

Thermodynamic and tunneling density of states of the integer quantum Hall critical state

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We examine the long wavelength limit of the self-consistent Hartree-Fock approximation irreducible static density-density response function by evaluating the charge induced by an external charge. Our results are consistent with the compressibility sum rule and inconsistent with earlier work that did not account for consistency between the exchange local field and the disorder potential. We conclude that the thermodynamic density of states is finite, in spite of the vanishing tunneling density of states at the critical energy of the integer quantum Hall transition.

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A two-dimensional (2D) electron gas in the strong magnetic-field limit has a zero-temperature insulator-metal-insulator transition in the presence of random disorder. When the strength of the magnetic field sweeps past a critical value, the off-diagonal (Hall) conductance changes abruptly by (e^2/h) while the diagonal (dissipative) conductance rises from zero to a finite value of order $e^2/2h$ and then returns to zero. This transition is often referred to as the integer quantum Hall transition. The filling factor range of the intermediate metallic phase is believed to collapse to a conducting critical point in the low-temperature thermodynamic limit. According to the noninteracting theory of the integer quantum Hall effect, this behavior occurs because single-electron states are extended at an isolated critical energy near the middle of each disorder broadened Landau level. The insulator-metal-insulator transition takes place when the Fermi level of the 2D electron system and one of the discrete critical energies cross.

Our current understanding of the integer quantum Hall transition¹ is far from complete, however, since Coulomb interaction between electrons are expected to provide a relevant perturbation at the noninteracting fixed point.² In addition there are several discrepancies between the results of the noninteracting theory and the experimental findings that may be due to the role of interactions. For example, recent experimental work has shown that the tunneling density of states (DOS) vanishes linearly at the Fermi energy³ in sharp contrast to the finite DOS of the noninteracting theory. To account for this discrepancy Yang and MacDonald⁴ carried out a numerical study in which disorder was treated exactly and Coulomb interaction was described by a self-consistent Hartree-Fock (HF) approximation. They found a linear Coulomb gap at all filling factors of the lowest Landau level, even at the critical energy. Wang and Xiong⁵ investigated the effects of dynamical screening in a systematic nonperturbative resummation of the most singular diagrams. They found that the DOS is linear at the critical energy, as in the Hartree-Fock theory, exhibiting quantum Coulomb gap behavior. In spite of the qualitative DOS change due to interactions, however, Yang, MacDonald, and Huckestein⁶ found that the

value of the localization length exponent ν_{loc} is identical to that of the noninteracting theory.⁷⁻¹¹

A second discrepancy between noninteracting electron theory and experimental findings is the value of the dynamical scaling exponent z . For noninteracting electrons, z equals the space dimension $d=2$. This remains true for short-ranged interactions since they are irrelevant at the noninteracting fixed point.^{2,12} However, experimental data are consistent with $z \approx 1$.¹³ Lee and Wang² conjectured that a change in z with no change in ν_{loc} is due to the noncritical linear suppression of the single-particle DOS induced by Coulomb interactions.^{4,6} Recently, Huckestein and Backhaus¹⁴ attempted to substantiate this conjecture by evaluating the density-density response function in the quantum Hall regime, including interaction effects within a time-dependent Hartree-Fock approximation (TDHFA). Their analysis of dynamic scaling gives $z=1$ and appears to be consistent with the result of a naive scaling argument incorporating the linearly vanishing DOS: the frequency-dependent length scale $L_\omega = \omega^{-1/z} = (1/\sqrt{\rho_0 \hbar \omega})$ with density of states $\rho_0 \propto \omega$ implies $z=1$. However, it is not clear that the ρ_0 that enters L_ω is the tunneling DOS. In fact, it has been argued¹⁵ that the relevant DOS here is the thermodynamics DOS or the compressibility $dn/d\mu$, which should be smooth and finite for a disordered system on general grounds. Huckestein and Backhaus, however, found that the thermodynamic DOS or the compressibility is zero at the transition. Since the compressibility is proportional to the inverse screening length, $q_{\text{sc}} = 2\pi e^2 dn/d\mu$, their result would imply an infinite screening length. The screening properties of the conducting critical state would, therefore, be highly unconventional. This result is plausible since the single particle states at a quantum Hall transition have the special property that only states at the critical energy are delocalized while all other states are localized. However, in order to make their calculation numerically manageable, Huckestein and Backhaus¹⁴ were forced to neglect correlations between the Hartree-Fock theory self-energy and the random potentials. Closely related approximations have long been common in a number of circumstances, for example, in addressing the properties of disordered superconductors where it has recently been demonstrated that they are unreliable.¹⁶

In order to evaluate the Hartree-Fock irreducible density-

density response function directly, it is necessary to solve a complicated integral equation. Huckestein and Backhaus circumvented this difficulty by disorder averaging the quasiparticle response function and the self-energy separately. We circumvent this difficulty without making approximations, by evaluating only the total charge response Q to the external potential from a point charge, solving the Hartree-Fock equations for the self-energy that is consistent¹⁷ with each disorder realization. In linear response, $Q = \lim_{q \rightarrow 0} \chi(q) 2\pi e^2 / \epsilon q$ where $\chi(q)$ is the full static response function, related to the irreducible response by $\chi(q) = \chi^{\text{irr}}(q) / [1 + (2\pi e^2 / \epsilon q) \chi^{\text{irr}}(q)]$, and ϵ is the dielectric constant. It follows that

$$Q^{-1} = 1 + \lim_{q \rightarrow 0} \frac{\epsilon q}{2\pi e^2 \chi^{\text{irr}}(q)}. \quad (1)$$

In the approximation studied by Huckestein and Backhaus,¹⁴ it was determined numerically (see their Fig. 3 and discussions) that $\chi_{\text{HB}}^{\text{irr}}(q) = s(\epsilon q / e^2) + O(l/L)$, where $s \approx 0.2$ is a numerical constant and l is the magnetic length. This would imply that the second term on the right-hand side of Eq. (1) has a finite value and $Q_{\text{HB}} \approx 0.56$. On the other hand, it is normally the case for metals that the screening wave vector $q_{\text{sc}} = 2\pi e^2 \lim_{q \rightarrow 0} \chi^{\text{irr}}(q)$, is related to the thermodynamic density of states according to $q_{\text{sc}} = 2\pi e^2 dn/d\mu$,¹⁸ a result often referred to as the compressibility sum rule, which would imply that $Q = 1$. (It is clear from experiment that $dn/d\mu$ must be finite at the integer quantum Hall transition.) Our numerical results support the applicability of the compressibility sum rule and are clearly inconsistent with the value of Q_{HB} that violates this sum rule by about 50%. When this rule is satisfied $Q = 1$, the external charge is perfectly screened, and the screening length q_{sc} is finite. Our results show that despite the linearly vanishing Coulomb gap in the tunneling DOS, $\chi^{\text{irr}}(q=0)$ is finite and imply that transport is indeed governed by diffusion, as for noninteracting electrons.

We perform our calculations in the Landau gauge [$\vec{A} = (0, Bx, 0)$] and apply quasiperiodic boundary conditions to the HF single-particle orbitals inside a square with area $A = a^2$. The basis states used to represent the HF Hamiltonian are related to elliptic theta functions and can be labeled by a set of guiding centers $|X_j\rangle$, $j = 1, \dots, N_\phi$ inside the fundamental cell of the finite system (N_ϕ is the total flux quantum passing through the area A). The wave functions of the basis states are $\langle \vec{r} | X_j \rangle = \sum_{k=-\infty}^{\infty} \phi_{X_j + ka}(\vec{r})$, where

$$\phi_X(\vec{r}) = \left(\frac{1}{a\sqrt{\pi}l} \right)^{1/2} \exp\left(i \frac{Xy}{l^2} - \frac{(X-x)^2}{2l^2} \right). \quad (2)$$

The HF eigenstate is given by a linear combination of the basis states. $|\alpha\rangle = \sum_i |X_i\rangle \langle X_i | \alpha \rangle$. The coefficients $\langle X_i | \alpha \rangle$ satisfy a set of matrix equations for the HF Hamiltonian, $H_{\text{HF}} = V_{\text{HF}} + V_I + V_{\text{ion}}$, which is a sum of the HF potential in the LLL, the impurity potential, and the ionic potential of the inserted test charge,

$$\begin{aligned} \sum_i [\langle X_j | V_I | X_i \rangle + \langle X_j | V_{\text{ion}} | X_i \rangle + \langle X_j | V_{\text{HF}} | X_i \rangle] \langle X_i | \alpha \rangle \\ = \epsilon_a \langle X_j | \alpha \rangle. \end{aligned} \quad (3)$$

The position of the ion is $\vec{R} = a(1/2, 1/2)$. The matrix elements of the ion potential is

$$\begin{aligned} \langle X_{j'} | V_{\text{ion}} | X_j \rangle = -\frac{1}{A} \sum_q \frac{2\pi e^2}{\epsilon q} e^{i\vec{q} \cdot \vec{R}} \\ \times \delta'_{j', j+m_y} \exp[iq_x(X_j + q_y l^2/2) - q^2 l^2/4], \end{aligned} \quad (4)$$

where $\vec{q} = (2\pi/a)(m_x + sN_\phi, m_y + tN_\phi)$ with $m_{x,y} = 1, \dots, N_\phi$ and s, t are integers. We have used a model disorder potential consisting of N_I δ -function scatters with strength uniformly distributed between $-\lambda$ and λ and scattering centers uniformly distributed inside the fundamental cell of the finite-size system. Hartree and Fock potentials can be expressed as a function of the electron density

$$\begin{aligned} \langle X_j | V_{\text{HF}} | X_i \rangle = \sum_q \Delta(\vec{q}) \exp\left[\frac{iq_x(X_i + X_j)}{2} \right] \\ \times \delta'(j, i + m_y) U_{\text{HF}}(\vec{q}). \end{aligned} \quad (5)$$

Here the sum over \vec{q} is over the discrete set of wave vectors consistent with the boundary conditions and $\delta'(n, n')$ is 1 if $n' = n \pmod{N_\phi}$ and 0 otherwise. $U_{\text{HF}}(\vec{q})$ is proportional to $e^2/\epsilon l$ and includes both Coulomb and exchange interactions. The quantity $\Delta(\vec{q})$ is proportional to the Fourier component of the charge density and is calculated self-consistently from the eigenvectors of the HF Hamiltonian by

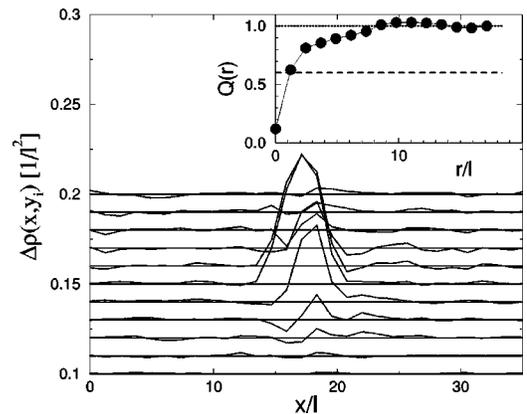


FIG. 1. Induced charge distribution for the critical state. Each curve represents one of $\Delta\rho(x, y_i)$ for $i = 10, 11, \dots, 19, 20$, where $y_i = [a(i-1)/29]$. These curves are shifted vertically for the sake of clear presentation. The parameters are $\nu = 1/2$, $\gamma = 0.6$, $N_D = 4$, and $N_\phi = 200$. The unit of length is l . Inset: Integrated induced charge as a function of the distance to the position of the ion. The dashed line corresponds to $Q_{\text{HB}} \approx 0.56$.

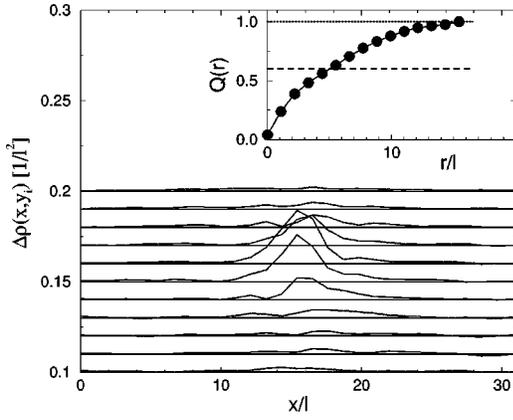


FIG. 2. Same as in Fig. 1 but for a more disorder broadened LLL with $\gamma=0.1$, $N_D=10$, and $N_\phi=162$.

$$\Delta(\vec{q}) = \frac{1}{N_\phi} \sum_{j=1}^{N_\phi} \sum_{j'=1}^{N_\phi} \delta'(j, i+m_y) \times \exp\left[\frac{iq_x(X_i+X_j)}{2}\right] \sum_{\alpha=1}^N \langle X_{j'} | \alpha \rangle \langle \alpha | X_j \rangle, \quad (6)$$

where N is the number of electrons in the system. For each disorder realization the matrix equation is solved for several values of $\gamma=(e^2/\epsilon l)/\Gamma$, where $\Gamma=(\lambda^2 N_I/l^2 A)^{1/2}$ is the characteristic disorder potential-energy scale and N_I is the number of impurities. For this model the self-consistent Born approximation (SCBA) DOS is nonzero for $|\epsilon| < (2/3\pi)^{1/2} \Gamma \sim 0.46\Gamma$.

For a system of N electrons in the k th realization of the random disorder potential, we write the electron density as $\rho_{P,N}^{(k)}(\vec{r}) = \sum_{\alpha=1, \dots, N} |\Psi_\alpha^{(k)}(\vec{r})|^2$ where $P=0, I$, depending on whether the ion is absent or present. Let $\rho_{0,N}$ be the density of the uniform background charge. When $\rho_{0,N}^{(k)}(\vec{r})$ is disorder averaged over N_D number of realizations we find $\overline{\rho_{0,N}^{(k)}(\vec{r})} \equiv (1/N_D) \sum_k \rho_{0,N}^{(k)}(\vec{r}) = \rho_{0,N}$. The charged density induced by the ion may be evaluated by computing $\overline{\rho_{I,N+1}(\vec{r}) - \rho_{0,N}(\vec{r})}$

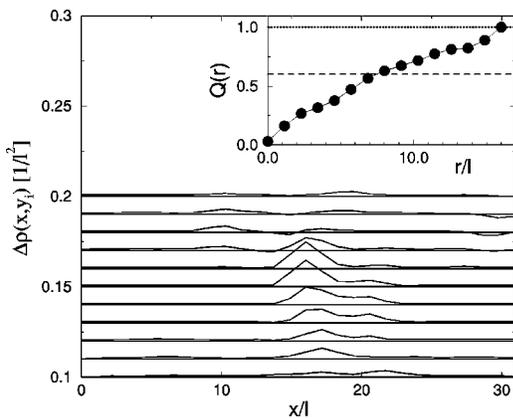


FIG. 3. Same as in Fig. 1 but for an insulating state with $\nu=1/7$, $\gamma=0.1$, $N_D=10$, and $N_\phi=175$.

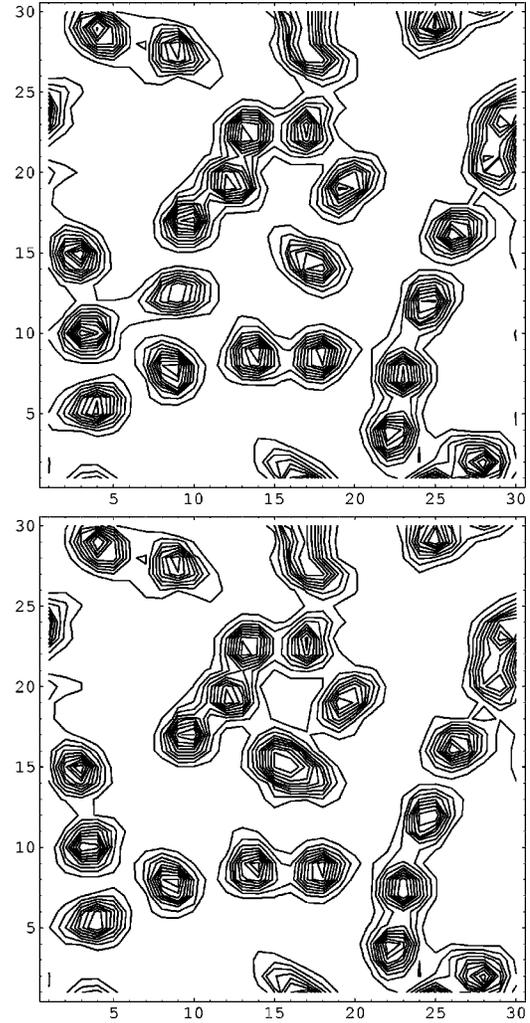


FIG. 4. Contour plots of the charge density in the presence of a strong the tendency towards crystallization for $\gamma=0.6$. at $\nu=1/7$ with $N_D=1$ and $N_\phi=175$. Plot (a) is without the ion while plot (b) is with the ion. Comparing these two plots we can infer that the ion captures an electron while the location of the other electrons are unchanged. The unit of length is l .

$= \overline{\rho_{I,N+1}(\vec{r})} - \overline{\rho_{0,N}}$. Since $\overline{\rho_{0,N}(\vec{r})} = \overline{\rho_{0,N+1}(\vec{r})} - 1/A$ the induced density $\Delta\rho(\vec{r}) = \overline{\rho_{I,N+1}(\vec{r})} - \overline{\rho_{0,N+1}(\vec{r})} + 1/A$.

Figure 1 displays the charge induced when an ion is placed at the center of the square at LLL filling fraction $\nu=1/2$ and for the parameters $\gamma=0.6$, $N_D=4$, and $N_\phi=200$. The inset in Fig. 1 shows how the total induced charge changes as the distance from the ion is varied. These results are consistent with $Q=1$ and a finite screening length $\lambda_{sc} \approx 5l$. They are clearly inconsistent with Huckestein and Backhaus's result $Q_{HF} \approx 0.56$. Using the DOS of the SCBA, we obtain a shorter Thomas-Fermi screening length, $\lambda_{TF} = 2\pi/(\gamma\epsilon) \approx 1l$. Figure 2 shows the disorder averaged $\Delta\rho(x, y_i)$ for a much broader Landau level with $\gamma=0.1$, $\nu=1/2$, $N_D=10$, and $N_\phi=162$. In this case we find that the screening length $\lambda_{sc} \approx 11l$ while $\lambda_{TF} \approx 6l$. Our results for the γ dependence of the screening length is in qualitative agreement with that implied by the DOS behavior in the SCBA.

For a more quantitative comparison for the disorder dependence of the screening length, a Hartree nonlinear screening treatment, in which disorder is treated in the SCBA,¹⁹ may be required instead of the simple SCBA.

We have also investigated the screening properties in the insulating regime at $\nu=1/7$. Figure 3 displays the induced densities for $\gamma=0.1$, $N_D=10$, and $N_\phi=175$. We observe from the plot that the induced density does not decay rapidly and that the fluctuations are spread over a wide range. In this case it is difficult to define a screening length. We believe this reflects the incomplete screening property²⁰ of an insulator. Indeed from the inset we see that the screening length appears to diverge with the finite system size. However, in the presence of a weaker random disorder with $\gamma=0.6$, where the tendency towards crystallization is strong, we find a different result. Contour plots of $\rho_0^{(k)}(\vec{r})$ and $\rho_I^{(k)}(\vec{r})$ are shown in Fig. 4. Plot (a) is without the ion while plot (b) is with the ion. Comparing these two plots we can infer that the ion captures an electron while the location of the other elec-

trons are unchanged. A similar abrupt reconfiguration of the ground state was found and studied in the pinning of Wigner crystals.²¹

We have shown here that despite the linearly vanishing Coulomb gap in the tunneling DOS, the thermodynamic DOS of the integer quantum Hall critical state remains finite, and transport is indeed governed by diffusion as for noninteracting electrons. It still remains an open question whether an improved implementation of the TDHFA, that takes into account self-consistently self-energy and vertex corrections, can give rise to the observed dynamic critical exponent for the integer quantum Hall transition.

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¹⁷In the calculation of the Thouless number by Yang, MacDonald, and Huckestein⁶ this selfconsistency is kept. When the Thouless number is calculated from the HF eigenvalues HF orbitals are used in effect. They calculated HF orbitals self-consistency *after* changing the boundary conditions. We believe that the change in

the exchange potential when the boundary conditions are changed is such as the ladder sum vertex corrections.

¹⁸We take $dn/d\mu$ to be defined by the static irreducible density response function in the long wavelength limit. In our numerical calculations, we (1) take the dc limit, (2) ensemble average, and (3) take the thermodynamic limit. In the metallic state we find that the full screening is achieved over an area that does not diverge with the system size, so that calculating the total induced charge is equivalent to taking the limit q to zero after taking the thermodynamic limit. We have used a unit test charge for our calculations because the response to fractional charges can be unphysical in the strongly localized limit well away from half-filling. Because of this choice we are not strictly in the linear response limit, even at half-filling. However, the response will be nonlinear only near the inserted charge. Since full screening is recovered over a finite area, we think that taking the linear response limit in the end would not change our conclusion of a finite $dn/d\mu$ at quantum Hall transition. Moreover, we expect the total induced charge in the long wavelength limit to follow linear response.

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²⁰It has frequently been argued that q_{sc} vanishes in an Anderson insulator, a view that is consistent with our numerical result. See for example, Y. Imry, Y. Gefen, and D. J. Bergman, Phys. Rev. B, **26**, 3436 (1982). This opinion appears not to be universally held, however, and some confusion appears to exist because of the glassy nature of localized interacting electron response functions.

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