## Recurrent-photon feedback in two-dimensional photonic-crystal lasers

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Theoretical studies are carried out for the close-to-lasing two-dimensional finite-size photonic crystals. On the basis of the simulations on the field-intensity distributions and the photon-energy flux distributions, this paper demonstrates the occurrence of a different type of positive feedback of light, which can be called *recurrent-photon feedback*. This feedback mechanism can be construed as an extension of the feedback mechanism in ordinary one-dimensional distributed-feedback lasers.

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Photonic crystals (PCs) provide an intriguing stage for controlling radiation fields. Considerable effort has been devoted to understanding and fabricating them, and exploring their novel applications.<sup>1</sup> However, most studies have focused on inactive PCs made from passive materials. A couple of years ago, in order to enrich PC research, the author proposed two-dimensional (2D) PCs made from active (especially, with respect to gain) materials.<sup>2–6</sup> In this kind of PCs optical gain was found to be greatly strengthened in the vicinity of the photonic band edge.<sup>2,5</sup> Moreover, laser oscillation was theoretically shown to occur in a 2D array of a finite number of gain rods, which did not possess any clear-cut cavity mirrors surrounding the array.<sup>4,6</sup> Experimental results on laser oscillation in these PCs have been reported<sup>7–9</sup> together with a discussion of their thresholds.<sup>6,10</sup>

Although 2D PC lasers have attracted much attention from the PC research community, it appears that the oscillation mechanisms of this kind of lasers are not well understood. It is basically true that the analogy taken from the ordinary 1D distributed-feedback (DFB) lasers may be useful for elucidating the feedback mechanisms in the 2D PC lasers. However, a clear difference exists between them in that the 2D PC lasers create waves propagating in a variety of directions because of the extra dimensionality of their structure while the 1D lasers create only waves in the two opposite directions. The resultant interference between these 2D waves becomes so complicated that the straightforward analogy from the 1D lasers appears to be unworkable. This emphasizes the need for detailed studies on the feedback mechanisms of 2D PC lasers.

This paper presents a possible different type of feedback mechanism on the basis of the numerical simulations on the close-to-lasing 2D PC lasers.

Prior to addressing the obtained results, we here briefly outline the method for calculating the characteristics of the 2D finite-size PC lasers: see the previous paper for some details of calculations.<sup>4,6</sup> We consider a two-dimensional array consisting of a finite number (*N*) of cylindrical rods with the triangular lattice. A single cylindrical gain rod (made from material *A*) is placed at every lattice point, while the background is made from material with no gain. Let us assume that material *A* has the optical amplitude gain  $K''_a$  (the ordinary power gain is given by  $2K''_a$ ).<sup>2–6</sup> Here, we focus on the electric field  $E_z(\mathbf{r})$ , whose polarization direction parallels the longitudinal axis of rods. This paper assumes as a light

source a plane light wave incident from the left side of the PC<sup>6</sup> (i.e., with the incident angle of 0) instead of the oscillating dipoles in the PC.<sup>4</sup> When this light wave enters the PC, it will be scattered by the array of rods and magnify its intensity as it propagates through the gain rods. Monitoring the light emitted from the PC enables us to investigate its lasing characteristics. Namely, we can determine the optical mode  $\omega_m$  for laser oscillation and the threshold amplitude gain  $K''_{am}$  by finding the point in the  $\omega$ - $K''_{a}$  space at which the intensity of the emitted light diverges to infinity.<sup>4,6</sup> These calculations have been carried out analytically with no approximations using Bessel functions.

In the present study, we used GaAs as material A (dielectric constant of 13.18) and the air as the background (1.00). Typically used structural parameters are l=138 nm for the period, f=0.3 for the filling factor of rods, and N=53 for the number of rods. The arrangement of these rods will be seen in Fig. 1(a).

The above calculations revealed many laser oscillation modes near the photonic band edge. For every mode obtained near the top of the first band, we carried out the investigations described below. Among these modes, we focus here on the mode  $\omega_m = 0.2019814420786$  (in the unit of  $2\pi c/l$ ) because it exhibited relatively distinct results. This mode occurred near the edge of the M point in the first Brillouin zone (BZ) and had the threshold amplitude gain of  $K''_{am} = 0.002\,902\,363\,960\,3$  (in the unit of  $2\pi/l$ ). That this mode is typified by laser oscillation is evidenced by the following fact. The light-output intensity increased by a factor of  $3 \times 10^{11}$  at the resonant mode  $\omega_m$  when the amplitude gain  $K''_{a}$  increased from 0 to a value just below  $K''_{am}$  (i.e.,  $K''_{a}$ =  $\eta K''_{am}$ ;  $\eta$ =0.999996), whereas at frequencies detuned slightly  $(\pm 0.1\%)$  the output intensity increased by a factor of only 6.5. Namely, the positive feedback of light occurs exactly at this  $\omega_m$  mode, which magnifies its intensity to nearly infinity within the framework of the linear-gain theory. All subsequent investigations focus on this mode ( $\omega = \omega_m$ ) and the optical amplitude gain  $(K''_a = \eta K''_{am})$  very close to the onset of laser oscillation.

Figure 1(a) shows the field-intensity distribution  $|E_z(\mathbf{r})|^2$ in the real space with mode  $\omega_m$  and amplitude gain  $\eta K''_{am}$ . The rod array is displayed as 53 circles that represent the cylindrical rods. In the colored figures in this paper, the intensity increases in the order blue, white, yellow, red, and black. As clearly shown in Fig. 1(a), the light field concen-



FIG. 1. (Color) (a) Field-intensity distribution  $|E_z(\mathbf{r})|^2$  in the real space with mode  $\omega_m$  and amplitude gain  $\eta K''_{am}$ . The rod array is displayed as 53 circles. The intensity increases in the order blue, white, yellow, red, and black. (b) Angle-resolved far-field intensity under the same conditions as for Fig. 1(a).

trates in several specific gain rods. The presence of repeated nodes and loops in the electric field in the PC suggests that light waves are present in the vertical directions. Actually, the light emitted into the air in upper and lower directions appears to be more intense than in other directions. This is clearly shown in Fig. 1(b), which shows the angle-resolved far-field intensity. According to Fig. 1(b), four **M**-point directions have preferentially far-field intensity and, in particular, two of them (vertical directions) have very strong intensities. The preferential emissions in the **M**-point directions are evidently caused by the enhanced optical gain for the light propagating along these directions.<sup>2,5</sup> Here, the slight deviations from the exact **M**-point directions come from the finite-size PC.

In order to investigate in more detail the light waves within the PC, we determined the Fourier transform of the electric field  $E_z(\mathbf{r})$ . Figure 2 indicates the field-intensity spectrum  $|E_z(\mathbf{g})|^2$  in the reciprocal space for mode  $\omega_m$  and amplitude gain  $\eta K''_{am}$ . The hexagon portrays the first BZ. Here, we should mention some technical issues. In determining the Fourier transform, we first used a real-space area that is 225 times the area shown in Fig. 1(a). This broad-area integration revealed, in addition to the spectra shown in Fig. 2, an intense spectrum band due to the waves elastically scattered by the PC; the circle (circumference) shown by the dotted line indicates the position of its spectrum and  $K_i$  is the wave number of the incident wave. This is caused by the fact that there are many elastically scattered waves outside the PC. However, the waves outside the PC are not important, because they are not involved in the feedback mechanisms of PC lasing. In order to remove the influence of the waves outside the PC, we next determined the Fourier transform in a restricted area. Namely, we assumed that the electric field vanishes outside the smallest circle that surrounds the entire PC. Figure 2 plots the resulting spectrum. The effects of the waves emitted from the PC as well as those of the elastically



FIG. 2. (Color) Field-intensity spectrum  $|E_z(\mathbf{g})|^2$  in the reciprocal space with  $\omega_m$  and  $\eta K''_{am}$  for the waves within the PC. The hexagon portrays the first BZ.  $K_i$  is the wave number of the incident wave and the circle (circumference) shown by the dotted line indicates the position of the spectrum for the waves elastically scattered by the PC.



FIG. 3. (Color) (a) Distribution of the photon-energy flow (Poynting vector) in the real space with  $\omega_m$  and  $\eta K''_{am}$ . The amplitudes of the Poynting vectors are shown by the contours and the vector directions are shown by the small arrows. The rod array is displayed as circles. The two (upward and downward) thick arrows indicate the main streams. (b) Simplified schematic view of the Recurrent-Photon Feedback (RPF) mechanism in 2D PC lasers. (c) Feedback mechanism in ordinary 1D DFB lasers.

scattered waves are eliminated in this diagram. Generally speaking, such kind of Fourier transform includes errors, because the transform tries to generate the spectra for the waves in the entire real space. However, we find no difference between those results obtained with and without the area restriction, except for the difference as to the absence and the presence, respectively, of the spectrum band due to the elastically scattered waves. Hence, we believe that this diagram exactly displays the waves present within the PC.

As clearly shown in Fig. 2, this PC exhibits two significant waves with the similar intensity, which appear to propagate in opposite directions (upper and lower **M**-point directions:  $\theta = \pm 90^{\circ}$ ). These waves can be regarded as the Bloch waves formed by the periodic rod array. The above results could also be foreseen from Fig. 1; this figure confirms that these two counterpropagating waves are the dominant waves at  $\omega_m$  and  $\eta K''_{am}$  in this PC. One might immediately conclude from this fact that a 1D standing wave is formed by the interference of these two waves and moreover that this standing wave formation is the basic origin of the feedback of light in the 2D PC lasers. This may be a natural consequence of drawing an analogy from 1D DFB lasers. In what follows, however, we demonstrate that a very different kind of feedback of light occurs in the 2D PC lasers.

Figure 3(a) shows the distribution of the photon-energy flow (Poynting vector) in the real space with mode  $\omega_m$  and amplitude gain  $\eta K''_{am}$ . The amplitudes of the Poynting vectors are shown by the contours and the vector directions are shown by the small arrows. The rod array is again portrayed as circles. Let us look at the streamlines of energy flow with high intensities [yellow to red regions in Fig. 3(a)]. We find the main streams indicated by the two (upward and downward) thick arrows. These two flows evidently correspond to the most intense waves in Fig. 2. These flows are part of the stream that appears to make a circle in the whole PC. Moreover, on the upper-left side of the PC, we see several small flows that join the main stream. Some light flows out of one stream, and joins another stream or leaves the PC. This kind of complicated pattern of photon-energy flow could not be predicted from Figs. 1 and 2. Actually, according to the standard laser textbook, the simple 1D standing wave in the presence of gain does not generate such a pattern of the photonenergy flow. As clearly shown in Fig. 3(a), we recognize intense upward and downward streams that flow in each separate passage (there are gain rods in this passage). We conclude from this fact that the two dominant waves in Fig. 2 represent primarily separate running waves rather than the components of a standing wave. Other minor waves with various wave-number vectors in Fig. 2 may serve to deflect the vertical main streams and so make a circle.

The lasing modes at other frequencies studied here did not always show the exactly same patterns as shown in Fig. 3(a). They were much more complicated than Fig. 3(a): some modes showed the flow patterns of several complicated loops and some other modes a lot of regularly arranged small vortices. At any rate, the flow pattern common to all modes in this PC was the existence of circular flows of photon energy. The appearance of the one-directional main flow shown in Fig. 3(a) may be caused by the lack of the symmetry of the PC structure. Namely, the degeneracy of the modes in the symmetric PC is lifted by introducing the structural asymmetry, and as a result the flow in one direction comes into existence. Actually, in the PC structure with the hexagonal symmetry, we found no circular photon-energy flows and its feedback mechanism was basically the same as that of the ordinary 1D lasers.

On the basis of the above simulations, we here propose a model for the feedback of light in the 2D PC lasers: *Recurrent-Photon Feedback* (RPF). Figure 3(b) shows an ex-

- <sup>1</sup>See, for example, *Photonic Band Gaps and Localization*, edited by C. M. Soukoulis (Plenum, NY, 1993); J. D. Joannopoulos, R. D. Meade, and J. N. Winn, *Photonic Crystals* (Princeton University Press, Princeton, NJ, 1995).
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tremely simplified schematic view of the RPF mechanism. In this model, the photon energy makes a circular flow as a result of the reflection by the surrounding interfaces (we symbolically denote them as virtual mirrors); the light that flows out of this circle enters the air. By continuing to flow in this circle, the photon stores optical energy in the resonator (PC) and increases the *O* value of the resonator. This is somewhat similar to the oscillation of the so-called whispering-gallery-mode lasers. The above model is not very odd, because it can be regarded as an extension of the feedback model for ordinary 1D DFB lasers. In DFB lasers, the two counterpropagating 1D Bloch waves form a standing wave [Fig. 3(c)]. The RPF mechanism may be construed as follows: the two 1D waves on the identical passage in the DFB laser come to possess the each separate passage in the 2D PC laser as a result of being given an extra dimensionality.

In conclusion, we have theoretically demonstrated the occurrence of a different type of feedback (recurrent-photon feedback) in 2D PC lasers. A variety of feedback mechanisms seem to exist depending on the PC structures, the photonic bands, and the optical modes, because 2D PC lasers have higher dimensionality than ordinary 1D lasers. We believe that the study of these feedback mechanisms will provide an interesting field in active-PC research and also prove to be useful for developing new PC lasers.

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