Phase structure and universality in two-dimensional disordered quantum antiferromagnets

E. C. Marino
Instituto de Física, Universidade Federal do Rio de Janeiro, Cx. P. 68528, Rio de Janeiro RJ 21945-970, Brazil

(Received 29 June 2001; published 10 January 2002)

Two-dimensional disordered quantum antiferromagnets are studied by means of a continuum description in which disorder is introduced by a random distribution of couplings (spin stiffnesses) in the ordered phase of the nonlinear sigma model. Quenched soliton (skyrmion) correlation functions are evaluated and used, along with quenched magnetization, to characterize the phase structure of the system. When magnetic dilution is exponentially suppressed, the introduction of disorder only modifies the subleading terms in the large distance behavior of the soliton correlation functions, yielding the same skyrmion energy as in the pure case. The system is in a "hard" disordered Néel phase similar to the ordered antiferromagnetic phase occurring in the pure case. Conversely, when magnetic dilution is not exponentially suppressed, the large distance behavior of the correlation functions is drastically changed. The system exists in a new phase in which the energy of quantum skyrmions is equal to zero in spite of the existence of a nonvanishing antiferromagnetic order parameter. This "soft" disordered Néel phase is characterized by universality classes, which are determined by the behavior of the distribution of random couplings in the small coupling region. The possible relation of this phase to spin glasses is briefly discussed.

DOI: 10.1103/PhysRevB.65.054418 PACS number(s): 75.10.Jm, 75.10.Nr, 75.70.-i

I. INTRODUCTION

The continuum description of two-dimensional quantum antiferromagnets has been the object of intense investigation for a long period of time.[1–4] The interest in this kind of description has been enhanced mostly because of its successful applications in the case of layered antiferromagnets such as high-temperature superconducting cuprates. For these compounds, the undoped parent materials can be very well described by the two-dimensional antiferromagnetic Heisenberg model on a square lattice.[5] In the continuum limit, this can be mapped into the ordered phase of the nonlinear sigma model (NLSM),[1] whose single coupling constant ρ_s, the spin stiffness, is directly related to the Heisenberg antiferromagnetic coupling J.

Site-diluted disordered antiferromagnets have been studied previously in the framework of the NLSM, leading to very interesting results.[4] The aim of this work is to investigate the consequences of disorder in the quantum Heisenberg antiferromagnet by considering a continuous random distribution of spin stiffness ρ_s in the NLSM. The effects of disorder are particularly interesting and, in fact, lead to unexpected results in the case of skyrmion correlation functions. Quantum skyrmion states $|sk\rangle$ are characterized by the property

$$Q|sk\rangle=|sk\rangle,$$

where Q is the topological charge operator. These excited states are created out of the ground state by an operator μ whose correlation functions and properties have been extensively studied in Ref. 6, for the case of the NLSM. The skyrmion energy, in particular, can be inferred from the large distance behavior of the skyrmion correlation function $\langle \mu\mu^\dagger\rangle$.

The pure NLSM at zero temperature is known to exist in two phases.[2] These can be characterized by an order parameter, which is the ground-state expectation value of the continuum limit of the sublattice spin operator, namely, $\langle\sigma\rangle$, along with a dual (disorder) parameter given by $\langle\mu\rangle$.[6] One of the phases of the pure system is ordered, having $\langle\sigma\rangle\neq 0$ and $\langle\mu\rangle=0$. In this phase $\langle\mu\mu^\dagger\rangle$ has an exponential large distance decay implying the existence of a nonzero creation energy for the skyrmions.[6] The other phase is a paramagnetic quantum disordered one, presenting $\langle\sigma\rangle=0$ and $\langle\mu\rangle\neq 0$. In this phase, the fact that the skyrmion states $|sk\rangle=\mu|0\rangle$ are not orthogonal to the vacuum means that there is actually no genuine skyrmion excitation in the system. A third possibility, which is not realized in the pure NLSM at zero temperature, would be a phase in which $\langle\mu\rangle=0$ with the skyrmion correlation function presenting a power-law decay at large distances. In this case, the system would have zero energy skyrmions in its excitation spectrum and $\langle\sigma\rangle=0$, the absence of order being closely related to the vanishing of the soliton energy. The possibility of both $\langle\mu\rangle\neq 0$ and $\langle\sigma\rangle\neq 0$, on the other hand, is forbidden by a duality relation existing between the spin and soliton operators, which has been rigorously demonstrated in one spatial dimension[8] and, for physical reasons, should also be valid in higher dimensions.

One can ask whether some different phases may occur when disorder is introduced in the system. Starting from the ordered phase of the pure NLSM, which corresponds to the Heisenberg antiferromagnet, we investigate this possibility by studying quenched averages in the presence of random couplings. When magnetic dilution is not exponentially suppressed, we conclude that the system exists in a new phase, where $\langle\langle\mu\rangle\rangle=0$ and $\langle\langle\sigma\rangle\rangle\neq 0$ (the quenched averages $\langle\langle\cdots\rangle\rangle$ are defined in Sec. III), but in which the skyrmion correlation function presents a power-law decay at large distances that implies the existence of zero energy skyrmions. This phase, which never occurs in the pure system, is a "soft" disordered Néel phase, possessing an order parameter $\langle\langle\sigma\rangle\rangle\neq 0$, in spite of the fact that the skyrmion energy vanishes. The power-law decay of correlation functions in this phase shows some characteristics of criticality. Universality classes, which are determined by the behavior of the distribution of random couplings at $\rho\to 0$, also can be clearly identified in this new phase, which bears some resemblance to a spin-glass phase. This is discussed in Sec. VI.

When the distribution function of random couplings is such that magnetic dilution is exponentially suppressed, on the other hand, the system is shown to exist in a phase with $\langle\langle\mu\rangle\rangle=0$ and $\langle\langle\sigma\rangle\rangle\neq 0$ presenting, at the same time, an exponential decay of the skyrmion correlation function, analogously to what happens in the ordered Néel phase of the pure system. The skyrmion excitation always possess a nonzero energy in this phase. Only the subleading behavior of the skyrmion and spin-correlation functions at large distances is modified by the introduction of disorder in this phase, which might be called a ''hard'' disordered Néel phase.

The paper is organized as follows. In Sec. II, we review some properties of the quantum NLSM relevant for the present work as well as the continuum limit of two-dimensional quantum antiferromagnets in the pure case. In Sec. III, we consider disorder in the NLSM, manifested in a random distribution of couplings in the Néel phase. The probability distribution functions for these couplings are also introduced. In Sec. IV, we study the quenched averages of skyrmion correlation functions in the case where magnetic dilution is exponentially suppressed. In Sec. V, we consider the same averages in situations in which magnetic dilution is not suppressed. We also show the occurrence of a new phase, presenting some characteristics of criticality, in which the system belongs to universality classes determined by the behavior of the distribution function at $\rho_s\to 0$. Discussion of the results, conclusions, and future perspectives are presented in Sec. VI.

II. THE QUANTUM NONLINEAR SIGMA MODEL AND THE PURE HEISENBERG ANTIFERROMAGNET

A. The quantum nonlinear sigma model

Let us start by reviewing the properties of the two-dimensional (2D) $O(3)$-symmetric quantum NLSM. Subsequently, we shall recall how it is mapped in the 2D Heisenberg antiferromagnet. The NLSM is defined by the action ($d^3x=d\tau d^2x$)

$$S=\int d^3x\frac{\rho_0}{2}\left[\frac{1}{c^2}(\partial_\tau\mathbf{n})^2+(\nabla\mathbf{n})^2\right], \tag{2.1}$$

where the field $\mathbf{n}$ is subject to the constraint $\mathbf{n}^2=1$. ρ_0 is a coupling parameter and c, a characteristic velocity. Henceforth, unless otherwise specified, we shall make $\hbar=1$ and $c=1$. Writing the nonlinear sigma field as $\mathbf{n}=(\sigma,\boldsymbol{\pi})$, the zero-temperature partition function can be expressed as

$$Z=\int D\sigma D\boldsymbol{\pi}D\lambda\exp\left\{-\int d^3x\left[\frac{1}{2}[(\partial_\mu\sigma)^2+|\partial_\mu\boldsymbol{\pi}|^2]+i\lambda[\sigma^2+|\boldsymbol{\pi}|^2-\rho_0]\right]\right\}, \tag{2.2}$$

where we rescaled the fields and introduced the constraint through the Lagrange multiplier field λ. We use the notation $\partial_\mu\equiv(\partial/\partial\tau,\nabla)$. Integrating on $\boldsymbol{\pi}$, we get the effective partition function

$$Z=\int D\sigma D\lambda\exp\left\{-\int d^3x\left[\frac{1}{2}(\partial_\mu\sigma)^2+i\lambda[\sigma^2-\rho_0]\right]+\mathrm{tr}\ \ln[-\Box+i\lambda]\right\}. \tag{2.3}$$

The constant saddle-point equations derived from the above expression are

$$\langle\lambda\rangle\langle\sigma\rangle=0,$$

$$\langle\sigma\rangle^2=\rho_0-\int\frac{d^3k}{(2\pi)^3}\frac{1}{k^2+m^2}, \tag{2.4}$$

where $m^2/2=i\langle\lambda\rangle$. At zero temperature, the system presents two phases:[2] an ordered Néel phase, for which $\langle\lambda\rangle=0$ and $\langle\sigma\rangle\neq 0$, and a (paramagnetic) quantum disordered phase, in which $\langle\lambda\rangle\neq 0$ and $\langle\sigma\rangle=0$. We explore below the physical properties of the basic excitations of the system and corresponding correlation functions in each of these two phases.

An important feature of the NLSM is the existence of topological excitations, called skyrmions. Classically, they are solutions of the field equations carrying the topologically conserved charge[9]

$$Q=\frac{1}{8\pi}\int d^2x\epsilon^{ij}\epsilon^{abc}n^a\partial_i n^b\partial_j n^c. \tag{2.5}$$

At quantum level, the skyrmion states $|sk\rangle$ are eigenstates of the Q operator with an eigenvalue equal to one and are created by an operator μ satisfying the commutation rule $[Q,\mu]=\mu$. The correlation functions of this operator have been studied in detail in the ordered phase of the NLSM, taking into account full quantum effects.[6] Together with $\langle\sigma\rangle$, the ground-state expectation value of the soliton creation operator μ is a convenient tool for the characterization of the phases of the system, which we are going to exploit.

B. The quantum disordered phase

We start with the quantum disordered phase, where $m\neq 0$. Evaluating the integral in Eq. (2.4) using the large k cutoff Λ and taking $\langle\sigma\rangle=0$, it is easy to see that

$$\frac{m}{4\pi}=\frac{\Lambda}{2\pi^2}-\rho_0>0. \tag{2.6}$$

Note that the large Λ behavior of Eq. (2.6), as usual, may be compensated by the bare coupling ρ_0, yielding a finite parameter m. Using the saddle-point solution $i\langle\lambda\rangle=m^2/2$, it becomes clear that, up to a constant, the effective σ-field action is given by

$$S_{\mathrm{eff}}[\sigma]=\int d^2x\frac{1}{2}[(\partial_\mu\sigma)^2+m^2\sigma^2]. \tag{2.7}$$

The spin-correlation function, in this phase, therefore, is given by (notice that we are working with imaginary time $\tau = it$)

$$\langle \sigma(\mathbf{x},\tau)\sigma(0,0)\rangle_{\rm qd} = \int_{-\infty}^{+\infty} \frac{d\omega}{d\pi} \int \frac{d^2k}{(2\pi)^2} \frac{e^{i\mathbf{k}\cdot\mathbf{x}} e^{i\omega\tau}}{\omega^2 + |\mathbf{k}|^2 + m^2} = \frac{e^{-m\sqrt{|\mathbf{x}|^2+\tau^2}}}{4\pi[|\mathbf{x}|^2+\tau^2]^{1/2}}. \tag{2.8}$$

The exponential decay reveals the presence of a correlation length $\xi = m^{-1}$. The large distance behavior $\langle\sigma\sigma\rangle_{\rm QD}\to 0$ confirms that $\langle\sigma\rangle_{\rm QD}=0$ in this phase. Conversely, no quantum soliton excitations are expected to be present in this phase and therefore we must have $\langle\mu\rangle \neq 0$, implying that the quantum skyrmion state $|sk\rangle$ is not orthogonal to the ground state. As we shall see this will be confirmed below.

C. The ordered phase

We now turn to the ordered phase. In this case, we have $m=0$. Evaluating the integral in Eq. (2.4), again using the large k cutoff Λ, we get

$$\langle\sigma\rangle^2_{\rm ord} = \left[\rho_0 - \frac{\Lambda}{2\pi^2}\right] > 0. \tag{2.9}$$

Once more, the large Λ behavior in Eq. (2.9) can be absorbed in a redefinition of the bare coupling ρ_0. Introducing the renormalized (finite) coupling ρ_s, the spin stiffness, as

$$\rho_s = \rho_0 - \frac{\Lambda}{2\pi^2} \geqslant 0, \tag{2.10}$$

we see that $\langle\sigma\rangle^2_{\rm ord} = \rho_s$, which is nonzero in the ordered phase. At the quantum critical point $\rho_s = 0$, the system enters the disordered phase, where $m \neq 0$. The sublattice magnetization M is given by $M = \langle\sigma\rangle$. In the ordered phase, we have $M_{\rm ord} = \sqrt{\rho_s}$ while in the quantum disordered phase, of course, $M_{\rm qd} = 0$.

Let us consider now the complete renormalization of the theory in the ordered phase. From Eq. (2.10), we can write

$$\rho_0 = Z\rho_s; \quad Z = \left(1 + \frac{\Lambda}{2\pi^2\rho_s}\right). \tag{2.11}$$

Introducing the renormalized fields $\mathbf{n}_R$, λ_R, and action S_R through

$$\mathbf{n} = Z^{-1/2}\mathbf{n}_R; \quad \lambda = Z\lambda_R; \quad S_R = S + \delta S, \tag{2.12}$$

where

$$\delta S = -i(Z^{-1}-1)\int d^3x Z\lambda_R, \tag{2.13}$$

it is easy to see that the renormalized action is given by

$$S_R = \int d^3x \left\{ \frac{\rho_s}{2} |\partial_\mu \mathbf{n}_R|^2 + i\lambda_R[|\mathbf{n}_R|^2 - 1] \right\}. \tag{2.14}$$

This is identical to the classical action, but with renormalized, physical quantities, replacing the bare ones. For this reason, this phase is known as "renormalized classical."[2] We see, in particular, that the physical coupling constant in this phase is the spin stiffness ρ_s.

Replacing S for S_R in Eq. (2.3), inserting the saddle-point value $\langle\lambda_R\rangle = 0$, and shifting the σ_R field around its vacuum expectation value $\langle\sigma_R\rangle = \sqrt{\rho_s}$, namely, defining $\eta \equiv \sigma_R - \sqrt{\rho_s}$, we get

$$S_{\rm eff}[\eta] = \int d^3x \frac{1}{2}(\partial_\mu \eta)^2. \tag{2.15}$$

This is the well-known Goldstone boson action and the corresponding correlation functions are

$$\langle \eta(\mathbf{x},\tau)\eta(0,0)\rangle = \langle\sigma_R(\mathbf{x},\tau)\sigma_R(0,0)\rangle_{\rm oaf} - \langle\sigma_R\rangle^2_{\rm oaf} = \frac{1}{4\pi[|\mathbf{x}|^2+\tau^2]^{1/2}}. \tag{2.16}$$

We now see that $\langle\sigma_R\sigma_R\rangle_{\rm oaf} \to \langle\sigma_R\rangle^2_{\rm oaf} \neq 0$, at large distances, thus confirming the fact that $\langle\sigma_R\rangle_{\rm oaf}\neq 0$ in this phase.

In the ordered antiferromagnetic phase, we have the occurrence of classical skyrmion excitations, possessing $Q=1$. These are given by[9]

$$\mathbf{n}_S(\mathbf{x}) = \rho_s[\sin f(r)\hat{r}, \cos f(r)] \tag{2.17}$$

with

$$f(r) = 2\arctan\frac{l}{r},$$

where l is an arbitrary scale and r is the radial distance in two-dimensional space. The energy of this classical skyrmion excitation in the ordered phase, described by the renormalized classical action (2.14) and measured with respect to the ordered antiferromagnetic background, is $E = 4\pi\rho_s$. This must be compared with the full quantum result obtained from a quantized skyrmion field theory. The two-point quantum skyrmion correlation function has been evaluated in the ordered phase of the quantum NLSM (Ref. 6) and the result is

$$\langle\mu(\mathbf{x},\tau)\mu^\dagger(0,0)\rangle_{\rm oaf} = \exp\{-2\pi\rho_s[|\mathbf{x}|^2+\tau^2]^{1/2}\}. \tag{2.18}$$

From this we can infer that the actual energy of the full quantum skyrmions in the ordered antiferromagnetic phase is $2\pi\rho_s$, that is, half of the classical value. From Eq. (2.18), we also see that $\langle\mu\mu^\dagger\rangle_{\rm oaf}\to 0$, at large distances, implying that $\langle\mu\rangle = 0$ in this phase. This means that the quantum skyrmion state $|sk\rangle$ is orthogonal to the vacuum and the quantum skyrmions are genuine excitations. For $\rho_s \to 0$, on the other hand, when we approach the quantum critical point leading to the disordered phase, we see from Eq. (2.18) that $\langle\mu\rangle \neq 0$, confirming therefore our anticipation for the ground-state expectation value of the skyrmion operator in the disordered phase.

D. Connection with the Heisenberg antiferromagnet

Two-dimensional antiferromagnets on a square lattice can be described by the $O(3)$-symmetric Heisenberg Hamiltonian, given by

$$H=\sum_{\langle ij\rangle} J_{ij}\mathbf{S}_i\cdot\mathbf{S}_j\,, \tag{2.19}$$

where the sum runs only over nearest-neighbor sites and $J_{ij}>0$. The "pure" case is characterized by the fact that the coupling constants J_{ij} are determined and fixed. In the homogeneous case, all the coupling constants are equal and we have $J_{ij}\equiv J>0$. At zero temperature, this system is known to exist only in an ordered Néel phase. The quantum fluctuations are not capable of destroying the long-range antiferromagnetic order for any value of the coupling constant.[7]

It has been shown that in the continuum limit, the above quantum Hamiltonian, in the homogeneous case, is mapped into the *ordered phase* of the quantum NLSM,[1,2] the nonlinear sigma field $\mathbf{n}(\mathbf{x},t)$ being the continuum limit of the sublattice spin operator. The spin stiffness ρ_s, which, as we saw, controls all the physical properties of the system in the ordered phase, is related to the Heisenberg antiferromagnetic coupling J as[2,5]

$$\rho_s=JS^2Z_{\rho_s}, \tag{2.20}$$

where S is the spin quantum number and Z_{ρ_s} is a constant accounting for quantum corrections to the classical continuum limit. For $S=\frac{1}{2}$, we have[2,5]

$$\rho_s\simeq 0.18J. \tag{2.21}$$

In the nonhomogeneous case, where the coupling constants J_{ij} are different for each link, we can derive the continuum limit by following the same procedure as in Ref. 1, provided that the configuration of coupling constants J_{ij} in Eq. (2.19) is slowly varying (this is going to be made precise in what follows). In this case, we obtain in the continuum limit, a nonlinear sigma model with the spin stiffness ρ_s replaced by a slowly varying configuration $\rho(\mathbf{r})$ that is related to J_{ij} in the same way that ρ_s is related to J, namely,

$$S=\int d^3x\left\{\frac{1}{2}|\partial_\mu\mathbf{n}|^2+i\lambda[|\mathbf{n}|^2-\rho_0(\mathbf{r})]\right\}. \tag{2.22}$$

This is equivalent to modifying the constraint from $\delta[|\mathbf{n}|^2-1]$ to $\delta[|\mathbf{n}|^2-f(\mathbf{r})]$, with $\rho_0(\mathbf{r})\equiv\rho_0 f(\mathbf{r})$. Integrating over $\boldsymbol{\pi}$, we get

$$S=\int d^3x\left\{\frac{1}{2}(\partial_\mu\sigma)^2+i\lambda[\sigma^2-\rho_0(\mathbf{r})]\right\}+\mathrm{tr}\,\ln[-\Box+i\lambda]. \tag{2.23}$$

Now translation invariance is lost and the saddle-point equations become

$$-\nabla^2\langle\sigma\rangle+2m^2(\mathbf{r})\langle\sigma\rangle=0,$$

$$\langle\sigma\rangle^2=\rho_0(\mathbf{r})-\int\frac{d^3k}{(2\pi)^3}\frac{1}{k^2+m^2(\mathbf{r})}\equiv\rho(\mathbf{r}). \tag{2.24}$$

We see that now $\langle\sigma\rangle=\langle\sigma\rangle\,(\mathbf{r})$ and consequently $m(\mathbf{r})\neq 0$, in spite of the fact that $\langle\sigma\rangle\neq 0$. The spin-correlation function becomes damped now, with a damping factor $m(\mathbf{r})$. This is in agreement with previous investigations of spin waves in similar situations.[4,10]

The soliton correlation function can be evaluated in a saddle-point approximation in the theory described by Eq. (2.22).[6] This has a simple expression in terms of $\langle\sigma\rangle$, which will be very convenient for obtaining quenched averages in the disordered version of the model, namely,

$$\begin{aligned}\langle\mu(\mathbf{x},\tau)\mu^\dagger(0,0)\rangle&=\exp\{-2\pi\langle\sigma\rangle^2[|\mathbf{x}|^2+\tau^2]^{1/2}\}\\&=\exp\{-2\pi\rho(\mathbf{r})[|\mathbf{x}|^2+\tau^2]^{1/2}\}.\end{aligned} \tag{2.25}$$

In the rest of this work, we consider the situation in which intrinsic disorder is introduced in the system and investigate its effects on the soliton correlation function.

III. THE CONTINUUM LIMIT OF DISORDERED ANTIFERROMAGNETS

A. The disordered system

Let us describe the presence of disorder in the two-dimensional Heisenberg quantum antiferromagnet, by considering a random distribution of couplings J_{ij} in Eq. (2.19) analogously to the Edwards-Anderson model.[11] Here, however, we will keep only antiferromagnetic couplings $J_{ij}>0$. This will allow us to easily obtain a continuum field-theory version for the disordered model, in the same way as in the pure case. The disorder is introduced in the continuum version by taking a random distribution $P[\rho(\mathbf{r})]$ for the slowly varying spin stiffness $\rho(\mathbf{r})$ appearing in Eq. (2.22). We require that $P[\rho(\mathbf{r})]=0$ for $\rho(\mathbf{r})<0$. This will ensure that, in spite of the presence of disorder, we are always in the ordered phase of the NLSM, for which the mapping to the Heisenberg antiferromagnet exists. We also impose the condition that the variance of this distribution is always much smaller than ρ_s, in order to ensure that the $\rho(\mathbf{r})$ configurations are slowly varying.

We are only going to consider the quenched case and take quantum averages at zero temperature using a fixed configuration for $\rho(\mathbf{r})$. Subsequently we shall evaluate the average over the $\rho(\mathbf{r})$ configurations using the $P[\rho(\mathbf{r})]$ distribution function. The relevant average for an operator $\mathbf{A}$ in the quenched random system will be therefore

$$\langle\langle\mathbf{A}\rangle_q\rangle_\rho=\frac{1}{\Lambda}\prod_{\mathbf{r}}\int_0^\infty d\rho(\mathbf{r})P[\rho(\mathbf{r})]\langle\mathbf{A}\rangle_q[\rho(\mathbf{r})], \tag{3.1}$$

where $\Lambda\equiv(\Pi_{\mathbf{r}}\cdot 1)$ is a normalization factor corresponding, in the lattice, to a product over all links (ij) and $\langle\mathbf{A}\rangle_q$ is the zero-temperature quantum average.

B. Distribution functions

Let us introduce now the distribution functions we are going to use, in order to describe the disorder. We shall consider basically two functions containing a Gaussian distribution centered around the pure spin stiffness ρ_s. The first one is (henceforth we omit the argument $\mathbf{r}$ in ρ)

$$P_1[\rho]=\begin{cases}\frac{1}{N_1}(\rho-\rho_s)^{\nu-1}e^{-(\rho-\rho_s)^2/2\Delta^2}, & \rho>\rho_s,\\ \frac{1}{N_1}(\rho_s-\rho)^{\nu-1}e^{-(\rho-\rho_s)^2/2\delta^2}, & 0\leqslant\rho<\rho_s,\\ 0, & \rho<0,\end{cases} \tag{3.2}$$

where $\nu>0$. In this expression, we assume that both $\Delta\ll\rho_s$ and $\delta\ll\rho_s$, thereby guaranteeing that the random ρ configurations are slowly varying. We also assume that

$$\left(\frac{L}{\hbar c}\right)\Delta\gg 1, \tag{3.3}$$

where L is the maximum dimension of the system and c, the spin-wave velocity, its characteristic velocity. Two regimes of disorder described by Eq. (3.2) can be distinguished and will produce a completely different behavior of the correlation functions, as we shall see. A first one is obtained by choosing a symmetric Gaussian, with $\delta=\Delta$. A second one is with the choice $\delta\ll\Delta$ and

$$\left(\frac{L}{\hbar c}\right)\delta\ll 1. \tag{3.4}$$

In the second case, there is a severe exponential suppression of values of the spin stiffness around $\rho_s=0$, that is, magnetic dilution is exponentially suppressed in a very strong way. In the first case, dilution is not so much suppressed and, as we shall see, the system has the same qualitative behavior as a diluted one.

Experimental values for the parameters of the above distribution, in the case of high-temperature superconducting cuprates in the ordered antiferromagnetic phase, which are typical examples of two-dimensional Heisenberg antiferromagnets, are[5] $\rho_s\simeq 10^{-1}$ eV, $\hbar c\simeq 1$ eV Å. For a sample of dimension $L\simeq 1$ mm, we choose $\Delta\simeq 10^{-3}$ eV, which satisfies Eq. (3.3). For δ, we have $\delta=\Delta$ in the case with dilution. In the case without dilution, the choice $\delta\simeq 10^{-9}$ eV will satisfy Eq. (3.4). In both cases, the condition Δ, $\delta\ll\rho_s$ is satisfied.

In the distribution function (3.2), the normalization factor is given by

$$N_1=2^{(\nu/2)-1}\Gamma\left(\frac{\nu}{2}\right)[\Delta^\nu+\delta^\nu]. \tag{3.5}$$

The average spin stiffness is

$$\overline{\rho}=\rho_s+\frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{2}\Gamma(\nu/2)}\Delta \tag{3.6}$$

in the case where dilution is not suppressed and

$$\overline{\rho}=\rho_s+\sqrt{2}\frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma(\nu/2)}\Delta\left[1-\left(\frac{\delta}{\Delta}\right)^\nu\right] \tag{3.7}$$

in the presence of exponential suppression of dilution $(\delta\ll\Delta)$.

The second distribution function we are going to use is

$$P_2[\rho]=\begin{cases}\frac{1}{N_2}\rho^{\nu-1}e^{-(\rho-\rho_s)^2/2\sigma^2}, & \rho\geqslant 0,\\ 0, & \rho<0,\end{cases} \tag{3.8}$$

where $\nu>0$. We assume $\sigma\ll\rho_s$, again to ensure that the ρ configurations are slowly varying. This condition can be experimentally satisfied, in the case of the high-temperature cuprates, with a choice of $\sigma\simeq 10^{-3}$ eV. Observe also that, using the same experimental values of the previous paragraph, this value of σ satisfies the condition

$$\left(\frac{L}{\hbar c}\right)\sigma\gg\left(\frac{\rho_s}{\sigma}\right)\gg 1, \tag{3.9}$$

which is similar to Eq. (3.3).

The normalization factor in Eq. (3.9) is now given by

$$N_2=\sigma^\nu\Gamma(\nu)D_{-\nu}\left(-\frac{\rho_s}{\sigma}\right)e^{-\rho_s^2/4\sigma^2}, \tag{3.10}$$

where $D_{-\nu}(x)$ is a parabolic cylinder function. For the $P_2[\rho]$ distribution function, the average spin stiffness is

$$\overline{\rho}=\rho_s+(\nu-1)\frac{\sigma^2}{\rho_s}. \tag{3.11}$$

IV. DISORDER WITH EXPONENTIALLY SUPPRESSED DILUTION

In this section, we consider the situation in which magnetic dilution is exponentially suppressed in the disordered system. As explained above, this corresponds to the choice of $P_1[\rho]$ as the distribution function, with the parameters Δ and δ satisfying Eqs. (3.3) and (3.4), respectively. We are going to evaluate the quenched skyrmion correlation functions, starting from the ordered antiferromagnetic phase of the pure system. In this case, the zero-temperature pure quantum averages $\langle\mu\mu^\dagger\rangle_{\rm oaf}$, given by Eq. (2.18), depend exponentially on the spin stiffness. For the disordered NLSM introduced in Sec. II B, in the regime where the spin stiffness $\rho(\mathbf{r})$ is slowly varying, the soliton correlation function is given by Eq. (2.25). Hence, when evaluating the quenched averages (3.1), we shall have a zero-temperature quantum average whose $\rho(\mathbf{r})$ dependence is of the form $\exp\{-\alpha\rho(\mathbf{r})\}$, where

$\alpha=2\pi X$, with $X\equiv[|\mathbf{x}|^2+\tau^2]^{1/2}$. The relevant integral for the evaluation of Eq. (3.1) is, therefore

$$A=A_1+A_2, \tag{4.1}$$

where

$$\begin{aligned}A_1&=\frac{1}{N_1}\int_0^{\rho_s}d\rho(\rho_s-\rho)^{\nu-1}e^{-\alpha\rho}e^{-(\rho-\rho_s)^2/2\delta^2}\\&=\frac{1}{N_1\alpha^\nu}\int_0^{\alpha\rho_s}dx(\alpha\rho_s-x)^{\nu-1}e^{-x}e^{-(x-\alpha\rho_s)^2/(2\alpha^2\delta^2)}\end{aligned} \tag{4.2}$$

and

$$\begin{aligned}A_2&=\frac{1}{N_1}\int_{\rho_s}^{\infty}d\rho(\rho-\rho_s)^{\nu-1}e^{-\alpha\rho}e^{-(\rho-\rho_s)^2/2\Delta^2}\\&=\frac{e^{-\alpha\rho_s}}{N_1\alpha^\nu}\int_0^{\infty}dx x^{\nu-1}e^{-x}e^{-x^2/(2\alpha^2\Delta^2)}.\end{aligned} \tag{4.3}$$

We shall be interested in the behavior of quenched averages of correlation functions, given by Eq. (3.1), at large distances $(X\to\infty)$. In this case, we have $\alpha\to\infty$ and we can, therefore, use conditions (3.4) and (3.3), respectively, in Eqs. (4.3) and (4.2) to obtain

$$A_1\overset{\alpha\to\infty}{\to}\frac{e^{-\alpha\rho_s}}{N_1\alpha^\nu}\int_{\alpha\rho_s-\alpha\delta}^{\alpha\rho_s}dx(\alpha\rho_s-x)^{\nu-1}=\frac{\delta^\nu}{N_1\nu}e^{-\alpha\rho_s} \tag{4.4}$$

and

$$A_2\overset{\alpha\to\infty}{\to}\frac{e^{-\alpha\rho_s}}{N_1\alpha^\nu}\int_0^{\infty}dx x^{\nu-1}e^{-x}=\frac{\Gamma(\nu)}{N_1}\frac{e^{-\alpha\rho_s}}{\alpha^\nu} \tag{4.5}$$

[we could also have obtained the result in Eq. (4.5) by exactly evaluating the last integral in Eq. (4.3) and subsequently taking the limit $\alpha\to0$, see Eqs. (5.7) and (5.8)]. We conclude from Eqs. (4.4) and (4.5) that

$$A\overset{\alpha\to\infty}{\to}\frac{e^{-\alpha\rho_s}}{N_1}\left[\frac{\Gamma(\nu)}{\alpha^\nu}+\frac{\delta^\nu}{\nu}\right]\overset{\alpha\to\infty}{\to}\frac{\Gamma(\nu)}{N_1}\frac{e^{-\alpha\rho_s}}{\alpha^\nu}, \tag{4.6}$$

where we used Eq. (3.4) in the second limit, in order to get the dominant behavior for large α.

From Eqs. (2.18) and (4.6) we can immediately obtain the expression for the long-distance behavior of the skyrmion quenched averages in the absence of magnetic dilution, namely,

$$\langle\langle\mu\mu^\dagger\rangle\rangle_{\text{oaf}}\overset{X\to\infty}{\to}\frac{\Gamma(\nu)}{(2\pi)^\nu N_1}\left(\frac{e^{-2\pi\rho_s X}}{X^\nu}\right). \tag{4.7}$$

Observe that the presence of disorder, in the case where dilution is exponentially suppressed, does not modify the dominant exponential large distance behavior. Only the subdominant power-law decay is modified by the introduction of an additional exponent ν.

V. DISORDER WITHOUT EXPONENTIAL SUPPRESSION OF DILUTION

In this section, we are going to consider the situation in which magnetic dilution is not exponentially suppressed by the random distribution of couplings. As we saw in Sec. II, this can happen either when we use the distribution $P_1[\rho]$ in the symmetric case when $\delta=\Delta$ or when we use $P_2[\rho]$. In what follows, we study the two cases separately.

A. Distribution function $P_1[\rho]$

In this case, the relevant integral for the evaluation of Eq. (3.1) is

$$B=B_1+A_2, \tag{5.1}$$

where A_2 is given by Eq. (4.3) and

$$\begin{aligned}B_1&=\frac{1}{N_1}\int_0^{\rho_s}d\rho(\rho_s-\rho)^{\nu-1}e^{-\alpha\rho}e^{-(\rho-\rho_s)^2/2\Delta^2}\\&=\frac{e^{-\alpha\rho_s}}{N_1\alpha^\nu}\int_0^{\alpha\rho_s}dx x^{\nu-1}e^{x}e^{-x^2/2(\alpha^2\Delta^2)}.\end{aligned} \tag{5.2}$$

In the large distance regime, when $\alpha\to\infty$, we can use Eq. (3.3) to obtain[12]

$$B_1\overset{\alpha\to\infty}{\to}\frac{e^{-\alpha\rho_s}}{N_1\alpha^\nu}\int_0^{\alpha\rho_s}dx x^{\nu-1}e^{x}=\frac{\rho_s^\nu e^{-\alpha\rho_s}}{\nu N_1}{}_1F_1(\nu;1+\nu;\alpha\rho_s), \tag{5.3}$$

where ${}_1F_1(\nu;1+\nu;\alpha\rho_s)$ is a confluent hypergeometric function. Using the large distance asymptotic behavior of this function, we get[12]

$$B_1\overset{\alpha\to\infty}{\to}\frac{\rho_s^{\nu-1}}{N_1\alpha}. \tag{5.4}$$

Combining Eqs. (5.4) with (4.6), we see that

$$B\overset{\alpha\to\infty}{\to}\frac{1}{N_1}\left[\frac{\rho_s^{\nu-1}}{\alpha}+\Gamma(\nu)\frac{e^{-\alpha\rho_s}}{\alpha^\nu}\right]\overset{\alpha\to\infty}{\to}\frac{\rho_s^{\nu-1}}{N_1\alpha}. \tag{5.5}$$

From this result, we can immediately infer the large distance behavior of the skyrmion quenched correlation function. This is given by

$$\langle\langle\mu\mu^\dagger\rangle\rangle_{\text{oaf}}\overset{X\to\infty}{\to}\frac{\rho_s^{\nu-1}}{2\pi N_1}\left(\frac{1}{X}\right). \tag{5.6}$$

Now the large distance behavior of the correlation function is drastically changed. The previously dominating exponential decay is completely washed out and we have, instead, a power-law decay. As we shall argue in the next subsection, the exponent of the power law is universally determined by the behavior of the distribution function at $\rho \to 0$.

B. Distribution function $P_2[\rho]$

Let us consider now the situation in which the disorder is described by the distribution function $P_2[\rho]$. The relevant integral for the evaluation of Eq. (3.1) is now[12]

$$C = \frac{1}{N_2}\int_0^\infty d\rho \rho^{\nu-1} e^{-\alpha\rho} e^{-(\rho-\rho_s)^2/2\sigma^2}$$
$$= \left[D_{-\nu}\left(-\frac{\rho_s}{\sigma} \right) \right]^{-1} \exp\left[-\frac{\rho_s \alpha}{2} \right]$$
$$\times \exp\left[\frac{\sigma^2\alpha^2}{4} \right] D_{-\nu}\left(\sigma\alpha - \frac{\rho_s}{\sigma} \right), \qquad (5.7)$$

where $D_{-\nu}(x)$ is the parabolic cylinder function. The large distance behavior of Eq. (5.7) can be obtained by considering the property[12]

$$D_{-\nu}(x) \overset{x\to\infty}{\to} e^{-x^2/4} x^{-\nu},$$

$$D_{-\nu}(-x) \overset{x\to\infty}{\to} \frac{\sqrt{2\pi}}{\Gamma(\nu)} e^{x^2/4} x^{\nu-1}. \qquad (5.8)$$

Using this and Eq. (3.9), we can immediately obtain the large distance behavior of the quenched skyrmion correlation functions for the distribution function $P_2[\rho]$. This is given by

$$\langle\langle \mu\mu^\dagger \rangle\rangle \overset{X\to\infty}{\to} \frac{\Gamma(\nu)}{(2\pi)^{\nu+1/2}} \left(\frac{\rho_s^{1-\nu}}{\sigma} \right) \left(\frac{1}{X^\nu} \right). \qquad (5.9)$$

We observe here that, as in the case of the distribution used in the previous subsection, the introduction of disorder completely modifies the large distance behavior of the correlation functions, eliminating the exponential decay. This fact can be generally understood by observing that the large distance (large α) behavior of Eq. (3.1) is determined by the behavior of the distribution $P[\rho]$ at $\rho \to 0$. This happens because, for the distribution functions $P[\rho]$ (with support in the region $\rho \geqslant 0$) and quantum averages $\langle A \rangle_q$ (ρ) (exponentially depending on ρ) considered in this work, the quenched average (3.1) is proportional to the α Laplace transform of $P[\rho]$. As a consequence, the asymptotic large distance behavior of Eq. (3.1) is universally determined by the behavior of $P[\rho]$ for $\rho \to 0$. We see, for instance, that the values of the parameter ν in $P_2[\rho]$ determine universality classes to which the disordered system belongs. This can be confirmed by the behavior of the correlator (5.6). Also, by comparing Eqs. (5.6) with (5.9) we conclude that distribution $P_1[\rho]$, in the symmetric case when $\delta = \Delta$, is in the $\nu = 1$ universality class of $P_2[\rho]$. This can be also verified by observing that both distributions have the same type of behavior at $\rho \to 0$. Conversely, in the asymmetric case when $\delta \ll \Delta$, the distribution function $P_1[\rho]$ is exponentially suppressed at $\rho \to 0$, a type of behavior that is never presented by $P_2[\rho]$, leading to an exponential decay of the quenched correlators.

VI. DISCUSSION AND CONCLUSIONS

Our continuum analysis of two-dimensional quantum antiferromagnets in the presence of a random distribution of couplings at zero temperature has shown that the quenched averages of soliton (skyrmion) correlation functions are modified with respect to the pure case. The modification is particularly drastic whenever magnetic dilution is not exponentially suppressed in the disordered system. This effect has quite interesting consequences in the physical properties of skyrmions. It is a well-known fact that an exponential decay of the soliton correlation function at large distances would indicate that the energy of the soliton excitations is nonzero and proportional to the coefficient of the exponent. In the pure NLSM in the ordered phase, this is given by $E_S = 2\pi\rho_s = 2\pi\langle\sigma\rangle^2$. A nonzero soliton energy, therefore, is associated to an ordered ground state with $\langle\sigma\rangle \neq 0$. Physically this can be understood as a consequence of the fact that in an ordered ground state there is an energy cost to make the spin flips necessary for the introduction of a soliton state. The exponential decay of $\langle \mu\mu^\dagger \rangle$ further implies through $\langle \mu\mu^\dagger \rangle \to |\langle\mu\rangle|^2$ that $\langle\mu\rangle = 0$, which means that the soliton states are orthogonal to the vacuum, that is to say, true excitations. A power-law decay, on the other hand, while still leading to $\langle\mu\rangle = 0$ and therefore meaning that quantum solitons are genuine excitations, would imply that the energy necessary for the creation of these solitons is equal to zero. In a *generic pure* system at zero temperature, this would correspond to a *quantum* disordered ground state ($\langle\sigma\rangle = 0$), because when the ground state is not an ordered one, there is no energy cost for introducing the spin flips necessary to create a soliton state. These zero energy quantum skyrmions, in spite of bearing minimal relation to their classical ancestors occurring in an ordered phase, would exist as true physical excitations in such a phase. It should be stressed, however, that the kind of quantum disorder occurring when we have a power-law decay of the soliton correlation function differs from the one found in a paramagnetic phase, such as the quantum disordered phase of the NLSM, in which we have $\langle \mu\mu^\dagger \rangle \to C \neq 0$, that is, $\langle\mu\rangle \neq 0$ and $\langle\sigma\rangle = 0$. Consequently, we conclude that a power-law decay of the soliton correlation function would imply some different type of disorder than the one found in a paramagnetic phase. In summary we have the following possibilities for the phases of a pure quantum antiferromagnet:

$$\langle \mu\mu^\dagger \rangle \overset{X\to\infty}{\to} C \neq 0; \quad \langle\mu\rangle \neq 0,$$
$$\langle\sigma\rangle = 0 - \text{paramagnetic quantum disordered},$$
$$\langle \mu\mu^\dagger \rangle \overset{X\to\infty}{\to} e^{-E_S X}; \quad \langle\mu\rangle = 0,$$
$$\langle\sigma\rangle \neq 0 - \text{antiferromagnetic (Néel)},$$
$$\langle \mu\mu^\dagger \rangle \overset{X\to\infty}{\to} \frac{1}{X^\nu}; \quad \langle\mu\rangle = 0,$$
$$\langle\sigma\rangle = 0 - \text{nonparamagnetic quantum disordered}. \qquad (6.1)$$

The first two are realized in the pure NLSM (Ref. 2) at zero temperature and only the second one is realized in the pure two-dimensional Heisenberg quantum antiferromagnet with nearest-neighbor interactions, also at $T=0$.[7]

We then consider the presence of disorder. Starting from the ordered antiferromagnetic phase of the pure NLSM, which corresponds to the Heisenberg antiferromagnet, we have studied in this work the effects of disorder introduced through a continuum random distribution of couplings. In all cases considered here, the quenched magnetization is nonzero, namely, $M_Q=\langle\langle\sigma\rangle\rangle\simeq\sqrt{\overline{\rho}}$, where $\overline{\rho}$ is given by Eqs. (3.6), (3.8), and (3.11), respectively. The studied systems are always in an antiferromagnetic ''disordered'' Néel phase having a nonzero order parameter in spite of the fact that the couplings are random. The types of disorder considered here, *ab initio* cannot destroy the antiferromagnetic order of the pure system since they only allow the presence of positive or null couplings. There are, however, two possible types of such phases, which we call ''soft'' or ''hard,'' according to whether magnetic dilution is exponentially suppressed or not.

When dilution is exponentially suppressed, only the subleading term of the correlation functions at large distances is modified by disorder. The exponential decay of the pure system is preserved and the skyrmion energy in the disordered system is the same as in the pure case. The system exists in a phase corresponding to the second possibility in Eq. (6.1), which may be called a ''hard'' disordered Néel phase. Conversely, when dilution is not exponentially suppressed, the large distance behavior is drastically changed from an exponential to a power-law decay. The skyrmion energy consequently becomes zero in the disordered system, despite the fact that the magnetization is nonvanishing and $\langle\langle\sigma\rangle\rangle\neq 0$. To our best knowledge, this is a completely novel behavior for quantum skyrmions. Physically, we can understand it as follows: the disorder introduced by the random distribution of couplings is sufficient to reduce to zero the energy necessary for the creation of a skyrmion configuration, even though it cannot destroy the quenched magnetization. The soliton correlation functions have a universal behavior, characterized by universality classes that are determined by the behavior of the distribution function at $\rho_s\rightarrow 0$.

It would be interesting to find a physical realization for this phase in which skyrmion correlators have a power-law decay at large distances and which is associated to a diluted disordered antiferromagnet. One might be tempted to associate it with a spin-glass phase since spin glasses can be obtained by dilution of antiferromagnetic systems. A well-known example in three dimensions is the substance $ZnCr_{2-x}Ga_xO_4$, obtained by diluting the pure antiferromagnet $ZnCr_2O_4$ with nonmagnetic Ga atoms.[14,13] The possibility of the ''soft'' phase, obtained in the present work by dilution of a pure antiferromagnet being a spin glass, however, is ruled out by the fact that the quenched magnetization is nonvanishing. A key ingredient for this would be the presence of frustration, which is absent in the system studied here but, conversely, present in the case of $ZnSr_{2-x}Ga_xO_4$. The diluted phase studied here is actually one that could be added to the list (6.1) and is characterized by

$$\langle\mu\mu^\dagger\rangle \overset{X\rightarrow\infty}{\rightarrow} \frac{1}{X^\nu};\quad \langle\mu\rangle=0,\quad \langle\sigma\rangle\neq 0 - \text{soft disordered Néel.} \tag{6.2}$$

In this phase, the skyrmion energy vanishes, not as a consequence of quantum fluctuations, as in the third phase listed in Eq. (6.1), but rather, because of the type of disorder introduced when magnetic dilution is not exponentially suppressed. This, in spite of not being capable of destroying the order parameter, forces the skyrmion energy to vanish. The physical properties of the zero energy quantum solitons occurring in these kind of phases may play an important role in planar antiferromagnetic systems such as high-temperature superconducting cuprates and shall be exploited elsewhere. For this purpose, one should investigate the effects of disorder in the continuum models for doped antiferromagnetic planar systems used to describe materials such as LSCO and YBCO.[15]

Let us stress, finally, that the kind of magnetic dilution considered in this work, even in the cases where it is not exponentially suppressed, is always softer than a delta-function-type of dilution containing a piece $P[\rho]=x\,\delta(\rho)$, which would be suitable for describing the type of dilution occurring in compounds such as $La_2Cu_{1-x}Zn_xO_4$.[4] As future extensions of this work, we intend to consider more general disorder distributions including these and ferromagnetic couplings as well. In the latter case, we can expect to describe spin-glass phases as well. It is likely that in a spin-glass phase the behavior of soliton correlation functions shall be similar to the one found in the soft Néel phase of our model. We are presently investigating this point.

ACKNOWLEDGMENTS

This work was supported in part by CNPq, FAPERJ, and PRONEX Grant No. 66.2002/1998-9.

[1] F. D. M. Haldane, Phys. Rev. Lett. **50**, 1153 (1983); Phys. Lett. A **93**, 464 (1983).

[2] S. Chakravarty, B. Halperin, and D. Nelson, Phys. Rev. B **39**, 2344 (1989).

[3] N. Read and S. Sachdev, Phys. Rev. B **42**, 4568 (1990); A. V. Chubukov, S. Sachdev, and J. Ye, *ibid.* **49**, 11 919 (1994); O. Starykh, *ibid.* **50**, 16 428 (1994); A. V. Chubukov and O. Starykh, *ibid.* **52**, 440 (1995); V. Y. Irkhin and A. A. Katanin, *ibid.* **55**, 12 318 (1997).

[4] Y.-C. Chen and A. H. Castro Neto, Phys. Rev. B **61**, 3772 (2000).

[5] A. P. Kampf, Phys. Rep. **249**, 219 (1994).

[6] E. C. Marino, Phys. Rev. B **61**, 1588 (2000).

[7] E. Manousakis, Rev. Mod. Phys. **63**, 1 (1991).

[8] R. Köberle and E. C. Marino, Phys. Lett. B **126**, 475 (1983).

[9] A. A. Belavin and A. M. Polyakov, Pisma Zh. Eksp. Teor. Fiz. **22**, 503 (1975) [JETP Lett. **22**, 245 (1975)].
[10] C. Wan, A. B. Harris, and D. Kumar, Phys. Rev. B **48**, 1036 (1993).
[11] S. F. Edwards and P. W. Anderson, J. Phys. F: Met. Phys. **5**, 965 (1975).
[12] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products* (Academic, New York, 1980).
[13] K. Binder and A. P. Young, Rev. Mod. Phys. **58**, 801 (1986).
[14] D. Fiorani *et al.*, Phys. Rev. B **30**, 2776 (1984).
[15] E. C. Marino, Phys. Lett. A **263**, 446 (1999); E. C. Marino and M. B. Silva Neto, Phys. Rev. B **64**, 092511 (2001).