons move in a quasi-two-dimensional plane. We work in the "Landau gauge" by choosing the vector potential $\mathbf{A} = \{0, xB_z - zB_x, 0\}$. The single-electron Hamiltonian is then

$$\hat{H} = \frac{1}{2m_b} \left[(-i\hbar\partial_x)^2 + \left(-i\hbar\partial_y - \frac{e}{c}(xB_z - zB_x) \right)^2 + (-i\hbar\partial_z)^2 \right] + \frac{1}{2} m_b \Omega^2 z^2. \quad (1)$$

Take the length unit $l_0^2 = \hbar c / eB = 1$ and separate out the plane wave in the y direction,

$$\Psi(x, y, z) = \frac{1}{\sqrt{L_y}} e^{iky} \phi(x, z), \quad (2)$$

where $k = 2\pi j / L_y = k_0 \cdot j$ ($j = 1, 2, \dots$). Then the Hamiltonian can be equivalently rewritten as

$$\hat{H} = \frac{1}{2} \hbar \omega_c \left[-\partial_\xi^2 - \partial_z^2 + \xi^2 - 2\xi z \tan \theta + \left(\tan^2 \theta + \frac{\Omega^2}{\omega_c^2} \right) z^2 \right], \quad (3)$$

where $\xi = x - x_0$ with $x_0 = kl_0$.

To decouple the two coupled harmonic oscillators, we make a coordinate rotation in the $\xi-z$ plane:

$$\begin{cases} \xi = u \cos \alpha - v \sin \alpha \\ z = u \sin \alpha + v \cos \alpha. \end{cases} \quad (4)$$

Take $\tan \alpha = [\omega_c^2 / (\omega_+^2 - \omega_c^2)] \tan \theta$, then the Hamiltonian becomes

$$\hat{H} = \frac{1}{2} \hbar \omega_- \left(-\frac{\partial^2}{\partial \xi^2} + \xi^2 \right) + \frac{1}{2} \hbar \omega_+ \left(-\frac{\partial^2}{\partial \eta^2} + \eta^2 \right), \quad (5)$$

where $\xi = (l_0/l_-)u$ and $\eta = (l_0/l_+)v$ with $l_{\pm}^2 = \hbar/m_b \omega_{\pm}$, and

$$\omega_{\pm}^2 = \frac{1}{2} \left(\Omega^2 + \frac{\omega_c^2}{\cos^2 \theta} \right) \pm \frac{1}{2} \sqrt{\left(\Omega^2 - \frac{\omega_c^2}{\cos^2 \theta} \right)^2 + 4\Omega^2 \omega_c^2 \tan^2 \theta}. \quad (6)$$

Hence for ω_{\pm} the corresponding eigen wave functions are

$$\begin{aligned} \phi_n^{\omega_+} &= N_n^+ H_n \left(\frac{-(x-x_0) \sin \alpha + z \cos \alpha}{l_+} \right) \\ &\cdot e^{-[-(x-x_0) \sin \alpha + z \cos \alpha]^2 / 2l_+^2}, \end{aligned} \quad (7)$$

$$\begin{aligned} \phi_n^{\omega_-} &= N_n^- H_n \\ &\times \left(\frac{(x-x_0) \cos \alpha + z \sin \alpha}{l_-} \right) \cdot e^{-[(x-x_0) \cos \alpha + z \sin \alpha]^2 / 2l_-^2}, \end{aligned} \quad (8)$$

where $H_n(x)$ is the Hermitian polynomials and $N_n^{\pm} = 1/\sqrt{2^n n! \sqrt{\pi} l_{\pm}}$ is the normalization coefficient.

As we are concerning the lowest Landau level, the degenerate single-particle wave functions are

$$\begin{aligned} \Psi_j &= \frac{e^{iky}}{\sqrt{L_y}} \frac{1}{\sqrt{\pi l_- l_+}} e^{-[(x-x_0) \cos \alpha + z \sin \alpha]^2 / 2l_-^2 - [(x-x_0) \sin \alpha + z \cos \alpha]^2 / 2l_+^2} \\ &\propto e^{-(1/2)j^2 [(l_0^4/l_-^2) k_0^2 \cos^2 \alpha + (l_0^4/l_+^2) k_0^2 \sin^2 \alpha]} \cdot e^{j[ik_0 y + (l_0^2/l_-^2) k_0 \cos \alpha (x \cos \alpha + z \sin \alpha) - (l_0^2/l_+^2) k_0 \sin \alpha (-x \sin \alpha + z \cos \alpha)]} \\ &\cdot e^{-(1/2l_-^2)(x \cos \alpha + z \sin \alpha)^2 - (1/2l_+^2)(-x \sin \alpha + z \cos \alpha)^2} \\ &= e^{-\frac{1}{2}j^2 [(l_0^4/l_-^2) k_0^2 \cos^2 \alpha + (l_0^4/l_+^2) k_0^2 \sin^2 \alpha]} \cdot e^{ju - v^2}. \end{aligned} \quad (9)$$

with

$$\begin{aligned} u &= ik_0 y + \frac{l_0^2}{l_-^2} k_0 \cos \alpha (x \cos \alpha + z \sin \alpha) \\ &- \frac{l_0^2}{l_+^2} k_0 \sin \alpha (-x \sin \alpha + z \cos \alpha), \end{aligned} \quad (10)$$

$$v^2 = \frac{1}{2l_-^2} (x \cos \alpha + z \sin \alpha)^2 - \frac{1}{2l_+^2} (-x \sin \alpha + z \cos \alpha)^2. \quad (11)$$

Since the wave function is localized around x_0 in the x direction, the edge effect can be omitted in the limit of thermodynamics. The many-particle wave function for the filled lowest LL is expressed in the Slater determinant form

$$\begin{aligned} \Phi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N) &= \begin{vmatrix} \Psi_1(\mathbf{r}_1) & \Psi_1(\mathbf{r}_2) & \dots \\ \Psi_2(\mathbf{r}_1) & \Psi_2(\mathbf{r}_2) & \\ \vdots & & \end{vmatrix} \\ &\propto \begin{vmatrix} e^{u_1} & e^{u_2} & \dots \\ e^{2u_1} & e^{2u_2} & \\ \vdots & & \end{vmatrix} \cdot e^{-\sum_k v_k^2} \\ &= \prod_{j < k} (e^{u_j} - e^{u_k}) e^{\sum_k (ik_0 y_k - w_k^2)}, \end{aligned} \quad (12)$$

which simply reduces to a Vandermonde determinant. Here we denote

$$\begin{aligned} w^2 &\equiv [(x - k_0 l_0^2) \cos \alpha + z \sin \alpha]^2 / 2l_-^2 \\ &+ [-(x - k_0 l_0^2) \sin \alpha + z \cos \alpha]^2 / 2l_+^2. \end{aligned} \quad (13)$$

Until now our wave function has been equivalent to that of in the symmetric gauge except an unimportant phase factor. Let us now extend it to the fractional filling states ν

$=1/(2p+1)$. In analogy to Laughlin's analysis in the symmetric gauge in absence of in-plane field, the wave function must satisfy the following conditions: (i) it must be antisymmetric for exchanging any two electrons; (ii) the state $|m\rangle$ is the eigenstate of the momentum in the y direction.

One finds that the unique form of the wave function for FQHE state $\nu=1/m$ ($m=2p+1$ is an odd number) is

$$\Phi_{1/m} = \prod_{j < k} (e^{u_j} - e^{u_k})^m e^{imk_0} \sum_k y_k e^{-\sum_k w_k^2}. \quad (14)$$

The total momentum in the y direction is $K = \frac{1}{2}N(N+1) \cdot mk_0$ with N the number of electrons,

$$\hat{P}_y \Phi_{1/m} = K \Phi_{1/m}. \quad (15)$$

According to J. K. Jain,¹⁶ we rewrite Eq. (14) as

$$\begin{aligned} \Phi_{1/m} &= \prod_{j < k} (e^{u_j} - e^{u_k}) e^{ik_0 \sum_k y_k} \cdot e^{-\sum_k w_k^2} \\ &\cdot \prod_{j < k} (e^{u_j} - e^{u_k})^{2p} \cdot e^{2pik_0 \cdot \sum_k y_k} \\ &= \Phi_1 \cdot \chi_0^{2p}. \end{aligned} \quad (16)$$

Here Φ_1 is the wave function for one filled Landau level. This Jastrow-form wave function can be considered as each electron carries $2p$ flux quanta, therefore we recover Jain's concept of "composite fermions" in the tilted field. In this picture, the electrons "nucleate" an even number of flux to screen enough of the external magnetic field, so that the composite fermions exactly fill an integer number of Landau levels associated with the surplus part of unscreen field ($B^* = B - 2p\phi_0\bar{\rho}$, with $\phi_0 = hc/e$ the unit magnetic quanta

and $\bar{\rho}$ the average particle density). The wave functions of general fractional filling factors ($\nu = n/2pn \pm 1$) are explicitly expressed as

$$\Phi_{n/2pn \pm 1} = \mathcal{P}_{LLL} \Phi_n^* \cdot \chi_0^{2p}, \quad (17)$$

where Φ_n^* is the wave function for composite fermions filling n LL's. The operator \mathcal{P}_{LLL} projects the wave function onto the lowest Landau level.

In summary, we have written a many-particle wave function for fractional quantum Hall states with in-plane magnetic field. We worked in the Landau gauge due to the presence of an in-plane field, where the relative angular momentum between two electrons is not a good quantum number. Haldane¹⁷ had written a many-particle wave function in the torus geometry with the magnetic field perpendicular to the 2DES plane. It is difficult to reproduce Laughlin's wave function in the symmetric gauge in the presence of an in-plane field. Our result is obtained by the same reasoning employed by Laughlin. It shows that fractional quantum Hall states in the lowest Landau level survive even when the 2DES is applied by an in-plane field. Hall plateaus at these filling factors can be observed in experiments. We conclude that the concept of a composite fermion in tilted field is still valid by a way analogous to that of J. K. Jain. It should be noted that the explicit form of the composite fermion is different from that of Jain's.¹⁶ Our result provides a supplementary proof to the explanation for the $\nu=5/2$ state when the external magnetic field is tilted at an angle, where the competition between a paired state of composite fermions and a unidirectional charge-density-wave state leads to the destroy of the pairing gap and the anisotropic transport subsequently takes place.¹⁴

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