Nonequivalence of chemical and hydrostatic pressures: A_{1g} and E_g frequencies and Stokes shift of Mn²⁺-doped perovskites

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The frequency of A_{1g} and E_g modes of MnF_6^{4-} in cubic ABF_3 (*A*: K; B: Mg, Zn and *A*: Rb, Cs; *B*: Cd, Ca) perovskites has been derived through density functional calculations on $MnF_6A_8B_6^{16+}$ clusters which reproduce the experimental impurity-ligand distance R_e . Both frequencies are found to experience a drastic decrement on passing from $KMgF_3:Mn^{2+}$ $(\hbar \omega_A = 556 \text{ cm}^{-1}, \ \hbar \omega_E = 456 \text{ cm}^{-1})$ to CsCdF₃:Mn²⁺ $(\hbar \omega_A$ $=$ 317 cm⁻¹, $\hbar \omega_E$ = 239 cm⁻¹) despite the fact that R_e increases *only* by 5%, leading to *effective* Grüneisen constants (called γ_A^c and γ_E^c) around 3.0 along the series. This figure is 60% higher than the usual Grüneisen constant γ_A calculated for a *given system* like KMgF₃:Mn²⁺ or KZnF₃:Mn²⁺ when hydrostatic pressure is applied. For supporting this relevant result the value of γ_A for $\rm CrF_6^{3-}$ in a fluoroelpasolite has been calculated as well. The obtained value $\gamma_A = 2.1$ is close to the experimental one ($\gamma_A = 1.9$) derived in K₂NaGaF₆:Cr³⁺ by combined optical and Raman measurements. As a salient feature, the increase of the Stokes shift when *Re* increases observed along the ABF_3 : Mn^{2+} series is now well explained through the bigger variations of A_{1g} and E_g frequencies induced by the chemical pressure in comparison to those coming from an hydrostatic pressure on a given system. The difference between γ_A^c and γ_A is discussed through a simple model that emphasizes the role played by the coupling of the MnF_6^{4-} complex to the lattice. The influence of chemical and hydrostatic pressures upon the luminescence efficiency is also briefly discussed. Attention is addressed to the method of calculating impurity-associated Gruneisen constants using clusters. Recent results on Cr^{3+} in several cubic chloroelpasolites that support the present conclusions are briefly discussed as well.

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I. INTRODUCTION

The presence of a transition-metal (TM) impurity in an insulating material leads to the appearance of physicochemical properties which are absent in the pure host lattice and may be useful for applied devices such as solid-state lasers, storage phosphors, etc. Despite a doped material being certainly more complex than a pure one, it was first pointed out by Sugano and Shulman¹ that the *electronic properties* due to an impurity in an *insulating* lattice can be understood to a good extent *only* on the basis of the ML_N complex formed by the impurity M and the N nearest anions.² This relevant conclusion means that, in thermodynamic equilibrium, a pure electronic parameter P_{el} related to the impurity (like the energy of an optical transition or the g factor) depends, in a good first approximation, only on the equilibrium impurityligand distance R_e as well as on the temperature *T*. This idea is correct provided (i) the active electrons of the complex are localized inside the ML_N complex and (ii) the electrostatic potential V_R , due to the rest of the lattice upon the active electrons, is essentially flat inside the complex region. Both conditions are fulfilled for divalent and trivalent TM impurities placed substitutionally in cubic insulating lattices.³ When the TM impurity is located in sites of lower symmetry, it is, however, necessary to include the effects of V_R for a proper understanding of the center.⁴

If apart from temperature a parameter P_{el} only depends on R_e , ΔP_{el} variations can thus be obtained using either a hydrostatic pressure or a change of the host lattice (chemical pressure) producing the *same* variation ΔR_e in the impurityligand distance. The equivalence between a hydrostatic and a chemical pressure has been demonstrated experimentally as far as electronic properties are concerned. For instance, the dependence of 10*Dq* upon *Re* has been measured experimentally either by means of a hydrostatic pressure on a given doped lattice or through different host lattices containing the *same* ML_N complex.⁵⁻¹¹

Writing

$$
10Dq = KR_e^{-n},\tag{1}
$$

it turns out that *similar* values of the exponent *n* are derived through *both* procedures.

As regards the local vibrations associated with a given impurity, they cannot, however, be understood *only* in terms of the ML_N complex. For instance, in a σ mode of vibration the ligands do *also* interact with the first host lattice cations (placed along the metal-ligand directions) whose nature is only modified when the host lattice changes. Therefore the effects of the chemical pressure on a ML_N complex are not necessarily equivalent to those arising from the hydrostatic pressure on a given doped compound when the vibrations associated with an impurity are concerned. This idea can also be of importance in the case of parameters (such as the Stokes shift or the Huang-Rhys factors) depending on the coupling between an excited electronic state and the local vibrations.12–17 Along this line the effects of both kind of pressures upon the luminescence quenching also deserve an investigation.

Searching to explore these attractive questions, the variation of the A_{1g} and E_g frequencies associated with the

 $\text{MnF}_6{}^{4-}$ complex in cubic ABF_3 ($B = \text{divalent element}$) fluoroperovskites is derived in the present work through density functional theory (DFT) calculations on clusters centered at Mn^{2+} . Results for $KBF_3: Mn^{2+}$ (*B*: Mg, Zn) and ABF_3 : Mn²⁺ (A: Rb, Cs; *B*: Cd, Ca) are reported. For comparison purposes the effects of a hydrostatic pressure on the systems $\widehat{KMgF_3}$: Mn^{2+} and $KZnF_3$: Mn^{2+} are also explored.

The ensemble of *cubic* fluoroperovskites doped with Mn^{2+} can be considered as *model systems*^{18–20} where the impurity-ligand distance has been determined through *three* independent methods.^{2,22,6} Moreover, recent DFT calculations on 21-atom clusters²³ lead to values of the equilibrium impurity-ligand distance R_e , which are coincident, within experimental uncertainties, with those derived from electron paramagnetic resonance $(EPR),^{22}$ optical,⁶ and extended x-ray absorption fine structure $(EXAFS)^{21}$ data at ambient pressure.

Experimental information on vibration modes coupled to the first excited state of MnF_6^{4-} in fluoroperovskites comes from the low-temperature optical spectra.²⁴ For instance, in $KMgF_3: Mn^{2+}$ a mode of 570 cm⁻¹ frequency has been revealed in the high-resolution emission spectrum. Although that frequency is close to that of the $LO₃$ mode of the pure lattice, that mode has been associated²⁵ with a local or resonant mode because its value is shifted to 540 cm^{-1} in the absorption spectrum.

The substitution of a host lattice by another one (chemical pressure on the complex) leads to variations of both ω_i (*i* $=$ *A*,*E*) and *R_e* parameters (termed as $\Delta_c \omega_i$ and $\Delta_c R_e$, respectively), and thus it allows one to define an *effective* Grüneisen parameter γ_i^c through the expression

$$
3\,\gamma_i^c = \frac{\Delta_c \omega_i}{\Delta_c R_e} \frac{R_e}{\omega_i}.\tag{2}
$$

Comparison of this *effective* constant γ_i^c with the actual Grüneisen parameter γ_i corresponding to *a given* doped compound can thus be used for describing in a simple way the differences between chemical and hydrostatic pressures.

Particular attention is paid through this work to the variation of the Stokes shift (E_S) associated with the ⁴ $T₁ \rightarrow {}^{6}A₁$ luminescent transition of MnF_6^{4-} along the fluoroperovskite series that has been studied experimentally.¹⁹ Such experimental results show that E_S increases as far as the actual value of R_e increases along the series. Moreover, an analysis of such results indicates that the average value of γ_i^c (*i* $=$ *A*,*E*) along the series can be around 3 indeed. This figure is higher than the *usual* values of γ _i measured for a given compound. In this sense the *average* $\bar{\gamma}$ value derived from thermal expansion data^{26,27} for pure KZnF₃ and CsCaF₃ is equal to 1.1. Moreover, the Grüneisen parameters associated with the local A_{1g} and E_g modes of $K_2N_4GaF_6$: Cr^{3+} have been derived through combined optical and Raman measurements under pressure.²⁸ The obtained results (γ_A = 1.85 and γ_E =2.1) point out that even in the case of the stretching modes the Grüneisen parameters are clearly smaller than 3. Therefore, for supporting the study carried out on MnF_6^{4-} in

FIG. 1. Cluster of 21 atoms used in the ADF calculation of ABF_3 :Mn²⁺ systems.

fluoroperovskites, the Grüneisen constant γ_A for CrF₆³⁻ in the elpasolite lattice K_3 CrF₆ has been calculated in the present work as well.

II. THEORETICAL

The properties due to Mn^{2+} -doped fluoroperovskites have been calculated by means of finite clusters. As the boundary condition, the surface atoms of the cluster are fixed at their experimental host lattice positions. This procedure provides a reasonable insight into the properties associated with an *impurity* provided (i) the active electrons are localized in a small region compared to the cluster size and (ii) impurityinduced local relaxations are important only in a domain whose size is again smaller than that of the cluster. For the present work $\text{MnF}_6A_8B_6^{16+}$ clusters (Fig. 1) have been used. As previously discussed, 23 this cluster reasonably fulfills the two conditions for the six fluoroperovskites. In particular, less than 5% of the charge associated with unpaired electrons is found to reside outside the MnF_6^{4-} complex

Calculations of the ground-state energy for different values of the metal-ligand distance *R* have been carried out using the Amsterdam density functional (ADF) code²⁹ and the generalized gradient approximation (GGA) by means of the Becke-Perdew functional.^{30,31} The effects of the electrostatic potential due to the rest of the ions not included in the $\text{MnF}_6A_8\text{M}_6^{16+}$ cluster have been considered in all the calculations. The basis set of the best quality in the ADF code has been employed. More details can be found elsewhere.²³

It has recently been shown that using $\text{MnF}_6A_8B_6^{16+}$ clusters the obtained R_e values for the series of Mn^2 ⁺-doped fluoroperovskites coincide, within the experimental uncertainties, with those derived from the analysis of the experimental isotropic superhyperfine constant *As* , the 10*Dq* parameter, and available EXAFS results. 23 For calculating the local A_{1g} and E_g frequencies through the $MnF_6A_8B_6^{16+}$ cluster, it has been assumed, in a first approximation, that only the ligand displacements are involved in the corresponding normal coordinates. This assumption is quite reasonable in the case of elpasolite lattices (where A_{1g} and E_g modes of the complex are also¹³ lattice modes at $\mathbf{k}=0$), but can only be taken as a first step in the case of perovskites. A description of A_{1g} and E_g modes is given in Ref. 32.

In the self-consistent solution of Kohn-Sham equations³³ the initial potential used in the *n* step, V_n^i , leads to a final potential V_n^f once such equations are solved. The initial potential V_{n+1}^i employed in the $n+1$ step is taken as

$$
V_{n+1}^i = (1 - \varepsilon) V_n^i + \varepsilon V_n^f, \tag{3}
$$

where the mixing coefficient ε is usually taken equal to 0.2. In the calculations on $KBF_3 : Mn^{2+} (B: Mg, Zn)$ and $ACdF_3$: Mn^{2+} (A: Rb, Cs) the convergence was perfect and reached for an error parameter of the ADF code equal to 10^{-6} . Problems were found however, to achieve a proper convergence in the case of $ACaF_3$: Mn^{2+} (A: Rb, Cs) systems. In these two systems convergence was obtained using an error parameter bigger than 30×10^{-6} and ε lying between 0.06 and 0.35. Moreover, for $R \le R_e$ convergence was reached only when $R_e - R \leq 8$ pm. This situation could be related to the presence of an a_1 "ghost orbital" lying in the antibonding $3d$ orbital region of Mn^{2+} as pointed out by other workers.34,35 In view of these facts the present analysis is based mainly on results obtained for the $KBF_3 : Mn^{2+}$ (*B*: Mg, Zn) and $ACdF_3$: Mn^{2+} (A: Rb; Cs) systems. The main trend emerging from these results is, however, followed by those coming from calcium perovskites as indicated later.

For the whole analyzed systems, we have verified that the fitting of the $E(R)$ curve by polynomials of different degree leads essentially to the *same* values of second and third derivatives at $R = R_e$. Here *R* is the fluoride coordinate along the $\langle 100 \rangle$ directions.

The volume dependence of a vibrational frequency ω_i of a given system is quantified through the associated Grüneisen parameter γ _{*i*}. For an octahedral molecule the volume is just determined by the metal-ligand distance R_e , and then γ_i is defined by the relation

$$
\gamma_i = \frac{\partial L \omega_i}{\partial LV} = \frac{1}{3} \frac{\partial L \omega_i}{\partial LR_e}.
$$
\n(4)

Thus the Grüneisen parameter γ_i plays a key role for understanding the effects of an hydrostatic pressure upon ω_i . Attention has to be paid, however, to the calculation of γ_i (*i* $=$ *A*,*E*) for MnF₆^{4 –} embedded in a lattice. To get an insight into γ_i ($i = A, E$) by means of calculations on a cluster, one has to determine the changes experienced by R_e and the ω_i $(i=4, E)$ when the cluster size is modified, but its geometry is frozen. In the cluster depicted in Fig. 1 the size is controlled by the distance R_S between the impurity and six $B²⁺$ ions lying in $\langle 100 \rangle$ directions. In the perfect host lattice at zero pressure, R_S is equal to the perovskite lattice parameter *a*. When the cluster size is changed, for a given R_S value the position of the 14 ''surface'' ions of the cluster is taken to be that corresponding to a perovskite lattice with $a=R_S$. Through this approximation, the ground state energy *E* of the cluster depends only on the R and R_S *variables*. From the $E(R, R_S)$ function the equilibrium metal-ligand distance R_e or the ω_A frequency can be derived for *each value* of the

cluster size. At *a given* R_S value, termed R_S^e , the equilibrium *Re* distance is just determined by the condition

$$
\left(\frac{\partial E(R, R_S = R_S^e)}{\partial R}\right)_{R=R_e} = 0
$$
\n(5)

and then the force constant k_A and frequency ω_A , *corresponding* to a given equilibrium distance R_e , are just given by

$$
k_A = 6M_L\omega_A^2 = \left[\frac{\partial^2 E(R; R_S = R_S^e)}{\partial R^2}\right]_{R=R_e}
$$
 (6)

where M_L means the mass of one of six ligand ions. Once the value of ω_A for different R_e distances is known, γ_A is easily derived. At zero pressure, R_S^e is taken equal to the experimental value in the perfect host lattice. For the present systems this assumption is reasonable as the relaxation of the ligand shell is smaller than 4.5% and, as discussed in Ref. 23, an estimation based on the theory of elasticity leads to a relative variation of the third neighbors distance smaller than 0.7%.

A similar procedure to that followed for MnF_6^{4-} in fluoroperovskites has been used for calculating γ_A in the case of CrF_6^{3-} in the K_3CrF_6 elpasolite lattice where two close complexes do not share any common ligand. According to a previous work,³⁶ calculations are carried out on a $\text{CrF}_6\text{K}_8\text{K}_6^{11+}$ cluster centered at the chromium ion.

It is worth noting that a different (and, in general, incorrect) value of γ_A is obtained when the surface atoms are kept fixed at a distance R_f (which can correspond to the perfect host lattice) from the central cation and only the ligands are allowed to follow an external pressure. $37,36$ The constant calculated by this *artificial* procedure will be hereafter denoted by $\gamma_A(f)$. The difference between $\gamma_A(f)$ and γ_A is discussed in Sec. III D.

III. RESULTS AND DISCUSSION

A. Vibrational frequencies for MnF6 ⁴^À in perovskites

The *R* dependence of the ground-state energy, *E*(*R*), obtained for the $MnF_6K_8Mg_6^{16+}$ cluster in $KMgF_3$ and the $\text{MnF}_6\text{Cs}_8\text{Cd}_6^{-16+}$ cluster in CsCdF₃ are both portrayed in Fig. 2. It can be noticed that the curvature around the minimum at R_e =2.057 Å for the former system is much higher than that corresponding to the latter one whose minimum appears at R_e =2.156 Å.

The values of calculated ω_A and ω_E frequencies for $KBF_3: Mn^{2+}$ (*B*: Mg, Zn) and $ACdF_3: Mn^{2+}$ (*A*: Rb, Cs) together with the corresponding equilibrium distances are displayed in Table I. For comparison purposes the R_e values derived 22 from the experimental isotropic superhyperfine constant A_s are also shown. As previously pointed out,²³ the calculated R_e values by means of 21-atom clusters agree with those derived from the analysis of experimental data within experimental uncertainties.

The reported values in Table I indicate that the replacement of a fluoroperovskite lattice by another one leads to

FIG. 2. Calculated ground-state energy for $KMgF_3: Mn^{2+}$ and $CsCdF_3$: Mn^{2+} for different values of the $Mn^{2+}-F^-$ distance *R*. Note that the origin of the *R* coordinate corresponding to $CsCdF₃:Mn²⁺$ is shifted by 0.10 Å with respect to that for $KMgF_3:Mn^{2+}.$

remarkable changes of ω_A and ω_E frequencies. For instance, R_e for CsCdF₃:Mn²⁺ is found to be only 5% higher than that for KMgF₃:Mn²⁺, while the associated ω_A frequency is calculated to be 43% smaller. A similar variation is found for the Jahn-Teller frequency ω_E . Systems for which R_e $>$ 2.12 Å are found to possess ω_A and ω_E values much smaller than those corresponding to KBF_3 : Mn^{2+} (*B*:Mg, Zn). This trend is not altered by the results on $RbCaF_3$: Mn^{2+} (R_e =2.125 Å) and $CsCaF_3$: Mn^{2+} (R_e $=$ 2.155 Å) obtained, however, under worse technical conditions. In both cases $\hbar \omega_A$ would lie in the 310–350 cm⁻¹ domain.

Comparing the results displayed in Table I for $CsCdF_3: Mn^{2+}$ and $KMgF_3: Mn^{2+} \gamma_A^c = 3.2$ and $\gamma_E^c = 2.85$ values are derived. This result already points out that the effective Grüneisen parameters γ_A^c and γ_E^c can attain values very close to 3 indeed.

The calculated variation of ω_A along the series of ABF_3 : Mn^{2+} systems is certainly bigger than that experienced by the $LO₃$ frequency of the corresponding host lattice.²⁷ For instance, the LO_3 frequency for $KMgF_3$ and

 $CsCaF₃$ is equal to 551 and 449 cm⁻¹, respectively, which means a variation of *only* 18%.

For the studied systems in Table I the frequency ω_A of the symmetric mode is found to be bigger than that of the Jahn-Teller mode, which also exhibits a stretching character. Qualitatively, this trend can simply be understood looking at the vibrations in a $ML₆$ molecule assuming two-body central forces.^{38,39} Let us consider the Q_{ϵ} ($\sim x^2 - y^2$) coordinate of the E_g mode. The two adjacent ligands placed initially at $(R_e, 0, 0)$ and $(0, R_e, 0)$ are moved to $(R_e + \delta, 0, 0)$ and $(0, R_e, 0, 0)$ $-\delta$,0), respectively. Thus the distance between both ions is kept *in first order* if $|\delta| \ll R_e$ at variance with what happens for the A_{1g} mode. Therefore, there is no contribution to ω_E from this ligand-ligand force constant.

Vibrations associated with the impurity are revealed²⁴ in high-resolution luminescence spectra of $KMgF_3$: Mn^{2+} and $KZnF_3$: Mn^{2+} . In both cases a clear replica of a mode whose frequency is 570 cm⁻¹ for KMgF₃:Mn²⁺ can be observed. In view of the results of Table I, that mode could be ascribed in a first approximation to the symmetric vibration of $F^$ ligands. On passing from $KMgF_3:Mn^{2+}$ to $KZnF_3:Mn^{2+}$ the experimental frequency of such a mode experiences a 30 -cm⁻¹ redshift which is smaller than that calculated in Table I. It is worth noting that the increase in R_e (termed ΔR_e) on going from KMgF₃:Mn²⁺ to KZnF₃:Mn²⁺ is calculated to be equal to 3 pm. This figure is consistent with *Re* values derived the experimental *As* provided the *experimental uncertainties* are taken into account. If ΔR_e is actually closer to 1 pm, one would expect a redshift for $\hbar \omega_A$ around 30 cm⁻¹ when $KMgF_3$ is replaced by $KZnF_3$.

No direct measurements have been carried out in the case of the Jahn-Teller frequency ω_E . The calculated $\hbar \omega_E$ values in Table I exhibit, however, a trend which is certainly comparable to the estimations reported^{12,24} for KMgF₃:Mn²⁺ $(R_e=207 \text{ pm}, \quad \hbar \omega_E=390 \text{ cm}^{-1})$ and RbMnF₃ $(R_e$ $=$ 212 pm, $\hbar \omega_E$ = 280 cm⁻¹).

B. Grüneisen constant γ_A for KBF_3 : Mn^{2+} (B :Mg, Zn) and K_3 CrF₆

Changes of the A_{1g} frequency of *a given* $KBF_3:Mn^{2+}$ system (*B*:Mg, Zn) produced by a modification of the equilibrium distance have also been studied. The main results collected in Fig. 3 indicate that $d\omega_A/dR_e$ is negative and thus γ_A is positive. This situation is usually found for stretching modes of molecules and crystals.^{40–42} Acoustic bending modes in silicon lead, however, to negative values

TABLE I. A_{1g} and E_g vibration frequencies for Mn^{2+} -doped fluoroperovskites. The equilibrium distances obtained from ADF calculations and from the experimental *As* value are also included.

Lattice	Lattice parameter (\AA)	Equilibrium distance R_e (Å)		Frequencies $\text{(cm}^{-1})$	
		From ADF calc.	From expt. A_s	A_{1g} mode ω_A	E_o mode ω_F
$KMgF_3$	3.973	2.057	2.07	556	456
KZnF ₃	4.054	2.086	2.08	454	387
RbCdF ₃	4.400	2.127	2.12	353	285
CsCdF ₃	4.464	2.156	2.14	317	239

FIG. 3. Variation of $\hbar \omega_A$ along the series of cubic fluoroperovskites doped with Mn^{2+} . Calculated hydrostatic pressure effect on $\hbar \omega_A$ for KBF₃:Mn²⁺ (*B*=Mg,Zn) systems is also showed for comparison.

of the Grüneisen constant, $4^{1,42}$ which is in turn responsible for the negative thermal expansion coefficient below 150 K.

As a salient feature, it can be noticed that for the same ΔR_e value the variation $\Delta \hbar \omega_A$ obtained for a given compound doped with Mn^{2+} under hydrostatic pressure is certainly smaller than through the replacement of the host lattice. More precisely, the Grüneisen constant γ_A is found to be practically equal to 1.8 for both $KMgF_3:Mn^{2+}$ and $KZnF_3$: Mn²⁺ systems. It is worth noting that using the same procedure as for $KMgF_3:Mn^{2+}$ and $KZnF_3:Mn^{2+}$ the Grüneisen constant γ_A for CrF_6^{3-} in the K₃CrF₆ elpasolite has also been calculated. The obtained value $\gamma_A = 2.1$ is close to the experimental one (γ_A =1.9) derived by Dolan *et al.*²⁸ in K_2NaGaF_6 : Cr^{3+} . It is worthwhile to remark that if γ_A is derived from the calculated potential curve for the cluster with the six surface ions *fixed* at the lattice positions at zero pressure, then it is found that $\gamma_A(f) = 1.1$. Why the γ_A value obtained through a reasonable calculation is higher than $\gamma_A(f)$ is analyzed in Sec. III. D.

In the present calculations on $KMgF_3:Mn^{2+}$ or $KZnF_3$: Mn^{2+} , it is found that when the cluster size is changed by a quantity ΔR_s , then the relative variation undergone by the metal-ligand distance is given by $\Delta R_e/R_e$ $= b(\Delta R_s / R_s)$, where $b = 0.87$. This figure means that ligands follow the motion of outer ions rather closely. In the case of the K_3CrF_6 elpasolite, *b* is found to be 0.4, suggesting that the local compressibility of CrF_6^{3-} is bigger than that of the whole lattice. A similar situation has been found 43 in MgO: Cr^{3+} .

C. Stokes shift for Mn2¿-doped fluoroperovskites

The luminescence due to MnF_6^{4-} complexes in fluorides comes only^{44,12,6} from the first excited state ${}^4T_{1g}$. The cor-

FIG. 4. Calculated Stokes shift values for two different γ values, 1.8 $(B \text{ line})$ and 3 $(C \text{ line})$, respectively. Experimental E_S values for different host lattices are also shown for comparison. \blacklozenge : KMgF₃. \blacktriangle : KZnF₃. ∇ : RbCdF₃. **i**: RbCaF₃. \blacktriangleright : CsCaF₃.

responding Stokes shift E_s strongly depends upon the ω_i (*i* $=$ *A*,*E*) frequencies.^{14,15} Therefore, one can expect that the big difference between γ_A and γ_A^c obtained through the present calculations plays an important role for explaining the *variation* of E_s measured through the series of cubic fluoroperovskites doped with Mn^{2+} . As shown in Fig. 4, the experimental Stokes shift¹⁹ increases as far as R_e increases in the fluoroperovskites series.

Theoretically, the Stokes shift for d^5 or d^3 ions placed in lattices like elpasolites or perovskites can simply by written $13,15$ as

$$
E_S = E_S(A) + E_S(E) \tag{7}
$$

where $E_s(A)$ and $E_s(E)$ denote the contribution arising from the coupling of the first electronic excited state with the symmetric and the Jahn-Teller mode, respectively. The expression of $E_s(i)$ ($i=A,E$) in terms of the corresponding coupling constant, V_i , and the ω_i ($i = A, E$) frequency is formally the same for both modes: $45,12,15$

$$
E_S(i) = \frac{V_i^2}{M_L \omega_i^2} = 2S_i \hbar \omega_i,
$$
\n(8)

where S_i is the associated Huang-Rhys factor. As to V_i for the ${}^{4}T_{1g}$ state of MnF₆⁴⁻, it has been shown^{15,14} that its dependence on R_e around 212 pm can be described by

$$
V_i = C_i R_e^{-n_i}.\tag{9}
$$

The exponents n_A and n_E are found to be very similar¹⁵ for MnF_6^{4-} as well as for CrF_6^{3-} . Moreover, $n_A = n + 1$, where the exponent *n* reflects the dependence of 10*Dq* upon *Re* through Eq. (1) . Microscopically, the strong dependence of $10Dq$ upon R_e in complexes like MnF₆⁴⁻ has been shown to be related to the $3d-2s(F)$ hybridization in the antibonding e_g ($-x^2-y^2,3z^2-r^2$) orbital.^{46,47}

On these bases, the *Re* dependence of *Es* found experimentally¹⁹ has to be related to the variations of frequencies along the series and also to the R_{ρ} dependence of the coupling constants V_A and V_E . Calculated values of E_s as a function of R_e by means of Eqs. (6) – (8) are also included in Fig. 4. According to the analysis performed in Ref. 15, the employed values of V_A and V_E for $KMgF_3:Mn^{2+}$ $(R_e = 206 \text{ pm})$ are $V_A = 95 \text{ cm}^{-1} \text{/pm}$ and V_E $=75$ cm⁻¹/pm. As to the exponents n_A and n_E involved in the R_e dependence of E_s , the values $n_A = 6$ and $n_E = 5.5$ are used.¹⁵ The frequencies for $KMgF_3$: Mn^{2+} are taken equal to $\hbar \omega_A$ =550 cm⁻¹ and $\hbar \omega_E$ =450 cm⁻¹. For simulating their variation along the fluoroperovskite series, it has first been assumed that γ_A^c and γ_E^c are equal to the Gruneisen constant γ_A = 1.8 calculated for KBF₃:Mn²⁺ (*B*:Mg, Zn) systems. It can be seen in Fig. 4 that the calculated values of E_s derived from the latter assumption fail to reproduce the trend experimentally observed.¹⁹ In fact, the derived E_s values, obtained assuming $\gamma_A^c \approx \gamma_E^c = 1.8$, do not give rise to a significant increase of the Stokes shift when *Re* increases, but rather to a slow decrement. As shown also in Fig. 4, this big discrepancy is, however, overcome by putting $\gamma_A^c \cong \gamma_E^c = 3$, as derived from the present calculations in Sec. III A. This relevant result can easily be rationalized through Eqs. (7) and (8) , implying that a contribution like $E_s(A)$ should be proportional to $R_e^{p_A}$ where the exponent p_A is given by

$$
p_A = 6\gamma_A^c - 2n_A \tag{10}
$$

if the host lattice chemical pressure is modified, while

$$
p_A = 6\gamma_A - 2n_A \tag{11}
$$

when a hydrostatic pressure is applied on a given compound. Therefore, if $n_A \approx 6$ and $\gamma_A = 1.8$, then $p_A = -1.2$, which means that an applied hydrostatic pressure on a system like $KMgF_3: Mn^{2+}$ would lead to a *slow increase* of $E_s(A)$. By contrast, when the chemical pressure along the fluoroperovskite series is varied it turns out that the calculated value γ_A^c $=$ 3 leads to $p_A \cong 6$ and thus to an important increase of E_s when *R* increases. This agreement can thus be viewed as an experimental proof, though indirect, of the conclusions in Secs. III A and III B. Therefore, for MnF_6^{4-} in fluoroperovskite lattices, the changes in A_{1g} and E_g frequencies due to the chemical pressure appear to be bigger indeed than those coming from an hydrostatic pressure on a given compound.

The results gathered in Fig. 4 also predict that the Stokes shift of a system like $KMgF_3:Mn^{2+}$ would be little affected by an applied hydrostatic pressure. Further experimental work is desirable to explore this prediction.

FIG. 5. Simple model representing three atoms $(M,$ impurity metal atom: *L*, ligand atom; and *S*, "surface atom") connected by two springs with parameters (see text) k, g and k', g' , respectively. x_i ($i=M,L,S$) represents the position of atom *i*.

D. Difference between γ_A^c , γ_A , and $\gamma_A(f)$: Insight through **a simple model**

The main differences between the three quantities γ_A^c , γ_A , and $\gamma_A(f)$ discussed in this work can be rationalized by means of a simple unidimensional model described in Fig. 5. In it, the atom at the origin representing the impurity is joined by an anharmonic spring to the ligand *L*, which in turn is joined to the surface atom by a different spring. The potential energy of the system including an external force acting on the surface atom is simply written as

$$
E = \frac{1}{2}k(x_L - x_M - R_0)^2 + \frac{1}{2}k'(x_S - x_L - R_0')^2 + Fx_S
$$

$$
-\frac{1}{6}g(x_L - x_M - R_0)^3 - \frac{1}{6}g'(x_S - x_L - R_0')^3. \tag{12}
$$

Here x_L and x_S mean the position of the ligand and surface atom, respectively, while the position of the impurity atom is fixed at $x_M=0$. The constants *k* and *g* (*k'* and *g'*) are involved in the harmonic and anharmonic terms of the first (second) spring, while R_0 and R'_0 are the equilibrium positions associated with the two isolated springs. For each value of the force *F*, the equilibrium x_L^e and x_S^e values are determined in a first approximation by the conditions

$$
k'(x_S^e - x_L^e - R'_0) + F \approx 0,\t(13)
$$

$$
k(x_L^e - R_0) - k'(x_S^e - x_L^e - R_0') \approx 0.
$$
 (14)

The force constant k_A associated with such an equilibrium position is given by

$$
k_A = \left(\frac{\partial^2 E}{\partial x_L^2}\right)_{\substack{x_L = x_L^e = k + k' - (g + g' \frac{k}{k'})}} \left(s + g' \frac{k}{k'}\right) (x_L^e - R_0). \quad (15)
$$

Therefore the change Δk_A experienced by the force constant k_A when x_L^e is varied is just given by

$$
\Delta k_A = -\left(g + g' \frac{k}{k'}\right) \Delta x_L^e. \tag{16}
$$

In the calculation of $\gamma_A(f)$ the only variable is x_L and the force *F* has to be applied on the atom *L* in order to modify x_L^e while the atom *S* is kept fixed at the perfect host lattice position termed x_S^h . In such a case Eq. (13) is not valid and the involved variation of k_A termed $\Delta k_A(f)$ is simply given by

$$
\Delta k_A(f) = -(g - g') \Delta x_L^e. \tag{17}
$$

This fact stresses that if $\Delta x_L^e > 0$, then the anharmonic energy of the first spring decreases while that of the second one increases. Comparison between Eqs. (16) and (17) clearly shows that for the same Δx_L^e variation $\Delta k_A(f) < \Delta k_A$ provided g' is not negligible.

Let us now focus on the variation undergone by k_A (called $\Delta_c k_A$) when the host lattice is modified. As a first approximation, we can assume that relaxation effects are important only for the ligand shell, while the surface atom remains at its host lattice position x_S^h . In this case the value of

$$
k_A = \left(\frac{\partial^2 E}{\partial x_L^2}\right)_{\substack{x_L = x_L^e\\ x_S = x_S^h}}
$$

for each lattice is also given by Eq. (14) . Therefore, if the host lattice is changed, then

$$
\Delta_c k_A = \Delta_c k' - g \Delta_c \left\{ \left(1 + \frac{g'}{g} \frac{k}{k'} \right) (x_L^e - R_0) \right\},\qquad(18)
$$

which means that *even in the harmonic approximation* the replacement of a host lattice by another one can lead to variations of the force constant k_A as k' is modified. If the complex (simbolized by atoms M and L) is, however, only weakly coupled to the rest of the lattice and thus $k' \le k$, it is possible that, for the same Δx_L^e , $\Delta_c k_A$ is comparable to Δk_A defined in Eq. (16). This would occur when $\Delta_c k' < g \Delta x_L^e$ assuming that $g'k/(gk')$ can be taken as a constant. A simple analysis of experimental $Mn^{2+}-F^-$ distances in fluoroperovskites through Eq. (14) leads, however, to k'/k ≈ 0.5 , pointing out the existence of a significant coupling between the complex and the lattice for the present cases. In the case of alkali halides or perovskites, k' decreases^{26,27} when the lattice parameter *a* increases. As x_L^e also increases following a , it turns out that the two contributions in Eq. (18) have the same sign and thus $\Delta_c k_A > \Delta k_A$.

E. Quenching of the luminescence in Mn2¿-doped fluorides

The influence of chemical or hydrostatic pressures upon the luminescence efficiency deserves some discussion. Since the first work by Dexter *et al.*, ⁴⁸ it is known that the luminescence quenching is partially governed by the Λ parameter defined by

$$
\Lambda = \frac{E_s}{2E_G},\tag{19}
$$

where E_G means the energy difference between the emitting first excited state and the ground one. In the case of Mn^{2+} in an octahedral field E_G is given in a first approximation by

$$
E_G = 10B + 6C - 10Dq, \t(20)
$$

where *B* and *C* are the usual Racah parameters.

An increase of Λ favors the lack of luminescence.^{48,49} The critical value Λ_c where luminescence disappears depends, however, on the electronic structure of the excited state. In the case of Mn^{2+} -doped fluorite lattices^{50,51} where the first excited state has a long lifetime the critical value is Λ_c =0.07, while for *F* centers⁵² Λ_c =0.25. In the case of Mn^{2+} -doped fluoroperovskites when R_e increases both E_s and E_G increase, leading to a small increase of Λ along the series. More precisely,^{19,50} Λ = 0.033 for KMgF₃:Mn²⁺, while Λ = 0.039 for RbCdF₃ :Mn²⁺. Therefore, in these systems, an increase of the chemical pressure leads to a slight decrement of Λ favoring the existence of luminescence through the whole series as the Λ variation is *only* of 15%.

According to the present results, the effects of a hydrostatic pressure upon Λ would also be different to those coming from chemical pressures. For instance, considering the results on E_s gathered in Fig. 3, an *increase* of Λ up to Λ ≈ 0.05 can be expected for KMgF₃:Mn²⁺ when the applied pressure makes $\Delta R_e / R_e = -10\%$. Although this figure is higher than Λ =0.033 measured at ambient pressure, it is, however, smaller than Λ_e = 0.07 as derived⁵⁰ from the results on Mn^{2+} -doped SrF₂ and BaF₂, and thus luminescence would be in principle not fully quenched. Further experimental work about this matter is desirable.

IV. FINAL REMARKS

Variations of the A_{1g} and E_g frequencies of MnF₆⁴⁻ along the series of cubic ABF_3 perovskites are calculated to be much bigger than those coming from a hydrostatic pressure applied on a given system like $KMgF_3$: Mn^{2+} and those corresponding to the LO_3 frequency of the host lattice.²⁷ These results allow one to understand the experimental increase of the Stokes shift associated with the MnF $_6^{4-}$ unit when the host lattice parameter a increases¹⁹ and also the nature of vibrations observed 24 in the high-resolution optical spectra on KBF_3 : Mn^{2+} (*B*:Mg, Zn). According to the present findings, such vibrational modes have a dominant local character similar to that found for Cr^{3+} in elpasolites.^{13–15} It was early pointed out by Sturge⁵³ that vibrational modes involved in progressions seen in optical spectra of transition-metal impurities exhibit a local or quasilocal character. The conclusions reached in this work thus support that view. The distinct role played by local and host lattice modes is also well seen in experiments⁵⁴ on the nonradiative decay in $Mg_2SiO_4:Cr^{4+}$. In a first step such a decay is found to involve the excitation of local modes. This energy is *later* transferred to the vibrational modes of host lattice.

Despite the present study accounting for the main trends displayed by optical spectra of ABF_3 : Mn^{2+} systems, the influence of cluster size and the addition of further shells^{55–57} to the description of A_{1g} and E_g modes deserve a supplementary investigation.

The present findings can also be useful for understanding the Stokes shift (and its dependence on hydrostatic or chemical pressures) of other 3*d* impurities in high-symmetry lattices where the impurity is sixfold coordinated.^{58,59} Recent experimental data on Cr^{3+} in several cubic chloroelpasolites⁶⁰ reveal that the Jahn-Teller energy E_{JT} in the ${}^{4}T_2$ first excited state decreases when R_e does. As E_{JT} is just equal^{45,12,13,15} to $S_E \hbar \omega_E$, such a variation is thus qualitatively similar to that derived for *cubic* fluoroperovskites doped with Mn^{2+} .

The relevance of knowing the R_e dependence of parameters like E_s or S_i ($i = A, E$) in order to explain the pressureinduced variations of optical properties has recently been pointed out.⁶¹ According to the analysis carried out in Sec. III C, one would expect^{14,15} that S_A and S_E would decrease when a hydrostatic pressure is applied provided pressure does not favor a distortion of the initial geometry. In $Cs₂NaScCl₆: Cr³⁺$ where $CrCl₆³⁻$ remains cubic under pressure, S_A and S_E are recently found⁶² to experience a decrement when pressure increases. By contrast, in $Mg_2SiO_4:Cr^{4+}$ the average Huang-Rhys factor of the emission band is found to increase 63 when pressure goes from to 9 to 12 GPa. This increase has been ascribed 63 to the appearance of bigger distortions on the $CrO₄^{4–}$ unit induced by the hydrostatic pressure.

Usually, when two *pure* compounds contain isoelectronic complexes with the same ligand, vibrational frequencies increase with the metal oxidation state.⁶⁴ Nevertheless, the ω_A frequency derived for $KMgF_3:Mn^{2+}$ is quite comparable to that for compounds^{62,47} involving Fe $F_6^{3^{\frac{1}{2}}}$ units. It is worth noting, however, that when R_e is closer to the sum of ionic radii of F⁻ and Mn²⁺ (212 pm), then $\hbar \omega_A$ is certainly smaller and close to \sim 400 cm⁻¹ as estimated¹² for RbMnF₃. This comparison stresses once more the relevant effects that chemical pressures can produce upon the properties of an impurity. The comparison between the experimental²⁸ and calculated γ_A for CrF₆³⁻ in elpasolites supports the use of simple clusters for deriving reasonable values of Grüneisen constants in the case of impurities in insulating lattices. When the Grüneisen constant of the A_1 mode is computed from the potential energy curve calculated with all ions fixed except the ligands, then the obtained value could, however, be significantly smaller than the right one. It is worth noting that using Eqs. (16) and (17) a ratio $\gamma_A(f)/\gamma_A$ close to 0.5 is obtained even if $g' \ll g$ provided $g'/g \approx k'/k$ holds. This idea can thus explain in a simple way the results discussed in Sec. III B and also the value of $\gamma_A = 0.91$ reported in Ref. 35 for CrF_6^{3-} in K₂NaGa_{F6}: Cr^{3+} , which can be compared to the experimental value²⁸ $\gamma_A = 1.9$.

From the present results it can be expected that in systems containing impurities like Cu^{2+} , Ag^{2+} , or Rh^{2+} where the Jahn-Teller coupling can occur in the *ground state*⁶⁵ chemical and hydrostatic pressures can also lead to different situations. Work along this line is now under way.

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