Relation between dipole moment and radiative lifetime in interface fluctuation quantum dots

A. Thränhardt, C. Ell, G. Khitrova, and H. M. Gibbs
Optical Sciences Center, University of Arizona, Tucson, Arizona 85721-0094

(Received 21 August 2001; published 2 January 2002)

A relation between dipole moment and radiative lifetime of quantum dots for arbitrary indices of refraction of the quantum dot and surrounding medium is derived. As limiting cases several formulas already in use are obtained and their correct use is discussed. We proceed to discuss the relation between dipole moment and radiative lifetime in interface fluctuation quantum dots (IFQD's) taking into account nonsphericity effects. This introduces a dependence on the light propagation direction. We calculate how to account for the nonsphericity of an interface fluctuation quantum dot depending on its diameter and well thickness and find that the lifetime of a typical IFQD with a 50 nm diameter is reduced by about 10% due to nonsphericity effects.

DOI: 10.1103/PhysRevB.65.035327 PACS number(s): 78.67.Hc, 78.60.−b, 78.70.−g

I. INTRODUCTION

There has been some confusion about the correct relation between dipole moment and radiative lifetime in semiconductor quantum dots. For atoms, this relation is well established. The radiative broadening Γ_{vac} of an atom emitting into vacuum is derived from its dipole moment through[1]

$$\Gamma_{\mathrm{vac}}=\frac{\omega^3|\mathcal{D}|^2}{3\pi\epsilon_0\hbar c_{\mathrm{vac}}^3}, \tag{1}$$

where $\mathcal{D}$ is the dipole moment, ω is the frequency of the optical transition, ϵ_0 is the permittivity of free space, and c_{vac} is the vacuum speed of light. We use MKS units throughout. When immersing the emitting object in a medium the radiative lifetime is changed, as has been shown by Yablonovitch for the case of quantum wells.[2] In quantum dots, the index of refraction n has often been introduced as a factor to account for the effects of the medium,[3,4]

$$\Gamma=n\Gamma_{\mathrm{vac}}. \tag{2}$$

Another approach uses a more complicated factor to account for the semiconductor medium,[5]

$$\Gamma=\frac{9n^5}{(2n^2+1)^2}\Gamma_{\mathrm{vac}}. \tag{3}$$

Here, we show that these different approaches can be viewed as special cases of a general formula including both the index of refraction of the quantum dot n_{QD} and the index of refraction of the surrounding medium n explicitly. Equation (2) assumes that the quantum dot has the same index of refraction as the medium, while Eq. (3) is based on an index of refraction of 1 for the emitting particle and applies to the case of an atom in a medium. The approach has been used for semiconductor quantum dots,[6,7] where, however, it is not justified and Eq. (2) should be used. The result may be modified by local field effects in an absorptive and dispersive host medium.[8]

Semiconductor quantum dots in general have dielectric constants very similar to the surrounding semiconductor medium. The complex interplay between about 10^6 atoms in a quantum dot results in a complex band structure. When reducing the band structure to a two-band model, all other potential transitions are taken into account approximately by the so-called background relative dielectric constant $\epsilon=n^2$ (Ref. 9). Its introduction is the basis on which a semiconductor quantum dot may be considered to have only one primary transition and thus act as a two-level system. Therefore, although a semiconductor quantum dot may often be considered analogously to an atom and as a two-level system, one may do so only when using the proper dielectric constants.

Interface fluctuation quantum dots (IFQD) are large monolayer islands naturally formed when growing structures with growth interruption.[10–12] Recently, interest has focused on them due to their large dipole moment,[13] which may make the observation of quantum entanglement in a semiconductor structure possible. In order to achieve this, a single quantum dot has to be coupled to an electromagnetic field such that the Rabi splitting is larger than the energy associated with all broadening mechanisms.[7,14] The Rabi energy is proportional to the dipole moment $\mathcal{D}$ which in turn is proportional to the square root of the area of an interface quantum dot. Except for very-high-Q whispering-gallery-mode cavities, the photon decay rate exceeds the dipole dephasing rate so the larger the dipole moment the better for achieving strong coupling. Therefore, large IFQD's seem ideally suited to achieve quantum entanglement.

The dipole moment is a function of the wave vector of the light it couples to, i.e., $\mathcal{D}=\mathcal{D}(\mathbf{K})$. The magnitude K of the light wave vector is determined by the transition energy. For spherical quantum dots, the dependence on the wave vector direction disappears and the dipole moment may thus be written as a simple number $\mathcal{D}(\mathbf{K})=\mathcal{D}$. For quantum dots that are small compared to the wavelength of light in the medium the direction dependence also disappears. Interface fluctuation quantum dots, however, are highly nonspherical, having a thickness of a few nanometers in the growth direction and a radius in the quantum well plane of several tens to hundreds of nanometers, which is quite an appreciable fraction of the transition wavelength in the medium. The $\mathbf{K}$ dependence must thus not be neglected. However, it is still possible to use Eq. (2) for the calculation of the radiative lifetime when using an average dipole moment $\mathcal{D}_{\mathrm{av}}$. In measurements of the dipole moment, the measured quantity

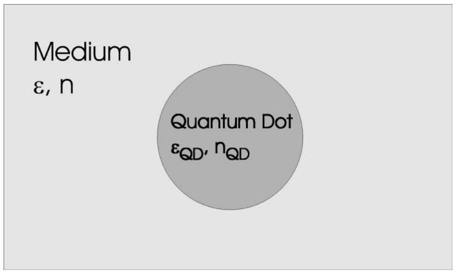

FIG. 1. Schematic picture of the quantum dot and surrounding medium. The quantum dot and medium have dielectric constant ϵ_{QD} and ϵ, respectively.

is often the dipole moment for coupling to light traveling in the growth direction $\mathcal{D}(K\mathbf{e}_z)$.[15] In this paper, we numerically calculate the form factor $F(R_0)=|\mathcal{D}_{av}|^2/|\mathcal{D}(K\mathbf{e}_z)|^2$ depending on the radius of the interface quantum dot R_0. The nonsphericity of interface quantum dots can then be corrected for by the simple factor $F(R_0)$ in lifetime calculations.

We have assumed the IFQD's to have spherical symmetry in the xy plane. In reality, IFQD's can be elongated in the $[1\bar{1}0]$ direction.[11] However, since relatively little is known about the shape of IFQD's we have not attempted a more exact description.

II. CALCULATION OF THE RADIATIVE LIFETIME

Here, we derive the radiative lifetime of a quantum dot for an arbitrary dielectric constant of the medium ϵ and dielectric constant of the quantum dot ϵ_{QD} as shown schematically in Fig. 1. For simplicity, in this section we assume a sperical quantum dot with radius R that is much smaller than the wavelength of light in the quantum dot, $R\ll\lambda_{QD}$. We start from the Hamiltonian of a two-level system interacting with a quantized light field,[1] thus restricting ourselves to the lowest possible transition:

$$H=\hbar\sum_s \Omega_s b_s^\dagger b_s+\hbar\omega_c c^\dagger c+\hbar\omega_v v^\dagger v+\hbar\sum_s (g_s b_s c^\dagger v+g_s^* b_s^\dagger v^\dagger c). \tag{4}$$

Here, b_s denotes photons in mode s and c $(c^\dagger)$ and v $(v^\dagger)$ are the annihilation (creation) operators for an electron in the conduction (valence) band, respectively. g_s is the coupling constant between electrons and photons, and $s=\{\mathbf{k},\sigma\}$ characterizes the photon modes by their wave vector $\mathbf{k}$ and polarization σ.

We write the state vector $|\psi(t)\rangle$ as an expansion into the excited state $|E\{0\}\rangle$ and the ground state $|G\{1s\}\rangle$:

$$|\psi(t)\rangle=C_{E\{0\}}(t)e^{-i\omega_e t}|E\{0\}\rangle+\sum_s C_{G\{1s\}}(t)e^{-i(\omega_h+\Omega_s)t}|G\{1s\}\rangle \tag{5}$$

with expansion coefficients $C_{A\{P\}}$ where A is the QD state and P denotes the photon states. As initial configuration we use

$$C_{E\{0\}}(t=0)=0, \tag{6}$$

where the quantum dot is in the excited state $|E\{0\}\rangle$ and all photon modes are empty. Thus, the interaction potential between electrons and photons [last term in Eq. (4)] only connects this state $|E\{0\}\rangle$ to states $|G\{1s\}\rangle$ where the quantum dot is in the ground state while mode s is occupied by one photon, which is why the state vector can be written in the simple form of Eq. (5).

Inserting Hamiltonian and state vector into the Schrödinger equation $i\hbar(\partial/\partial t)|\psi(t)\rangle=H|\psi(t)\rangle$ and projecting onto each of the states, we derive

$$\dot{C}_{E\{0\}}(t)=-\sum_s |g_s|^2\int_{t_0}^t dt' e^{-i(\Omega_s-\omega)t'}C_{E\{0\}}(t'), \tag{7}$$

where $\omega=\omega_c-\omega_v$. Let us consider a quantum dot in a dielectric medium with index of refraction n. The sum over the wave numbers included into the sum over the photon modes s may be converted into an integral as follows:

$$\sum_{\mathbf{K}}\rightarrow\frac{V}{(2\pi)^3}\int d^3K=\frac{Vn^3}{(2\pi)^3c_{vac}^3}\int d^3\Omega$$
$$=\frac{Vn^3}{(2\pi)^3c_{vac}^3}\int_0^\infty d\Omega\,\Omega^2\int_0^\pi d\theta\sin\theta\int_0^{2\pi}d\phi, \tag{8}$$

where V is the quantization volume. Inserting this into Eq. (7), we obtain

$$\dot{C}_{E\{0\}}(t)=-\frac{Vn^3}{(2\pi)^3c_{vac}^3}\int_0^\infty d\Omega\,\Omega^2\int_0^\pi d\theta\sin\theta\times\int_0^{2\pi}d\phi|g(\Omega,\theta)|^2\int_{t_0}^t dt'\,e^{-i(\Omega_s-\omega)t'}C_{E\{0\}}(t'). \tag{9}$$

We solve the time integral by the method of coarse graining, i.e., we assume that $C_{E\{0\}}(t')$ is slowly varying such that $C_{E\{0\}}(t')\approx C_{E\{0\}}(t)$:

$$\lim_{t\to\infty}\int_{t_0}^t dt' e^{-i(\Omega_s-\omega)t'}C_{E\{0\}}(t')$$
$$=C_{E\{0\}}(t)\int_{t_0}^t dt' e^{-i(\Omega_s-\omega)t'}$$
$$=C_{E\{0\}}(t)\left\{\pi\delta(\Omega-\omega)-\mathcal{P}\left[\frac{i}{\Omega-\omega}\right]\right\}. \tag{10}$$

Neglecting the term $\mathcal{P}[i/(\Omega-\omega)]$ which leads to a frequency shift related to the Lamb shift, we obtain

$$\dot{C}_{E\{0\}}(t) = -\frac{Vn^3\pi\omega^2}{(2\pi)^3 c_{\mathrm{vac}}^3} \times \int_0^\pi d\theta \sin\theta \int_0^{2\pi} d\phi |g(\omega,\theta)|^2 C_{E\{0\}}(t). \tag{11}$$

Let us now evaluate the coupling constant $g(\omega,\theta)$ entering Eq. (11):

$$|g(\omega,\theta)|^2 = \sum_{\sigma=1}^{2} \left(\frac{\mathcal{E}_\omega}{\hbar}\right)^2 |\langle E\{0\}|e\mathbf{r}\cdot\mathbf{e}_\sigma|G\{1s\}\rangle|^2 = \sum_{\sigma=1}^{2} \left(\frac{\mathcal{E}_\omega}{\hbar}\right)^2 \left| \int d^3r\, u_c^*(\mathbf{r})\xi_c^*(\mathbf{r}) e\mathbf{r}\cdot\mathbf{e}_\sigma u_v(\mathbf{r})\xi_v(\mathbf{r}) \right|^2, \tag{12}$$

where the electric field per photon $\mathcal{E}_\omega$ describes the strength of the electric field at the location of the coupling, i.e., the quantum dot. The electron wave functions have been written as products of the lattice periodic Bloch function $u_v(\mathbf{K},\mathbf{r})$ $[u_c(\mathbf{K},\mathbf{r})]$ in the valence [conduction] band and the envelope wave function $\xi(\mathbf{r})$.[9] The weak wave vector dependence of the Bloch functions has been neglected. $\mathbf{e}_\sigma$ is the polarization vector of the light with polarization σ.

The integral is then split into an integral over a lattice cell and a sum over all the lattice cells with the envelope functions varying slowly over a lattice cell,

$$|g(\omega,\theta)|^2 = \sum_{\sigma=1}^{2} \left(\frac{\mathcal{E}_\omega}{\hbar}\right)^2 \left| \sum_{n=1}^{N} \xi_c^*(\mathbf{R}_n)\xi_v(\mathbf{R}_n) \times \int_{V_0} d^3r\, u_c^*(\mathbf{r}) e\mathbf{r}\cdot\mathbf{e}_\sigma u_v(\mathbf{r}) \right|^2. \tag{13}$$

Here, V_0 denotes the volume of a unit cell. We introduce the interband optical dipole matrix element

$$\mathbf{d}_{cv} = \frac{1}{V_0}\int_{V_0} d^3r\, u_c^*(\mathbf{r}) e\mathbf{r}\, u_v(\mathbf{r}) \tag{14}$$

and derive

$$|g(\omega,\theta)|^2 = \sum_{\sigma=1}^{2} \left(\frac{\mathcal{E}_\omega}{\hbar}\right)^2 \left| \int d^3R\, \xi_c^*(\mathbf{R})\xi_v(\mathbf{R}) \mathbf{d}_{cv}\cdot\mathbf{e}_\sigma \right|^2. \tag{15}$$

With σ labeling the two linear polarization directions, we get

$$|g(\omega,\theta)|^2 = \left(\frac{\mathcal{E}_\omega}{\hbar}\right)^2 \left| \int d^3R\, \xi_c^*(\mathbf{R})\xi_v(\mathbf{R}) d_{cv} \right|^2 \times \sin^2\theta(\cos^2\phi + \sin^2\phi) = \left(\frac{\mathcal{E}_\omega}{\hbar}\right)^2 \left| \int d^3R\, \xi_c^*(\mathbf{R})\xi_v(\mathbf{R}) d_{cv} \right|^2 \sin^2\theta. \tag{16}$$

The coupling constant is thus independent of the azimuthal coordinate ϕ. Although the dipole moment $\mathbf{d}_{cv}$ is a vector quantity, the direction dependence of the dot product $\mathbf{d}_{cv}\cdot\mathbf{e}_\sigma$ is lifted by the sum over all possible polarizations.

The electric field per photon $\mathcal{E}_\omega$ is determined by the eigenmodes $\mathbf{f}_{\mathbf{K}\sigma}(\mathbf{r})$ of the light. We consider a quantum dot with dielectric constant $\epsilon_{\mathrm{QD}} = n_{\mathrm{QD}}^2$ in a dielectric medium of dielectric constant $\epsilon = n^2$. For $\epsilon_{\mathrm{QD}} = \epsilon$, the eigenmodes would be plane waves $\mathbf{f}_{\mathbf{K}\sigma}(\mathbf{r}) = (\mathbf{e}_\sigma/\sqrt{\epsilon V})e^{-i\mathbf{K}\cdot\mathbf{r}}$. For the case of different dielectric constants in quantum dot and medium, the eigenfunctions must fulfill the wave equation

$$\epsilon\frac{\Omega_K^2}{c_{\mathrm{vac}}^2}\mathbf{f}_{\mathbf{K}\sigma}(\mathbf{r}) - \boldsymbol{\nabla}\times[\boldsymbol{\nabla}\times\mathbf{f}_{\mathbf{K}\sigma}(\mathbf{r})] - (\epsilon - \epsilon_{\mathrm{QD}})\Theta(R_0 - r)\frac{\Omega_K^2}{c_{\mathrm{vac}}^2}\mathbf{f}_{\mathbf{K}\sigma}(0) = \mathbf{0}, \tag{17}$$

where the quantum dot has been taken as a sphere with radius R_0 around the origin. Here, we have assumed $\Theta(R_0 - r)\mathbf{f}_{\mathbf{K}\sigma}(\mathbf{r}) \approx \Theta(R_0 - r)\mathbf{f}_{\mathbf{K}\sigma}(0)$, which is valid for $R_0 \ll \lambda$. Generalizing the derivation of Glauber and Lewenstein,[5] we obtain

$$\mathbf{f}_{\mathbf{K}\sigma}(0) = \frac{\mathbf{e}_\sigma}{\sqrt{\epsilon V}}\frac{3\epsilon}{2\epsilon + \epsilon_{\mathrm{QD}}}. \tag{18}$$

Thus the electric field per photon $\mathcal{E}_\omega$ entering Eq. (16) is

$$\mathcal{E}_\omega = \sqrt{\frac{\hbar\omega}{\epsilon_0\epsilon V}}\frac{3\epsilon}{2\epsilon + \epsilon_{\mathrm{QD}}}. \tag{19}$$

We remark that nonsphericity of a semiconductor quantum dot does not make the dipole moment $\mathbf{K}$ dependent as long as the quantum dot is small compared with the wavelength of light in the medium. Although for a nonspherical quantum dot the wave equation, Eq. (17), cannot be written in this simple form, the eigenmodes are given by Eq. (18) for any small QD. Thus, as a result of the fact that only the optical dipole matrix element $\mathbf{d}_{cv}$ is a vector quantity and its directional dependence is obliterated by the sum over all possible polarizations σ, the dipole moment is wave vector dependent only for large quantum dots.

Inserting Eq. (16) into Eq. (11) yields

$$\dot{C}_{E\{0\}}(t) = -\frac{9\epsilon^{5/2}}{(2\epsilon + \epsilon_{\mathrm{QD}})^2}\frac{\omega^3|\mathcal{D}|^2}{6\pi\epsilon_0\hbar c_{\mathrm{vac}}^3} C_{E\{0\}}(t), \tag{20}$$

where we have introduced the QD dipole moment

$$\mathcal{D} = \int d^3R\, \xi_c^*(\mathbf{R})\xi_v(\mathbf{R}) d_{cv}. \tag{21}$$

We thus find

$$\dot{C}_{E\{0\}}(t) = -\frac{\Gamma}{2}C_{E\{0\}}(t), \tag{22}$$

where

$$\Gamma=\frac{9\epsilon^{5/2}}{(2\epsilon+\epsilon_{\mathrm{QD}})^2}\frac{\omega^3|\mathcal{D}|^2}{3\pi\epsilon_0\hbar c_{\mathrm{vac}}^3}=\frac{9\epsilon^{5/2}}{(2\epsilon+\epsilon_{\mathrm{QD}})^2}\Gamma_{\mathrm{vac}}. \quad (23)$$

For the case of equal dielectric constants for quantum dot and medium, $\epsilon_{\mathrm{QD}}=\epsilon$, we retrieve Eq. (2). In contrast, for a quantum dot with a dielectric constant of 1 ($\epsilon_{\mathrm{QD}}=1$) we obtain Eq. (3). We also point out that were a quantum dot placed into vacuum, its radiative decay rate would be

$$\Gamma=\frac{9}{(2+\epsilon_{\mathrm{QD}})^2}\Gamma_{\mathrm{vac}} \quad (24)$$

due to its dielectric constant ϵ.

In a real (finite) semiconductor structure, the QD lifetime may change due to etalon effects. The magnitude of this change is calculated by comparing the eigenmodes of the structure at the QD location to a plane wave eigenfunction with an absolute value of $1/\sqrt{V}$ at the QD location where V is the quantization volume. Depending on whether the norm of the eigenmodes is greater or smaller than $1/\sqrt{V}$ at the QD location spontaneous emission may be enhanced or suppressed. The eigenmodes of any semiconductor structure can be calculated in a transfer matrix calculation.

III. LIFETIME OF ASPHERIC QUANTUM DOTS

In this section, we calculate the lifetime of interface fluctuation quantum dots. These generally have a dielectric constant very similar to the semiconductor medium around them, such that we can safely assume $\epsilon_{\mathrm{QD}}=\epsilon$. The radiative lifetime is then given by Eq. (2). However, we can no longer uphold the approximations made in Sec. II, namely spherical symmetry and a QD dimension which is much smaller than the wavelength of light in the medium. An interface quantum dot is highly aspherical, having a thickness of a few nanometers in the growth direction and a radius of several tens to hundreds of nanometers in the quantum well plane.

Omitting the assumption $R\ll\lambda$, the eigenmodes are plane waves $\mathbf{f}_{\mathbf{K}\sigma}(\mathbf{r})=(\mathbf{e}_\sigma/\sqrt{\epsilon V})e^{-i\mathbf{K}\cdot\mathbf{r}}$ for a homogeneous dielectric constant in medium and quantum dot. Thus, the electric field per photon becomes

$$\mathcal{E}_\omega=\sqrt{\frac{\hbar\omega}{\epsilon_0\epsilon V}}. \quad (25)$$

Taking the light wave vector into account, we rewrite Eq. (12) as

$$|g(\omega,\theta)|^2=\sum_{\sigma=1}^{2}\left(\frac{\mathcal{E}_\omega}{\hbar}\right)^2|\langle E\{0\}|e\mathbf{r}\cdot\mathbf{e}_\sigma e^{-i\mathbf{K}\cdot\mathbf{r}}|G\{1s\}\rangle|^2. \quad (26)$$

This leads to a dipole moment

$$\mathcal{D}(\mathbf{K})=\int d^3r\,\xi_c^*(\mathbf{r})\xi_v(\mathbf{r})e^{-i\mathbf{K}\cdot\mathbf{r}}d_{cv} \quad (27)$$

and a radiative broadening

$$\Gamma=\frac{n\omega^3}{3\pi\epsilon_0\hbar c_{\mathrm{vac}}^3}\frac{1}{4\pi K_0^2}\int d^3K|\mathcal{D}(\mathbf{K})|^2\delta(K-K_0), \quad (28)$$

where $K_0=n\omega/c_{\mathrm{vac}}$ is determined by the transition energy. In order to derive a correction factor accounting for the nonequivalence between growth and in-plane direction, we need to calculate the **K**-dependent dipole moment in Eq. (27). We assume a localized basis consisting of wave functions $\phi(\mathbf{r}_\parallel)$ in the in-plane (xy) direction and the confinement functions of an infinitely high quantum well in the growth (z) direction with barriers at $z=\pm L_z/2$. For the quantum well ground state, we get

$$\xi(\mathbf{r})=\phi(\mathbf{r}_\parallel)\zeta_z(z) \quad (29)$$

where

$$\zeta_z(z)=\sqrt{\frac{2}{L_z}}\cos\frac{\pi z}{L_z}. \quad (30)$$

Thus,

$$\mathcal{D}(\mathbf{K})=\int d^2r_\parallel\phi_c^*(\mathbf{r}_\parallel)\phi_v(\mathbf{r}_\parallel)e^{-i\mathbf{K}_\parallel\cdot\mathbf{r}_\parallel}\times\sin x\left(\frac{1}{x}+\frac{x}{\pi^2-x^2}\right)d_{cv}, \quad (31)$$

where $x=K_zL_z/2$.

For the wave function in the in-plane direction $\phi(\mathbf{r}_\parallel)$ we compare results obtained using two different trial functions, one being a step function

$$\phi(\mathbf{r}_\parallel)=\frac{1}{\sqrt{\pi R_0^2}}\Theta(R_0-r_\parallel) \quad (32)$$

for a quantum dot with radius R_0, the other being a Gaussian

$$\phi(\mathbf{r}_\parallel)=\sqrt{\frac{2}{\pi}}e^{-r_\parallel^2/R_0^2}. \quad (33)$$

We define a form factor

$$F(R_0)=\frac{\int d^3K|\mathcal{D}(\mathbf{K})|^2\delta(K-K_0)}{4\pi K_0^2|\mathcal{D}(K_0\mathbf{e}_z)|^2}, \quad (34)$$

where $\mathbf{e}_z$ is the unit vector in the growth direction. Briefly, this form factor is the ratio between the dipole moment averaged over all directions and the dipole moment for coupling to light propagating in the growth direction. Using this factor, we can rewrite the lifetime of an interface quantum dot as

$$\Gamma=\frac{n\omega^3|\mathcal{D}|^2F(R_0)}{3\pi\epsilon_0\hbar c_{\mathrm{vac}}^3}, \quad (35)$$

where $\mathcal{D}=\mathcal{D}(K_0\mathbf{e}_z)$ is the dipole moment for coupling to light propagating in the growth direction which is usually referred to as *the* dipole moment of an IFQD (e.g., in normal

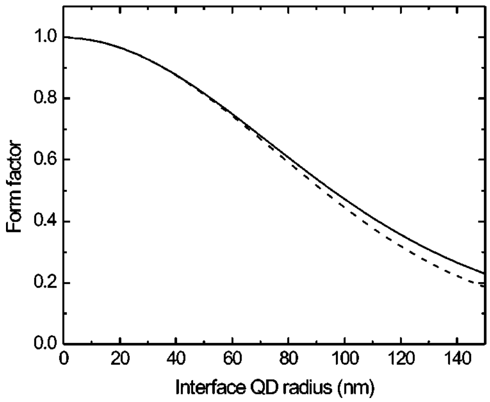

FIG. 2. Form factor for an interface quantum dot using a step function (solid line) and a Gaussian (dashed line) as eigenfunctions in the growth direction as a function of the QD radius. We observe that the results for both sets of trial functions deviate little from each other, indicating that the result is not sensitive to the choice. For a typical QD size of 50 nm the radiative lifetime is reduced by about 10% by the form factor.

incidence absorption measurements[15] or normal-incidence coupling to an IFQD in a microcavity).

Figure 2 shows the form factor for the two in-plane trial functions ($\epsilon = 13.71$, $L_z = 4$ nm). For small IFQD's, the effects of nonsphericity are negligible, yielding a form factor $F(0) = 1$ as expected. For larger radii, the form factor decreases strongly. We also observe that the results for the two sets of trial functions deviate little from each other, indicating that the result is insensitive to the choice of trial function. For a typical dot size of 50 nm,[11,12] we find a form factor of about 0.9. The lifetime is thus reduced to 90% of its value for a spherical QD with the same dipole moment $\mathcal{D}(K_0\mathbf{e}_z)$. Since L_z is much smaller than the wavelength of light, the results depend very little on its choice.

The dipole moment of a large quantum dot depends strongly on the inclusion of the Coulomb interaction which has not been taken into account in the one-particle trial wave functions discussed so far. In order to estimate its effects, we consider a two-dimensional Coulomb potential. We also assume that the relative in-plane motion remain unaffected by the quantum dot confinement potential. This approximation is valid only when the quantum dot radius is large compared with the 2D exciton Bohr radius a_0. We write the electron-hole wave function as

$$\Psi(\mathbf{r}_e,\mathbf{r}_h) = \frac{1}{\sqrt{\pi R_0^2}}\Theta(R-R_0)\Phi_{1s}(\mathbf{r})\zeta_z(z_e)\zeta_z(z_h), \tag{36}$$

taking into account only $1s$ states and assuming a step function for the center of mass motion. $\mathbf{R}$ and $\mathbf{r}$ are the in-plane center of mass and relative motion coordinates, respectively,

$$\mathbf{R} = \frac{m_e}{m_e+m_h}\mathbf{r}_e + \frac{m_h}{m_e+m_h}\mathbf{r}_h, \tag{37}$$

$$\mathbf{r} = \mathbf{r}_e - \mathbf{r}_h \tag{38}$$

with electron (hole) mass m_e (m_h). Since the wave function is no longer separable in electron and hole wave functions, the dipole moment is[16]

$$\begin{aligned}\mathcal{D}(\mathbf{K}) &= d_{cv}\int d^3R\,\Psi(\mathbf{R},\mathbf{R})e^{-i\mathbf{K}\cdot\mathbf{R}}\\ &= d_{cv}\int d^3r_e\int d^3r_h\Psi(\mathbf{r}_e,\mathbf{r}_h)e^{-i\mathbf{K}\cdot\mathbf{r}_e}\delta(\mathbf{r}_e-\mathbf{r}_h)\\ &= \frac{d_{cv}\Phi_{1s}(\mathbf{r}=\mathbf{0})}{\sqrt{\pi R_0^2}}\int d^2Re^{-i(\mathbf{K}\cdot\mathbf{R})}\Theta(R-R_0)\\ &\quad\times\int dz\,|\zeta_z(z)|^2\,e^{-iK_z z}.\end{aligned} \tag{39}$$

Inserting the wave function, Eq. (36), into Eq. (39), we derive for the form factor the same expression as for the single particle case when using a step function, Eq. (32). We thus conclude that the Coulomb interaction has little or no effect on the form factor. An elongation of the IFQD's in $[1\bar{1}0]$ direction[11] could be accounted for by choosing elliptical wave functions in the xy plane and would result in an average form factor whose value is somewhere in between the values for the two axes. However, since relatively little is known about the shape of IFQD's we have not attempted a more exact description.

IV. CONCLUSION

We have derived a formula for the calculation of the radiative lifetime of a quantum dot for arbitrary dielectric constants of quantum dot ϵ_{QD} and surrounding medium ϵ,

$$\Gamma = \frac{9\epsilon^{5/2}}{(2\epsilon+\epsilon_{\mathrm{QD}})^2}\Gamma_{\mathrm{vac}}. \tag{40}$$

For the case of semiconductor quantum dots, we can in general assume $\epsilon_{\mathrm{QD}} = \epsilon$ and thus the formula reduces to the well-known equation

$$\Gamma = \sqrt{\epsilon}\,\Gamma_{\mathrm{vac}}. \tag{41}$$

As for interface quantum dots, the dot is neither spherical nor is its size in general small compared with the wavelength of light in the semiconductor medium. This introduces a form factor $F(R_0)$ into Eq. (41) that we have calculated as a function of the quantum dot size. The form factor relates the dipole moment measured in IFQD's coupling to light propagating in the growth direction to their total radiative lifetime. Due to their nonsphericity, we derive a reduction in lifetime of about 10% in a typical IFQD with a 50 nm diameter.

ACKNOWLEDGMENTS

This work is supported by NSA/ARO, AFOSR, NSF (EPDT), JSOP (AFOSR and ARO), and COEDIP. A.T. acknowledges support by the Humboldt Foundation through the Feodor-Lynen program.

[1] P. Meystre and M. Sargent III, *Elements of Quantum Optics*, 2nd ed. (Springer-Verlag, Berlin, 1991).
[2] E. Yablonovitch, T. J. Gmitter, and R. Bhat, Phys. Rev. Lett. **61**, 2546 (1998).
[3] L. C. Andreani, G. Panzarini, and J.-M. Gérard, Phys. Rev. B **60**, 13 276 (1999).
[4] U. Bockelmann, Phys. Rev. B **48**, 17 637 (1993).
[5] R. J. Glauber and M. Lewenstein, Phys. Rev. A **43**, 467 (1991).
[6] G. Björk, S. Machida, Y. Yamamoto, and K. Igeta, Phys. Rev. A **44**, 669 (1991).
[7] C. Ell, H. M. Gibbs, G. Khitrova, E. S. Lee, S. Park, D. G. Deppe, and D. L. Huffaker, LEOS Newsletter **13(4)**, 8 (1999).
[8] For an atomic system see G. Juzeliūnas, Phys. Rev. A **55**, R4015 (1997); M. E. Crenshaw, and C. M. Bowden, Phys. Rev. Lett. **85**, 1851 (2000).
[9] H. Haug and S. W. Koch, *Quantum Theory of the Optical and Electronic Properties of Semiconductors*, 3rd ed. (World Scientific, Singapore, 1994).
[10] A. Zrenner, L. V. Butov, M. Hagn, G. Abstreiter, G. Böhm, and G. Weimann, Phys. Rev. Lett. **72**, 3382 (1994).
[11] D. Gammon, E. S. Snow, B. V. Shanabrook, D. S. Katzer, and D. Park, Phys. Rev. Lett. **76**, 3005 (1996).
[12] Qiang Wu, R. D. Grober, D. Gammon, and D. S. Katzer, Phys. Rev. Lett. **83**, 2652 (1999).
[13] L. C. Andreani, G. Panzarini, and J.-M. Gérard, Phys. Status Solidi A **178**, 145 (2000).
[14] G. Khitrova, H. M. Gibbs, F. Jahnke, M. Kira, and S. W. Koch, Rev. Mod. Phys. **71**, 1591 (1999).
[15] J. R. Guest, T. H. Stievater, Xiaoqin Li, D. G. Steel, D. Gammon, D. S. Katzer, D. Park, C. Ell, A. Thränhardt, G. Khitrova, and H. M. Gibbs (unpublished).
[16] Y. Z. Hu, M. Lindberg, and S. W. Koch, Phys. Rev. B **42**, 1713 (1990).