Unitary limit of spin-orbit scattering in two-dimensional *s*- and *d*-wave superconductors

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Nonmagnetic impurities affect the paramagnetic response of superconductors via the associated spin-orbit interaction that, when the nonmagnetic impurity is close to the unitary limit, must be treated beyond the classical Born approximation. Here the Zeeman response of two-dimensional *s*- and *d*-wave superconductors is calculated within the self-consistent *T*-matrix formulation for both impurity and spin-orbit scatterings. It is shown that at the unitary limit, for which the spin-orbit scattering is maximum, the spin-up and spin-down channels become decoupled implying full Zeeman splitting of the quasiparticle excitations. These results could be used to test the unitary scattering hypothesis in high-T_c superconductors.

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I. INTRODUCTION

The response of a superconductor to defects and/or impurities brings important information on the nature of the superconducting state and it has been the subject of an enormous amount of theoretical and experimental research.[1] The effect of impurities on the superconducting state depends crucially on the nature of the impurity (magnetic or nonmagnetic) and on the symmetry of the order parameter. For isotropic *s*-wave superconductors, the response to a nonmagnetic impurity is feeble,[2] while the effect of a magnetic one is dramatic.[3,4] Instead when the order parameter is anisotropic, as in *d*-wave high-T_c superconductor, the response to disorder is always dramatic,[5,6] leading to Kondo-like effects in the case of magnetic scattering potentials,[7] or resonant behaviors for nonmagnetic impurities close to the unitary limit.[8] For high-T_c superconductors, therefore, it is experimentally more difficult to establish whether the impurity acts effectively as a magnetic or a nonmagnetic scattering potential. For example, recent scanning-tunneling-microscope images of the local tunneling conductivity around a Zn impurity in $Bi_2SR_2CuO_2$,[9] have been fitted by both nonmagnetic[10] and magnetic impurity models.[7]

A topic that could be helpful to clarify the effective nature of disorder in high-T_c oxides is the analysis of the response to some external applied perturbation. The aim of this paper is to show some important consequences of having strong nonmagnetic impurities on the Zeeman response of a *s*- or *d*-wave superconductor. The way in which nonmagnetic scattering centers affect the spin degrees of freedom is via the associated spin-orbit interaction as described by the so-called Elliott-Yafet theory.[11,12] Hence, if v is the nonmagnetic impurity potential,[13] the corresponding spin-orbit scattering is proportional to $v_{\rm so}=v\,\delta g$, where δg is the shift of the g factor. The actual value of δg depends on the wave-function penetration into the ions and the Fermi-surface topology and it is a rather difficult problem.[12] For copper oxides the main contribution to δg should come from the d orbital of Cu atoms for which $\delta g \simeq 0.1$. Within the Born approximation, the spin-orbit scattering rate in the normal state is therefore $1/\tau_{\rm so}\simeq(\delta g)^2/\tau_{\rm imp} \ll 1/\tau_{\rm imp}$, where $1/\tau_{\rm imp}$ is the scattering rate due to v alone. This is also known as the Elliott-Yafet relation. However, when the impurity scattering is close to the unitary limit, as often advocated for impurity-doped high-T_c superconductors, the Elliott-Yafet formula must be generalized in order to include multiscattering processes. For a two-dimensional system with sufficiently diluted impurity concentrations n_i, the solution of the normal state T-matrix equations for both v and $v_{\rm so}$ leads to[14]

$$\frac{1}{\tau_{\rm so}}=2\frac{1+c^2}{1+(2c/\delta g)^2}\frac{1}{\tau_{\rm imp}}, \tag{1}$$

where $c=1/\pi N_0 v$, $1/\tau_{\rm imp}=2\Gamma/(1+c^2)$, $\Gamma=n_i/\pi N_0$, and N_0 is the density of states per spin direction at the Fermi level. In the weak scattering limit $c\gg 1$, Eq. (1) reproduces the result of the Born approximation: $1/\tau_{\rm so}=(\delta g)^2/2\tau_{\rm imp} \ll 1/\tau_{\rm imp}$. However for $c=0.1$, that is the value estimated in Ref. 9, Eq. (1) leads to $1/\tau_{\rm so}\simeq 0.4/\tau_{\rm imp}$ when $\delta g=0.1$, and in the extreme unitary limit: $\lim_{c\to 0}1/\tau_{\rm so}=2/\tau_{\rm imp}$ as long as $\delta g\neq 0$. Hence, when the impurity potential is strong, or more generally as long as $c \sim< \delta g$, inevitably the spin-orbit interaction becomes as important as the spin-independent coupling to the impurity.

The above discussion suggests, therefore, that if nonmagnetic impurities in high-T_c superconductors are close to the unitary limit, the effect of spin-orbit coupling should be large. In particular, the Zeeman response to an applied magnetic field should be deeply altered by the spin-mixing processes associated with $v_{\rm so}$ and eventually, for sufficiently strong spin-orbit scattering, the Zeeman splitting should vanish. Here it is shown that for two-dimensional systems this conclusion is actually wrong: the Zeeman splitting resulting from the solution of the T-matrix equation is much more robust than that obtained within the Born approximation, and at the unitary limit $(c/\delta g=0)$ both *s*- and *d*-wave superconductors are fully Zeeman splitted by an applied in-plane magnetic field.

II. SPIN-ORBIT *T* MATRIX

For quasi-two-dimensional systems the Zeeman response to an external magnetic field **H** should be best observed when **H** is directed parallel to the conducting plane (for example, the Cu-O plane in copper oxides), since in this case the coupling of **H** to the orbital motion of the electrons is minimized.[15] Hence, the total Hamiltonian is $\mathcal{H}=\mathcal{H}_0+\mathcal{H}_{\rm imp}+\mathcal{H}_{\rm so}$, where

$$\mathcal{H}_0=\sum_{\mathbf{k},\alpha}\epsilon(\mathbf{k})c^\dagger_{\mathbf{k}\alpha}c_{\mathbf{k}\alpha}-h\sum_{\mathbf{k},\alpha}\alpha c^\dagger_{\mathbf{k}\alpha}c_{\mathbf{k}\alpha}-\sum_{\mathbf{k}}\Delta(\mathbf{k})(c^\dagger_{\mathbf{k}\uparrow}c^\dagger_{-\mathbf{k}\downarrow}+c_{-\mathbf{k}\downarrow}c_{\mathbf{k}\uparrow}), \tag{2}$$

where $\epsilon(\mathbf{k})$ is the electron dispersion measured with respect to the chemical potential, α is a spin index, $h=\mu_{\rm B}H$, and $\mu_{\rm B}$ is the Bohr magneton. In the following it is assumed that the charge carriers are confined to move in the x-y plane so that $\mathbf{k}\equiv(k_x,k_y)$ and that the spins are directed along and opposite to the direction of the magnetic field, fixed to lie along the x direction: $\mathbf{H}=H\hat{x}$. For s-wave superconductors $\Delta(\mathbf{k})=\Delta$ while for d-wave superconductors $\Delta(\mathbf{k})=\Delta\cos(2\phi)$, where ϕ is the polar angle in the k_x-k_y plane. Without loss of generality, here Δ is used as an input parameter although it should be calculated self-consistently from a suitable gap equation. Moreover, for simplicity, local variations of the order parameter are neglected. The impurity and spin-orbit Hamiltonians, $\mathcal{H}_{\rm imp}$ and $\mathcal{H}_{\rm so}$, are given by

$$\mathcal{H}_{\rm imp}=v\sum_{\mathbf{k},\mathbf{k}',i}\sum_{\alpha}\exp[-i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{R}_i]c^\dagger_{\mathbf{k}\alpha}c_{\mathbf{k}'\alpha}, \tag{3}$$

$$\mathcal{H}_{\rm so}=i\frac{\delta g v}{k_F^2}\sum_{\mathbf{k},\mathbf{k}',i}\exp[-i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{R}_i]([\mathbf{k}\times\mathbf{k}']\cdot\hat{\mathbf{z}})\times(c^\dagger_{\mathbf{k}\uparrow}c_{\mathbf{k}'\downarrow}+c^\dagger_{\mathbf{k}\downarrow}c_{\mathbf{k}'\uparrow}), \tag{4}$$

where $\mathbf{R}_i$ denotes the random positions of the impurities and k_F is the Fermi momentum. Note that due to two dimensionality, the spin-orbit matrix element of Eq. (4) is proportional to σ_z and, since the spins are quantized along the x direction, the spin-orbit scattering is, therefore, always accompanied by spin-flip processes. The Zeeman response for $\mathcal{H}_{\rm imp}=0$ and $\mathcal{H}_{\rm so}=0$ has already been considered in Ref. 16 for d-wave superconductors and in Ref. 17 for mixed symmetries of the order parameter. The inclusion of $\mathcal{H}_{\rm imp}$, which, however, does not mix the spin states, has been studied in Ref. 18. The total Hamiltonian $\mathcal{H}=\mathcal{H}_0+\mathcal{H}_{\rm imp}+\mathcal{H}_{\rm so}$ for d-wave symmetry has been considered in Ref. 19 within the Born approximation for both impurity and spin-orbit scatterings. Here, instead, the problem is generalized beyond the Born approximation by solving the self-consistent T-matrix equation for both $\mathcal{H}_{\rm imp}$ and $\mathcal{H}_{\rm so}$.

The generalized Matsubara Green's function $G(\mathbf{k},n)$ in the particle-hole spin space resulting from Eqs. (2)–(4) satisfies the Dyson equation $G^{-1}(\mathbf{k},n)=G_0^{-1}(\mathbf{k},n)-\Sigma(\mathbf{k},n)$, where $G_0^{-1}(\mathbf{k},n)=i\omega_n-\rho_3\epsilon(\mathbf{k})-\rho_2\tau_2\Delta(\mathbf{k})-h\rho_3\tau_3$ is the propagator resulting from $\mathcal{H}_0$. The Pauli matrices ρ_i and τ_i $(i=1,2,3)$ act on the particle-hole and spin subspaces, respectively. Within the self-consistent T-matrix approach, the self energy is $\Sigma(\mathbf{k},n)=n_iT(\mathbf{k},\mathbf{k},n)$, where the T matrix is the solution of the following equation:

$$T(\mathbf{k},\mathbf{k}',n)=u(\mathbf{k},\mathbf{k}')+\sum_{\mathbf{k}''}u(\mathbf{k},\mathbf{k}'')G(\mathbf{k}'',n)T(\mathbf{k}'',\mathbf{k}',n), \tag{5}$$

and $u(\mathbf{k},\mathbf{k}')=\rho_3 v+i\,\delta g v[\hat{\mathbf{k}}\times\hat{\mathbf{k}}']_z\tau_1$. Giving to the momentum dependence of the spin-orbit part of $u(\mathbf{k},\mathbf{k}')$, the T matrix can be splitted into the impurity and spin-orbit contributions for both s- and d-wave symmetries of the order parameter. Hence, $T(\mathbf{k},\mathbf{k}',n)=T_{\rm imp}(n)+T_{\rm so}(\mathbf{k},\mathbf{k}',n)$, where $T_{\rm imp}(n)=\rho_3 v+\rho_3 v\Sigma_{\mathbf{k}}G(\mathbf{k},n)T_{\rm imp}(n)$ is the usual impurity T matrix, and

$$T_{\rm so}(\mathbf{k},\mathbf{k}',n)=i\,\delta g v[\hat{\mathbf{k}}\times\hat{\mathbf{k}}']_z\tau_1+i\,\delta g v\sum_{\mathbf{k}''}[\hat{\mathbf{k}}\times\hat{\mathbf{k}}'']_z\tau_1 G(\mathbf{k}'',n)T_{\rm so}(\mathbf{k}'',\mathbf{k}',n) \tag{6}$$

is the spin-orbit T matrix. The solution of Eq. (6) is of the form[14]

$$T_{\rm so}(\mathbf{k},\mathbf{k}',n)=i\,\delta g v[\hat{\mathbf{k}}\times\mathbf{t}(\hat{\mathbf{k}}',n)]\tau_1, \tag{7}$$

where

$$\mathbf{t}(\hat{\mathbf{k}},n)=\hat{\mathbf{k}}+i\,\delta g v\sum_{\mathbf{k}'}\hat{\mathbf{k}}'\tau_1 G(\mathbf{k}',n)[\hat{\mathbf{k}}'\times\mathbf{t}(\hat{\mathbf{k}},n)]_z. \tag{8}$$

The above equation can be easily solved in terms of the components $t_x(\hat{\mathbf{k}},n)$ and $t_y(\hat{\mathbf{k}},n)$ of the vector operator $\mathbf{t}(\hat{\mathbf{k}},n)$,

$$t_x(\hat{\mathbf{k}},n)=A^{-1}_{xy}(n)\left[\hat{k}_x+i\,\delta g v\hat{k}_y\sum_{\mathbf{k}'}(\hat{k}_x)^2\tau_1 G(\mathbf{k}',n)\right], \tag{9}$$

$$t_y(\hat{\mathbf{k}},n)=A^{-1}_{yx}(n)\left[\hat{k}_y-i\,\delta g v\hat{k}_x\sum_{\mathbf{k}'}(\hat{k}_y)^2\tau_1 G(\mathbf{k}',n)\right], \tag{10}$$

where $A^{-1}_{xy}(n)$ and $A^{-1}_{yx}(n)$ are the inverse of the following 4×4 matrices:

$$A_{xy}(n)=1-(\delta g v)^2\left[\sum_{\mathbf{k}}(\hat{k}_x)^2\tau_1 G(\mathbf{k},n)\right]\times\left[\sum_{\mathbf{k}}(\hat{k}_y)^2\tau_1 G(\mathbf{k},n)\right], \tag{11}$$

$$A_{yx}(n)=1-(\delta g v)^2\left[\sum_{\mathbf{k}}(\hat{k}_y)^2\tau_1 G(\mathbf{k},n)\right]\times\left[\sum_{\mathbf{k}}(\hat{k}_x)^2\tau_1 G(\mathbf{k},n)\right]. \tag{12}$$

Finally, from Eq. (7), $T_{\rm so}(\mathbf{k},\mathbf{k},n)$ reduces to

$$T_{\rm so}(\mathbf{k},\mathbf{k},n)=i\,\delta g v \hat{k}_x\hat{k}_y[A_{yx}^{-1}(n)-A_{xy}^{-1}(n)]\tau_1 + (\delta g v)^2\sum_{\mathbf{k}'}[A_{yx}^{-1}(n)(\hat{k}_x\hat{k}'_y)^2 + A_{xy}^{-1}(n)(\hat{k}_y\hat{k}'_x)^2]\tau_1 G(\mathbf{k}',n)\tau_1. \quad (13)$$

Further analysis of the T-matrix problem requires the explicit inclusion of the symmetry of the order parameter. This is done in the following sections, where Eq. (13) is solved for both s- and d-wave symmetries.

A. *s*-wave symmetry

The usual procedure to evaluate self-consistently the electron propagator is to guess the form of $G(\mathbf{k},n)$ that, after being substituted into $T_{\rm imp}(n)$ and $T_{\rm so}(\mathbf{k},\mathbf{k},n)$, generates only combinations of ρ_i and τ_j matrices already contained in $G(\mathbf{k},n)$. The direct substitution of $G_0(\mathbf{k},n)$ into $T_{\rm imp}(n)$ and $T_{\rm so}(\mathbf{k},\mathbf{k},n)$ is a practical way to guess the correct form of $G(\mathbf{k},n)$ via the Dyson equation. When this is done, it is easy to realize that when the symmetry of the order parameter is s wave, $\Delta(\mathbf{k})=\Delta$, the two matrices A_{xy} and A_{yx} defined in Eqs. (11) and (12) become equal. Hence, the form of the electron propagator for an s-wave symmetry of the order parameter reduces to

$$G^{-1}(\mathbf{k},n)=i[\tilde{\omega}-i\tilde{h}\rho_3\tau_3]-\rho_3[\tilde{\epsilon}(\mathbf{k})-i\tilde{\Lambda}\rho_3\tau_3] - \rho_2\tau_2[\tilde{\Delta}-i\tilde{\Gamma}\rho_3\tau_3], \quad (14)$$

where the frequency dependence of the tilded quantities is implicit. The tilded quantities are obtained by substituting Eq. (14) into the equations for the impurity and spin-orbit T matrices and requiring self-consistency via the Dyson equation. In general, the solution is very complicated, but a considerable simplification arises if infinite electron bandwidth and particle-hole symmetry of the normal-state electron dispersion are assumed. In this case, in fact, several integrals over $\mathbf{k}$ average to zero,[20] leading to the following self-consistent equations:

$$i\tilde{\omega}_\pm = i(\omega_n \pm ih)+\frac{\Gamma}{1+c^2}g_\pm + 2\Gamma\frac{(2c/\delta g)^2 g_\mp + g_\pm}{1+(2c/\delta g)^4+2(2c/\delta g)^2(f_+f_- - g_+g_-)}, \quad (15)$$

$$\tilde{\Delta}_\pm = \Delta+\frac{\Gamma}{1+c^2}f_\pm + 2\Gamma\frac{(2c/\delta g)^2 f_\mp + f_\pm}{1+(2c/\delta g)^4+2(2c/\delta g)^2(f_+f_- - g_+g_-)}, \quad (16)$$

where $\tilde{\omega}_\pm=\tilde{\omega}\pm i\tilde{h}$, $\tilde{\Delta}_\pm=\tilde{\Delta}\pm i\tilde{\Gamma}$ and $g_\pm = i\tilde{\omega}_\pm/\sqrt{\tilde{\Delta}_\pm^2+\tilde{\omega}_\pm^2}$, $f_\pm=\tilde{\Delta}_\pm/\sqrt{\tilde{\Delta}_\pm^2+\tilde{\omega}_\pm^2}$. To display the spin-mixing effect of the spin-orbit interaction, Eqs. (15) and (16) are more conveniently rewritten in terms of $u_\pm=\tilde{\omega}_\pm/\tilde{\Delta}_\pm$,

$$u_\pm=\frac{\omega_n\pm ih}{\Delta}+2\frac{\Gamma}{\Delta}\left(\frac{c}{\delta g}\right)^2 \times\frac{\dfrac{u_\mp-u_\pm}{(1+u_\mp^2)^{1/2}}}{1+\left(\dfrac{2c}{\delta g}\right)^4+2\left(\dfrac{2c}{\delta g}\right)^2\dfrac{1+u_\pm u_\mp}{(1+u_\pm^2)^{1/2}(1+u_\mp^2)^{1/2}}}. \quad (17)$$

Apart for the trivial limit $u_\pm=(\omega_n\pm ih)/\Delta$ that holds true in the absence of spin-orbit interaction ($\delta g=0$), the two spin channels u_+ and u_- are coupled together. Within the Born approximation, $c/\delta g\gg 1$, Eq. (17) reduces to the two-dimensional version of the $u_\pm$ formula found in classic literature,[15,21]

$$u_\pm=\frac{\omega_n\pm ih}{\Delta}+\frac{1}{2}\Gamma\left(\frac{\delta g}{c}\right)^2\frac{u_\pm-u_\mp}{(1+u_\mp^2)^{1/2}}. \quad (18)$$

The characteristic feature displayed by the more general expression (17) is that, as the unitary limit $c/\delta g=0$ is approached, u_+ and u_- becomes decoupled and the full Zeeman splitting $u_+-u_-=2ih/\Delta$ is recovered. In such a limit therefore, the s-wave superconductor is fully Zeeman splitted as if the spin-orbit scattering would be spin conserving. The same conclusion can be obtained by calculating the zero-temperature spin susceptibility $\chi_{\rm s}$ as inferred by the linear-response theory. In fact, by including the spin-vertex function consistent with the T-matrix formulation it is possible to show that

$$\frac{\chi_{\rm s}}{\chi_{\rm n}}=1-\frac{\pi T}{\Delta}\sum_n\frac{1}{1+(\omega_n/\Delta)^2}\,\frac{1}{[1+(\omega_n/\Delta)^2]^{1/2}+\rho_{\rm so}}, \quad (19)$$

where $\chi_{\rm n}=2\mu_{\rm B}^2N_0$, T is the temperature and $\rho_{\rm so}=(\Gamma/\Delta)(\delta g/c)^2/[1+(\delta g/2c)^2]^2$. At zero temperature and for $\rho_{\rm so}<1$, Eq. (19) reduces to

$$\frac{\chi_{\rm s}}{\chi_{\rm n}}=1-\frac{1}{\rho_{\rm so}}\left[\frac{\pi}{2}-\frac{\arccos(\rho_{\rm so})}{\sqrt{1-\rho_{\rm so}^2}}\right]. \quad (20)$$

For $\delta g/c\ll 1$, $\rho_{\rm so}\simeq(\Gamma/\Delta)(\delta g/c)^2$ and Eq. (20) becomes equal to the two-dimensional version of the Abrikosov-Gorkov expression based on the Born approximation.[22] Instead, for $c/\delta g\ll 1$, $\rho_{\rm so}\simeq(\Gamma/\Delta)(2c/\delta g)^2$ and $\chi_{\rm s}/\chi_{\rm n}\simeq 2\pi(\Gamma/\Delta)(2c/\delta g)^2\ll 1$ that vanishes when $c/\delta g=0$.

The absence of spin-mixing contributions at the unitary limit can be interpreted as a consequence of the fact that $\mathcal{H}_0+\mathcal{H}_{\rm imp}$ commutes with S_x while $\mathcal{H}_{\rm so}$ commutes with S_z. Therefore, for weak spin-orbit scattering ($c/\delta g\gg 1$) S_x is a rather good quantum number and $\mathcal{H}_{\rm so}$ induces weak spin-flip processes leading to coupled $u_{\pm}$ equations. The spin decoupling at the unitary limit $c/\delta g \ll 1$, could be explained by arguing that for very strong spin-orbit interaction S_z rather than S_x is a good quantum number. The Cooper pairs are then formed by electrons with opposite spins in the z direction and the spin rigidity of the superconducting condensate is efficient against spin-flip transitions induced by the magnetic field $\mathbf{H}=H\hat{\mathbf{x}}$. In the limiting case of infinitely strong spin-orbit interaction, therefore, H can only induce polarization of the quasiparticle excitations. Note that, in case the magnetic field is directed along the z direction, the total Hamiltonian then commutes with S_z and the Zeeman response of a s-wave superconductor becomes independent of the spin-orbit interaction for whatever value of $c/\delta g$. Of course, for a three-dimensional system, the above reasoning does no longer apply because the spin-orbit interaction does not commute with any component of $\mathbf{S}$.

B. *d*-wave symmetry

For the above considerations to be valid, only two dimensionality and a singlet superconducting condensate are required. Therefore, in principle, also a two-dimensional d-wave superconductor should exhibit spin-channels decoupling as $c/\delta g \to 0$. This is indeed so even if there are qualitative differences with respect to s-wave superconductors since for d-wave symmetry the spin-orbit scattering becomes pair breaking.[23] Again assuming particle-hole symmetry and an infinite electron bandwidth, for $\Delta(\mathbf{k}) \equiv \Delta(\phi) = \Delta\cos(2\phi)$ the electron propagator is of the form

$$G^{-1}(\mathbf{k},n)=i(\tilde{\omega}-i\tilde{h}\rho_3\tau_3)-\rho_3[\tilde{\epsilon}(\mathbf{k})-i\tilde{\Lambda}\rho_3\tau_3] -\rho_2\tau_2[\tilde{\Delta}(\phi)-i\tilde{\Gamma}(\phi)\rho_3\tau_3+i\tau_1\tilde{\Omega}(\phi)], \tag{21}$$

where $\tilde{\Delta}(\phi)=\tilde{\Delta}\cos(2\phi)$, $\tilde{\Gamma}(\phi)=\tilde{\Gamma}\cos(2\phi)$, and $\tilde{\Omega}(\phi)=\tilde{\Omega}\sin(2\phi)$. The origin of $\tilde{\Omega}(\phi)$ (absent in the s-wave case) stems from the fact that, for d-wave symmetry, the two matrices A_{xy} and A_{yx} in Eqs. (11) and (12) are no longer equal, so that the term proportional to $\hat{k}_x\hat{k}_y=\sin(2\phi)$ in $T_{\mathrm{so}}(\mathbf{k},\mathbf{k},n)$, Eq. (13), is nonzero. As for the s-wave case the self-consistent Dyson equation can be expressed in terms of $\tilde{\omega}_{\pm}$ and $\tilde{\Delta}_{\pm}$, but now there is an additional equation for $\tilde{\Omega}$,

$$i\tilde{\omega}_{\pm}=i(\omega_n\pm ih)+\frac{\Gamma}{c^2-(g^0_{\pm})^2}g^0_{\pm}+2\Gamma\frac{(2c/\delta g)^2 g_{\mp}+(f^2_{\mp}-g^2_{\mp})g_{\pm}}{[(2c/\delta g)^2-g_+g_--f_+f_-]^2-(g_+f_-+g_-f_+)^2}, \tag{22}$$

$$\tilde{\Delta}_{\pm}=\Delta+2\Gamma\frac{(2c/\delta g)^2 f_{\mp}-(f^2_{\mp}-g^2_{\mp})f_{\pm}}{[(2c/\delta g)^2-g_+g_--f_+f_-]^2-(g_+f_-+g_-f_+)^2}, \tag{23}$$

$$\tilde{\Omega}=-2\Gamma\frac{(2c/\delta g)(g_+f_-+g_-f_+)}{[(2c/\delta g)^2-g_+g_--f_+f_-]^2-(g_+f_-+g_-f_+)^2}, \tag{24}$$

$$g_{\pm}=\frac{2}{\pi N_0}\sum_{\mathbf{k}}\frac{\sin(\phi)^2}{2\pi}\frac{i\tilde{\omega}_{\pm}[\tilde{\omega}^2_{\mp}+E_{\mp}(\mathbf{k})^2]\mp 2i(\tilde{\omega}_+-\tilde{\omega}_-)\tilde{\Omega}(\phi)^2}{[\tilde{\omega}^2_++E_+(\mathbf{k})^2][\tilde{\omega}^2_-+E_-(\mathbf{k})^2]-\tilde{\Omega}(\phi)^2\{(\tilde{\omega}_+-\tilde{\omega}_-)^2+[\tilde{\Delta}_+(\phi)-\tilde{\Delta}_-(\phi)]^2\}}, \tag{25}$$

$$f_{\pm}=\frac{2}{\pi N_0}\sum_{\mathbf{k}}\frac{\sin(\phi)^2}{2\pi}\frac{\tilde{\Delta}_{\pm}(\phi)[\tilde{\omega}^2_{\mp}+E_{\mp}(\mathbf{k})^2]\mp[\tilde{\Delta}_+(\phi)-\tilde{\Delta}_-(\phi)]\tilde{\Omega}(\phi)^2}{[\tilde{\omega}^2_++E_+(\mathbf{k})^2][\tilde{\omega}^2_-+E_-(\mathbf{k})^2]-\tilde{\Omega}(\phi)^2\{(\tilde{\omega}_+-\tilde{\omega}_-)^2+[\tilde{\Delta}_+(\phi)-\tilde{\Delta}_-(\phi)]^2\}}, \tag{26}$$

where $E_{\pm}(\mathbf{k})^2=\epsilon(\mathbf{k})^2+\tilde{\Delta}_{\pm}(\phi)^2+\tilde{\Omega}(\phi)^2$ and $g^0_{\pm}$ can be obtained from Eq. (25) by setting $\sin(\phi)^2 \to 1/2$. The off-diagonal contribution $\tilde{\Omega}$ defined in Eq. (24) is responsible for spin-mixing terms appearing in $g_{\pm}$, $f_{\pm}$, and $g^0_{\pm}$. However, at the unitary limit $c/\delta g=0$, $\tilde{\Omega}$ vanishes and the above self-consistent equations reduce to

$$i\tilde{\omega}_{\pm}=i(\omega_n\pm ih)-\frac{\Gamma}{g^0_{\pm}}+2\Gamma\frac{g_{\pm}}{f^2_{\pm}-g^2_{\pm}}, \tag{27}$$

$$\tilde{\Delta}_{\pm}=\Delta-2\Gamma\frac{f_{\pm}}{f^2_{\pm}-g^2_{\pm}}, \tag{28}$$

where

$$g_{\pm}=2\int\frac{d\phi}{2\pi}\frac{i\tilde{\omega}_{\pm}\sin(\phi)^2}{[\tilde{\Delta}_{\pm}(\phi)^2+\tilde{\omega}^2_{\pm}]^{1/2}}, \tag{29}$$

$$f_{\pm}=2\int\frac{d\phi}{2\pi}\frac{\tilde{\Delta}_{\pm}(\phi)\sin(\phi)^2}{[\tilde{\Delta}_{\pm}(\phi)^2+\tilde{\omega}^2_{\pm}]^{1/2}}. \tag{30}$$

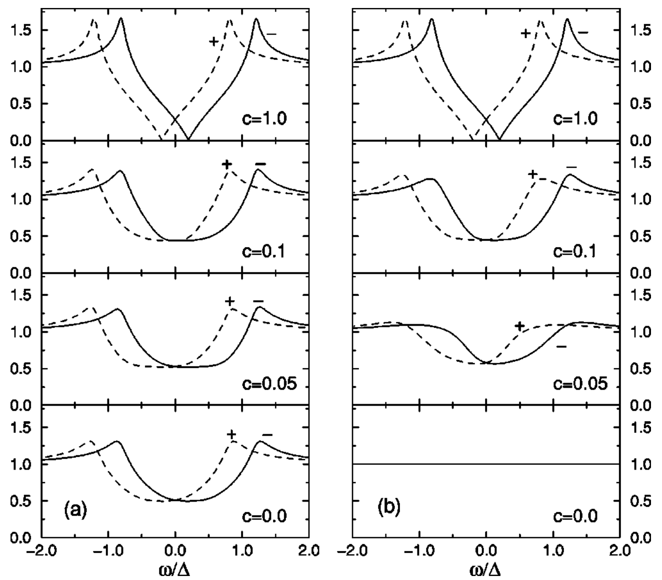

FIG. 1. Zeeman-split quasiparticle density of states $N_+(\omega)/N_0$ (dashed lines) and $N_-(\omega)/N_0$ (solid lines) for a *d*-wave superconductor with $h=H/\Delta=0.2$, $\Gamma=0.1$, $\delta g=0.1$, and different values of the scattering parameter *c*. (a): solution of the complete *T*-matrix equations. (b): solution for the Born approximation to the spin-orbit coupling.

The two spin channels $+$ and $-$ in Eqs. (27) and (28) are now completely decoupled in analogy therefore with the *s*-wave case treated before. However, now even at $T=0$ the spin susceptibility is expected to remain nonzero (as long as $\Gamma\neq 0$). This is due to the fact that, although there are no spin-mixing processes at $c/\delta g=0$, the pair-breaking effect of both impurity and spin-orbit scatterings leads to a finite density of states at the Fermi level.[24] In this situation, therefore, the Zeeman-splitted density of states is a more direct evidence for the spin-decoupling effect at $c/\delta g\to 0$. This is shown in Fig. 1 where the two spin channels density of states, $N_\pm(\omega)$, are plotted for $\Gamma=0.1$, $\delta g=0.1$, $h=0.2$ and for different values of *c*. $N_\pm(\omega)$ is calculated numerically from

$$\frac{N_\pm(\omega)}{N_0}=-\,\mathrm{sgn}(\omega)\,\mathrm{Im}[g^0_\pm(\omega)], \tag{31}$$

where $g^0_\pm(\omega)$ is the analytic continuation on the real axis ($i\omega_n\to\omega+i\delta$) of Eq. (25) (with $\sin(\phi)^2\to 1/2$).[25] In Fig. 1(a), $N_\pm(\omega)$ is calculated from the solution of the general equations (22)–(24), while, for comparison, the result for the Born approximation applied to the spin-orbit part of Eqs. (22)–(24) is shown in Fig. 1(b).[26] Up to $c=0.1$ ($\delta g/c=1$) the general *T*-matrix solution and the Born approximation agree quite well, while already for $c=0.05$ ($\delta g/c=2$) the splitted coherence peaks at $\omega/\Delta\simeq\pm(1\pm h)$ are still quite visible in Fig. 1(a) and completely suppressed in Fig. 1(b). Since, at the unitary limit $c/\delta g=0$, the spins are completely decoupled, Eqs. (27) and (28), the two spin density of states are identical and shifted by $\pm h$, one with respect to the other. This is in contrast to the Born solution, Fig. 1(b), for which the strong spin-mixing terms lead to a flat density of states. It should be stressed that before reaching the $\delta g/c \gg 1$ limit, the Born approximation predicts that *d*-wave superconductivity is already completely destroyed.[27] Therefore, the results for $c=0$ in Fig. 1(b) should be considered just as a mathematical limit to be compared with the solutions of the *T*-matrix approach of Fig. 1(a).

III. CONCLUSIONS

In summary, it has been shown that when the nonmagnetic impurity scattering is close to the unitary limit, the associated spin-orbit interaction is not small provided $c\sim<\delta g$, and it must be treated beyond the simple Born approximation. Within the self-consistent *T*-matrix approach, it has been demonstrated that the Zeeman splitting of both *s*- and *d*-wave two-dimensional superconductors is much more robust than that obtained by the Born approximation. At the unitary limit $c/\delta g=0$, for which the spin-orbit coupling is maximum, a two-dimensional *s*-wave superconductor does not show any spin-mixing processes and the Zeeman response coincides with that of a pure superconductor. Also for *d*-wave superconductors the spin-orbit coupling becomes effectively spin conserving at $c/\delta g=0$, but, in addition, it induces pair-breaking effects that must be added to those caused by the scalar impurities.[14]

Let us comment now on the possible limitations of the present theory. The calculation method used here is a standard one based on a *T*-matrix approximation for diluted impurities.[5,6,20] However, when it is applied to two-dimensional systems, such as, the copper oxides, this standard procedure is complicated by the appearance of singularities in the electron self-energy.[28] Contrary to the results based on the *T*-matrix solution and self-consistent approaches to deal with the singularities,[29] nonperturbative methods suggest that the density of states of a *d*-wave superconductor actually vanishes nonanalytically at the Fermi level.[30] The low-energy behavior appears to be heavily modified by the level spacing of a localization volume that leads to the opening of a pseudogap in the low-lying single-electron excitations.[31] It should, however, be noted that discrepancies between different approaches affect only the very low-energy excitations, while for energies not much smaller than Δ the *T*-matrix approach is quite reliable (for finite but small impurity concentrations). In this respect, the main result of Fig. 1(a) (i.e., the persistence of the Zeeman splitting of the coherence peaks in the density of states even when $c/\delta g$ is zero) should not be an artificial feature of the *T*-matrix approximation. This conclusion is also sustained by the quite general physical explanation of the Zeeman-splitting persistence at the unitary limit proposed in Sec. II A and from an analysis of the single spin-orbit impurity problem not reported here. Note that, for these same reasons, some standard simplifications employed in the present calculations (infinite bandwidth, particle-hole symmetric electron dispersion and absence of local suppressions of the order parameter) should not affect too seriously the main result.

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[26] For $\delta g/c \ll 1$, the last terms of Eqs. (22) and (23) are replaced by $2\Gamma(\delta g/2c)^2 g_\mp$, and $2\Gamma(\delta g/2c)^2 f_\mp$, respectively (Ref. 19). Note that in the same approximation $\tilde{\Omega}\propto\Gamma(\delta g/2c)^3$ and can be neglected.
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