Thermoelectric transport of composite fermions at $\nu=\frac{1}{2}$ and $\nu=\frac{3}{2}$: A simple way of evaluating p

V. C. Karavolas and G. P. Triberis
University of Athens, Physics Department, Solid State Section, Panepistimiopolis, 15784 Zografos, Athens, Greece
(Received 10 August 2001; published 13 December 2001)

We propose a simple and generic way of evaluating p directly from the ratio of the experimental values of the diagonal components of the resistivity ρ_{xx} at filling factors $\nu=3/2$ and $\nu=1/2$ in the fractional quantum Hall regime. The p value determines the energy dependence of the scattering time, and the diffusion thermopower of the system. We use the idea of parallel conduction of two gases. One gas, composed of electrons, fully occupies one of the two spin levels of the lowest Landau level, and a second, composed of composite fermions, partially occupies the other spin level. The analysis is free of limitations connected with the specific scattering mechanisms and the nature of the carriers. The validity of the method is tested successfully, using the available experimental data, for electron and hole gases.

DOI: 10.1103/PhysRevB.65.033307 PACS number(s): 73.21.−b, 71.10.Pm, 73.43.Cd

The quantization of the Hall effect, discovered in 1980, is a remarkable macroscopic quantum phenomenon that occurs in two-dimensional electron systems and strong perpendicular magnetic fields. In 1982, working with much higher mobility samples, Tsui *et al.*[1] discovered the fractional quantization of the Hall conductivity. At very low temperatures and high magnetic fields an increasing number of Hall plateaus were observed corresponding to fractional filling factors with odd denominators.

A very promising approach to understand a system near even denominators is to attach to each particle an even number of ''flux quanta.'' In this way, a quasiparticle named ''composite fermion'' (CF) was created. Jain,[2] by following this idea, constructed successfully the hierarchy of the fractional quantum Hall effect (FQHE).

In previous investigations we presented calculations for the resistivity and diffusion thermopower of a two-dimensional electron gas (2DEG) and hole gas (2DHG) at low temperatures near $\nu=1/2$ and for the resistivity of a 2DEG near $\nu=3/2$ in a wide magnetic-field range.[3,4] In the present work, we focus upon the relation between the diagonal components of the resistivity ρ_{xx} at $\nu=3/2$ and $\nu=1/2$. The analysis reveals a simple way of evaluating the p value. This p value determines the energy dependence of the scattering time τ_t through the relation[5,6] $\tau_t=\tau_0 E^p$, where τ_0 is a constant and p is directly connected with the scattering mechanisms.The p value also determines the diffusion thermopower S^d of the system.[5–7] The validity of the method is tested successfully, using the recent experimental data, for electron and hole gases.

The difference of the electrochemical potential of the electrons $\Delta\mu$, and of the composite fermions $\Delta\mu^*$ between the edges of the sample are connected through the following equation:[8]

$$\Delta\mu=\Delta\mu^*+\int_\beta^\alpha (2h/e)\mathbf{J}\cdot\hat{\mathbf{B}}\times\mathbf{dr}=\Delta\mu^*+2hJ_N, \quad (1)$$

where α and β label different edges of the Hall bar, $\mathbf{B}$ is the magnetic field, $\mathbf{J}$ is the current density, and J_N is the particle current.

The resistivity is defined from the basic transport equations as

$$\rho=\frac{\nabla\mu}{e^2J_N}=\frac{\nabla\mu^*+\int_\beta^\alpha (2h/e)\mathbf{J}\cdot\hat{\mathbf{B}}\times\mathbf{dr}}{e^2J_N}=\frac{\nabla\mu^*}{e^2J_N}+\frac{\int_\beta^\alpha (2h/e)\mathbf{J}\cdot\hat{\mathbf{B}}\times\mathbf{dr}}{e^2J_N}=\rho^{qp}+\rho^{CS} \quad (\nabla T=0) \quad (2)$$

where

$$\rho^{qp}=\frac{\nabla\mu^*}{e^2J_N} \quad (\nabla T=0), \quad \rho_{CS}=\frac{2\pi\hbar s}{e^2}. \quad (3)$$

ρ_{CS} is the nondiagonal term of the resistivity arising from the statistical potential, ρ^{qp} is the quasiparticle CF's integer quantum Hall effect (IQHE) resistivity term, and s is the number of flux quanta attached to each carrier.

A very important result from the theory is that the conductivities for the electrons in the FQHE and for the CF's in the IQHE are *added in parallel*.[9,10] The resistivity tensor of a system near $\nu=1/2$ is expressed in terms of the quasiparticle $\rho_{1/2}$, resistivity components ρ_{xx}^{qp} and ρ_{xy}^{qp}, and the Chern-Simmons term ρ_{CS}.[4] For a wide magnetic-field range they have been calculated elsewhere.[3,4]

The thermopower is defined from the basic transport equations by

$$S=\frac{1}{e}\frac{\nabla\mu}{\nabla T}=\frac{1}{e}\frac{\nabla\mu^*+\int_\beta^\alpha (2h/e)\mathbf{J}\cdot\hat{\mathbf{B}}\times\mathbf{dr}}{\nabla T}=\frac{1}{e}\frac{\nabla\mu^*}{\nabla T}+\frac{1}{e}\frac{\int_\beta^\alpha (2h/e)\mathbf{J}\cdot\hat{\mathbf{B}}\times\mathbf{dr}}{\nabla T}=S^{qp} \quad (J_N=0) \quad (4)$$

where

$$S^{qp}=\frac{1}{e}\frac{\nabla\mu^*}{\nabla T} \quad (J_N=0). \tag{5}$$

It is important to notice that the Chern-Simmons term that appears in the resistivity is absent in the thermopower tensor. Thus, taking into account the fact that for $B_{eff}=0$, $S^{qp}_{xy}=0$, the total diffusion thermopower of the system is given by[7]

$$S^{qp}_{xx}=-\frac{\pi^2 k_B^2 T(p+1)}{3eE_F}=-\frac{\pi k_B^2 Tm^*(p+1)}{6e\hbar^2 n_{CF}}. \tag{6}$$

The system under study consists of N carriers moving on a (x,y) plane in the presence of an external magnetic field $B=(0,0,B_z)$ perpendicular to the plane. We will consider only the case when the magnetic field is so high that all the carriers populate the two lowest spin levels of the lowest Landau level.

The $\nu=3/2$ case is different than the $\nu=1/2$. This is due to the fact that at $\nu=3/2$ the magnetic field is not inadequate enough to transform all the carriers to CF's as in the $\nu=1/2$ case. We will treat the system under study as two different gases, showing parallel conduction. One of electrons fully occupies one of the two spin levels of the lowest Landau level ($i=1$), and a second set of electrons that occupies the other spin level ($i=2$) (Ref. 4) have been transformed to composite fermions. The total current in the case of a parallel conducting multilayer system is the sum of the currents in the different layers. Consequently the total sheet conductivity is the sum of the sheet conductivities of the separate layers,[11,4] given by

$$\tilde{\sigma}_i=\begin{pmatrix}\sigma_{i,xx} & -\sigma_{i,xy}\\ \sigma_{i,xy} & \sigma_{i,xx}\end{pmatrix}, \tag{7}$$

where $\sigma_{i,xx}$ and $\sigma_{i,xy}$ are the diagonal and nondiagonal components of layer i conductivity, respectively.

Then the total conductivity of the two-layer system is

$$\tilde{\sigma}_{tot}=\tilde{\sigma}_1+\tilde{\sigma}_2=\begin{pmatrix}(\sigma_{1,xx}+\sigma_{2,xx}) & -(\sigma_{1,xy}+\sigma_{2,xy})\\ (\sigma_{1,xy}+\sigma_{2,xy}) & (\sigma_{1,xx}+\sigma_{2,xx})\end{pmatrix}. \tag{8}$$

The conductivity of the electrons fully occupying the lowest spin level of the lowest Landau level ($i=1$) is given by

$$\tilde{\sigma}_{el}=\begin{pmatrix}\sigma_{1,xx} & -\sigma_{1,xy}\\ \sigma_{1,xy} & \sigma_{1,xx}\end{pmatrix}=\begin{pmatrix}0 & -\dfrac{e^2}{h}\\ \dfrac{e^2}{h} & 0\end{pmatrix}. \tag{9}$$

The conductivity of the second gas ($i=2$) can be obtained from the inversion of the resistivity tensor $\rho_{1/2}$.[4]

The total resistivity of the system at $\nu=3/2$ is given by

$$\rho_{xx,3/2}=\frac{(\rho_{xx}^{qp^2}+N^2)\rho_{xx}^{qp}}{\rho_{xx}^{qp^2}+M^2}, \tag{10a}$$

$$\rho_{xy,3/2}=\frac{(\rho_{xx}^{qp^2}+N^2)M}{\rho_{xx}^{qp^2}+M^2}, \tag{10b}$$

where $M=\rho_{xy}^{qp}+2h/e^2+\sigma_{1,xx}[\rho_{xx}^{qp^2}+(\rho_{xy}^{qp}+2h/e^2)^2]$ and $N=(\rho_{xy}^{qp}+2h/e^2)$.

Equation (10a) for $B_{eff}=0$, i.e., $\rho_{xx}^{qp}\ll 2h/e^2$ and $\rho_{xy}^{qp}=0$ transforms to

$$\rho_{xx,3/2}=\frac{\rho_{xx}^{qp}}{9}, \tag{11}$$

where

$$\rho_{xx}^{qp}=\frac{m^*_{3/2}}{n_{CF,3/2}e^2\tau_{t,3/2}}. \tag{12}$$

$m^*_{3/2}$ is the effective mass, $\tau_{t,3/2}$ is the scattering time, and $n_{CF,3/2}$ is the concentration of the composite fermions at $\nu=3/2$. At $\nu=3/2$ only the one-third of the carriers are transformed to composite fermions,[4] i.e., $n_{CF,3/2}=n/3$.

However, at $\nu=1/2$, all the carriers are transformed to composite fermions,[4] i.e., $n_{CF,1/2}=n$. On the other hand,

$$\rho_{xx,1/2}=\rho_{xx}^{qp}=\frac{m^*_{1/2}}{n_{CF,1/2}e^2\tau_{t,1/2}}, \tag{13}$$

where, $m^*_{1/2}$ is the effective mass, $\tau_{t,1/2}$ is the scattering time, and $n_{CF,1/2}$ is the concentration of the composite fermions at $\nu=1/2$.

Halperin *et al.*[10] who calculated the transport time using the lowest Born approximation as well as Khveshchenko,[6] who calculated the transport time beyond the lowest Born approximation, found that the transport time is proportional to the effective mass. We can approximate the transport time τ_t by[5]

$$\tau_t=\tau_0 m^* n_{CF}^p. \tag{14}$$

Thus, Eq. (12) implies that the resistivity is independent of the effective mass. The composite fermion gasses at both $\nu=3/2$ and $\nu=1/2$ correspond to carriers connected with two magnetic fluxes. They differ only in their concentrations and the value of their effective masses. At low temperatures we may assume that Eq. (14) holds for both gases, with the same τ_0, because τ_0 is independent of the filling factor[6,10] and the effective mass of the composite fermions. The resistivity is connected to the filling factor only through the composite fermion concentration. Given that the p value does not depend on the composite fermion concentration,[6]

$$\frac{\rho_{xx,3/2}}{\rho_{xx,1/2}}=\frac{3^{p+1}}{9}. \tag{15}$$

Equation (15) permits the straightforward evaluation of the p value just from the ratio of the $\rho_{xx,3/2}/\rho_{xx,1/2}$ available from the experiment.

TABLE I. Experimental data for $\rho_{xx,3/2}/\rho_{xx,1/2}$.

Group	$\rho_{xx,3/2}/\rho_{xx,1/2}$	Carriers	T (mK)	$n_s(p_s)$ ($\times 10^{15}$ m^{-2})
Sajoto *et al.* (Ref. 19)	0.35	Electrons	28	0.22
Sajoto *et al.* (Ref. 19)	0.43	Electrons	28	0.50
Sajoto *et al.* (Ref. 19)	0.33	Electrons	62	1.3
Willet *et al.* (Ref. 20)	0.30	Electrons	85	3
Yeh *et al.* (Ref. 21)	0.34	Electrons	40	1.3
Nicholas *et al.* (Ref. 22)	0.39	Electrons	710	4.8
Nicholas *et al.* (Ref. 22)	0.38	Electrons	590	3.0
Zeitler *et al.* (Ref. 23)	0.34	Electrons		
Pan *et al.* (Ref. 24)	0.35	Electrons		2.2
Tieke *et al.* (Ref. 12)	0.2	Electrons	28	1.76
Crump (Ref. 13)	0.19	Holes	299	(0.93)
Ying *et al.* (Ref. 14)	0.14	Holes	50	(0.65)
Bayot *et al.* (Ref. 16)	0.14	Holes	100	(0.4)

Substituting in Eq. (15) the values for p from Halperin *et al.*[10] ($p=0.5$), we obtain $\rho_{xx,3/2}=0.65\rho_{xx,1/2}$. Using Khveshchenko's value[6] for p ($p=0.13$), we obtain $\rho_{xx,3/2}=0.39\rho_{xx,1/2}$. In Table I we present a series of experimental data for the ratio $\rho_{xx,3/2}/\rho_{xx,1/2}$. The vast majority of the data are in remarkable agreement with our result for $p=0.13$. There are only two exceptions. The first concerns electrons and it has been reported by Tieke *et al.*[12] for which $\rho_{xx,3/2}=0.2\rho_{xx,1/2}$. The second concerns holes and it is referred to the data reported by Crump,[13] Ying *et al.*,[14] and Bayot *et al.*[15]

Crump's[13] data for holes with concentration $p_s=0.93\times10^{15}$ m^{-2} give a ratio $\rho_{xx,3/2}/\rho_{xx,1/2}=0.19$. From Eq. (15) we obtain $p=-0.5$. Ying *et al.*[14] and Bayot *et al.*[15] data, also for holes with concentrations, respectively, $p_s=0.65\times10^{15}$ m^{-2} and $p_s=0.4\times10^{15}$ m^{-2} give a ratio $\rho_{xx,3/2}/\rho_{xx,1/2}=0.14$. From Eq. (15) we obtain $p=-0.8$. This p value deviates significantly from the value obtained ($p=0.13$) if the random magnetic field scattering mechanism[6] is dominant. However, these hole concentrations are quite smaller than those of other samples studied by Bayot *et al.*[16] There, $p_s=1.35\times10^{15}$ m^{-2}, and in a previous work[3] we have shown that Khveshchenko's value for p is applicable. Thus, comparing the p values as the hole concentration diminishes we expect p to decrease. If this is the case, and p becomes lower than -1, according to Eq. (6) a change of sign in the diffusion component of the thermopower should be observed. Recently, Moldovan *et al.*[17] reported thermopower data for holes of concentrations in the range of 0.2×10^{15} m$^{-2}<p_s<1.2\times10^{15}$ m^{-2}. For low enough concentrations 0.18×10^{15} m$^{-2}<p_s<0.33\times10^{15}$ m^{-2} their data showed a change in the sign of the thermopower. These concentrations are very close to those reported by Bayot *et al.*[15] ($p_s=0.4\times10^{15}$ m^{-2}) and consequently, the negative p value obtained ($p=-0.8$), using Eq. (15) looks reasonable. Even for higher hole concentrations ($p_s=0.86\times10^{15}$ m^{-2}), Moldovan *et al.*[17] reported negative p values.

Tieke *et al.*[12] data although concern electrons of comparable electron sheet densities with other experiments listed at Table I seem to deviate from the 0.39 value of the resistivities' ratio. Applying Eq. (15) to their data we obtain $p=-0.4$. This p value together with Tieke *et al.* conclusion that the dominant contribution to the thermopower is due to phonon drag, suggests that the effective mass of the carriers involved is comparable with the free electron mass, i.e., $m^*\simeq m_0$ and not $m^*=5m_0$ that is the value they used in their report. This is in agreement with Shubnikov-de Haas analyses that yield $m^*\simeq m_0$.[18] Consequently, the scattering mechanism in the specific samples needs further investigation.

In summary, in the present work, we propose a simple way of evaluating p directly from the ratio of the experimental values of the diagonal component of the resistivity at $\nu=3/2$ and $\nu=1/2$ for electron or hole gases, in the fractional quantum Hall effect regime. The analysis is not subjected to limitations such as the specific scattering mechanism involved and the nature of the carriers present in the device. For electron gases our result justifies the p value proposed by Khveshchenko.[6] However, for hole gases, our result suggests negative p values, in agreement with recently reported experimental data on thermopower.[17]

V.C.K. acknowledges the support of the Public Benefit Foundation "Alexander S. Onassis."

[1] D.C. Tsui, H.L. Stormer, and A.C. Gossard, Phys. Rev. Lett. **48**, 1559 (1982).

[2] J.K. Jain, Phys. Rev. Lett. **63**, 199 (1989); Phys. Rev. B **40**, 8079 (1989).

[3] V.C. Karavolas, G.P. Triberis, and F.M. Peeters, Phys. Rev. B **56**, 15 289 (1997).

[4] V.C. Karavolas and G.P. Triberis, Phys. Rev. B **63**, 035313 (2001).

[5] V.C. Karavolas and P.N. Butcher, J. Phys.: Condens. Matter **3**, 2597 (1991).
[6] D.V. Khveshchenko, Phys. Rev. B **54**, R14 317 (1996).
[7] N.R. Cooper, B.I. Halperin, and I.M. Ruzin, Phys. Rev. B **55**, 2344 (1997).
[8] G. Kirczenow and B.L. Johnson, Phys. Rev. B **51**, 17 579 (1995).
[9] A. Lopez and E. Fradkin, Phys. Rev. B **44**, 5246 (1991).
[10] B.I. Halperin, P.A. Lee, and N. Read, Phys. Rev. B **47**, 7312 (1993).
[11] M.J. Kane, N. Apsley, D.A. Anderson, L.L. Taylor, and T. Kerr, J. Phys. C **18**, 5629 (1985).
[12] B. Tieke, U. Zeitler, R. Fletcher, S.A.J. Wiegers, A.K. Geim, J.C. Maan, and M. Henini, Phys. Rev. Lett. **76**, 3630 (1996).
[13] P.A. Crump, Ph.D. thesis, University of Nottingham (1995).
[14] X. Ying, V. Bayot, M.B. Santos, and M. Shayegan, Phys. Rev. B **50**, 4969 (1994).
[15] V. Bayot, X. Ying, M.B. Santos, and M. Shayegan, Europhys. Lett. **25**, 613 (1994).
[16] V. Bayot, E. Grivei, H.C. Manoharan, X. Ying, and M. Shayegan, Phys. Rev. B **52**, R8621 (1995).
[17] L. Moldovan, S. Melinte, V. Bayot, S.J. Papadakis, E.P. De Poortere, and M. Shayegan, Phys. Rev. Lett. **85**, 4369 (2000).
[18] R.R. Du, H.L. Stormer, D.C. Tsui, L.N. Pfeiffer, and K.W. West, Phys. Rev. Lett. **70**, 2944 (1993); D.R. Leadley, R.J. Nicholas, C.T. Foxon, and J.J. Harris, *ibid.* **72**, 1906 (1994).
[19] T. Sajoto, Y.W. Suen, L.W. Engel, M.B. Santos, and M. Shayegan, Phys. Rev. B **41**, 8449 (1990).
[20] R. Willett, J.P. Eisenstein, H.L. Stormer, D.C. Tsui, A.C. Gossard, and J.H. English, Phys. Rev. Lett. **59**, 1776 (1987).
[21] A.S. Yeh, H.L. Stormer, D.C. Tsui, L.N. Pfeiffer, K.W. Baldwin, and K.W. West, Phys. Rev. Lett. **82**, 592 (1999).
[22] R.J. Nicholas, D.R. Leadley, M.S. Daly, M. van der Burgt, P. Gee, J. Singleton, D.K. Maude, J.C. Portal, J.J. Harris, and C.T. Foxon, Semicond. Sci. Technol. **11**, 1477 (1996).
[23] U. Zeitler, J.C. Maan, P. Wyder, R. Fletcher, C.T. Foxon, and J.J. Harris, Phys. Rev. B **47**, 16 008 (1993).
[24] W. Pan, R.R. Du, H.L. Stormer, D.C. Tsui, L.N. Pfeiffer, K.W. Baldwin, and K.W. West, Phys. Rev. Lett. **83**, 820 (1999).