Ground- and first-excited state energies of impurity magnetopolaron in an anisotropic quantum dot

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The polaronic effect on the low-lying energy levels of an electron bound to a hydrogenic impurity in a three-dimensional (3D) anisotropic harmonic potential subjected to a uniform magnetic field is investigated by introducing a trial wave function constructed as a direct product form of an electronic part and a part of coherent phonons. Binding energies of impurity magnetopolarons corresponding to each level are analyzed in terms of the effects of both quantum confinement and magnetic field. Furthermore, a detailed discussion of the effects due to the electron–LO-phonon interaction and the effects of both magnetic field and quantum confinements on cyclotron masses asssociated with the transitions between ground and first-excited states of the bound electron is also given in this paper. Our results show that the polaron effect arising from the electron–LO-phonon interaction and confining effects together with the effect of magnetic field have a great influence on the impurity binding energies and on cyclotron masses associated with transitions between the relevant states.

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I. INTRODUCTION

In recent years, there has been a remarkable interest shown by both experimental and theoretical physicists in the study of low-dimensional nanostructures, not only from the point of view of fundamental physics but also from their potential use in designing electronic and optoelectronic devices.[1,2] Such low-dimensional systems, depending on their ways of confining electrons, can be classified into three types; quasi-two-dimensional quantum wells (QW's), quasi-one-dimensional quantum wires or quantum-well wires (QWW's) and quasi-zero-dimensional quantum dots (QD's). Due to the fact that reduction of dimensionality of the system to two, one, or zero dimensions, by a spatial confining potential that restricts the motions of electrons to a plane, line, or point, leads to a strong size quantization of energy levels of an electron, whereby optical and electronic properties of electrons are modified in these types of low-dimensional quantum structures compared with those of the bulk. Furthermore, it is well known that the polaron effects arising from the electron-phonon interaction are enhanced by confining the electron motion to two, one, or zero space dimensions, and thus strongly influence the above-noted physical properties of electrons in these systems as well. Consequently, studying the fundamental role of the effects due to electron-phonon interactions as well as of the effects connected with reducing the dimensionality of the system on electronic levels of an electron has become one of the main subjects of the current theoretical as well as experimental works in this field (see Ref. 3 and references therein).

In these works, much attention has been paid to the study of the properties of impurities, and particularly impurity-phonon interaction effects in low-dimensional structures, since the presence of an impurity influences the above-mentioned properties of electrons as well. There has been a number of theoretical investigations dealing with this problem in the literature. A review of the results of early work without quantum confinement effects in this problem can be found in Ref. 4. The first theoretical investigation connected with the calculation of binding energy of the ground state of a hydrogenic donor as a function of QW size has been worked out by Bastard[5] within the framework of variational approach. Later, some authors modified[6] his work and then extended their studies to cylindrical,[7] rectangular,[8] and finite cylindrical[9] QWW's, and also to spherical[10–12] QD's by applying variational procedure within the effective mass approximation. The problem of an electron bound to a hydrogenic impurity interacting with a LO-phonon field, called the impurity polaron, was studied initially by Erçelebi and Tomak[13] in QW structures within the adiabatic approach, and later by Degani and Hipólito[14] within the random phase approximation. The effects of electron-phonon coupling on impurity binding energies have also been considered in purely 2D polar semiconductors[15] and parabolic QW's[16,17] within the framework of Feynman-Haken path integral theory, and in infinite QW's,[18,19] and QW's with finite potential barrier[20] within the framework of variational approaches based on the one- or two-parameter trial wave function method. Many confinement models containing various electron-phonon interactions within different approximation schemes have been studied in QWW's,[21] and in spherical,[22] parabolic,[23,24] and cylindrical QD's.[25,26] The main result of these works are that the impurity binding energy increases from its bulk value as the dimension is reduced, and it is enhanced by the presence of the electron-phonon interaction. All the theoretical works referred to here have been performed in the absence of a magnetic field and were generally related to the calculation of ground-state binding energy of impurity polaron. There also exist a few studies concerned with the analysis of excited-state energies of impurity polaron subjected to a magnetic field in these low-dimensional structures, in particular, in QD systems.

In the presence of an external magnetic field that leads to a quantization of electron energy spectrum into Landau levels, at most, two excited states of impurity magnetopolaron have been studied in finite[27,28] and infinite[29] QW's, in parabolic QWW's,[30,31] and in 2D parabolic QDs.[32–34] Very recently, the low-lying spectrum of a two-electron QD with a confining parabolic potential in the presence of an on-center Coulomb impurity is studied by Lee *et al*.[35] within the exact numerical diagonalization scheme. From the results of these investigations, it has been concluded that binding energy of impurity magnetopolaron depends sensitively on the magnetic field strength, as well as on the confinement length.

Up to now, there have been also several experimental investigations[36] reporting that, in interpreting the cyclotron emission and absorption spectra, polaronic effects play an important role, particularly in such low-dimensional systems. The detailed theoretical calculations on the corrections to the cyclotron mass induced by the electron-phonon interaction for magnetopolaron and impurity magnetopolaron confined to low-dimensional structures have been previously carried out in the literature, for example, in purely two-dimensional (2D) systems and heterostructures by using the memory function approach[37] and in QW's by using random phase approximation[38] and perturbative methods.[39] Recent discussions have focused on the study of the cyclotron mass of magnetopolaron in parabolic QD's,[40] and impurity magnetopolaron in 2D parabolic QD's,[41] by using different approximation schemes. The main observation and outcome of these investigations are that polaronic effects are of great importance in determining the spectral properties of these low-dimensional materials.

The purpose of the present paper is to study the dependence of low-lying energy levels of impurity magnetopolaron subjected to a 3D parabolic potential on both magnetic field and spatial confinement length. To achieve this, we restrict ourselves to the case of bulk LO phonons and introduce a trial wave function taken to be the direct product of an electronic part and a part of coherent phonons. We are also concerned with the study of the effects due to the electron–LO-phonon interaction, quantum confinement, and magnetic field on the cyclotron masses associated to the transitions between the ground and first-excited states of an electron bound to a hydrogenic impurity in a 3D parabolic potential. We have found an analytical expression for the impurity magnetopolaron energy that allows us to perform a systematical analysis of the effects of both magnetic field and spatial confinement on the binding energies of impurity magnetopolaron in QD's, QWW's, and QW's. We establish a unified treatment that allows us to make comparisons between the results of binding energies of impurity magnetopolarons in these three systems. The paper is organized as follows. In the next section, we construct a trial wave function as a direct product of the electronic and phonon parts, and then obtain the energy levels of impurity magnetopolaron in terms of certain variational parameters. In Sec. III, we present the results and discussions, together with a conclusion.

II. THEORY

We consider an electron bound to a Coulomb impurity, that is interacting with bulk LO phonons and subjected to a 3D anisotropic harmonic potential and a uniform magnetic field along the z direction. The Fröhlich Hamiltonian of the system is given by

$$H=H_E+\sum_{\mathbf{q}} \hbar\omega_0 b_{\mathbf{q}}^{\dagger} b_{\mathbf{q}}+\sum_{\mathbf{q}} (V_{\mathbf{q}} b_{\mathbf{q}} e^{i\mathbf{q}\cdot\mathbf{r}}+\text{H.c.}), \tag{1}$$

where

$$H_E=\frac{1}{2\mu}\left(\mathbf{p}+\frac{e}{c}\mathbf{A}\right)^2-\frac{e^2}{\epsilon_0|\mathbf{r}|}+\frac{1}{2}\mu\omega_{\perp}^2\mathbf{r}_{\perp}^2+\frac{1}{2}\mu\omega_{\parallel}^2 z^2 \tag{2}$$

is the electronic part, and

$$|V_{\mathbf{q}}|^2=(\hbar\omega_0)^2\left(\frac{4\pi\alpha}{V}\right)\frac{r_0}{q_{\perp}^2+q_z^2} \tag{3}$$

is the electron-phonon interaction amplitude. In Eq. (1), $b_{\mathbf{q}}^{\dagger}$ ($b_{\mathbf{q}}$) is the creation (annihilation) operator of an optical phonon with a wave vector $\mathbf{q}=(\mathbf{q}_{\perp},q_z)$ and an energy $\hbar\omega_0$, and $\mathbf{p}$ and $\mathbf{r}\equiv(r_{\perp},z)$ denote the electron momentum and position operators, respectively. α and r_0 are the electron-phonon coupling constant and polaron radius, respectively. By choosing the symmetrical Coulomb gauge $\mathbf{A}=B(-y,x,0)/2$ for the vector potential, Eq. (2) can then be written as a sum of Hamiltonians for an isotropic 2D harmonic oscillator in the lateral plane with mass μ and the frequency $\omega^2=(\omega_c/2)^2+\omega_{\perp}^2$ plus a term of $(\omega_c/2)L_z$, and a 1D oscillator along the z axis with mass μ and frequency $\omega_{\parallel}$, in the absence of an impurity. In our variational scheme, we choose the electronic part of the trial wave function as a direct product of the basis of these two oscillators in the presence of impurities, since the impurity magnetopolaron we consider is subjected to not only the magnetic confinement in the lateral plane but also to a 3D anisotropic spatial confinement in all directions. Consequently, the variational state vector for the Hamiltonian in Eq. (1) is composed of direct products of electronic and phonon contributions

$$|\Psi_{n,m,l}\rangle=|n,\mp m,l\rangle\otimes D(f)|0\rangle_{\text{ph}}, \tag{4}$$

where $|n,\mp m,l\rangle=|n,\mp m\rangle\otimes|l\rangle$ represent the basis of an electron in a 3D anisotropic harmonic potential subjected to a uniform magnetic field in the z direction, and its coordinate representation is given by

$$\begin{aligned}\langle\mathbf{r}|n,\mp m,l\rangle&=\psi_{n,\mp m}(\mathbf{r}_{\perp})\psi_l(z)\\&=N_{n,\mp m,l}(\gamma,\eta)e^{-\gamma^2\mathbf{r}_{\perp}^2/2}(x\mp iy)^m L_n^m(\gamma^2\mathbf{r}_{\perp}^2)\\&\quad\times e^{-\eta^2z^2/2}H_l(\eta z),\end{aligned} \tag{5}$$

where η and γ are variational parameters. In Eq. (5), $N_{n,\mp m,l}$ is the normalization constant, and the functions L_n^m and H_l are associated Laguerre polynomials and Hermite polynomials, respectively. In Eq. (4), $D(f)$ is the well-known Lee-Low-Pines transformation, by which coherent boson states are generated through the application on the zero-phonon state $|0\rangle_{\text{ph}}$ and is given by

$$D(f)=\exp\left[\sum_{\mathbf{q}} (b_{\mathbf{q}}^{\dagger} f_{\mathbf{q}}-b_{\mathbf{q}} f_{\mathbf{q}}^{*})\right], \tag{6}$$

where $f_{\mathbf{q}}$ is the variational function. With this choice of trial wave function, the expectation value of the impurity magnetopolaron Hamiltonian given by Eq. (1) can then be written in the form $E_{n,\mp m,l}=E^{E}_{n,\mp m,l}+E^{I}_{n,\mp m,l}$, where the first term

$$E^{E}_{n,\mp m,l}(\gamma,\eta)=\left(\frac{\hbar^2}{2\mu}\gamma^2+\frac{1}{2}\mu\omega^2\frac{1}{\gamma^2}\right)(2n+m+1)\mp m\frac{\hbar\omega_c}{2}+\left(\frac{\hbar^2}{2\mu}\eta^2+\frac{1}{2}\mu\omega_{\|}^2\frac{1}{\eta^2}\right)\left(l+\frac{1}{2}\right)-\frac{e^2}{2\pi^2\epsilon_0}\int\frac{d^3\mathbf{k}}{\mathbf{k}^2}\rho_{n,\mp m}(\mathbf{k}_{\perp},\gamma)\rho_l(k_z,\eta) \quad (7)$$

is the electronic part, in which

$$\frac{1}{|\mathbf{r}|}=\frac{1}{2\pi^2}\int\frac{d^3\mathbf{k}}{\mathbf{k}^2}e^{i\mathbf{k}\cdot\mathbf{r}}$$

is used, and the last term

$$E^{I}_{n,\mp m,l}=\sum_{\mathbf{q}}\left[\hbar\omega_0|f_{\mathbf{q}}|^2+V_{\mathbf{q}}f_{\mathbf{q}}\rho_{n,\mp m}(\mathbf{q}_{\perp},\gamma)\rho_l(q_z,\eta)+V^*_{\mathbf{q}}f^*_{\mathbf{q}}\rho^*_{n,\mp m}(\mathbf{q}_{\perp},\gamma)\rho^*_l(q_z,\eta)\right] \quad (8)$$

is the part including the electron-phonon interaction term with the matrix elements $\rho_{n\mp m}(\mathbf{q}_{\perp},\gamma)$ and $\rho_l(q_z,\eta)$, whose full description and various values for different sets of quantum numbers $(n,\mp m,l)$ can be found in Ref. 3. Minimization of $E_{n,\mp m,l}$ with respect to $f^*_{\mathbf{q}}$ yields

$$f_{\mathbf{q}}=-\frac{V^*_{\mathbf{q}}}{\hbar\omega_0}\rho^*_{n,\mp m}(\mathbf{q}_{\perp},\gamma)\rho^*_l(q_z,\eta). \quad (9)$$

After inserting $f_{\mathbf{q}}$ back into Eq. (8), converting the sum over $\mathbf{k}$ into an integral on $\mathbf{k}$ and then changing the integral variables $q_{\perp}/\sqrt{2}\gamma=x$ and $q_z/\sqrt{2}\eta=y$, and similarly $k_{\perp}/\sqrt{2}\gamma=x$ and $k_z/\sqrt{2}\eta=y$ in Eq. (7), the impurity magnetopolaron energy becomes

$$\overline{E}_{n,\mp m,l}(\overline{\gamma},\overline{\eta})=\left(\frac{1}{2\overline{\gamma}^2}+\frac{1}{2}\overline{\omega}^2\overline{\gamma}^2\right)(2n+m+1)\mp m\frac{\overline{\omega}_c}{2}+\left(\frac{1}{2\overline{\eta}^2}+\frac{1}{2}\overline{\omega}_{\|}^2\overline{\eta}^2\right)\left(l+\frac{1}{2}\right)-\frac{2}{\pi}\mathcal{G}_{n,\mp m,l}(\alpha,\beta;\overline{\Omega})\frac{\overline{\eta}}{\overline{\gamma}^2}. \quad (10)$$

By introducing new dimensionless parameters $(\hbar/\mu\omega_0)^{1/2}\gamma=1/\overline{\gamma}$ and $(\hbar/\mu\omega_0)^{1/2}\eta=1/\overline{\eta}$, energy and other parameters are made dimensionless and expressed in terms of the LO-phonon frequency ω_0. Accordingly the dimensionless confinement frequencies $\overline{\omega}_{\perp(\|)}$ are directly related to the dimensionless confinement lengths $u_{\perp(z)}=l_{\perp(z)}/r_0=\sqrt{2/\overline{\omega}_{\perp(\|)}}$. In Eq. (10), the function $\mathcal{G}_{n,\mp m,l}(\alpha,\beta;\overline{\Omega}^2)=\beta\mathbf{I}^{(1)}_{n,\mp m,l}(\overline{\Omega}^2)+\alpha\mathbf{I}^{(2)}_{n,\mp m,l}(\overline{\Omega}^2)$, which contains the electron-phonon coupling strength, and the binding energy parameter $\beta=e^2/\epsilon_0\hbar\omega_0 r_0$ is defined through the relevant integrals

$$\mathbf{I}^{(1)}_{n,\mp m,l}(\overline{\Omega}^2)=\int_0^\infty x\,dx\,\rho_{n,\mp m}(x^2/2)\int_0^\infty dy\frac{e^{-y^2/2}}{\overline{\Omega}^2x^2+y^2}L_l(y^2) \quad (11)$$

and $\mathbf{I}^{(2)}_{n,\mp m,l}(\overline{\Omega}^2)$ whose definition is given by Eq. (24) of Ref. 3. These integrals can be calculated according to values of $\overline{\Omega}^2=\overline{\eta}^2/\overline{\gamma}^2$ in three different cases, for various values of quantum numbers $(n,\mp m,l)$. It should be pointed out that $\overline{\Omega}$ plays an important role in the determination of the features of low-dimensional systems. In order to see this, it is sufficient to consider Eq. (10) in the absence of impurity and electron-phonon interactions, that is, when $\alpha=0$ and $\beta=0$. First, one minimizes the resultant energy with respect to $\overline{\gamma}$ and $\overline{\eta}$, and then obtains $\overline{\gamma}^4=1/\overline{\omega}^2$ and $\overline{\eta}^4=1/\overline{\omega}_{\|}^2$. Finally, substituting these results back into the related energy yields

$$\overline{E}^{(0)}_{n,\mp m,l}=\overline{\omega}(2n+m+1)+\overline{\omega}_{\|}\left(l+\frac{1}{2}\right)\mp m\frac{\overline{\omega}_c}{2}, \quad (12)$$

which defines the well-known Fock-Darwin energy levels[2] in a confining potential and a magnetic field. Thus, the case $\overline{\Omega}^2=1$ $(\overline{\omega}=\overline{\omega}_{\|})$ defines a QD that represents a 3D confinement, that is, quasi-zero-dimensional motion, embedded in a three-dimensional material, whereas $\overline{\Omega}^2>1$ $(\overline{\omega}>\overline{\omega}_{\|})$ and $\overline{\Omega}^2<1$ $(\overline{\omega}<\overline{\omega}_{\|})$ correspond to a QWW and QW that are a 2D confinement (quasi-one-dimensional motion) and a 1D confinement (quasi-two-dimensional motion), respectively, where all confinements are embedded in a 3D material.

The quantity of interest for cyclotron resonance experiments is the cyclotron mass, defined as

$$\overline{m}^*_{n\mp ml}=\overline{\omega}_c/[\overline{E}_{n,\mp m,l}(\overline{\omega}_c,\overline{\omega})-\overline{E}_{0,0,0}(\overline{\omega}_c,\overline{\omega})], \quad (13)$$

which is renormalized in terms of the electron band mass. In Eq. (13), only $(0,0,0)\rightarrow(n,\mp m,l)$ transitions allowed by the selection rules are taken into account, although one can consider the cycylotron masses arising from the other allowed transitions between the excited states. Another important quantity we calculate is the polaronic contribution to the impurity binding energies by defining the variation energy as

$$\Delta E_{n,\mp m,l}=\overline{E}_{n,\mp m,l}(\alpha,\beta;\overline{\omega})-\overline{E}_{n,\mp m,l}(0,\beta;\overline{\omega}), \quad (14)$$

which is simply the difference between the energies in the presence and absence of LO phonons for three types of confinements, the QD's, QWW's, and QW's, respectively.

III. RESULTS AND DISCUSSION

We now consider Eq. (10) in order to obtain the ground- and first-excited-state energies of impurity magnetopolarons in three different cases of $\overline{\Omega}$, each of which corresponds to a physical case on certain conditions as pointed out in the preceding section. First, one needs to evaluate integrals

$\mathbf{I}^{(1)}_{n,\mp m,l}(\overline{\Omega}^2)$ and $\mathbf{I}^{(2)}_{n,\mp m,l}(\overline{\Omega}^2)$, respectively. The integral $\mathbf{I}^{(2)}_{n,\mp m,l}(\overline{\Omega}^2)$ was explicitly evaluated in detail according to the values of $\overline{\Omega}$ in three different cases, for different sets of quantum numbers in Ref. 3. The remaining integral $\mathbf{I}^{(1)}_{n,\mp m,l}(\overline{\Omega}^2)$ can also be evaluated by using the same techniques described in Ref. 3, for the same sets of quantum numbers $(n,\mp m,l)$ we choose, so one obtains

$$\mathbf{I}^{(1)}_{000}(\overline{\Omega}^2)=\sqrt{\frac{\pi}{2}}\frac{1}{\overline{\Omega}^2}\mathcal{F}^0(\overline{\Omega}^2),$$

$$\mathbf{I}^{(1)}_{0\mp 10}(\overline{\Omega}^2)=\sqrt{\frac{\pi}{2}}\frac{1}{\overline{\Omega}^2}\left[\mathcal{F}^0(\overline{\Omega}^2)-\frac{1}{\overline{\Omega}^2}\mathcal{F}^1(\overline{\Omega}^2)\right], \quad (15)$$

$$\mathbf{I}^{(1)}_{001}(\overline{\Omega}^2)=\sqrt{\frac{\pi}{2}}\frac{1}{\overline{\Omega}^2}[\mathcal{F}^0(\overline{\Omega}^2)+\mathcal{F}^1(\overline{\Omega}^2)-\overline{\Omega}^2],$$

with

$$\mathcal{F}^0(\overline{\Omega}^2)=\begin{cases}\dfrac{\overline{\Omega}}{\sqrt{\overline{\Omega}^2-1}}\ln(\overline{\Omega}+\sqrt{\overline{\Omega}^2-1}), & \overline{\Omega}^2>1\\ 1, & \overline{\Omega}^2=1\\ \dfrac{\overline{\Omega}}{\sqrt{1-\overline{\Omega}^2}}\arctan\dfrac{\sqrt{1-\overline{\Omega}^2}}{\overline{\Omega}}, & \overline{\Omega}^2<1\end{cases} \quad (16)$$

and

$$\mathcal{F}^1(\overline{\Omega}^2)=\begin{cases}\dfrac{1}{2}\dfrac{\overline{\Omega}^3}{\overline{\Omega}^2-1}\left[\overline{\Omega}-\dfrac{1}{2}\dfrac{1}{\sqrt{\overline{\Omega}^2-1}}\ln\dfrac{\overline{\Omega}+\sqrt{\overline{\Omega}^2-1}}{\overline{\Omega}-\sqrt{\overline{\Omega}^2-1}}\right], & \overline{\Omega}^2>1\\ 1/3, & \overline{\Omega}^2=1\\ \dfrac{1}{2}\dfrac{\overline{\Omega}^3}{1-\overline{\Omega}^2}\left[\dfrac{1}{\sqrt{1-\overline{\Omega}^2}}\arctan\dfrac{\sqrt{1-\overline{\Omega}^2}}{\overline{\Omega}}-\overline{\Omega}\right], & \overline{\Omega}^2<1.\end{cases} \quad (17)$$

Thus, inserting Eq. (15), together with Eqs. (16) and (17) and the values of $\mathbf{I}^{(2)}_{n,\mp m,l}(\overline{\Omega}^2)$ corresponding to the same sets of quantum numbers, into Eq. (10) provides an explicit expression for the impurity magnetopolaron energy in a 3D anisotropic confining potential, so one can finally perform the minimization of $\overline{E}_{n,\mp m,l}(\overline{\gamma},\overline{\eta})$ with respect to $\overline{\gamma}$ and $\overline{\eta}$, which requires a numerical treatment.

A. $\overline{\Omega}^2=\overline{\eta}^2/\overline{\gamma}^2=1$

This is the case that corresponds to taking $\overline{\omega}=\overline{\omega}_\parallel$ in Eq. (10), since $\overline{\eta}=\overline{\gamma}$. Therefore, this condition defines a box type confinement and represents a QD embedded in a three dimensional material, so one obtains the result

$$\overline{E}_{n,\mp m,l}(\overline{\gamma})=\left(\frac{1}{2\overline{\gamma}^2}+\frac{1}{2}\overline{\omega}^2\overline{\gamma}^2\right)\left(2n+m+l+\frac{3}{2}\right)\mp m\frac{\overline{\omega}_c}{2}-\frac{2}{\pi}\mathcal{G}_{n,\mp m,l}(\alpha,\beta;1)\frac{1}{\overline{\gamma}}, \quad (18)$$

which has to be minimized with respect to $\overline{\gamma}$. Before treating the ground- and first-excited-state energies of impurity magnetopolaron in QD, we first discuss the situation with $\alpha=0$ and $\beta=0$. In this case, it can be easily seen that the variation with respect to $\overline{\gamma}$ gives $\overline{\gamma}^2=1/\overline{\omega}$, which yields $\overline{E}^{(0)}_{n,\mp m,l}=[2n+m+l+(3/2)]\overline{\omega}\mp m(\overline{\omega}_c/2)$, as found in Eq. (12); for the zero magnetic field case, $\overline{\omega}_\perp$ becomes equal to $\overline{\omega}_\parallel$, so that $\overline{E}^{(0)}_{n,\mp m,l}$ reduces to the $\overline{E}^{(0)}_{n,l}=(2n+1)\overline{\omega}_\perp+[l+(1/2)]\overline{\omega}_\perp$, which is the well-known energy eigenvalues of a 3D isotropic oscillator. In the presence of a magnetic field, and further restricting ourselves to the case of $\overline{\omega}_\perp=0$, we come to the result $\overline{E}^{(0)}_{n,\mp m,l}=[n+(m\mp m)/2+(1/2)]\overline{\omega}_c+[l+(1/2)](\overline{\omega}_c/2)$, which is the energy eigenvalue of an electron moving in a homogenous magnetic field and 1D parabolic potential.

Finally, in the presence of an impurity and the electron-phonon interaction, one can minimize Eq. (18) with respect to $\overline{\gamma}$ with appropriate choices for the values of α, β, and the confinement parameter $u_\perp=u_z=u_\bullet$. In Fig. 1(a), we plot the ground- and first-excited-state energies of impurity magnetopolarons in QD's as a function of dimensionless cyclotron frequency $\overline{\omega}_c$, for some values of α and β, at fixed confinement length, i.e., $u_\bullet=2$, which corresponds to the case $l_\bullet=2r_0$. It is obvious from the results presented in Fig. 1(a) that the magnetic field lifts the degeneracy of levels with positive and negative values of m, when no impurity and electron-phonon interaction are present (dashed curves), since the magnetic field dependence of the relevant Fock-Darwin energy levels is given by Eq. (12), that is, $\overline{E}^{(0)}_{000}=3\overline{\omega}/2$, $\overline{E}^{(0)}_{0\mp 10}=(5\overline{\omega}\mp\overline{\omega}_c)/2$, and $\overline{E}^{(0)}_{001}=5\overline{\omega}/2$, and they become $\overline{E}^{(0)}_{000}=3\overline{\omega}_\bullet/2$, $\overline{E}^{(0)}_{0\mp 10}=\overline{E}^{(0)}_{001}=5\overline{\omega}_\bullet/2$ in the absence of magnetic field, i.e., $\overline{\omega}_c=0$. One also notices, by comparing the thin solid curves $(\alpha=0,\beta=1)$ and dashed ones in Fig. 1(a), that the presence of impurity shifts the energy spectrum to lower energies. Additionally, switching the electron-phonon interaction on (thick solid curves) yields each level to further shift down to much deeper values. In order to understand better the influence of electron-phonon interaction on impurity electronic levels, we have also plotted the binding energies of impurity magnetopolaron in a QD as a function of magnetic field in Fig. 1(b), at a fixed confinement length of the QD. It can be seen that, by comparing solid and

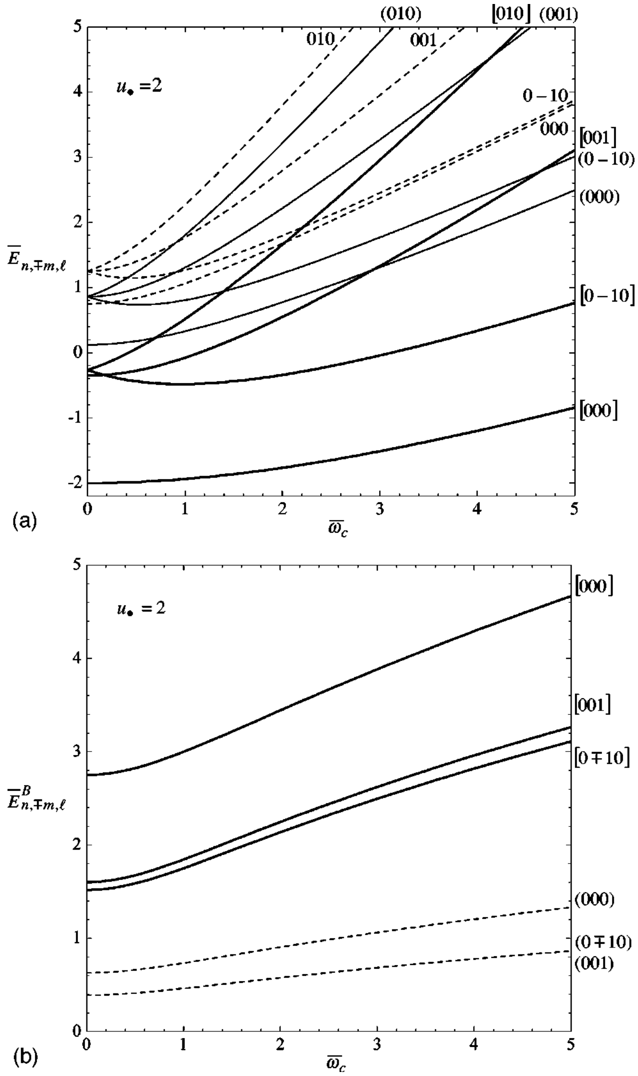

FIG. 1. (a) Cyclotron frequency dependence of impurity magnetopolaron energy levels $\overline{E}_{n,\mp m,l}$ in a QD with $u_{\bullet}=2$. The dashed ($\beta=0$) and thin lines ($\beta=1$) represent the unperturbed energy levels ($\alpha=0$), and they are denoted by the labels $n\mp ml$ and ($n\mp ml$), respectively. The thick lines represents the perturbed energy levels, denoted by $[n\mp ml]$, with $\beta=1$, at $\alpha=3$. (b) Binding energies of impurity magnetopolaron as a function of cyclotron frequency at $u_{\bullet}=2$ and $\beta=1$. The dashed and solid lines again represent the unperturbed ($\alpha=0$) and perturbed ($\alpha=3$) levels.

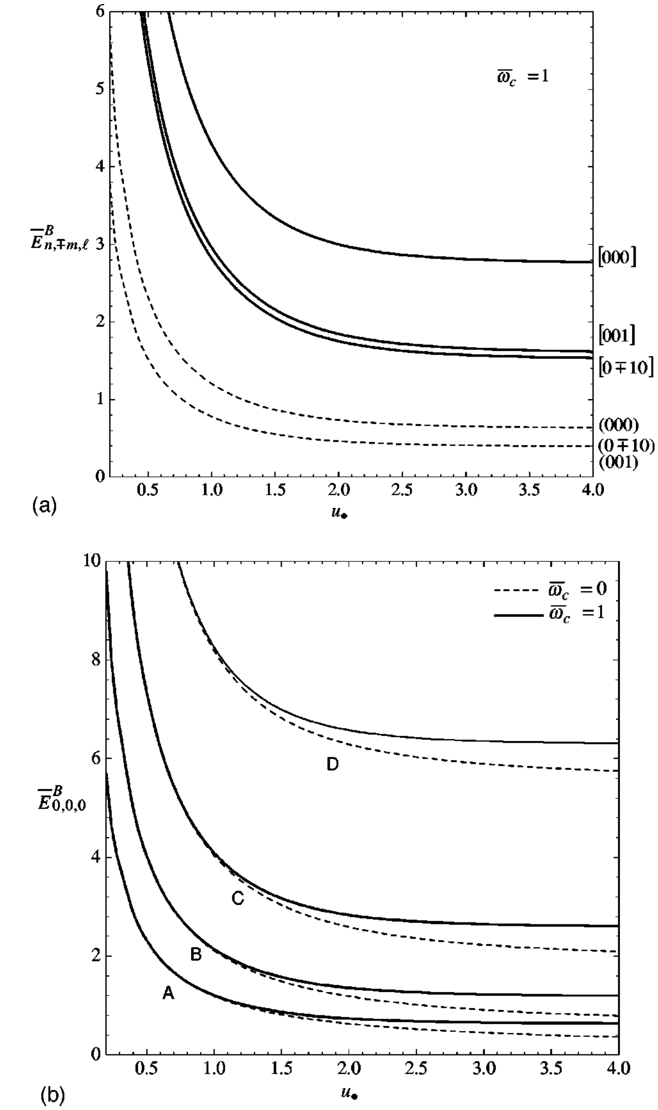

FIG. 2. (a) Binding energies of the impurity magnetopolaron as a function of confinement length $u_{\bullet}$, at fixed cyclotron frequency, i.e., $\overline{\omega}_c=1$, and at $\beta=1$. Again, the dashed and solid curves represent the unperturbed ($\alpha=0$) and perturbed ($\alpha=3$) cases, respectively. (b) Ground-state binding energy of impurity magnetopolaron as a function of confinement length $u_{\bullet}$ in the absence and the presence of magnetic field; for $\alpha=0$ and $\beta=1$ (A), for $\alpha=1$ and $\beta=1$ (B), for $\alpha=0$ and $\beta=3$ (C), and for $\alpha=3$ and $\beta=3$ (D).

dashed curves, switching the electron-phonon interaction on results in very large binding energies, and they increase with increasing magnetic field. Figure 2(a) displays the variation of binding energies of the impurity magnetopolaron with the confinement length of the QD for the same values of the parameters used in Fig. 1(b), but at fixed magnetic field, i.e., $\overline{\omega}_c=1$. We observe from the figure that the binding energies of impurity magnetopolaron increases with decreasing confinement length of the QD, which is consistent with the uncertainty principle, i.e., more localization of the particle results in an increase of its momentum and consequently a greater energy. To observe the combined effects of both spatial and magnetic confinements together with the binding parameter β on ground-state binding energy of impurity magnetopolaron in the QD, we also give a plot displaying the behavior of ground-state binding energy of impurity magnetopolaron as a function of the confinement length of the QD, with the same parameters of Fig. 1(a), but at two different values of magnetic field, i.e., $\overline{\omega}_c=0$ and 1 [Fig. 2(b)]. We can see from the figure that, by comparing the dashed curves with solid ones, the effect of magnetic field on the binding energy of impurity magnetopolaron begins to dominate with increasing values of electron-phonon coupling strength and binding parameter.

Before presenting the numerical results on the cyclotron mass by means of the results obtained above, we shall give a simple analysis of Eq. (18) by discussing its asymptotic expressions to provide better insight into the dependencies of $\overline{E}_{n,\mp m,l}$ on α and $\overline{\omega}$, from which cyclotron masses of an impurity magnetopolaron subjected to a 3D parabolic confinement potential can be obtained analytically. To do this, we minimize Eq. (18) with respect to $\overline{\gamma}$, and find a fourth-order equation in $\overline{\gamma}$,

$$\overline{\omega}^2\left(2n+m+l+\frac{3}{2}\right)\overline{\gamma}^4+2\mathcal{G}_{n,\mp m,l}(\alpha,\beta;1)\overline{\gamma}-\left(2n+m+l+\frac{3}{2}\right)=0,$$

whose asymptotic analysis can be made in some certain cases. First, one can consider the case of strong confinement and small values of α and β, i.e., $\overline{\omega}\gg\alpha,\beta$, and then obtains $\overline{\gamma}^2=1/\overline{\omega}$. In this case, the excited-state energies are obtained as

$$\overline{E}_{n,\mp m,l}=\left(2n+m+l+\frac{3}{2}\right)\overline{\omega}\mp m\frac{\overline{\omega}_c}{2}-\frac{2}{\pi}\sqrt{\overline{\omega}}\mathcal{G}_{n,\mp m,l}(\alpha,\beta;1). \tag{19}$$

Thus, the renormalized cyclotron mass associated with the transitions between the ground and excited states are found to be in the form

$$\overline{m}^*_{n\mp ml}=\overline{\omega}_c\left\{(2n+m+l)\overline{\omega}\mp m\frac{\overline{\omega}_c}{2}-\frac{2}{\pi}\sqrt{\overline{\omega}}[\mathcal{G}_{n,\mp m,l}(\alpha,\beta;1)-\mathcal{G}_{0,0,0}(\alpha,\beta;1)]\right\}^{-1}, \tag{20}$$

which holds approximately for $\overline{\omega}\gg\alpha,\beta$. The other asymptotic expression for the excited-state energies can be obtained in the case of large values of α and β and small confinements, i.e., $\overline{\omega}\ll\alpha,\beta$. In this limit, the variational parameter minimizing the energy is found as $\overline{\gamma}=\pi(2n+m+l+\frac{3}{2})/2\mathcal{G}_{n,\mp m,l}(\alpha,\beta;1)$, so that by substituting this root back into Eq. (18), the excited-state energies of an impurity magnetopolaron in a QD become

$$\overline{E}_{n,\mp m,l}=-\frac{2}{\pi^2}\frac{\mathcal{G}^2_{n,\mp m,l}(\alpha,\beta;1)}{\left(2n+m+l+\frac{3}{2}\right)}\mp m\frac{\overline{\omega}_c}{2}+\frac{1}{8}\overline{\omega}^2\left(2n+m+l+\frac{3}{2}\right)^3\frac{\pi^2}{\mathcal{G}^2_{n,\mp m,l}(\alpha,\beta;1)}, \tag{21}$$

from which, by substituting this back into the definition of the renormalized cyclotron mass given by Eq. (13), one can obtain an explicit analytical expression for the relevant renormalized cyclotron masses, depending on $\overline{\omega}$, α, and β.

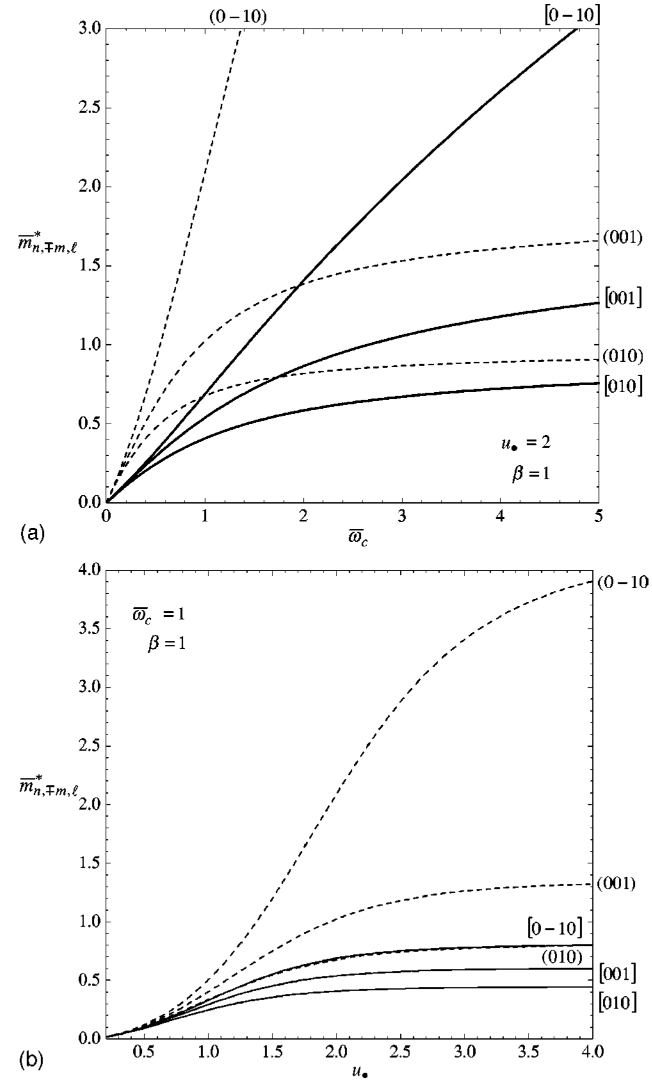

FIG. 3. The renormalized cyclotron mass $\overline{m}^*_{n,\mp m,l}$ of the impurity magnetopolaron in a QD as a function of (a) dimensionless cyclotron frequency at $u_\bullet=2$ and (b) dimensionless confinement length at $\overline{\omega}_c=1$, for $\beta=1$. The dashed and solid lines represent the transitions $(0,0,0)\rightarrow(n,\mp,m,l)$ with $\alpha=0$ and the transitions $[0,0,0]\rightarrow[n,\mp,m,l]$ with $\alpha=3$, for $\beta=1$, respectively.

Figures 3(a) and 3(b) show the variation of the renormalized cyclotron masses associated with the first three transitions $(0,0,0)\rightarrow(n,\mp m,l)$ of impurity magnetopolaron in a QD with both dimensionless cyclotron frequency $\overline{\omega}_c$ for the fixed value of dimensionless confinement length $u_\bullet=\sqrt{2/\overline{\omega}_\bullet}$ [Fig. 3(a)], and dimensionless confinement length for the resonance value of cyclotron frequency, i.e., $\overline{\omega}_c=1$ [Fig. 3(b)], respectively. For comparison, cyclotron masses for those transitions in the absence and the presence of an electron–LO-phonon interaction are represented by dashed and solid curves and are abbreviated as $(n,\mp m,l)$ and $[n,\mp m,l]$, respectively. Both the dotted and solid curves are for $\beta=1$. One notices, by comparing the solid and dotted curves in Fig. 3(a), that for the transition $(0,0,0)\rightarrow(0,-1,0)$ the influence of the electron-phonon interaction on the related cyclotron mass becomes more enhanced with increasing magnetic field, compared to those of other transitions, and that the cyclotron mass of the transition $(0,0,0)\rightarrow(0,1,0)$ tends to the bare band mass. As can be seen from Fig. 3(b), increasing the degree of confinement yields a decrease in relevant cyclotron masses.

In the framework of this approach, one can also analyze the polaronic contribution to the energy levels of the impurity magnetopolaron in the case of strong confinement by using Eq. (14), and finds its approximate expression as

$$\Delta E_{n,\mp m,l}=-\frac{2}{\pi}\sqrt{\bar{\omega}}[\mathcal{G}_{n,\mp m,l}(\alpha,\beta;1)-\mathcal{G}_{n,\mp m,l}(0,\beta;1)],$$

which is independent of β. To see this, one can easily calculate the polaronic correction to the ground-state energy and find $\Delta E_{0,0,0}=-\alpha\sqrt{\bar{\omega}/\pi}$. In the contrary case, i.e., in the case of large values of α and β and small confinements, one uses Eq. (21) for the definition of variation energy, and then obtains the polaronic correction to the ground-state energy as $\Delta E_{0,0,0}=-(\alpha+2\sqrt{2}\beta)/3\pi$, by neglecting the contribution from the last term in Eq. (21) which is proportional to $\bar{\omega}^2$.

B. $\bar{\Omega}^2=\bar{\eta}^2/\bar{\gamma}^2>1$

This condition, $\bar{\eta}^2>\bar{\gamma}^2$, allows us to keep the confining potential only in the lateral plane and to take $\bar{\omega}_\parallel=0$, so that electrons are free to move along the z axis. Therefore, this case defines a 2D confinement, quasi-one-dimensional motion, and represents a QWW embedded in a 3D material. Therefore, one needs to minimize

$$\bar{E}_{n,\mp m,l}(\bar{\gamma},\bar{\eta})=\left(\frac{1}{2\bar{\gamma}^2}+\frac{1}{2}\bar{\omega}^2\bar{\gamma}^2\right)(2n+m+1)\mp m\frac{\bar{\omega}_c}{2}+\left(\frac{1}{2\bar{\eta}^2}+\frac{1}{2}\bar{\omega}_\parallel^2\bar{\eta}^2\right)\left(l+\frac{1}{2}\right)-\frac{2}{\pi}\mathcal{G}_{n,\mp m,l}(\alpha,\beta;\bar{\Omega}^2>1)\frac{\bar{\eta}}{\bar{\gamma}^2} \quad (22)$$

with respect to both $\bar{\eta}$ and $\bar{\gamma}$, with an appropriate choice of the parameters α and β and the confinement parameter $u_\perp$. In Fig. 4(a), we plot the ground- and first-excited-state energies of an impurity magnetopolaron in QWW as a function of magnetic field, i.e., $\bar{\omega}_c$, whose behavior is, at a first glance, similar to that obtained in a QD. But, it is apparent from the comparison of Figs. 4(a) and 1(a) that (001) is not split in the absence of both the impurity and electron-phonon interaction, since, from Eq. (12), magnetic field dependencies of those energy levels are now given as $\bar{E}_{000}^{(0)}=\bar{E}_{001}^{(0)}=\bar{\omega}$ and $\bar{E}_{0\mp10}^{(0)}=2\bar{\omega}\mp(\bar{\omega}_c/2)$. Additionally, two points should be noted, by comparing the thin solid curves with dashed ones, and dashed curves with thick solid ones; the first is that (000) and (001) levels have no longer the same energies in the presence of an impurity, i.e., they are split (thin solid curves) and further shifted down to lower energy values. The second point concerns the presence of the electron-phonon interaction, in which each level is further shifted down (thick solid curves).

In Figs. 4(b) and 4(c), cyclotron masses for the transitions $(0,0,0)\rightarrow(n,\mp m,l)$ of the impurity magnetopolaron calculated from Eq. (10) are plotted as a function of both dimensionless cyclotron frequency [Eq. 4(b)] and dimensionless confinement length [Fig. 4(c)], for the same values of the parameters used in Fig. 1. Again, the solid and dashed curves correspond to the presence and the absence of the electron-phonon interaction, respectively. The behavior of cyclotron masses for the relevant transitions under the variation of both magnetic field and confinement length of the QWW is different from those found in the QD even in the absence of electron–LO-phonon coupling. Although their behavior is nearly the same at higher magnetic fields, they show quite different behavior at low magnetic fields, particularly, around $\bar{\omega}_c=1$. In particular, the cyclotron mass for the transition $(0,0,0)\rightarrow(0,0,1)$ behaves rather differently even in the absence of the electron–LO-phonon interaction as seen from Fig. 4(b)

C. $\bar{\Omega}^2=\bar{\eta}^2/\bar{\gamma}^2<1$

The characteristic condition of this case is $\bar{\eta}^2<\bar{\gamma}^2$. Here, it is possible to take $\bar{\omega}_\perp=0$, that is, a confining potential along the z axis keeps the electrons moving in the lateral plane freely. Therefore, this case defines a quasi-two-dimensional motion with 1D confinement and represents a QW embedded in a 3D material. Hence, one can easily find the ground- and first excited-state energies of the impurity magnetopolaron energy by using Eq. (22), provided that one now inserts integrals $\mathbf{I}_{n,\mp m,l}^{(1)}(\bar{\Omega}^2<1)$ and $\mathbf{I}_{n,\mp m,l}^{(2)}(\bar{\Omega}^2<1)$ in the related equation and then performs the minimization with respect to both $\bar{\eta}$ and $\bar{\gamma}$. In Fig. 5(a), we plot the ground- and first-excited-state energies of impurity magnetopolaron in a QW as a function of magnetic field $\bar{\omega}_c$. The behavior of energy spectra of impurity magnetopolaron in a QW under the variation of magnetic field is quite different from those in the QD and QWW, since now the magnetic field dependence of the Fock-Darwin energy levels are given in the form $\bar{E}_{000}^{(0)}=(\bar{\omega}_c+\bar{\omega}_\parallel)/2$, $\bar{E}_{0\mp10}^{(0)}=(2\bar{\omega}_c\mp\bar{\omega}_c+\bar{\omega}_\parallel)/2$ and $\bar{E}_{001}^{(0)}=(\bar{\omega}_c+3\bar{\omega}_\parallel)/2$, and they are linear in $\bar{\omega}_c$, in the absence of the electron-phonon interaction and impurity. The presence of an impurity yields a splitting of (0 – 10) and (000) levels, and energy levels shift down. The presence of an electron-phonon interaction makes each level shift further down as well.

Figures 5(b) and 5(c) show the relevant cyclotron masses of the first three transitions as a function of both dimensionless cyclotron frequency [Fig. 5(b)] and dimensionless confinement length [Fig. 5(c)], respectively, with the same parameters used in Figs. 3 and 4. The characteristic features of the cyclotron mass for the transitions $(0,0,0)\rightarrow(n,\mp m,l)$ in the QW are quite different from those found in both the QD and QWW. In particular, their variation with the confinement length at fixed magnetic field is noticeable, as seen from Fig. 5(c).

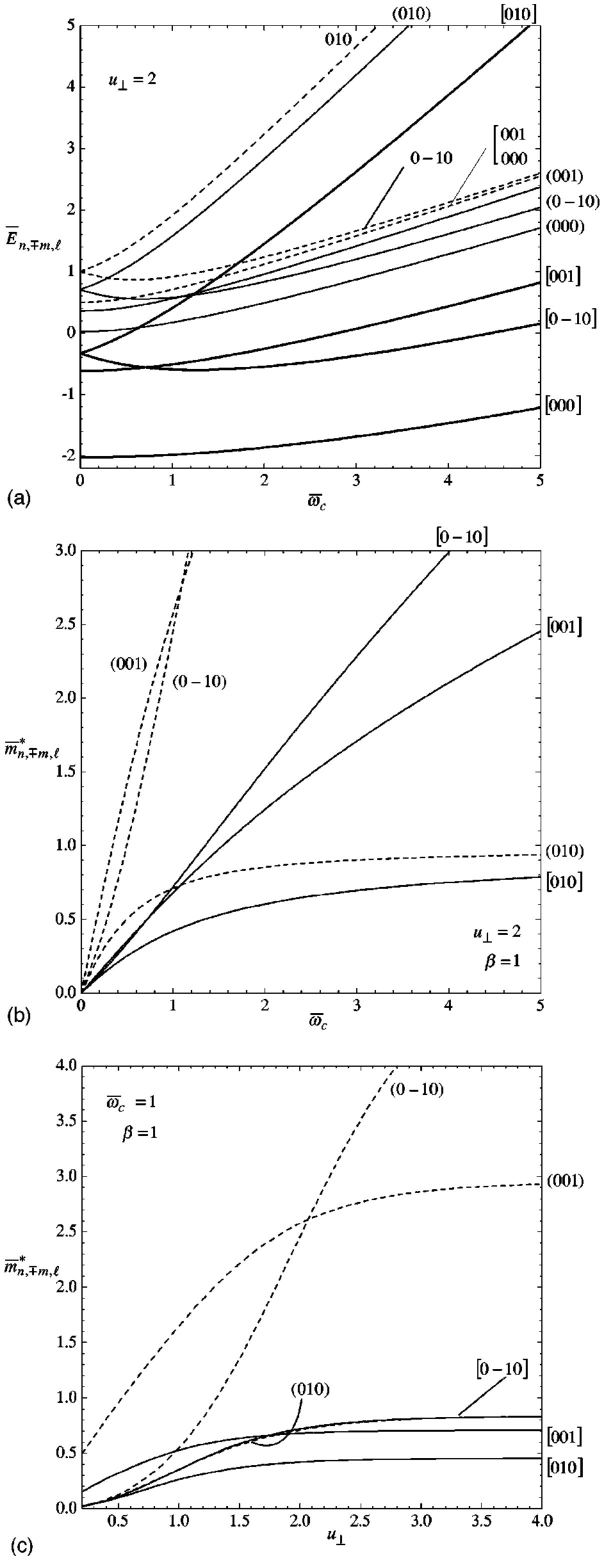

FIG. 4. (a) Cyclotron frequency dependence of impurity magnetopolaron energy levels, $\overline{E}_{n,\mp m,l}$, in a QWW with $u_\perp = 2$. Same parameters are used for each case as in Fig. 1(a). The renormalized cyclotron mass $\overline{m}^*_{n,\mp m,l}$ of the impurity magnetopolaron in a QWW as a function of (b) dimensionless cyclotron frequency at $u_\perp = 2$ and (c) dimensionless confinement length at $\overline{\omega}_c = 1$, for $\beta = 1$. The dashed and solid lines represent the transitions $(0,0,0) \to (n,\mp,m,l)$ with $\alpha = 0$ and the transitions $[0,0,0] \to [n,\mp,m,l]$ with $\alpha = 3$, for $\beta = 1$, respectively.

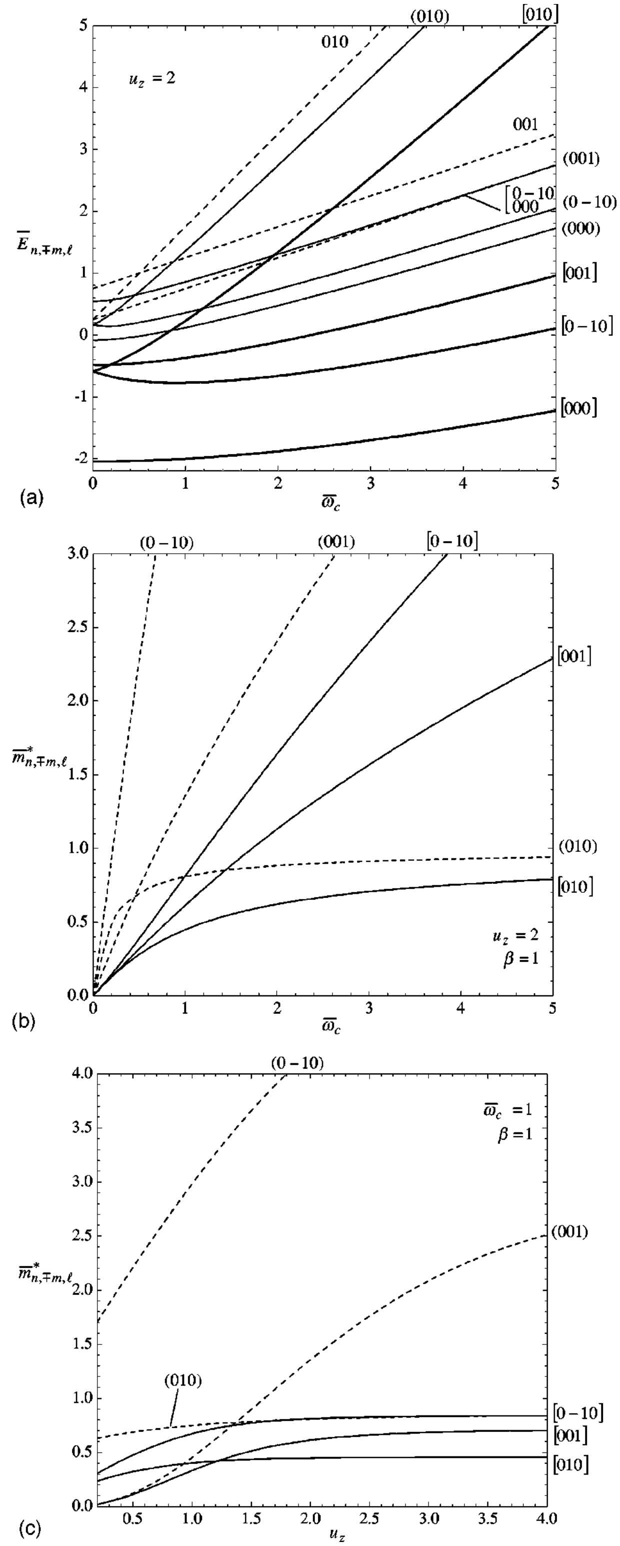

FIG. 5. (a) Cyclotron frequency dependence of impurity magnetopolaron energy levels $\overline{E}_{n,\mp m,l}$ in a QW with $u_\parallel = 2$. The same parameters are used for each case as in Fig. 1(a). The renormalized cyclotron mass $\overline{m}^*_{n,\mp m,l}$ of the impurity magnetopolaron in a QW as a function of (b) dimensionless cyclotron frequency at $u_z = 2$ and (c) dimensionless confinement length at $\overline{\omega}_c = 1$, for $\beta = 1$. The dashed and solid lines represent the transitions $(0,0,0) \to (n,\mp,m,l)$ with $\alpha = 0$ and the transitions $[0,0,0] \to [n,\mp,m,l]$ with $\alpha = 3$, for $\beta = 1$, respectively.

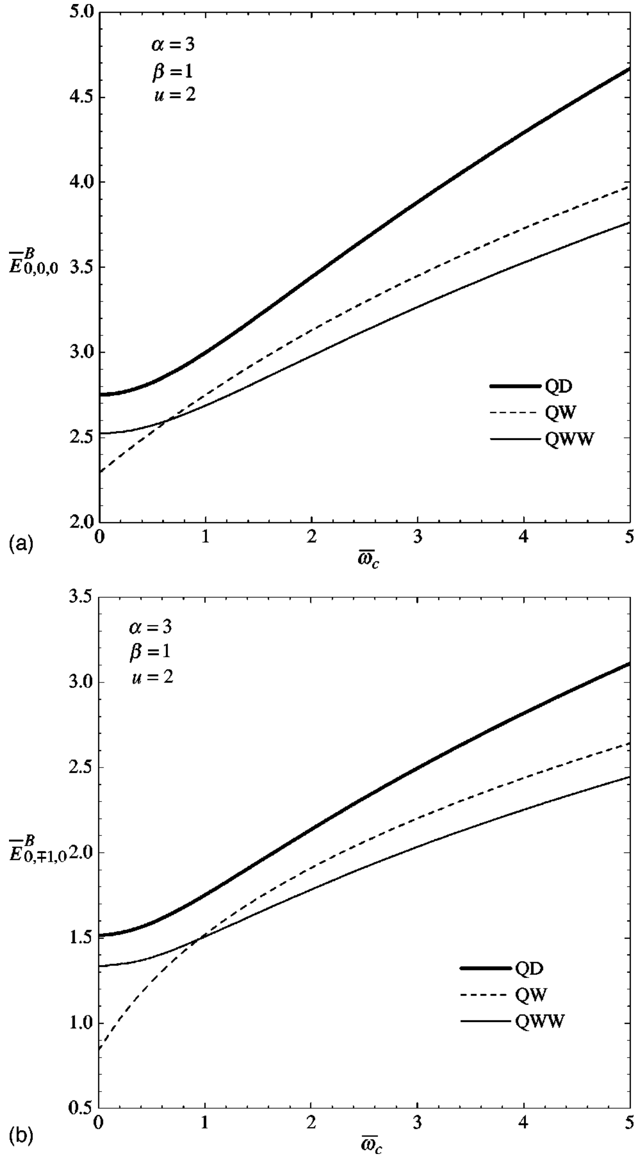

FIG. 6. (a) Ground-state binding energy of the impurity magnetopolaron in QD, QW, and QWW structures as a function of dimensionless cyclotron frequency. (b) First two excited-state binding energies of the impurity magnetopolaron as a function of the dimensionless cyclotron frequency in the same structures.

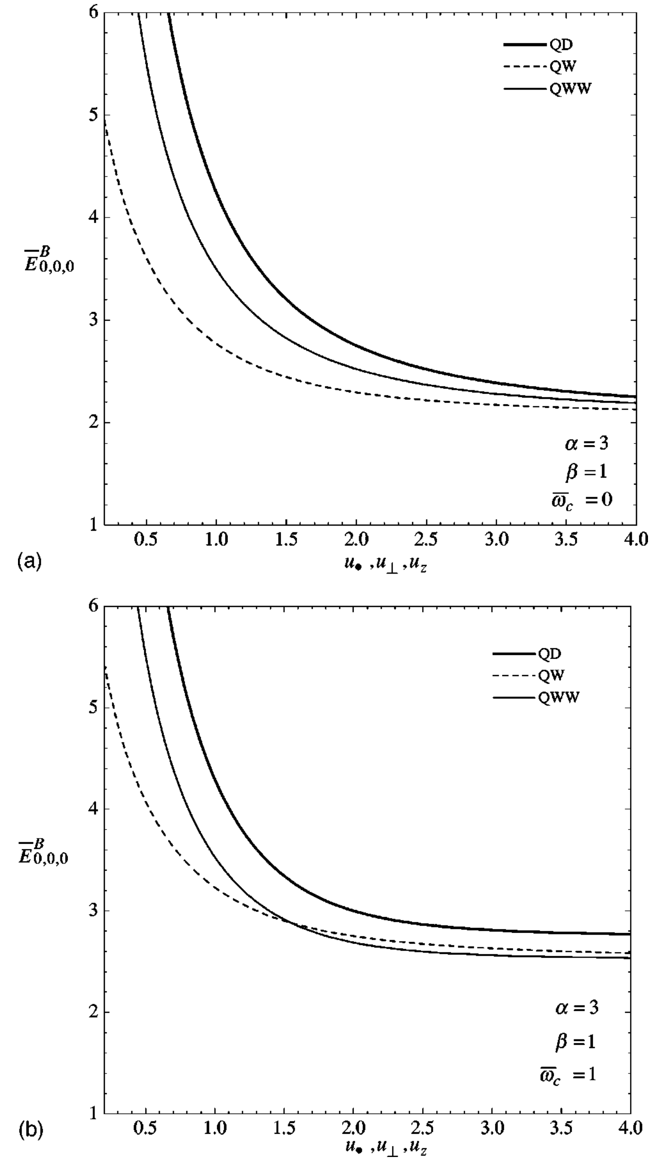

FIG. 7. Ground-state binding energy of the impurity magnetopolaron as a function of confinement length (a) in the absence and (b) the presence of a magnetic field.

Figure 6(a) presents a comparison of the magnetic field dependence of the ground-state binding energy of impurity magnetopolaron in a QD with the results of those found in a QW and QWW at fixed values of α and β and $u_\perp = u_z = u_\bullet = 2$. It is clear from this graph that the contribution of LO phonons to the impurity ground-state binding energy is greater than those found in a QW and QWW, up to a critical value of the magnetic field. At this value of the magnetic field, the binding energy of the impurity magnetopolaron in a QW becomes larger than that in a QWW. This phenomenon arises from the existence of an additional magnetic confinement term in the lateral plane; in other words, the QW case becomes a 3D confinement in quasi-zero-dimensional motion similar to a QD, whereas the QWW system still has 2D spatial confinement together with an extra magnetic confinement in the lateral plane. This feature can also be observed for excited states, as seen from Fig. 6(b).

The comparison of the ground-state binding energy of impurity magnetopolaron for three types of confinement in the absence ($\overline{\omega}_c = 0$) and in the presence ($\overline{\omega}_c = 1$) of a magnetic field as a function of confinement length is shown in Figs. 7(a) and 7(b), respectively. An inspection of Fig. 7(a) reveals that the ground-state binding energy of the impurity magnetopolaron is more enhanced in a more confined system in the absence of the magnetic field, and they coalesce smoothly into a bulk value when the confinement degree decreases, as expected. However, the existence of a magnetic field causes the phenomena described above, i.e., due to the additional magnetic confinement in the lateral plane, the ground-state binding energy of the impurity magnetopolaron in a QW is larger than that in a QWW, up to a certain value of the confinement length; as the confinement length increases, this is reversed as seen from Fig. 7(b).

To conclude the discussion, we summarize the qualitative aspects of present work. Our present investigations demonstrate that the complicated dependencies of the ground- and first-excited-state energies of impurity magnetopolaron subjected to an anisotropic confining potential on spatial confinement parameters and magnetic field together with the electron-phonon coupling strength α and binding energy parameter β can be examined with the use of variational scheme given in the present paper, which allows one to study the effect of electron-phonon interaction on energy levels of the impurity magnetopolaron, not only for a QD system but also for QW and QWW systems. As is expected, by setting $\beta=0$, Eq. (10) can be reduced to the magnetopolaron energy found in Ref. 3, whose authors have checked their results with various asymptotic limits and conclude that they unify all expressions for the ground-state energy of the magnetopolaron found by other authors cited therein. Accordingly, we conclude that the binding energies of the impurity magnetopolaron have more pronounced effects when both magnetic and spatial confinement take place. Additionally, in the absence of magnetic field, the analysis has shown that the most pronounced effects due to the electron–LO-phonon interaction in the binding energies of the impurity magnetopolaron have been found in the most confined structure, i.e., in the QD, and then in the QWW. But, in the presence of a magnetic field, the binding energies of the impurity magnetopolaron in a quasi-2D system with a 1D confinement, i.e., in a QW, begin to be more enhanced around $\overline{\omega}_c=1$ compared with those found in a quasi-1D system with 2D confinement, i.e., in a QWW. This phenomenon is due to the fact that there exists an additional term in the Hamiltonian that leads to magnetic confinement in the lateral plane and induces a QW system, in a sense, to become a 3D confinement in quasi-zero-dimensional motion similar to a QD, whereas the QWW system is still a quasi-two dimensional confined structure even in the presence of such a term. We have also investigated the polaronic correction, associated with electron–LO-phonon coupling, to the cyclotron mass of the impurity magnetopolaron confined in a 3D parabolic potential, and concluded that the presence of electron–LO-phonon interaction leads to important changes in the behavior of cyclotron mass under the variation of magnetic field in confined structures. In summary, by presenting a detailed comparative analysis of the effects of quantum confinements and magnetic field on the ground- and first-excited-state energies of the impurity magnetopolaron in a parabolic QD, QW, and QWW, we have shown that the binding energies of the impurity magnetopolaron increase with increasing degree of spatial confinement and there also occurs an additional increase in the related binding energies in the presence of a magnetic field.

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