Effective conductivity of two-dimensional electrons in heterostructures with nonuniform doping

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The frequency and temperature dependences of the effective conductivity are calculated in selectively doped heterostructures with long-range variations of energy levels caused by nonuniform donor distribution. The random electric field induced by lateral redistribution of the two-dimensional concentration modifies the frequency dispersion of response in comparison with the standard Drude mechanism. The spectral and temperature dependences of the conductivity are investigated and the interplay between the Drude mechanism and the random contribution is analyzed.

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I. INTRODUCTION

It is generally accepted that, the linear response of free carriers in the classical frequency region (if $\hbar\omega$ is smaller than the mean energies and the interband gap) shows the Drude dispersion. This fact has no analogies with a gas plasma response for the nonuniform case, where the contribution of random electric fields (in the space-time domain) plays an essential role due to the Landau damping[1] and spectral dependences of response are more complicated even for the classical frequencies. A long-range potential is screened effectively in the three-dimensional (3D) or 2D solid-state plasma so that the nonuniform electric field is of little importance and only the concentration redistribution is essential. Since the mean free path is usually shorter than the characteristic scale of the nonuniform doping, the response can be described in the framework of the effective-media approach (see Refs. 2 and 3). But this condition is violated in selectively doped heterostructures where the donors are separated from 2D electrons by the distance Z (see Fig. 1; for sake of simplicity we consider the δ-doping case). Several variants of the lateral inhomogeneities of donors have been discussed. A large-scale potential appears even with the δ-correlated donors because the Fourier components with wave vectors less than Z^{-1} do not act on 2D electrons [see Ref. 4 and Eqs. (4) and (5) below]. The modifications on mobility due to the spatial correlations of donors were calculated in Refs. 5 and 6 while the mechanisms of formation of the in-plane inhomogeneous distribution of donors (due to nonuniform occupation of donor states or due to impurity diffusion) have been discussed in Refs. 7 and 8. In the present paper, we examine the frequency-dependent conductivity of the in-plane nonuniform selectively doped heterostructure taking into account a lateral electron redistribution under the long-range potential. The interplay between the Drude dispersion, caused by the standard relaxation processes, and the contribution from the random electric fields, due to the planar large-scale inhomogeneities, is analyzed when the scale of random variations of potential l_c is shorter than the relaxation length l_F, i.e., in the *nonlocal regime* of response.

The Drude dispersion of 2D conductivity has been examined at far-infrared spectral region as far back as the silicon inversion layers were started to be investigated.[9] Some results concerning AlGaAs-based heterostructures have also been published (see Ref. 10 that mainly deals with the 2D plasmon dispersion). A clear demonstration of the Drude dispersion of conductivity at high-frequency region was presented for the high-mobility selectively doped structures.[11] The measurements were performed by use of high-quality structures with the wide spacer Z when the variations of potential are essential only for $l_c \simeq Z$. The peculiarities of the low-frequency dispersion of conductivity under the large-scale potential due to a ''long-time tail'' of the velocity correlations were considered beyond the kinetic approximations (see Ref. 12 and references therein). To the best of our knowledge, the effect of a random electric field on the frequency dispersion of conductivity is not considered previously even for the kinetic regime of response, $l_c \lesssim l_F$. The calculations below are based on the electrostatic description of the long-ranged variations of potentials in selectively-doped heterostructures and on the linearized kinetic equation that is solved for the quasielastic scattering of electrons by acoustic phonons. The random electric field is due to the electron redistribution induced by the current in the nonuniform heterostructure and is determined from the frequency-dependent component of electric neutrality condition that is solved self-consistently. We analyze the frequency and temperature dependences of the effective conductivity in selectively doped heterostructures and find that the ratio l_c/l_F is strongly affected by these dependences.

The paper is organized as follows. In Sec. II, we describe the potential induced by nonuniform donor sheet and evaluate the general expression for effective conductivity tensor based on the linearized kinetic equation. We also perform the self-consistent calculation of the random electric field based on the electric neutrality condition. In Sec. III, we discuss the modifications of frequency dependences and temperature dependences of the effective conductivity in the nonuniform heterostructures with the dominant mechanism of relaxation due to scattering by acoustic phonons. The assumptions used

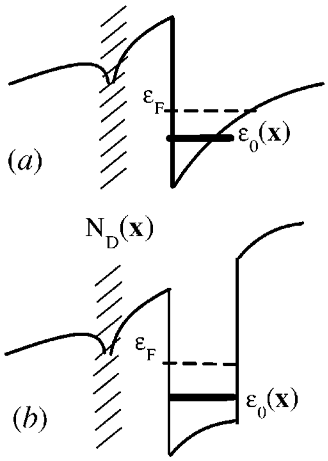

FIG. 1. Band diagrams along z axis for selectively doped heterojunction (a) and quantum well (b) with the nonuniform ground level $\varepsilon_o(\mathbf{x})$ due to the nonuniform donor distribution $N_D(\mathbf{x})$.

and concluding remarks are presented in the last section. In the Appendix we briefly describe the parameters of quasi-elastic collision integral.

II. BASIC EQUATIONS

In order to describe the effective response, we evaluate the energy level variations due to the in-plane nonuniform δ doping. We next consider the general expression for the induced current and evaluate the averaged contribution of the random electric field to this current performing the self-consistent solution of the integral equation obtained from the Poisson equation.

A. Long-range disorder

The self-consistent description of the 2D electrons in selectively doped structures with long-range variations of donor distribution over 2D plane (see Fig. 1) is based the Schrödinger and the Poisson equations. Since the Schrödinger equation contains the self-consistent potential $V_{\mathbf{x}z}$, which is parametrically dependent on the 2D coordinate $\mathbf{x}$ for the large-scale variations of levels, we write the concentration of electrons as $n_{\mathbf{x}z}=\rho_{2\mathrm{D}}(\varepsilon_F-w_{\mathbf{x}})\psi_{\mathbf{x}z}^2$. Here $\rho_{2\mathrm{D}}$ is the 2D density of states, ε_F the Fermi energy, $\psi_{\mathbf{x}z}$ the wave function of the ground state, and $w_{\mathbf{x}}=\int dz\, V_{\mathbf{x}z}\psi_{\mathbf{x}z}^2$ is the long-range random addendum to the energy of the 2D electrons. This system of equations can be simplified in the case of weak variations of the ground level by using the first-order perturbation theory. Neglecting the correction to the wave function, we obtain $w_{\mathbf{x}}=\int dz\, \delta V_{\mathbf{x}z}\psi_z^2$, where $\delta V_{\mathbf{x}z}$ is a nonuniform part of the self-consistent potential. Performing the in-plane Fourier transformation, we write the linearized Poisson equation as follows:

$$\left(\frac{d^2}{dz^2}-q^2\right)\delta V_{\mathbf{q}z}=\frac{4\pi e^2}{\epsilon}(\rho_{2\mathrm{D}}w_{\mathbf{q}}\psi_z^2+\delta\mathcal{N}_{\mathbf{q}z}), \tag{1}$$

where ϵ is the dielectric constant supposed to be uniform, and $\delta\mathcal{N}_{\mathbf{q}z}$ is the nonuniform part of the donor distribution over δ layer.

The solution of Eq. (1) is written through the Green's function, $-\exp(-q|z-z'|)/(2q)$, as

$$w_{\mathbf{q}}=-\int dz\,\psi_z^2\int dz'\,e^{-q|z-z'|}\left(\frac{2}{a_Bq}q_{\mathbf{q}}\psi_{z'}^2+\frac{4\pi e^2}{\epsilon q}\delta\mathcal{N}_{\mathbf{q}z'}\right), \tag{2}$$

where a_B is the Bohr radius. Assuming that $q^{-1}\gtrsim l_c$ is larger than the width of the 2D electron layer, we write the coefficients in Eq. (2) in the form

$$\int dz\,\psi_z^2\int dz'\,e^{-q|z-z'|}\psi_{z'}^2\simeq 1,$$

$$\int dz\,\psi_z^2\int dz'\,e^{-q|z-z'|}\delta\mathcal{N}_{\mathbf{q}z'}\simeq\delta n_{\mathbf{q}}e^{-qZ}, \tag{3}$$

so that under the condition $l_c\gg a_B$ the solution of Eq. (2) is given by

$$w_{\mathbf{q}}\simeq-\frac{\delta n_{\mathbf{q}}}{\rho_{2\mathrm{D}}}e^{-qZ}. \tag{4}$$

Here Z is the distance between the 2D electrons and the δ layer of donors and $\delta n_{\mathbf{q}}=\int dz\,\delta\mathcal{N}_{\mathbf{q}z}$ stands for the nonuniform part of donor concentration.

For further numerical estimations we assume the Gaussian correlations of variations of the donor concentration over the δ layer. These variations are described by the correlation function $W(q)=\int d\Delta\mathbf{x}\exp(-i\mathbf{q}\cdot\Delta\mathbf{x})\langle\langle w_{\mathbf{x}}w_{\mathbf{x}'}\rangle\rangle$, which is written as

$$W(q)=\pi l_c^2\left(\frac{\overline{\delta n}}{\rho_{2\mathrm{D}}}\right)^2\exp[-(ql_c/2)^2-2qZ], \tag{5}$$

where $\overline{\delta n}$ is the typical variation of the 2D donor concentration, l_c determines the lateral scale of inhomogeneities, and $\langle\langle\cdots\rangle\rangle$ stands for the averaging over 2D plane. Note that $W(q)$ is the Gaussian correlator if $Z\ll l_c$ and $W(q)\propto\exp[-2qZ]$ if $Z\gg l_c$. Below we use the general correlator in Eq. (5). The specific cases discussed in Refs. 5–8 can be considered using different $\overline{\delta n}$ and l_c.

B. Average current

The linearized kinetic equation for the 2D electrons with the dispersion law $\varepsilon_{p\mathbf{x}}=\varepsilon_p+w_{\mathbf{x}}$ ($\varepsilon_p=p^2/2m$, m is the effective mass) under a weak electric field $\mathbf{E}_{\mathbf{x}}\exp(-i\omega t)$ is represented as

$$(-i\omega+\mathbf{v}\cdot\nabla_{\mathbf{x}}+\delta\mathbf{f}_{\mathbf{x}}\cdot\nabla_{\mathbf{p}})\delta f_{\mathbf{px}}-I_c(\delta f|\mathbf{px})$$
$$=(e\mathbf{E}_{\mathbf{x}}\cdot\mathbf{v})\delta(\varepsilon_F-\varepsilon_{p\mathbf{x}}). \tag{6}$$

Here $\delta f_{\mathbf{px}}$ is the quasiclassical distribution function, $\mathbf{v}=\mathbf{p}/m$ is the in-plane velocity, $\delta\mathbf{f}_{\mathbf{x}}=-\nabla_{\mathbf{x}}w_{\mathbf{x}}$ a random force due to nonscreened variations of level, and ε_F the Fermi energy for the low-temperature case. The contribution from the random force in Eq. (6) is estimated as $\overline{w}/l_c p_F$ while the drift contribution is of the order of v_F/l_c. Thus, the random force can be discarded under the condition $\overline{w} \ll \varepsilon_F$. Separating the symmetric part of the distribution function $\phi_{\varepsilon\mathbf{x}} = \int_0^{2\pi} d\varphi\, \delta f_{\mathbf{px}}/2\pi$, we search for the high-frequency distribution of the form $\delta f_{\mathbf{px}} = \phi_{\varepsilon\mathbf{x}} + \chi_{\mathbf{px}}$. Here $\chi_{\mathbf{px}}$ is the nonsymmetric part of the distribution. Consequently, $\langle \chi_{\mathbf{px}} \rangle = 0$, where $\langle \cdots \rangle = \int_0^{2\pi} (d\varphi/2\pi) \cdots$ means the averaging over angle. The symmetric part of Eq. (6) takes the form

$$(i\omega + \hat{I}_{\text{qel}})\phi_{\varepsilon\mathbf{x}} = \langle \mathbf{v} \cdot \boldsymbol{\nabla}_{\mathbf{v}} \chi_{\mathbf{px}} \rangle, \tag{7}$$

where $\hat{I}_{\text{qel}}$ stands for the quasielastic collision operator (see the Appendix). Using Eqs. (6) and (7), we obtain the nonsymmetric part of kinetic equation in the form

$$(-i\omega + \mathbf{v} \cdot \boldsymbol{\nabla}_{\mathbf{x}} + \nu_e)\chi_{\mathbf{px}} - \langle \mathbf{v} \cdot \boldsymbol{\nabla}_{\mathbf{x}} \chi_{\mathbf{px}} \rangle$$
$$= (e\mathbf{E}_{\mathbf{x}} \cdot \mathbf{v})\delta(\varepsilon_{F\mathbf{x}} - \varepsilon) - \mathbf{v} \cdot \boldsymbol{\nabla}_{\mathbf{x}} \phi_{\varepsilon\mathbf{x}}. \tag{8}$$

The elastic collision integral $I_c(\chi|\mathbf{px})$ is replaced here by $-\nu_e \chi_{\mathbf{px}}$ with the relaxation frequency ν_e that is not dependent on energy for the two-dimensional limit [see Eq. (A4)]. We have also introduced the nonuniform Fermi energy according to $\varepsilon_{F\mathbf{x}} = \varepsilon_F + w_{\mathbf{x}}$.

Performing the spatial Fourier transformation, we write Eqs. (7) and (8) as a system of integro-differential equations

$$(i\omega + \hat{I}_{\text{qel}})\phi_{\varepsilon\mathbf{q}} = i\langle (\mathbf{q} \cdot \mathbf{v})\chi_{\mathbf{pq}} \rangle,$$
$$[i(\mathbf{q} \cdot \mathbf{v} - \omega) + \nu_e]\chi_{\mathbf{pq}} - i\langle (\mathbf{q} \cdot \mathbf{v})\chi_{\mathbf{pq}} \rangle = \Psi_{\mathbf{pq}} - i(\mathbf{q} \cdot \mathbf{v})\phi_{\varepsilon\mathbf{q}}, \tag{9}$$

where the Fourier component of electric field $\mathbf{E_q}$ appears in the nonhomogeneous term $\boldsymbol{\Psi}_{\mathbf{pq}} = \Sigma_{\mathbf{q}'}(e\mathbf{E}_{\mathbf{q}'} \cdot \mathbf{v})\Delta_{\mathbf{q}-\mathbf{q}'}(\varepsilon_F - \varepsilon)$ with the random factor $\boldsymbol{\Delta}_{\mathbf{q}}(\varepsilon_F - \varepsilon) \equiv \int (d\mathbf{x}/L^2)\exp(-i\mathbf{qx})\delta(\varepsilon_{F\mathbf{x}} - \varepsilon)$; L^2 is the normalization area. The contribution averaged over angle in Eq. (9) is determined by the nonsymmetric equation and can be expressed through the formulas

$$\langle (\mathbf{q} \cdot \mathbf{v})\chi_{\mathbf{pq}} \rangle = \frac{1 + (\omega + i\nu_e)\langle (\mathbf{q} \cdot \mathbf{v} - \omega - i\nu_e)^{-1} \rangle}{\langle (\mathbf{q} \cdot \mathbf{v} - \omega - i\nu_e)^{-1} \rangle}$$
$$\times (\phi_{\varepsilon\mathbf{q}} + i\Psi_{\varepsilon\mathbf{q}}),$$
$$\Psi_{\varepsilon\mathbf{q}} = \sum_{\mathbf{q}'} \frac{(e\mathbf{E}_{\mathbf{q}'} \cdot \mathbf{q})}{q^2} \Delta_{\mathbf{q}-\mathbf{q}'}(\varepsilon_F - \varepsilon), \tag{10}$$

where $\boldsymbol{\Psi}_{\mathbf{pq}}$ was replaced by $\Psi_{\varepsilon\mathbf{q}}(\mathbf{q} \cdot \mathbf{v})$ under the average over angle. We transform Eq. (10) performing the integration over angle[13] in this expression so that the right side of symmetric Eq. (9) and $\chi_{\mathbf{pq}}$ take the forms

$$\langle (\mathbf{q} \cdot \mathbf{v})\chi_{\mathbf{pq}} \rangle = (\omega - R_{\varepsilon\omega})(\phi_{\varepsilon\mathbf{q}} + i\Psi_{\varepsilon\mathbf{q}}),$$
$$\chi_{\mathbf{pq}} = i\frac{(\omega - R_{\varepsilon\omega})\Psi_{\varepsilon\mathbf{q}} - i\Psi_{\mathbf{pq}}}{\mathbf{q} \cdot \mathbf{v} - \omega - i\nu_\varepsilon} - \left(1 + \frac{R_{\varepsilon\omega} + i\nu_e}{\mathbf{q} \cdot \mathbf{v} - \omega - i\nu_e}\right)\phi_{\varepsilon\mathbf{q}}. \tag{11}$$

Here we introduced the function $R_{\varepsilon\omega} = \sqrt{(\omega + i\nu_e)^2 - (qv)^2} - i\nu_e$, which has dimension of frequency.

The induced current density is given by the standard formula $\mathbf{j_q} = (2e^2/L^2)\Sigma_{\mathbf{p}} \mathbf{v} \chi_{\mathbf{pq}}$, so that the averaged current $\langle\langle \mathbf{j_q} \rangle\rangle$ is expressed through $\langle\langle \phi_{\varepsilon\mathbf{q}} \rangle\rangle$, $\langle\langle \Psi_{\varepsilon\mathbf{q}} \rangle\rangle$, and $\langle\langle \Psi_{\mathbf{pq}} \rangle\rangle$ according to Eq. (11). Since the averaged values are proportional to $\delta_{\mathbf{q}0}$, we obtain $\langle\langle \mathbf{j_q} \rangle\rangle = \delta_{\mathbf{q}0}\mathbf{j}$ with the single nonzero contribution to $\mathbf{j}$ given by $\langle\langle \Psi_{\mathbf{pq}=0} \rangle\rangle$. Using curl$\mathbf{E_x} = 0$, we can separate the uniform and random parts of electric field and $\mathbf{E_q}$ is written as $\mathbf{E}\delta_{\mathbf{q}0} - i\mathbf{q}\Phi_{\mathbf{q}}$, where $\mathbf{E}$ is the uniform applied field and $\Phi_{\mathbf{q}}$ determines the random part of $\mathbf{E_q}$. After the substitution of $\langle\langle \boldsymbol{\Psi}_{\mathbf{pq}=0} \rangle\rangle$ and the integration over $\mathbf{p}$, we transform the induced current as

$$\mathbf{j} = \frac{e^2}{mL^2} \sum_{\mathbf{q}} \langle\langle E_{\mathbf{q}} n_{-\mathbf{q}} \rangle\rangle \frac{i}{\omega - i\nu_e}$$
$$= i\frac{e^2 n_{2\text{D}}}{m(\omega - i\nu_e)}\left(\mathbf{E} - \frac{i}{L^2}\sum_{\mathbf{q}} \mathbf{q}\frac{\langle\langle \Phi_{\mathbf{q}} w_{-\mathbf{q}} \rangle\rangle}{\varepsilon_F}\right), \tag{12}$$

where the right side is written through the Fourier component of concentration $n_{\mathbf{q}} = n_{2\text{D}} + \rho_{2\text{D}} w_{\mathbf{q}}$ and $n_{2\text{D}} = \rho_{2\text{D}}\varepsilon_F$.

C. Random electric field

The scalar potential $\Phi_{\mathbf{q}}$ in Eq. (12) should be determined from the high-frequency Poisson equation. The general solution of such equation can be written in analogy with the static case [see Eq. (2)] through the Fourier component of charge density $\rho_{\mathbf{q}} = (e\rho_{2\text{D}}/2)\int_0^\infty d\varepsilon\, \phi_{\varepsilon\mathbf{q}}$.[14] For the long-scale inhomogeneities case, when $l_c \gg a_B$, we use below the electric neutrality condition $\rho_{\mathbf{q}} = 0$ in order to determine $\Phi_{\mathbf{q}}$. Thus, the random part of field is determined by the symmetric part of distribution. Using Eqs. (11) and (A6) in the upper Eq. (9) we obtain equation for $\phi_{\varepsilon\mathbf{q}}$ in the form

$$\nu_e \overline{\varepsilon}^2 \frac{d}{d\varepsilon}\left[\frac{d\phi_{\varepsilon\mathbf{q}}}{d\varepsilon} + \frac{\phi_{\varepsilon\mathbf{q}}}{T}\right] + iR_{\varepsilon\omega}\phi_{\varepsilon\mathbf{q}} = (R_{\varepsilon\omega} - \omega)\Psi_{\varepsilon\mathbf{q}}, \tag{13}$$

with the characteristic energy transfer $\overline{\varepsilon}$ is as introduced in the Appendix and T is the phonon temperature; here we consider the case $\overline{\varepsilon} \ll T$. After the substitution $\phi_{\varepsilon\mathbf{q}} = \exp(-\varepsilon/2T)\overline{\phi}_{\varepsilon\mathbf{q}}$, we write equation for $\overline{\phi}_{\varepsilon\mathbf{q}}$ as

$$\overline{\varepsilon}^2 \frac{d^2\overline{\phi}_{\varepsilon\mathbf{q}}}{d\varepsilon^2} - \frac{R_{\varepsilon\omega}}{i\nu_e}\overline{\phi}_{\varepsilon\mathbf{q}} = e^{\varepsilon/2T}\frac{R_{\varepsilon\omega} - \omega}{\nu_e}\Psi_{\varepsilon\mathbf{q}} \equiv e^{\varepsilon/2T}\Theta_{\varepsilon\mathbf{q}}, \tag{14}$$

where we have neglected the contributions of the order of $(\overline{\varepsilon}/2T)^2$. This equation is considered below with the zero boundary conditions at $\varepsilon \to 0$ and ∞ because the electrons only in the vicinity of the Fermi energy contribute to the response. Since $\overline{w} \ll \varepsilon_F$, the fundamental solutions of Eq. (14) are written in the quasiclassical approximation as $\varphi_\varepsilon^{(\pm)} = \exp[\pm\int d\varepsilon \sqrt{R_{\varepsilon\omega}/i\nu_e}/\overline{\varepsilon}]$. The general nonhomogeneous solution takes form

$$\phi_{\varepsilon\mathbf{q}}=-\int_0^{\varepsilon}\frac{d\varepsilon'}{2\overline{\varepsilon}}\frac{\varphi_{\varepsilon}^{(-)}\varphi_{\varepsilon'}^{(+)}}{\sqrt{R_{\varepsilon'\omega}/i\nu_e}}e^{(\varepsilon'-\varepsilon)/2T}\Theta_{\varepsilon'\mathbf{q}}-\int_{\overline{\varepsilon}}^{\infty}\frac{d\varepsilon'}{2\overline{\varepsilon}}\frac{\varphi_{\varepsilon}^{(+)}\varphi_{\varepsilon'}^{(-)}}{\sqrt{R_{\varepsilon'\omega}/i\nu_e}}e^{(\varepsilon'-\varepsilon)/2T}\Theta_{\varepsilon'\mathbf{q}}. \tag{15}$$

Substituting this solution with $\Psi_{\varepsilon\mathbf{q}}$ from Eq. (10) to the condition $\rho_{\mathbf{q}}=0$, we obtain an integral equation for $\Phi_{\mathbf{q}}$ as follows:

$$\sum_{\mathbf{q}'}\mathcal{K}_{\mathbf{qq}'}\Phi_{\mathbf{q}'}+i\frac{(\mathbf{E}\cdot\mathbf{q})}{q^2}\mathcal{K}_{\mathbf{qq}'=0}=0, \tag{16}$$

where the random kernel is given by

$$\mathcal{K}_{\mathbf{qq}'}=\int_0^{\infty}d\varepsilon\Bigg[\int_0^{\varepsilon}\frac{d\varepsilon'}{2\overline{\varepsilon}}\frac{\varphi_{\varepsilon}^{(-)}\varphi_{\varepsilon'}^{(+)}}{\sqrt{R_{\varepsilon'\omega}/i\nu_e}}e^{(\varepsilon'-\varepsilon)/2T}\times\frac{R_{\varepsilon'\omega}-\omega}{\nu_e}\Delta_{\mathbf{q}-\mathbf{q}'}(\varepsilon_F-\varepsilon),+\int_0^{\varepsilon}\frac{d\varepsilon'}{2\overline{\varepsilon}}\frac{\varphi_{\varepsilon}^{(+)}\varphi_{\varepsilon'}^{(-)}}{\sqrt{R_{\varepsilon'\omega}/i\nu_e}}e^{(\varepsilon'-\varepsilon)/2T}\times\frac{R_{\varepsilon'\omega}-\omega}{\nu_e}\Delta_{\mathbf{q}-\mathbf{q}'}(\varepsilon_F-\varepsilon)\Bigg]. \tag{17}$$

Separating $\langle\langle\mathcal{K}_{\mathbf{qq}'}\rangle\rangle=\delta_{\mathbf{qq}'}\mathcal{K}_q$ and the random part of kernel, $k_{\mathbf{qq}'}=\delta_{\mathbf{qq}'}\mathcal{K}_q-\mathcal{K}_{\mathbf{qq}'}$, we write equation for $\Phi_{\mathbf{qq}'}\equiv\Phi_{\mathbf{q}}w_{\mathbf{q}'}$ in the form

$$\mathcal{K}_q\Phi_{\mathbf{qq}'}=\mathcal{F}_{\mathbf{qq}'}+\sum_{\mathbf{q}_i}k_{\mathbf{qq}_i}\Phi_{\mathbf{q}_i\mathbf{q}'}, \tag{18}$$

with the nonhomogeneous term given by $\mathcal{F}_{\mathbf{qq}'}=\mathcal{K}_{\mathbf{qq}'\infty 0}w_{\mathbf{q}'}(\mathbf{E}\cdot\mathbf{q})/q^2$.

The current density given by Eq. (12) contains the factor $\langle\langle\Phi_{\mathbf{qq}'}\rangle\rangle$ at $\mathbf{q}'=-\mathbf{q}$ and the averaging procedure for $\langle\langle\Phi_{\mathbf{qq}'}\rangle\rangle$ is similar to the consideration of the single-particle Green's function. As a result, Eq. (18) is transformed into

$$\mathcal{K}_q\langle\langle\Phi_{\mathbf{qq}'}\rangle\rangle=\delta_{\mathbf{q}'-\mathbf{q}}\langle\langle\mathcal{F}_{\mathbf{q},-\mathbf{q}}\rangle\rangle+\sum_{\mathbf{q}_1}\mathcal{M}_{\mathbf{qq}_2}\langle\langle\Phi_{\mathbf{q}_1\mathbf{q}'}\rangle\rangle,$$

$$\mathcal{M}_{\mathbf{qq}_1}=\sum_{\mathbf{q}_2}\frac{\langle\langle k_{\mathbf{qq}_0}k_{\mathbf{q}_0\mathbf{q}_1}\rangle\rangle}{\mathcal{K}_{q2}+ia_Bq/3}+\cdots=\delta_{\mathbf{qq}_1}\mathcal{M}_q, \tag{19}$$

where the series $\mathcal{M}_{\mathbf{qq}_1}$ has the same structure as the self-energy function in the Dyson equation (see also Refs. 2 and 3) and the nonuniform part of Eq. (19) is $\propto\delta_{\mathbf{q}'-\mathbf{q}}$ under the average. Finally, Eq. (12) contains the factor $\langle\langle\Phi_{\mathbf{q}_1-\mathbf{q}}\rangle\rangle=\langle\langle\mathcal{F}_{\mathbf{q}_1-\mathbf{q}}\rangle\rangle/(\mathcal{K}_q+ia_Bq/2-\mathcal{M}_q)$. Thus the induced current is obtained in the explicit form.

III. EFFECTIVE CONDUCTIVITY

In this section, we consider the modifications of the effective conductivity caused by the second addendum in Eq. (12) and we examine the temperature dependence of the static conductivity as well as the frequency dispersion in nonideal heterostructure under consideration.

A. Averaging procedure

We have performed the average over random potential in $\langle\langle\Phi_{\mathbf{q}_1-\mathbf{q}}\rangle\rangle$ using the relations

$$\langle\langle\delta(\varepsilon_{F\mathbf{x}}-\varepsilon)\rangle\rangle=\delta_{\overline{w}}(\varepsilon_F-\varepsilon'),$$

$$\langle\langle w_{-\mathbf{q}}\Delta_{\mathbf{q}}(\varepsilon_F-\varepsilon)\rangle\rangle=W(q)\delta_{\overline{w}}'(\varepsilon_F-\varepsilon') \tag{20}$$

with the broadening δ function $\delta_{\overline{w}}(E)=\exp[-(E/\overline{w})^2/2]/(\sqrt{2\pi}\overline{w})$ and with the correlation function $W(q)$ given by Eq. (5). These results are obtained after the Fourier expansion of δ function by using the well-known exact formula for averaging of a random functional in the exponent [see Eq. (15) in Ref. 14]. Next, we express $\mathbf{j}$ through the effective conductivity $\sigma_{\omega}^{\mathrm{eff}}$ (it is a scalar due to the in-plane isotropy of the averaged system) according to $\mathbf{j}=\sigma_{\omega}^{\mathrm{eff}}\mathbf{E}$. Replacing $\langle\langle\mathcal{K}_{\mathbf{qq}'=0}w_{-\mathbf{q}}\rangle\rangle$ in $\langle\langle\mathcal{F}_{\mathbf{q},-\mathbf{q}}\rangle\rangle$ through $W(q)\mathcal{A}_q$, we obtain an explicit expression for $\sigma_{\omega}^{\mathrm{eff}}$ in the form

$$\eta_{\omega}^{\mathrm{eff}}=\sigma_{\omega}\left[1-\frac{1}{L^2}\sum_{\mathbf{q}}\frac{W(q)}{2\varepsilon_F}\frac{\mathcal{A}_q}{\mathcal{K}_q-\mathcal{M}_q}\right], \tag{21}$$

where $\sigma_{\omega}=ie^2n_{2D}/m(\omega+i\nu_e)$ is the conductivity of uniform 2D electrons. The coefficients $\mathcal{A}_q$ and $\mathcal{K}_q$ are given by

$$\begin{pmatrix}\mathcal{A}_q\\ \mathcal{K}_q\end{pmatrix}=\int_0^{\infty}d\varepsilon\Bigg[\int_0^{\varepsilon}\frac{d\varepsilon'}{2\overline{\varepsilon}}\frac{\varphi_{\varepsilon}^{(-)}\varphi_{\varepsilon'}^{(+)}}{\sqrt{R_{\varepsilon'\omega}/i\nu_e}}e^{(\varepsilon'-\varepsilon)/2T}\times\frac{R_{\varepsilon'\omega}-\omega}{\nu_e}\begin{pmatrix}\delta_{\overline{w}}'(\varepsilon_F-\varepsilon')\\ \delta_{\overline{w}}(\varepsilon_F-\varepsilon')\end{pmatrix},+\int_0^{\varepsilon}\frac{d\varepsilon'}{2\overline{\varepsilon}}\frac{\varphi_{\varepsilon}^{(+)}\varphi_{\varepsilon'}^{(-)}}{\sqrt{R_{\varepsilon'\omega}/i\nu_e}}e^{(\varepsilon'-\varepsilon)/2T}\times\frac{R_{\varepsilon'\omega}-\omega}{\nu_e}\begin{pmatrix}\delta_{\overline{w}}'(\varepsilon_F-\varepsilon')\\ \delta_{\overline{w}}(\varepsilon_F-\varepsilon')\end{pmatrix}\Bigg]. \tag{22}$$

Using the inequality $\overline{\varepsilon}\ll\overline{w}$ once again, we perform the integrations in Eq. (22) with the rapidly decreasing exponential factors $\varphi_{\varepsilon}^{(-)}\varphi_{\varepsilon}^{(+)}=\exp(-\int_{\varepsilon'}^{\varepsilon}d\varepsilon'\sqrt{R_{\varepsilon'\omega}/i\nu_e}/\overline{\varepsilon})$ and $\varphi_{\varepsilon}^{(+)}\varphi_{\varepsilon}^{(-)}=\exp(-\int_{\varepsilon}^{\varepsilon'}d\varepsilon'\sqrt{R_{\varepsilon'\omega}/i\nu_e}/\overline{\varepsilon})$. As a result,

$$\mathcal{K}_q\simeq i\int_0^{\infty}d\varepsilon\,\delta_{\overline{w}}(\varepsilon_F-\varepsilon)\frac{R_{\varepsilon\omega}-\omega}{R_{\varepsilon\omega}}\simeq i\frac{R_{\varepsilon_F\omega}-\omega}{R_{\varepsilon_F\omega}}, \tag{23}$$

with the right-hand side of equation written under the condition $\overline{w}<\varepsilon_F$. The similar consideration of $\mathcal{A}_q$ after the integration by parts provides the coefficient

$$\mathcal{A}_q \simeq i\int_0^\infty d\varepsilon\, \frac{\delta_{\overline{w}}(\varepsilon_F-\varepsilon)}{\sqrt{R_{\varepsilon\omega}}}\,\frac{d}{d\omega}\,\frac{R_{\varepsilon\omega}-\omega}{\sqrt{R_{\varepsilon\omega}}}$$

$$\simeq \left|\frac{i}{\sqrt{R_{\varepsilon_F\omega}}}\,\frac{d}{d\varepsilon}\,\frac{R_{\varepsilon\omega}-\omega}{\sqrt{R_{\varepsilon\omega}}}\right|_{\varepsilon_F} \tag{24}$$

and $\sigma_\omega^{\text{eff}}$ is obtained after the substitution of these coefficients into Eq. (21).

We now turn to the consideration of the self-consistent contribution $\mathcal{M}_q$. Within the second-order accuracy, the average $\langle\langle k_{\mathbf{q}\mathbf{q}_2} k_{\mathbf{q}_q\mathbf{q}}\rangle\rangle$ is expressed through

$$\left\langle\left\langle\left[\delta_{\mathbf{q}\mathbf{q}_2}\delta_{\overline{w}}(\varepsilon_F-\varepsilon)-\frac{(\mathbf{q}\cdot\mathbf{q}_2)}{q^2}\Delta_{\mathbf{q}-\mathbf{q}_2}(\varepsilon_F-\varepsilon)\right]\right.\right.$$

$$\left.\left.\times\left[\delta_{\mathbf{q}_2\mathbf{q}'}\delta\overline{w}(\varepsilon_F-\varepsilon')-\frac{(\mathbf{q}_2\cdot\mathbf{q}')}{q_2^2}\boldsymbol{\Delta}_{\mathbf{q}_2-\mathbf{q}'}(\varepsilon_F-\varepsilon')\right]\right\rangle\right\rangle$$

$$=\frac{\delta_{\mathbf{q}\mathbf{q}'}}{L^2}W(|\mathbf{q}-\mathbf{q}_2|)\,\delta_w'(\varepsilon_F-\varepsilon)\,\delta_w'(\varepsilon_F-\varepsilon')\,\frac{(\mathbf{q}_2\cdot\mathbf{q}_2)^4}{q^2q_2^2}, \tag{25}$$

Next, using Eq. (17) and performing a straightforward calculation, we transform $\mathcal{M}_q$ into the form

$$\mathcal{M}_q \simeq \frac{4\mathcal{A}_q}{L^2}\sum_{\mathbf{q}_1}\frac{W(|\mathbf{q}-\mathbf{q}_1|)(\mathbf{q}\cdot\mathbf{q}_1)}{q^2q_1^2}\,\frac{4\mathcal{A}_{\mathbf{q}_1}}{\mathcal{K}_{q_1}}, \tag{26}$$

Finally, $\sigma_\omega^{\text{eff}}$ determined by Eq. (21) takes form

$$\sigma_\omega^{\text{eff}}=\sigma_\omega\left\{1-\frac{1}{L^2}\sum_{\mathbf{q}}\frac{W(q)\mathcal{F}_{q\omega}}{4\varepsilon_F}\right.$$

$$\left.\times\left[1-\frac{\mathcal{F}_{q\omega}}{L^2}\sum_{\mathbf{q}_1}\frac{W(|\mathbf{q}-\mathbf{q}_1|)(\mathbf{q}\cdot\mathbf{q}_1)^4}{q^2q_1^2}\mathcal{F}_{q_1\omega}\right]^{-1}\right\} \tag{27}$$

with the function $\mathcal{F}_{q\omega}=[(dR_{\varepsilon\omega}/d\varepsilon)/R_{\varepsilon\omega}](R_{\varepsilon\omega}+\omega)/(R_{\varepsilon\omega}-\omega)$ where q-dependent factor $R_{\varepsilon\omega}$ is appeared in Eq. (11).

B. Static conductivity

For the static case $\omega=0$, we replace $R_{\varepsilon\omega}$ by $i[\sqrt{\nu_e^2+(qv)^2}-\nu_e]$ in Eq. (27). Using the variable $x=ql_c$ and performing the integrations over the angle, we transform Eq. (27) into

$$\frac{\sigma^{\text{eff}}}{\sigma_e}=1-\left(\frac{\overline{\delta n}\,\eta}{4n_{2D}}\right)^2\times\text{Re}\int_0^\infty dx\,\frac{W_x x^3}{V_\eta(x)[V_\eta(x)-1]-M_\eta(x)+i0},$$

$$M_\eta(x)=2\left(\frac{\overline{\delta n}\,\eta^2}{4n_{2D}}\right)^2x^2\int_0^\infty dx_1\frac{w(x,x_1)x_1^3}{V_\eta(x_1)[V_\eta(x_1)-1]}, \tag{28}$$

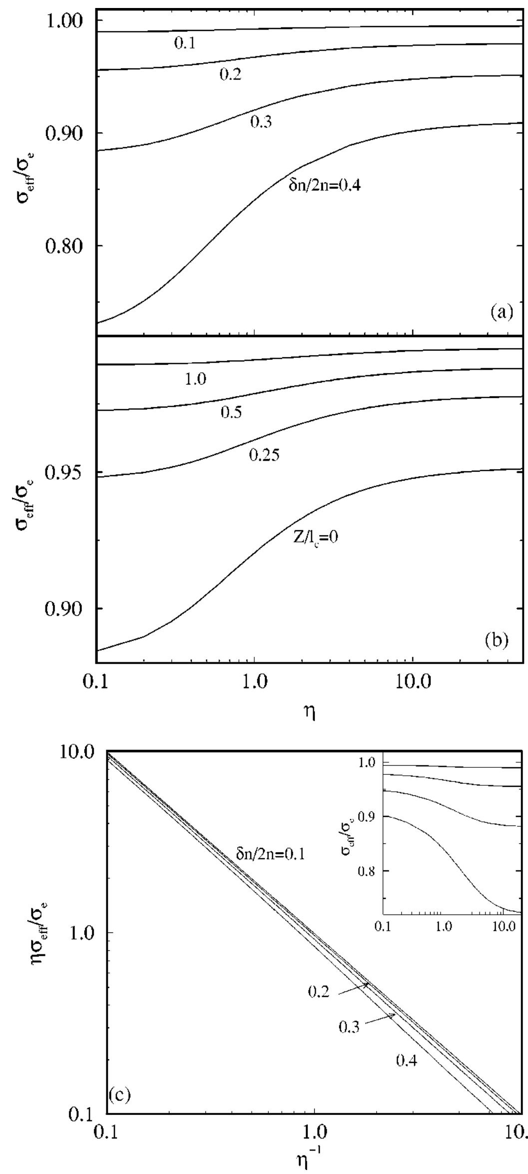

FIG. 2. Temperature-dependent corrections to the static conductivity given by Eq. (28) vs η for $\overline{\delta n}/2n_{2D}=0.1$, 0.2, 0.3, and 0.4 at $Z/l_c\ll 1$ (a); the same for $Z/l_c=0$, 0.25, 0.5, 1 at $\overline{\delta n}/2n_{2D}=0.3$ (b). Temperature dependence of conductivity, given by $\eta\sigma^{\text{eff}}/\sigma_e$ vs $\eta^{-1}\propto T$ is shown in (c). The inset in (c) shows $\sigma^{\text{eff}}/\sigma_e$ vs η^{-1} for clearness.

where $\sigma_\varepsilon=e^2n_{2D}/(m\nu_e)$ is the static conductivity for the in-plane uniform structures, $V_\eta(x)=\sqrt{1+(\eta x)^2}$, and $\eta=l_F/l_c$ are dimensionless parameters. We have also used the angle-averaged correlation functions: $W_x=\exp(-x^2/4-2xZ/l_c)$ and $w(x,x_1)=\int_0^{2\pi}d\varphi/(2\pi)\exp(-|\mathbf{x}-\mathbf{x}_1|^2/4-2|\mathbf{x}-\mathbf{x}_1|Z/l_c)$. According to Eq. (A7), $\eta\propto\nu_\varepsilon^{-1}\propto T^{-1}$ for the acoustic phonon scattering under the condition $T\gg\overline{\varepsilon}$. So Eq. (28) describes the temperature dependence of the conductivity.

In Figs. 2(a) and 2(b) we plot the static conductivity as a function of η given by Eq. (28) with different parameters $\overline{\delta n}/2n_{2D}$ and Z/l_c. Such a relation determines the temperature-dependent corrections to the effective conductivity due to disorder. Figure 2(c) shows the dependence of $\sigma^{\rm eff}$ vs $\eta^{-1}\propto T$, thus the inverse proportional dependence does not essentially change even for big disorder. The case of the samples with the far-separated δ layer, when $Z/l_c\sim 1$, is illustrated in Fig. 2(b). The shapes of the dependences are not changed but the effect of disorder is suppressed due to additional cutoff factor in the correlation function. Let us estimate the typical values of η used above. The mean free path in the structures with $n_{2D}\simeq 2\times 10^{11}\,{\rm cm}^{-2}$ and with the mobility about $10^6\,{\rm cm}^2/{\rm V\,s}$ is around 10 μm. Since the Bohr radius and the width of 2D layer are about 100 Å, the quasiclassical description of long-range disorder is applicable if $l_c>0.1\,\mu$m, so η appears to be up to a hundred. As the mobility increases, η increases proportionally.

C. Frequency dispersion

The spectral dependences of the effective conductivity $\sigma_\omega^{\rm eff}/\sigma_\varepsilon\equiv F(\Omega,\eta)$ are determined by the product of the Drude factor $\sigma_D=i/(\Omega+i)$ and the brackets from Eq. (27) with the dimensionless frequency $\Omega=\omega/\nu_\varepsilon$. The function $F(\Omega,\eta)$ is given by

$$F(\Omega,\eta)=\frac{i}{\Omega+i}\left[1+\left(\frac{\overline{\delta n}\,\eta}{4n_{2D}}\right)^2\int_0^\infty dx\,\frac{W_x x^3[V_{\eta\Omega}(x)-i+\Omega]}{V_{\eta\Omega}(x)[V_{\eta\Omega}(x)-i][V_{\eta\Omega}(x)-i-\Omega]-M_{\eta\Omega}(x)}\right],$$

$$M_{\eta\Omega}(x)=2[V_{\eta\Omega}(x)-i+\Omega]\left(\frac{\overline{\delta n}\,\eta^2}{4n_{2D}}\right)^2 x^2\int_0^\infty dx_1\frac{w(x,x_1)x_1^3[V_{\eta\Omega}(x_1)-i+\Omega]}{V_{\eta\Omega}(x_1)[V_{\eta\Omega}(x_1)-i][V_{\eta\Omega}(x_1)-i-\Omega]}, \tag{29}$$

where $V_{\eta\Omega}(x)=\sqrt{(\Omega+i)^2-(\eta x)^2}$.

The real and imaginary part of the frequency-dependent conductivity of Eq. (29) is plotted in Fig. 3(a) and 3(b), respectively, for the structure with $Z/l_c\to 0$ at different η. To demonstrate the effects of nonuniform doping on the conductivity spectra more transparently, the insets in Fig. 3(a) and 3(b) give the ratios ${\rm Re}\,F(\Omega,\eta)/{\rm Re}\,\sigma_D$ and ${\rm Im}\,F(\Omega,\eta)/{\rm Im}\,\sigma_D$ correspondingly. In contrast to ${\rm Re}\,F(\Omega,\eta)$, which is mainly modified at $\Omega\gg 1$, the imaginary contribution is essentially modified at $\Omega\sim 1$. The value $\Omega=10$ corresponds to the centimeter wavelength range in the samples with the mobility about $10^6\,{\rm cm}^2/{\rm V\,s}$. Thus, under a variation of temperature one can realize the radio frequency or microwave measurements of the response in the samples with mobilities $\gtrsim 10^6\,{\rm cm}^2/{\rm V\,s}$. The case of the samples with the separated δ layer for $Z/l_c>0$ is illustrated in Fig. 4 for $\overline{\delta n}/2n_{2D}=0.3$ and $\eta=20$. It is seen that the distance between the 2D electron layer and the doping layer strongly affects the frequency-dependent conductivity when $Z/l_c<0.5$. However, such an effect is not significant for the imaginary part of the conductivity.

IV. CONCLUSION

In this paper, we have obtained and analyzed the effective conductivity of δ-doped heterostructure with the long-ranged disorder caused by the nonuniform donor distribution. We show that the nonuniform contributions to the electron dispersion law $\varepsilon_{p\mathbf{x}}$ is essential in the selectively doped structures. The presence of nonscreened large-scale variations of levels qualitatively change the character of response due to the nonuniform part of electric field proportional to applied field **E**. This new contribution to the effective conductivity changes both the temperature dependence and the frequency dispersion of conductivity. A substantial variation of conductivity with temperature (up to 20% at $\overline{\delta n}/2n_{2D}=0.3$) is obtained. The frequency dispersion of the real and imaginary parts of conductivity is mainly modified at $\Omega\sim 1$ and $\Omega\sim\eta$, respectively.

Now we discuss the main assumptions used in the calculations above presented. The quasiclassical kinetic equation is generally accepted for the description both static and high-frequency responses of 2D electrons if $\hbar\omega\ll\varepsilon_F$. Furthermore, we consider the electron-phonon scattering only. This is the main channel of momentum relaxation in the high-mobility structures[15] while the electron-electron collisions can determine the symmetric part of distribution in the heavily doped samples (consequently, we used $n_{2D}\simeq 2\times 10^{11}\,{\rm cm}^{-2}$ in the numerical estimations). The effects under consideration should be enhanced for the low-temperature region ($T<4$ K) but one needs to consider the Bloch-Gruneisen regime of relaxation. These complications of scattering process are required a special consideration. The screening plays an essential role in the high-frequency response. We do not consider these peculiarities here because this frequency region corresponds to the sub-millimeter excitation case when a more complicated electrodynamical description is necessary. Note also, that the averaging over nonuniformities with the scale l_c is applicable to the samples with the size $>l_c$; otherwise a classical mesoscopical regime can be realized. The quantum corrections to the frequency dispersion of conductivity[16] appears to be weak in the samples with high-mobility electrons. A validity of the standard self-consistent procedure for the Poisson equation is discussed in Refs. 2 and 3 and the assumption of uniform dielectric permittivity neglects the image-charge effects.

To conclude, the obtained modified frequency and temperature dependences of the effective conductivity will stimulate an experimental study of response taking into account the lateral nonuniformities of the high-mobility 2D

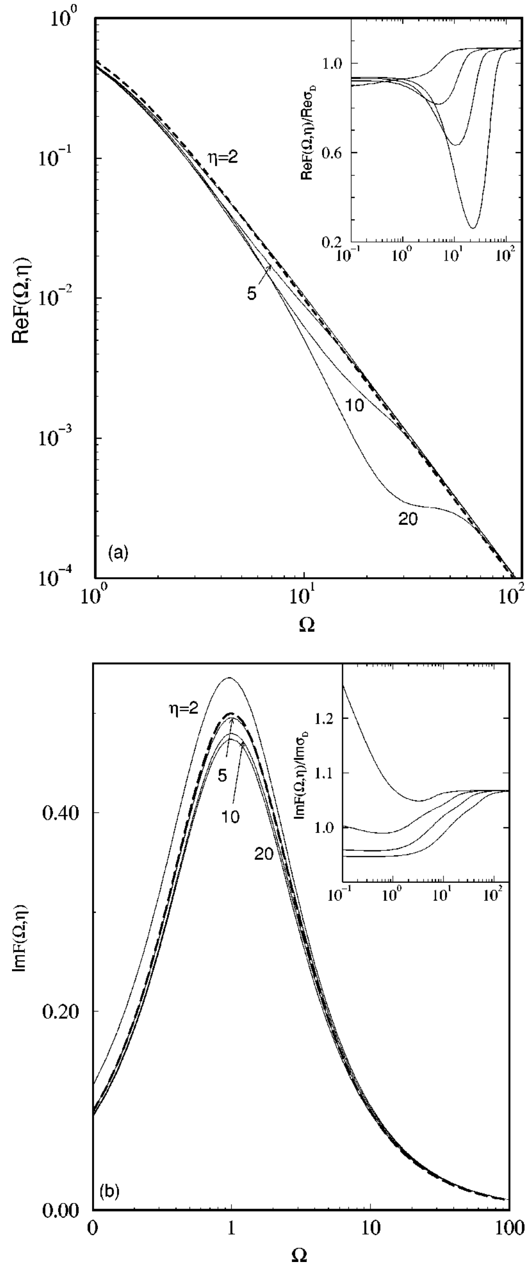

FIG. 3. The real (a) and imaginary (b) parts of $F(\Omega,\eta)$ in the structures with $Z/l_c \to 0$ vs Ω for different $\eta = l_F/l_c$. All curves were calculated for $\overline{\delta n}/2n_{2D} = 0.3$. The thick-dashed curve correspond to the ideal case, $\overline{\delta n} = 0$. The insets in (a) and (b) show $\mathrm{Re}\,F(\Omega,\eta)/\mathrm{Re}\,\sigma_D$ and $\mathrm{Im}\,F(\Omega,\eta)/\mathrm{Im}\,\sigma_D$, respectively.

electrons in selectively doped structures. Along with the above-discussed measurements of conductivity, other characteristics of nonideal structures under consideration, such as magnetotransport coefficients, 2D plasmon dispersion, and cyclotron resonance, are influenced by the nonuniform variations of the ground level; these phenomena require a special consideration.

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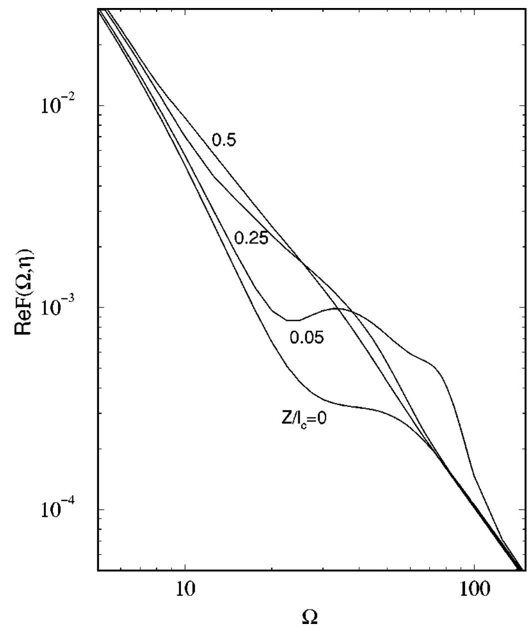

FIG. 4. The real part of the effective conductivity $\mathrm{Re}\,F(\Omega,\eta)$ vs Ω in the structures with $Z/l_F = 0$, 0.05, 0.25, and 0.5 for $\overline{\delta n}/2n_{2D} = 0.3$ and $\eta = 20$.

APPENDIX: QUASIELASTIC COLLISION INTEGRAL

Below we evaluate the quasielastic collision integral that appears in Eq. (13). It is convenient to write the general expression for collision integral through the sum $\overline{W}_{\mathbf{pp}'}$ and difference $\Delta W_{\mathbf{pp}'}$ of the transition probabilities for emission and absorption of phonons:

$$I_c(f|\mathbf{px}t) = \sum_{\mathbf{p}'} \overline{W}_{\mathbf{pp}'}(f_{\mathbf{p}'\mathbf{x}t} - f_{\mathbf{px}t}) - \sum_{\mathbf{p}'} \frac{\Delta W_{\mathbf{pp}'}}{2}[f_{\mathbf{p}'\mathbf{x}t}(1-f_{\mathbf{px}t}) - f_{\mathbf{px}t}(1-f_{\mathbf{p}'\mathbf{x}t})], \tag{A1}$$

$$\overline{W}_{\mathbf{pp}'} = \frac{2\pi}{\hbar}\sum_{q_\perp} |C_Q|^2 |\langle 0|e^{iq_\perp z}|0\rangle|^2 \left(N_Q + \frac{1}{2}\right) \times [\delta(\varepsilon_{p'} - \varepsilon_p + \hbar\omega_Q) + \delta(\varepsilon_{p'} - \varepsilon_p - \hbar\omega_Q)],$$

$$\Delta W_{\mathbf{pp}'} = \frac{2\pi}{\hbar}\sum_{q_\perp} |C_Q|^2 |\langle 0|e^{iq_\perp z}|0\rangle|^2 \times [\delta(\varepsilon_{p'} - \varepsilon_p + \hbar\omega_Q) - \delta(\varepsilon_{p'} - \varepsilon_p - \hbar\omega_Q)].$$

Here C_Q is the bulk matrix element for deformation electron-phonon interaction, $Q = \sqrt{|\mathbf{p} - \mathbf{p}'|^2/\hbar^2 + q_\perp^2}$ is the 3D wave vector, and N_Q is the Planck distribution with temperature T. The phonon frequency, $\omega_Q = sQ$, is expressed through the sound velocity s and the overlap factor,[17] $\langle 0|e^{iq_\perp z}|0\rangle = \int dz\, \rho_z^2 e^{iq_\perp z}$, is determined through the ground-level wave function, $q_\perp$ is the transverse component of phonon wave vector. Following the quasielastic approach, we expand the transition probabilities in Eq. (A1) as follows:

$$\overline{W}_{\mathbf{pp}'}\simeq\frac{\mathcal{L}_{|\mathbf{p}-\mathbf{p}'|}}{L^2}\delta(\varepsilon_{p'}-\varepsilon_p)+\frac{\overline{\mathcal{L}}_{|\mathbf{p}-\mathbf{p}'|}}{L^2}\delta''(\varepsilon_{p'}-\varepsilon_p),$$

$$\Delta W_{\mathbf{pp}'}\simeq\frac{\Delta\mathcal{L}_{|\mathbf{p}-\mathbf{p}'|}}{L^2}\delta'(\varepsilon_{p'}-\varepsilon_p). \tag{A2}$$

For the two-dimensional case, $p_F\gg\hbar/d$, the coefficients of (A2) appear to be independent of $|\mathbf{p}-\mathbf{p}'|$ and below we use

$$\mathcal{L}\simeq\frac{2\pi}{\hbar}L^2\sum_{q_\perp}|C_{q_\perp}|^2|\langle 0|e^{iq_\perp z}|0\rangle|^2\coth\left(\frac{\hbar\omega_{q_\perp}}{T}\right),$$

$$\overline{\mathcal{L}}\simeq\frac{2\pi}{\hbar}L^2\sum_{q_\perp}|C_{q_\perp}|^2|\langle 0|e^{iq_\perp z}|0\rangle|^2(\hbar\omega_{q_\perp})^2\coth\left(\frac{\hbar\omega_{q_\perp}}{T}\right),$$

$$\Delta\mathcal{L}\simeq\frac{2\pi}{\hbar}L^2\sum_{q_\perp}|C_{q_\perp}|^2|\langle 0|e^{iq_\perp z}|0\rangle|^2\hbar\omega_{q_\perp}. \tag{A3}$$

Separating the distribution function on the symmetric and nonsymmetric parts, $f_{\varepsilon\mathbf{x}t}$ and $\Delta f_{\mathbf{px}t}$, we obtain the nonsymmetric contribution to collision integral through the momentum relaxation time ν_e:

$$I_c(\Delta f|\mathbf{px}t)\simeq-\nu_e\Delta f_{\mathbf{px}t},\quad \nu_e=\mathcal{L}\rho_{2\mathrm{D}}. \tag{A4}$$

The symmetric part of collision integral is transformed to the differential form

$$I_c(f|\varepsilon\mathbf{x}t)\simeq\nu_e\overline{\varepsilon^2}\frac{d}{d\varepsilon}\left[\frac{df_{\varepsilon\mathbf{x}t}}{d\varepsilon}+\beta f_{\varepsilon\mathbf{x}t}(1-f_{\varepsilon\mathbf{x}t})\right],\quad \nu_e=\mathcal{L}\rho_{2\mathrm{D}} \tag{A5}$$

with the characteristic energies $\overline{\varepsilon}$ and β^{-1} determined by $\nu_e\overline{\varepsilon^2}=\overline{\mathcal{L}}\rho_{2\mathrm{D}}/2$ and $\beta=\Delta\mathcal{L}/(2\overline{\mathcal{L}})$. The linearized collision integral used in Eq. (13) is given by

$$\hat{I}_{\mathrm{qel}}\phi_{\varepsilon\mathbf{x}t}\simeq\nu_e\overline{\varepsilon^2}\frac{d}{d\varepsilon}\left[\frac{d\phi_{\varepsilon\mathbf{x}t}}{d\varepsilon}+\beta_{\varepsilon\mathbf{x}}\phi_{\varepsilon\mathbf{x}t}\right] \tag{A6}$$

and $\beta_{\varepsilon\mathbf{x}}=\beta(1-2f_{\varepsilon\mathbf{x}})=\beta\,\mathrm{sgn}(\varepsilon_{F\mathbf{x}}-\varepsilon)$ for the low-temperature limit.

For the "equipartition" phonon distribution $T\gg\hbar\omega_{q\perp}$ we use $\coth(\hbar\omega_{q\perp}/T)\approx T/\hbar\omega_{q\perp}$, so that $\beta=T^{-1}$. The elastic relaxation frequency in Eq. (A4) takes the form

$$\nu_e=\frac{\pi D^2T}{\hbar s^2\rho}\rho_{2\mathrm{D}},\quad k=\int dz\,\varphi_z^4, \tag{A7}$$

where D is the deformation potential, ρ is the material density, and $k=3/(2d)$ for the hard-wall model of QW. The characteristic energy is given by

$$k\overline{\varepsilon^2}=L^{-2}\sum_{q_{nt}}|\langle 0|e^{iq_\perp z}|0\rangle|^2(\hbar\omega_{q_\perp})^2=(\hbar s)^2\int dz\left(\frac{d\varphi_z^2}{dz}\right)^2 \tag{A8}$$

and for the hard-wall model of QW we obtain $\overline{\varepsilon}=(2\pi/\sqrt{3})s\hbar/d$. Since $q_\perp$ is of the order of d^{-1}, Eqs. (A7) and (A8) are valid if $T\gg\overline{\varepsilon}$ (or $\beta\overline{\varepsilon}\ll 1$). For the hard-wall model of QW with the width d, we obtain $\overline{\varepsilon}=(2\pi/\sqrt{3})s\hbar/d$ and $k=3/(2d)$. For the selectively doped heterojunction we use the trial function $\psi_z=(z/\sqrt{2a^3})\exp(-z/2a)$ with the characteristic length $a=(a_B/2)\sqrt[3]{R/6\varepsilon_F}$ and the effective Rydberg $R=(\hbar/a_s)^2/m$. The straightforward integrations of Eqs. (A7) and (A8) result in $\overline{\varepsilon}=\sqrt{29/6}s\hbar/a$ and $k=3/(16a)$.

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