Spin susceptibility of the superfluid ^{3}He-B in aerogel

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(Received 30 July 2001; published 3 December 2001)

The temperature dependence of paramagnetic susceptibility of superfluid ^{3}He-B in aerogel is found. Calculations have been performed for an arbitrary phase shift of *s*-wave scattering in the framework of BCS weak-coupling theory and the simplest model of aerogel as an aggregate of homogeneously distributed ordinary impurities. Both limiting cases of the Born and unitary scattering can be easily obtained from the general result. The existence of gapless superfluidity starting at the critical impurity concentration depending on the value of the scattering phase has been demonstrated. While larger than in the bulk liquid the calculated susceptibility of the B phase in aerogel proves to be conspicuously smaller than that determined experimentally in the high pressure region.

DOI: 10.1103/PhysRevB.65.024501 PACS number(s): 67.57.Pq, 67.57.Bc

I. INTRODUCTION

Superfluid ^{3}He is an ideal object for a study of physical properties of a system with nontrivial pairing. In particular, it is interesting to investigate how a superfluid state could be influenced by the presence of impurities. Direct contamination of ^{3}He with any atomic impurities is impossible. During the last years, instead of this, experimentalists have used liquid ^{3}He to fill up aerogel which is a matrix of randomly arranged silica filaments of nanometer diameter. Commonly used aerogels occupy about 2% of space volume, the rest being taken with liquid helium.

Experiments[1–5] established decrease of the superfluid transition temperature and the density of the superfluid component of ^{3}He in aerogel. These effects are related to the scattering of quasiparticles from surface, which suppresses Cooper pairing in the p state. Corresponding theoretical treatments have been proposed in Refs. 6–10.

NMR measurements[2] pointed to the existence of the A phase of ^{3}He in aerogel in relatively high magnetic fields as well as of the B phase[11] in lower fields. Recently, evidence of the phase transition between the A and B phases has been demonstrated.[12–14]

The A phase as an equal-spin pairing state possesses practically the same spin susceptibility as the normal phase. This property remains intact in the A phase in aerogel. On the other hand, in the B phase where all the three spin states of the Cooper pairs with the spin projections $S_z = 0, \pm 1$ are equally populated, spin susceptibility is partly suppressed compared to its normal-state value. This suppression in aerogel was experimentally found[12] to constitute about 45% of the value in bulk ^{3}He-B.

The purpose of the present article is to calculate spin susceptibility of the ^{3}He-B in aerogel. We shall work in the framework of BCS weak-coupling theory, aerogel being approximated as homogeneously distributed ordinary impurities. At high pressures this model gives values of Δ and ρ_s roughly by a factor of 2 larger[6,8] than those observed experimentally. At the same time, as it was discussed in Ref. 15, at low pressures, when the coherence length is larger than the average pore size, the homogeneous scattering model is more likely to be justified.

A similar conclusion follows from the present derivation. The calculated susceptibility is less suppressed compared to the bulk dependence. This suppression is less pronounced for larger values of the scattering phase but still even in the unitary limit noticeably larger than the measured suppression of the susceptibility at high pressures.[3,12]

The calculations presented in the paper have been done for an arbitrary value δ_0 of the phase shift of s-wave scattering. Both extremes—the Born approximation and the unitary limit—can be easily obtained from the general result.

A corresponding theory of changes of paramagnetic susceptibility in ordinary superconductors caused by impurities with ordinary, paramagnetic, and spin-orbital type of scattering has been developed in the Born approximation in Ref. 16. The present theory is not a simple generalization of the cited paper. The point is that usually to go beyond the Born approximation in the theory of dirty alloys it is sufficient to substitute the Born scattering amplitude by the exact one.[17] This substitution does not lead to the correct answer for a dirty metal in a magnetic field. The method developed in the present paper does not rely on the substitution.

The paper is organized as follows. After the introduction of the basics of the Abrikosov-Gor'kov theory in Sec. II, we write down its solution for an arbitrary impurity scattering phase without magnetic field in Sec. III, where we also establish the region of the existence of gapless superfluidity. Then by finding corrections in leading order in magnetic field we arrive at an expression for the susceptibility. In the last section we briefly discuss conclusions.

II. EQUATIONS

The Abrikosov-Gor'kov equations of an electrically neutral impure superfluid in an external magnetic field $\mathbf{B}$ are (see, e.g., Ref. 18)

$$[i\omega_n - \hat{H}(\mathbf{k}) - \hat{\Sigma}(\omega_n)]\hat{G}(\mathbf{k},\omega_n) = \hat{1}, \tag{1}$$

where $\omega_n = \pi T(2n+1)$ is the Matsubara frequency. The Green function $\hat{G}(\mathbf{k},\omega_n)$ is a 2×2 matrix in particle-hole space,

$$\hat{G}(\mathbf{k},\omega_n) = \begin{pmatrix} G_{\alpha\beta}(\mathbf{k},\omega_n) & F_{\alpha\beta}(\mathbf{k},\omega_n) \\ F^{+}_{\alpha\beta}(\mathbf{k},\omega_n) & \overline{G}_{\alpha\beta}(\mathbf{k},\omega_n) \end{pmatrix}, \tag{2}$$

made up of the normal $G_{\alpha\beta}(\mathbf{k},\omega_n)$ and anomalous $F_{\alpha\beta}(\mathbf{k},\omega_n)$ Green functions which are in their turn 2×2 matrices in spin space. Here

$$\overline{G}_{\alpha\beta}(\mathbf{k},\omega_n) = -G^{\mathrm{T}}_{\alpha\beta}(-\mathbf{k},-\omega_n) \tag{3}$$

and the superscript "T" implies transposition.

The Hamiltonian

$$\hat{H}(\mathbf{k}) = \begin{pmatrix} H_{0\alpha\beta}(\mathbf{k}) & \Delta_{\hat{k}\alpha\beta} \\ \Delta^{\dagger}_{\hat{k}\alpha\beta} & -H^{\mathrm{T}}_{0\alpha\beta}(-\mathbf{k}) \end{pmatrix} \tag{4}$$

consists of the one-particle part

$$H_{0\alpha\beta}(\mathbf{k}) = \xi_{\mathbf{k}}\delta_{\alpha\beta} - \mu\boldsymbol{\sigma}_{\alpha\beta}\mathbf{B} \tag{5}$$

including kinetic

$$\xi_{\mathbf{k}} \equiv \xi_{|\mathbf{k}|} = k^2/2m^* - \epsilon_F \tag{6}$$

and Zeeman energy, and of the pairing interaction $V_{\alpha\beta,\lambda\mu}(\mathbf{k},\mathbf{k}')$ via the order parameter $\Delta_{\hat{k}\alpha\beta}$, set by the self-consistency equation

$$\Delta_{\hat{k}\alpha\beta} = -T\sum_{n}\sum_{\mathbf{k}'} V_{\beta\alpha,\lambda\mu}(\mathbf{k},\mathbf{k}')F_{\mu\lambda}(\omega_n,\mathbf{k}'). \tag{7}$$

$\mathbf{B}$ is the magnetic flux, and $\boldsymbol{\sigma}_{\alpha\beta}$ is a vector of the three Pauli matrices in spin space.

The impurity scattering self-energy part comprises normal Σ_1 and anomalous Σ_2 parts

$$\hat{\Sigma} = \begin{pmatrix} \Sigma_1 & \Sigma_2 \\ \Sigma_2^{+} & \overline{\Sigma}_1 \end{pmatrix}, \tag{8}$$

and in the clean limit of small impurity concentrations n_{imp}, when one can neglect the interference of the scattering on different impurities, is just n_{imp} times the contribution of a single impurity,

$$\hat{\Sigma} = n_{\mathrm{imp}}\hat{T}, \tag{9}$$

where $\hat{T}$ is the scattering amplitude. It is related to the scattering potential $\hat{U}_{\mathbf{kk}'}$ by the Lippmann-Schwinger equation

$$\hat{T}_{\mathbf{kk}'} = \hat{U}_{\mathbf{kk}'} + \sum_{\mathbf{q}} \hat{U}_{\mathbf{kq}}\hat{G}_{\mathbf{q}}\hat{T}_{\mathbf{qk}'}. \tag{10}$$

In the simplest case of a short-range impurity potential $U(\mathbf{r}) = u\,\delta(\mathbf{r})$ its Fourier component is momentum-independent $U_{\mathbf{k}-\mathbf{k}'} = u$, so that

$$\hat{U}_{\mathbf{kk}'} \equiv \begin{pmatrix} U_{\mathbf{k}-\mathbf{k}'}\delta_{\alpha\beta} & 0 \\ 0 & -U_{\mathbf{k}'-\mathbf{k}}\delta_{\alpha\beta} \end{pmatrix} = u\,\delta_{\alpha\beta}\hat{\tau}_3, \tag{11}$$

where $\hat{\boldsymbol{\tau}}$ is a vector of the three Pauli matrices in particle-hole space. Parametrized in terms of the scattering phase δ_0 the impurity potential is

$$u = -\tan\delta_0/\pi N_0. \tag{12}$$

Equation (10) is then easily solved, and the scattering amplitude turns out to be independent of momenta:

$$\hat{T}_{\mathbf{kk}'}(\omega_n) \equiv \hat{T}(\omega_n) = \hat{U}\left(\hat{1} - \hat{U}\sum_{\mathbf{k}}\hat{G}(\mathbf{k},\omega_n)\right)^{-1}. \tag{13}$$

Equations (1), (4), (9), and (13) form a closed system of equations on the Green function (2).

The pairing interaction $V_{\beta\alpha,\lambda\mu}(\mathbf{k},\mathbf{k}')$ is usually reckoned nonzero only in a thin $\sim\epsilon_1 \ll \epsilon_F$ layer near the Fermi surface. In ^{3}He the spin-orbital interaction is weak and the Cooper pairs are formed in the spin-triplet, orbital p-wave state. So the pairing interaction can be factorized,

$$V_{\beta\alpha,\lambda\mu}(\mathbf{k},\mathbf{k}') = -\frac{3}{2}V_1\hat{k}\hat{k}'\mathbf{g}_{\beta\alpha}\mathbf{g}^{\dagger}_{\lambda\mu}, \tag{14}$$

where V_1 is the constant of the p-wave pairing attraction and

$$\mathbf{g}_{\alpha\beta} = i(\boldsymbol{\sigma}\sigma_y)_{\alpha\beta} \tag{15}$$

is a vector of the three basis symmetric matrices.

The order parameter

$$\Delta_{\hat{k}\alpha\beta} = \mathbf{d}_{\hat{k}}\mathbf{g}_{\alpha\beta}, \tag{16}$$

where $d_{\mu\hat{k}} = A_{\mu i}\hat{k}_i$ is the order-parameter vector.

Substituting Eqs. (14) and (16) into Eq. (7) yields

$$\mathbf{d}_{\hat{k}} = \frac{3}{2}V_1 T\sum_{n}\sum_{\mathbf{k}'}(\hat{k}\hat{k}')\mathrm{tr}_{\sigma}[\mathbf{g}^{\dagger}F(\omega_n,\mathbf{k}')]. \tag{17}$$

As one can see,

$$\mathbf{d}_{\hat{k}}\mathbf{d}^{*}_{\hat{k}} = \frac{1}{2}\mathrm{tr}_{\sigma}[\Delta^{+}_{\hat{k}}\Delta_{\hat{k}}], \tag{18}$$

where tr_{σ} denotes a trace over spin indices. For unitary phases, among which are the A and B phases actually being realized in pure ^{3}He, $(\Delta^{+}_{\hat{k}}\Delta_{\hat{k}})_{\alpha\beta} \propto \delta_{\alpha\beta}$, and so it is convenient to introduce the notation $\Delta^2_{\hat{k}} = \mathbf{d}_{\hat{k}}\mathbf{d}^{*}_{\hat{k}}$. We will consider only the unitary phases.

In the A phase,

$$\mathbf{d}_{\hat{k}} = \sqrt{\frac{3}{2}}\Delta\hat{d}[(\hat{\Delta}' + i\hat{\Delta}'')\hat{k}], \tag{19}$$

where $\hat{d}$, $\hat{\Delta}'$, and $\hat{\Delta}''$ are three unit vectors, and $\hat{\Delta}'$ and $\hat{\Delta}''$ are mutually orthogonal.

In the B phase,

$$\mathbf{d}_{\hat{k}} = \Delta\overleftrightarrow{R}\hat{k}e^{i\varphi}, \tag{20}$$

where $\overleftrightarrow{R}$ is the matrix of rotation on an arbitrary angle.

In both cases, the scalar Δ is introduced so as to fulfill the normalization $\langle\Delta_{\hat{k}}^2\rangle_{\hat{k}}=\langle\mathbf{d}_{\hat{k}}\mathbf{d}_{\hat{k}}^*\rangle_{\hat{k}}=\Delta^2$, where $\langle\cdots\rangle_{\hat{k}}$ stands for averaging over directions $\hat{k}$ of momentum.

III. ZERO MAGNETIC FIELD

In the absence of a magnetic field the solution of Eqs. (1), (4), (9), and (13) is (see elsewhere)

$$\hat{G}^{(0)}=\frac{1}{\tilde{\omega}_n^2+\tilde{\xi}_{\mathbf{k}}^2+\Delta_{\hat{k}}^2}\times\begin{pmatrix} -(i\tilde{\omega}_n+\tilde{\xi}_{\mathbf{k}})\delta_{\alpha\beta} & \Delta_{\hat{k}\alpha\beta} \\ \Delta^+_{\hat{k}\alpha\beta} & -(i\tilde{\omega}_n-\tilde{\xi}_{\mathbf{k}})\delta_{\alpha\beta}\end{pmatrix}. \quad (21)$$

The corresponding sum over momenta in the expression (13) for the scattering amplitude is

$$\sum_{\mathbf{k}}\hat{G}^{(0)}(\mathbf{k},\omega_n)=g(\omega_n)\hat{1}\delta_{\alpha\beta},$$

where we denoted

$$g(\omega_n)=\sum_{\mathbf{k}}G^{(0)}=-N_0\int\left\langle\frac{i\tilde{\omega}_n+\tilde{\xi}}{\tilde{\omega}_n^2+\tilde{\xi}^2+\Delta_{\hat{k}}^2}\right\rangle_{\hat{k}}d\tilde{\xi}$$
$$=-i\pi N_0\tilde{\omega}_n\left\langle\frac{1}{\sqrt{\tilde{\omega}_n^2+\Delta_{\hat{k}}^2}}\right\rangle_{\hat{k}}. \quad (22)$$

Here $N_0=m^*k_F/2\pi^2\hbar^2$ is the density of states on the Fermi surface.

So the scattering self-energy (9) equals

$$\hat{\Sigma}^{(0)}=\frac{n_{\text{imp}}u}{1-u^2g^2}[\hat{\tau}_3+ug\hat{1}]\delta_{\alpha\beta}. \quad (23)$$

In the above formulas both the Matsubara frequency $\omega_n=\pi T(2n+1)$ and the kinetic energy are renormalized:

$$i\tilde{\omega}_n=i\omega_n+\frac{1}{2}\text{tr}_\tau[\hat{\Sigma}^{(0)}(\omega_n)], \quad (24)$$

$$\tilde{\xi}_{\mathbf{k}}=\xi_{\mathbf{k}}-\frac{1}{2}\text{tr}_\tau[\hat{\tau}_3\hat{\Sigma}^{(0)}(\omega_n)]. \quad (25)$$

Here tr_τ is the trace in particle-hole space.

It should be noted that the term proportional to $\hat{\tau}_3$ in $\hat{\Sigma}=\hat{\Sigma}^{(0)}+\hat{\Sigma}^{(1)}+\cdots$ must be omitted. This term just produces a shift in the chemical potential (renormalization of $\xi_{\mathbf{k}}$) from introducing the impurities and disappears in the assumption of particle-hole symmetry after integration over ξ in the subsequent calculations.

Substituting Eqs. (22) and (23) into Eq. (24) yields the self-consistency equation on the renormalized Matsubara frequency $\tilde{\omega}_n$,

$$\tilde{\omega}_n=\omega_n+\Gamma\left\langle\frac{\tilde{\omega}_n\sqrt{\tilde{\omega}_n^2+\Delta_{\hat{k}}^2}}{\tilde{\omega}_n^2+\cos^2\delta_0\Delta_{\hat{k}}^2}\right\rangle_{\hat{k}}, \quad (26)$$

where Γ is the scattering rate:

$$\Gamma=\frac{n_{\text{imp}}}{\pi N_0}\sin^2\delta_0. \quad (27)$$

In the Born limit ($\delta_0\to 0$) it tends to the conventional half of the inverse free flight time:

$$\Gamma\to\frac{n_{\text{imp}}}{\pi N_0}\delta_0^2=n_{\text{imp}}\pi N_0u^2=\frac{1}{2\tau}. \quad (28)$$

In the B phase, $\Delta_{\hat{k}}^2\equiv\Delta^2$ does not depend on $\hat{k}$, and one can thus omit the angular brackets of the $\hat{k}$ averaging in Eq. (26). Using the overt form (21), the self-consistency equation (17) taking the trace over spin indices and averaging over $\hat{k}$ reduces to

$$\frac{1}{N_0V_1}=T\sum_n\int\frac{d\xi}{\tilde{\omega}_n^2+\xi^2+\Delta^2}=\pi T\sum_n\frac{1}{\sqrt{\tilde{\omega}_n^2+\Delta^2}}. \quad (29)$$

This equation determines the temperature and impurity concentration behavior of the order parameter Δ. To obtain an expression of value one should exclude the unobservable pairing constant V_1 off the left-hand side of Eq. (29). Using the standard procedure,[18] one obtains the self-consistency equation on $\Delta(T,\Gamma)$:

$$\ln\frac{T}{T_{c0}}=\pi T\sum_n\left(\frac{1}{\sqrt{\tilde{\omega}_n^2+\Delta^2}}-\frac{1}{|\omega_n|}\right). \quad (30)$$

The asymptotics of $\Delta(T,\Gamma)$ at $T=0$ and $T=T_c$ are found in the Appendix.

Expanding Eq. (26) in degrees of Δ^2 gives $\tilde{\omega}_n\approx\omega_n+\Gamma\text{sgn}\omega_n+O(\Delta^2)$. When inserted into Eq. (30) it yields in zeroth order in Δ^2 the Abrikosov-Gor'kov[19] implicit expression for the dependence of the critical temperature T_c on the impurity concentration,

$$\ln\frac{T_{c0}}{T_c}=\psi\left(\frac{\Gamma}{2\pi T_c}+\frac{1}{2}\right)-\psi\left(\frac{1}{2}\right), \quad (31)$$

where ψ is the digamma function. Note, however, that Γ corresponds to different impurity concentrations for various scattering phases [see Eq. (27)]. $T_c(\Gamma)$ vanishes at the critical concentration

$$\Gamma_c=\frac{1}{2}\Delta_{00}, \quad (32)$$

where

$$\Delta_{00}=\pi T_{c0}e^{-\gamma} \quad (33)$$

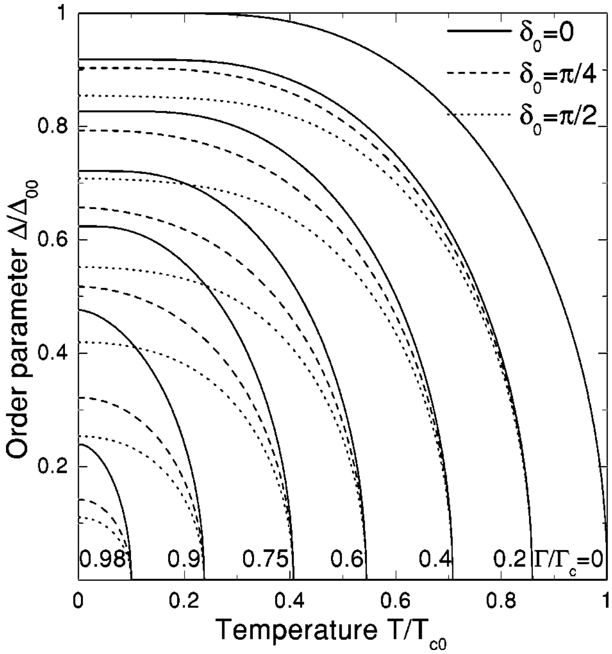

FIG. 1. Order parameter Δ vs temperature T for different scattering phases δ_0 and impurity concentrations obtained numerically. Solid lines correspond to the Born limit $\delta_0=0$, dashed lines to $\delta_0=\pi/4$ and dotted lines to the unitary scattering $\delta_0=\pi/2$. In order of the decrease of the superfluid transition temperature T_c the triples of the curves correspond to $\Gamma/\Gamma_c=0$, 0.2, 0.4, 0.6, 0.75, 0.9, 0.98. The first curve in the pure case $\Gamma=0$ is common for all phases.

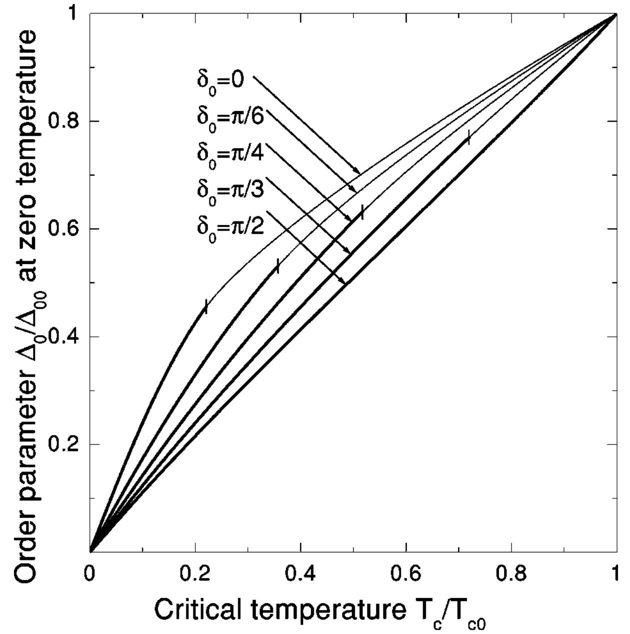

FIG. 2. Order parameter Δ_0 at zero temperature as a function of the suppression T_c/T_{c0} of the critical temperature for several values of the scattering phase δ_0. For each δ_0 the value of T_c/T_{c0} is pointed, the suppression below which makes the superfluid gapless in the whole range $0<T<T_c$. In the unitary limit ($\delta_0\rightarrow\pi/2$) the superfluid is gapless for whatever T_c/T_{c0}. Bold parts on each curve mark the regions of the existence of gapless superfluidity.

is the order parameter at zero temperature for a pure superfluid, and $\gamma\approx0.5772$ is Euler's constant.

Figure 1 shows temperature dependences of the order parameter for seven impurity concentrations and for three scattering phases: $\delta_0=0$, $\pi/4$, and $\pi/2$ obtained numerically from Eq. (30). Plots not shown for intermediate $0<\delta_0<\pi/2$ all sit in the interior of the belt between curves for $\delta_0=0$ (Born limit) and $\delta_0=\pi/2$ (unitary limit).

The order parameter Δ_0 at zero temperature as a function of the scattering rate Γ/Γ_c is presented in Fig. 2 for several values of the scattering phase δ_0. The curves for intermediate values of $0<\delta_0<\pi/2$ lie inside the strap between the curves for the Born ($\delta_0\rightarrow0$) and unitary ($\delta_0\rightarrow\pi/2$) limits.

From the Abrikosov-Gor'kov theory of paramagnetic impurities in alloys it is known that in the Born limit ($\delta_0\rightarrow0$) starting at the scattering rate of

$$2\Gamma_c e^{-\pi/4}\approx0.91\Gamma_c \tag{34}$$

the energy spectrum gap vanishes though the amplitude Δ of the order parameter remains finite until Γ_c. Ordinary impurities in a p-wave superfluid will give rise to the existence of an analogous gapless region of impurity concentrations. Indeed, the density of states of quasiparticles at an energy ϵ off the Fermi level is expressed in terms of the Green function as

$$N(\epsilon)=-\frac{1}{\pi}\mathrm{Im}\sum_{\mathbf{k}}G(\mathbf{k},\omega_n)|_{i\omega_n\rightarrow\epsilon+i0}. \tag{35}$$

Using Eq. (22) we get

$$N(\epsilon)=N_0\,\mathrm{Im}\frac{\tilde{\epsilon}}{\sqrt{\Delta^2-\tilde{\epsilon}^2}}, \tag{36}$$

where $\tilde{\epsilon}=i\tilde{\omega}_n|_{i\omega_n\rightarrow\epsilon+i0}$. From Eq. (26) we obtain an implicit equation on $\tilde{\epsilon}$ for a given ϵ:

$$\tilde{\epsilon}=\epsilon+\Gamma\frac{\tilde{\epsilon}\sqrt{\Delta^2-\tilde{\epsilon}^2}}{\Delta^2\cos^2\delta_0-\tilde{\epsilon}^2}. \tag{37}$$

For $\epsilon=0$ it gives three solutions in the complex plane: $\tilde{\epsilon}_1=0$ and

$$\tilde{\epsilon}_{2,3}^2=\frac{2\Delta^2\cos^2\delta_0-\Gamma^2-\Gamma\sqrt{\Gamma^2+4\Delta^2\sin^2\delta_0}}{2}. \tag{38}$$

It can be verified that for all the three solutions $\Delta^2-\tilde{\epsilon}^2$ is always (real) positive, and so $\sqrt{\Delta^2-\tilde{\epsilon}^2}$ always real. In order for $N(\epsilon=0)$ to be finite one than sees from Eq. (36) that $\tilde{\epsilon}^2$ should be negative. From Eq. (38) this gives

$$\Gamma>\Delta\cos^2\delta_0. \tag{39}$$

This inequality means that for a given phase shift δ_0 and a scattering rate Γ the superfluid is gapless at temperatures $T_{\mathrm{gl}}<T<T_c$, where T_{gl} is such that $\Gamma>\Delta(T_{\mathrm{gl}})\cos^2\delta_0$. The superfluid is gapless in the whole temperature range $0<T<T_c$ of the existence of superfluidity if inequality (39) is fulfilled already for $\Delta_0\equiv\Delta(T=0)$.

Making use of the implicit dependence (A7) of Δ_0 on δ_0, we get for the lower boundary of the onset of gapless superfluidity in the whole temperature range

$$\Gamma = 2\Gamma_c \cos^2\delta_0 \exp\left(-\frac{\pi}{2}\frac{\cos^2\delta_0}{1+\cos\delta_0}\right), \tag{40}$$

which is a generalization of the Abrikosov-Gor'kov value (34) on an arbitrary scattering phase δ_0. The corresponding value of the upper boundary of the order parameter at zero temperature is

$$\Delta_0 = \Delta_{00} \exp\left(-\frac{\pi}{2}\frac{\cos^2\delta_0}{1+\cos\delta_0}\right). \tag{41}$$

When the scattering phase is increased (impurity potential becomes "stronger") the boundary value of Γ decreases, meaning that starting from less impurity concentration the superfluidity becomes gapless. At last, in the unitary limit $\delta_0 \to \pi/2$ the boundary value is $\Gamma=0$, and the liquid is gapless for whatever small impurity concentration.

The points of the onset of gapless superfluidity are marked on the plot of the dependence of the order parameter at zero temperature Δ_0/Δ_{00} on the suppression T_c/T_{c0} of the superfluid transition temperature (see Fig. 2). The regions of the existence of gapless superfluidity are shown on the plots of $\Delta_0(T_c)$ with bold lines.

IV. MAGNETIC SUSCEPTIBILITY

Magnetization $\mathbf{M}$ can be calculated from the normal Green function in a magnetic field $\mathbf{B}$ as

$$\mathbf{M} = \mu T \sum_n \sum_{\mathbf{k}} G_{\alpha\beta} \boldsymbol{\sigma}_{\beta\alpha}. \tag{42}$$

To find the static susceptibility $\chi_{\alpha\beta}$ $(M_\alpha = \chi_{\alpha\beta} B_\beta)$ it is sufficient to solve Eqs. (1), (4), (9), and (13) in leading order in the field. Let the superscript (1) designate the quantities proportional to $\mathbf{B}$. From Eqs. (9) and (13) we get

$$\hat{\Sigma}^{(1)} = \frac{1}{n_{\text{imp}}} \hat{\Sigma}^{(0)} \hat{U} \left(\sum_{\mathbf{k}} \hat{G}^{(1)} \right) \hat{U}^{-1} \hat{\Sigma}^{(0)}. \tag{43}$$

One should plug here the expression for the Green function in first order in the field derived from Eq. (1):

$$\hat{G}^{(1)} = \hat{G}^{(0)} (\hat{H}^{(1)} + \hat{\Sigma}^{(1)}) \hat{G}^{(0)}. \tag{44}$$

We are now going to show that both the anomalous self-energy Σ_2 remains zero and the order parameter Δ unchanged in first order in magnetic field:

$$\Sigma_2^{(1)} = 0, \quad \Delta^{(1)} = 0. \tag{45}$$

Mathematically, this is due to the fact that self-consistent solution of Eqs. (43) and (44) produces (linear) homogeneous equations on $\Sigma_2^{(1)}$ and $\Delta^{(1)}$ which have regularly only trivial solutions. At the same time, the equation on the normal part of self-energy $\Sigma_1^{(1)}$ involves a right-hand side proportional to the magnetic field coming from

$$\hat{H}^{(1)}(\hat{k}) = \begin{pmatrix} -\mu \boldsymbol{\sigma}_{\alpha\beta}\mathbf{B} & \Delta^{(1)}_{\hat{k}\alpha\beta} \\ \Delta^{(1)\dagger}_{\hat{k}\alpha\beta} & \mu \boldsymbol{\sigma}^{\mathrm{T}}_{\alpha\beta}\mathbf{B} \end{pmatrix}. \tag{46}$$

To be more specific, when written out in components, Eq. (44) reads

$$G^{(1)} = G^{(0)}[1] + F^{(0)}[2], \tag{47}$$

$$F^{(1)\dagger} = F^{(0)\dagger}[1] + \overline{G}^{(0)}[2], \tag{48}$$

where we made use of the abbreviated notation

$$[1] = (\Sigma_1^{(1)} - \mu \boldsymbol{\sigma}\mathbf{B}) G^{(0)} + (\Sigma_2^{(1)} - \Delta^{(1)}) F^{(0)\dagger}, \tag{49}$$

$$[2] = (\Sigma_2^{(1)\dagger} - \Delta^{(1)\dagger}) G^{(0)} + (\overline{\Sigma}_1^{(1)} + \mu \boldsymbol{\sigma}^{\mathrm{T}}\mathbf{B}) F^{(0)\dagger}. \tag{50}$$

Let us first look at the equation on $\Delta^{(1)}$. Expanding Eq. (7) yields

$$\Delta_{\hat{k}}^{(1)\dagger} = \frac{3}{2} V_1 g_\mu^\dagger T \sum_n \sum_{\mathbf{k}'} (\hat{k}\hat{k}') \mathrm{tr}_\sigma [g_\mu F^{(1)\dagger}(\omega_n, \mathbf{k}')]. \tag{51}$$

Substituting here Eq. (48) for $F^{(1)\dagger}$ one obtains the equation on $\Delta^{(1)\dagger}$. It turns out that after taking a trace over spin indices as well as integrating over ξ on the supposition of electron-hole symmetry there remain only terms proportional to $\Delta^{(1)\dagger}$.

Indeed, consider the terms in Eq. (48) in pairs. The two terms involving the anomalous part $\Sigma_2^{(1)}(\omega_n)$ of the self-energy, which, we recall, does not depend on momenta, are proportional to either $\overline{G}^{(0)} G^{(0)}$, which does not depend on $\hat{k}$, or to $(F^{(0)\dagger})^2 \propto \hat{k}^2$. They vanish after averaging with $\hat{k}'$ over the direction of momenta in Eq. (51).

The two terms proportional to $\mathbf{B}$ when inserted into Eq. (51) give

$$\propto \mathrm{tr}_\sigma [g_\mu (F^{(0)\dagger} \boldsymbol{\sigma}\mathbf{B} G^{(0)} - \overline{G}^{(0)} \boldsymbol{\sigma}^{\mathrm{T}}\mathbf{B} F^{(0)\dagger})]$$

$$\propto d_\nu \mathrm{tr}_\sigma [g_\mu (g_\nu^\dagger (i\tilde{\omega}_n + \xi') \sigma_\lambda - \sigma_\lambda^{\mathrm{T}} (i\tilde{\omega}_n - \xi') g_\nu^\dagger)] B_\lambda.$$

The terms proportional to ξ' vanish on integrating over ξ' in the supposition of particle-hole symmetry. The terms proportional to $\tilde{\omega}_n$ vanish on summation over the Matsubara frequencies $\omega_n = (2n+1)\pi T$ from $n=-\infty$ to ∞ because of the oddness of $\tilde{\omega}_n(\omega_n)$: $\tilde{\omega}_n(-\omega_n) = -\tilde{\omega}_n(\omega_n)$.

Similar argumentation leads to the conclusion of vanishing of the terms involving $\Sigma_1^{(1)}$ because, as we shall see later, in the assumption of particle-hole symmetry $\Sigma_1^{(1)}$ is even in ω_n.

So we are left with a homogeneous linear equation on $\Delta^{(1)}$ which has normally only trivial solutions $\Delta^{(1)}=0$.

One obtains the equations to determine the normal $\Sigma_1^{(1)}$ and anomalous $\Sigma_2^{(1)}$ parts of the self-energy by plugging Eqs. (21) and (23) into Eq. (43):

$$\hat{\Sigma}^{(1)}=\begin{pmatrix}\Sigma_1^{(1)} & \Sigma_2^{(1)}\\ \Sigma_2^{(1)\dagger} & \overline{\Sigma_1^{(1)}}\end{pmatrix}=\frac{n_{\rm imp}u^2}{(1-u^2g^2)^2}\times\begin{pmatrix}(1+ug)^2\sum_{\mathbf{k}} G^{(1)} & (1-u^2g^2)\sum_{\mathbf{k}} F^{(1)}\\ (1-u^2g^2)\sum_{\mathbf{k}} F^{(1)\dagger} & (1-ug)^2\sum_{\mathbf{k}} \overline{G^{(1)}}\end{pmatrix}. \tag{52}$$

Since $\Delta^{(0)}_{\hat{k}}$ and, consequently, also $F^{(0)}(\omega_n,\mathbf{k})$ by virtue of Eq. (21) and $\Delta^{(1)}_{\hat{k}}$ are all proportional to the unit vector $\hat{k}$, any of the terms in Eqs. (47) and (48) comprising one or three of either of the quantities vanishes on summing over $\mathbf{k}$ in Eq. (43). For the anomalous part there remains only two terms giving a homogeneous equation on the four components of $\Sigma_2^{(1)}$:

$$\Sigma_2^{(1)\dagger}=\frac{n_{\rm imp}u^2}{1-u^2g^2}\sum_{\mathbf{k}}\left(F^{(0)\dagger}\Sigma_2^{(1)}F^{(0)\dagger}+\overline{G^{(0)}}\Sigma_2^{(1)\dagger}G^{(0)}\right). \tag{53}$$

Bar in exceptional cases, this equation has only a trivial solution $\Sigma_2^{(1)}\equiv 0$. However, one can verify separately for each component of the expansion of $\Sigma_2^{(1)}(\omega_n)$ in terms of the three Pauli matrices and the unit matrix,

$$\Sigma^{(1)}_{2\alpha\beta}(\omega_n)=\Sigma_2^{(1)}(\omega_n)\delta_{\alpha\beta}+\boldsymbol{\Sigma}_2^{(1)}(\omega_n)\boldsymbol{\sigma}_{\alpha\beta}, \tag{54}$$

that this homogeneous equation on $\Sigma_2^{(1)}(\omega_n)$ does indeed have only a trivial solution.

Taking into account the remark after Eq. (25) we omit the terms proportional to $\hat{\tau}_3$ in Eq. (52) and for the normal part $\Sigma_1^{(1)}$ get the equation

$$\Sigma_1^{(1)}=n_{\rm imp}u^2\frac{1+u^2g^2}{(1-u^2g^2)^2}\sum_{\mathbf{k}}\left[G^{(0)}(\Sigma_1^{(1)}-\mu\boldsymbol{\sigma}\mathbf{B})G^{(0)}+F^{(0)}(\overline{\Sigma_1^{(1)}}+\mu\boldsymbol{\sigma}^{\rm T}\mathbf{B})F^{(0)\dagger}\right]. \tag{55}$$

Making use of the explicit form of the zero-order Green function (21), we first calculate the sum over $\mathbf{k}$, introducing the auxiliary quantity

$$\Pi(\omega_n)=\frac{\pi N_0\Delta^2}{3(\tilde{\omega}_n^2+\Delta^2)^{3/2}}, \tag{56}$$

so that $\Sigma_{\mathbf{k}}(G^{(0)})^2=\frac{3}{2}\Pi(\omega_n)$ and $\Sigma_{\mathbf{k}}F^{(0)}F^{(0)\dagger}=\Pi(\omega_n)$. In addition, we introduce

$$\tilde{\Pi}(\omega_n)=n_{\rm imp}u^2\frac{1+u^2g^2}{(1-u^2g^2)^2}\Pi(\omega_n)=\frac{\Gamma\Delta^2}{3}\frac{\tilde{\omega}_n^2\cos 2\delta_0+\Delta^2\cos^2\delta_0}{\sqrt{\tilde{\omega}_n^2+\Delta^2}(\tilde{\omega}_n^2+\Delta^2\cos^2\delta_0)^2}. \tag{57}$$

The second term in Eq. (55) is proportional to the product of $F^{(0)}_{\alpha\beta}\propto\Delta\mathbf{g}_{\alpha\beta}\overleftrightarrow{R}\hat{k}$ and $F^{(0)\dagger}\propto\Delta\mathbf{g}^\dagger_{\alpha\beta}\overleftrightarrow{R}\hat{k}$. After averaging over $\hat{k}$ and integration over ξ we find that

$$\sum_{\mathbf{k}}F^{(0)}f(\omega_n)F^{(0)\dagger}=\frac{1}{2}\Pi(\omega_n)\mathbf{g}f(\omega_n)\mathbf{g}^\dagger. \tag{58}$$

So

$$\Sigma_1^{(1)}=\frac{3}{2}\tilde{\Pi}(\omega_n)\left[\Sigma_1^{(1)}-\mu\boldsymbol{\sigma}\mathbf{B}+\frac{1}{3}\mathbf{g}(\overline{\Sigma_1^{(1)}}+\mu\boldsymbol{\sigma}^{\rm T}\mathbf{B})\mathbf{g}^\dagger\right], \tag{59}$$

$$\overline{\Sigma_1^{(1)}}=\frac{3}{2}\tilde{\Pi}(\omega_n)\left[\overline{\Sigma_1^{(1)}}+\mu\boldsymbol{\sigma}^{\rm T}\mathbf{B}+\frac{1}{3}\mathbf{g}^\dagger(\Sigma_1^{(1)}-\mu\boldsymbol{\sigma}\mathbf{B})\mathbf{g}\right]. \tag{60}$$

Expanding $\Sigma_1^{(1)}$ and $\overline{\Sigma_1^{(1)}}$ in the basis of the three Pauli matrices and the unit matrix similarly to Eq. (54), one sees that for the coefficient of expansion over the unit matrix one obtains an uncoupled homogeneous equation, with trivial solution. So one may put simply

$$\Sigma_1^{(1)}=\boldsymbol{\Sigma}_1^{(1)}\boldsymbol{\sigma},\quad \overline{\Sigma_1^{(1)}}=\overline{\boldsymbol{\Sigma}_1^{(1)}}\boldsymbol{\sigma}^{\rm T}. \tag{61}$$

Using $\mathbf{g}=i\boldsymbol{\sigma}\sigma_y$, $\mathbf{g}^\dagger=-i\sigma_y\boldsymbol{\sigma}$, and the equalities

$$\sigma_y\boldsymbol{\sigma}\sigma_y=-\boldsymbol{\sigma}^{\rm T},\quad \sigma_\mu\boldsymbol{\sigma}\sigma_\mu=-\boldsymbol{\sigma}, \tag{62}$$

we find that

$$g_\mu\boldsymbol{\sigma}g_\mu^\dagger=g_\mu^\dagger\boldsymbol{\sigma}g_\mu=\boldsymbol{\sigma}^{\rm T}, \tag{63}$$

so that solution of the system (59) and (60) is

$$\Sigma_1^{(1)}=-\mu\boldsymbol{\sigma}\mathbf{B}\frac{\tilde{\Pi}(\omega_n)}{1-\tilde{\Pi}(\omega_n)},\quad \overline{\Sigma_1^{(1)}}=\mu\boldsymbol{\sigma}^{\rm T}\mathbf{B}\frac{\tilde{\Pi}(\omega_n)}{1-\tilde{\Pi}(\omega_n)}. \tag{64}$$

Since from Eq. (47) it follows that

$$G^{(1)}=G^{(0)}(\Sigma_1^{(1)}-\mu\boldsymbol{\sigma}\mathbf{B})G^{(0)}+F^{(0)}(\overline{\Sigma_1^{(1)}}+\mu\boldsymbol{\sigma}^{\rm T}\mathbf{B})F^{(0)\dagger}, \tag{65}$$

we find from Eq. (42) that

$$\mathbf{M}=-2\mu^2\mathbf{B}T\sum_n\frac{\Pi(\omega_n)}{1-\tilde{\Pi}(\omega_n)}. \tag{66}$$

Because we have first integrated over $\mathbf{k}$ and left the summation over n, this expression does not include[17] the normal-state Fermi gas Pauli susceptibility $\chi_n^0=2\mu^2N_0$. Adding and subtracting it from both sides of Eq. (42), we get eventually

$$\frac{\chi^0}{\chi_n^0}=1-\frac{\pi}{3}T\sum_n\frac{\dfrac{\Delta^2}{(\tilde{\omega}_n^2+\Delta^2)^{3/2}}}{1-\dfrac{\Gamma\Delta^2}{3}\dfrac{\tilde{\omega}_n^2\cos 2\delta_0+\Delta^2\cos^2\delta_0}{\sqrt{\tilde{\omega}_n^2+\Delta^2}(\tilde{\omega}_n^2+\Delta^2\cos^2\delta_0)^2}}. \tag{67}$$

This expression gives susceptibility as a function of temperature for a given scattering rate Γ and a scattering phase δ_0. The infinite sum over the Matsubara frequencies should be taken using self-consistent expressions for the renormalized Matsubara frequency $\tilde{\omega}_n$ Eq. (26), and the order parameter $\Delta(T)$ Eq. (30).

In the clean limit $\Gamma \to 0$ the Matsubara frequencies rest unrenormalized and susceptibility reduces to its conventional bulk value

$$\frac{\chi^0}{\chi_n^0} = 1 - \frac{\pi}{3} T \sum_n \frac{\Delta^2}{(\omega_n^2 + \Delta^2)^{3/2}} \equiv 1 - \frac{1}{3}[1 - Y_B(T)], \tag{68}$$

where

$$Y_B(T) = 1 - \pi T \sum_n \frac{\Delta^2}{(\omega_n^2 + \Delta^2)^{3/2}} \tag{69}$$

is the *B*-phase Yosida function.

At $T \to T_c$ susceptibility χ^0/χ_n^0 behaves linearly. Indeed, then $\Delta \to 0$ and $\tilde{\omega} \to \omega + \Gamma$. Hence

$$\frac{\chi^0}{\chi_n^0} \to 1 - \frac{\pi \Delta^2}{3} T_c \sum_n \frac{1}{|\tilde{\omega}_n|^3} \to 1 + \frac{2\Delta^2}{3(4\pi T_c)^2} \psi_c^{(2)}, \tag{70}$$

where $x_c = \Gamma/2\pi T_c$ and $\psi_c^{(n)} = \psi^{(n)}(\frac{1}{2} + x_c)$ is the polygamma function. Using the asymptotic expression (A2) for Δ at $T \to T_c$, we find eventually

$$\left. \frac{\chi^0}{\chi_n^0} \right|_{T \to T_c} \to 1 + 2\psi_c^{(2)} \frac{x_c \psi_c^{(1)} - 1}{3\psi_c^{(2)} + x_c \psi_c^{(3)} \cos 2\delta_0} \frac{T_c - T}{T_c}. \tag{71}$$

In the Born limit ($\delta_0 \to 0$) we arrive at the expression for the spin susceptibility:

$$\frac{\chi^0}{\chi_n^0} = 1 - \frac{\pi}{3} T \sum_n \frac{\dfrac{\Delta^2}{(\tilde{\omega}_n^2 + \Delta^2)^{3/2}}}{1 - \dfrac{\Gamma \Delta^2}{3} \dfrac{1}{(\tilde{\omega}_n^2 + \Delta^2)^{3/2}}}. \tag{72}$$

In the opposite unitary limit ($\delta_0 \to \pi/2$),

$$\frac{\chi^0}{\chi_n^0} = 1 - \frac{\pi}{3} T \sum_n \frac{\dfrac{\Delta^2}{(\tilde{\omega}_n^2 + \Delta^2)^{3/2}}}{1 + \dfrac{\Gamma \Delta^2}{3} \dfrac{1}{\tilde{\omega}_n^2 \sqrt{\tilde{\omega}_n^2 + \Delta^2}}}. \tag{73}$$

Susceptibilities for several impurity concentrations and three scattering phases [$\delta_0 = 0$ (Born limit), $\delta_0 = \pi/4$, and $\delta_0 = \pi/2$ (unitary limit)] are plotted as functions of the reduced temperature T/T_c in Fig. 3 with Fermi-liquid effects taken into account (see below).

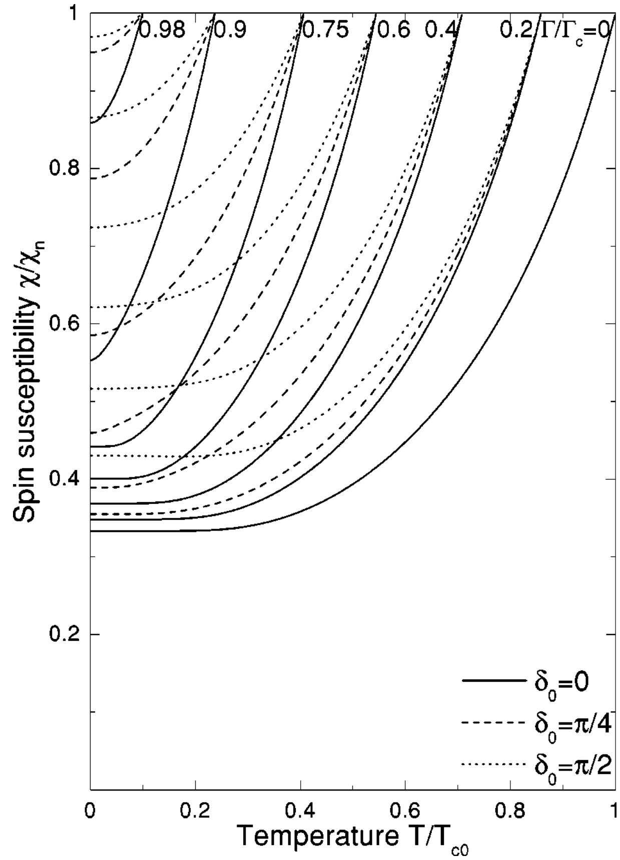

FIG. 3. Spin susceptibility of superfluid ^{3}He in aerogel vs temperature for several values of impurity concentration Γ and scattering phases δ_0 with the Fermi liquid corrections taken into account. Solid lines correspond to the Born limit $\delta_0 = 0$, dashed lines to $\delta_0 = \pi/4$, and dotted lines to the unitary scattering $\delta_0 = \pi/2$. In order of the decrease of the superfluid transition temperature T_c the triples of the curves correspond to $\Gamma/\Gamma_c = 0$, 0.2, 0.4, 0.6, 0.75, 0.9, 0.98. The first curve in the pure case $\Gamma = 0$ is common for all phases.

Fermi-liquid corrections

The Fermi-liquid interaction between quasiparticles leads to renormalization of the susceptibility. An external magnetic field is screened by a polarization of the liquid, which is quantitatively described by even terms of the expansion in Legendre polynomials of the antisymmetric (exchange) part F^{a} of the Fermi-liquid interaction.

It was argued in Ref. 20 on the basis of experimental data that only the zeroth term F_0^{a} is nonzero. Then the magnetization $M_\alpha = \chi_{\alpha\beta} B_\beta$ induced by magnetic field **B** in a liquid with an interaction equals the magnetization $M_\alpha = \chi_{\alpha\beta}^0 B_\beta^{\mathrm{eff}}$ that would be induced in an interaction-free liquid by an effective field

$$\mathbf{B}^{\mathrm{eff}} = \mathbf{B} - F_0^{\mathrm{a}} \mathbf{M}/\chi_n^0. \tag{74}$$

Here $\chi_n^0 = 2\mu^2 N_0$ is the Pauli paramagnetic susceptibility of a normal Fermi gas. This yields[21,22]

$$\chi_{\alpha\beta} = (\delta_{\alpha\beta} + F_0^{\mathrm{a}} \chi_{\alpha\beta}^0/\chi_n^0)^{-1} \chi_{\alpha\beta}^0. \tag{75}$$

In the *B* phase $\chi_{\alpha\beta}^0 \propto \delta_{\alpha\beta}$ and we get

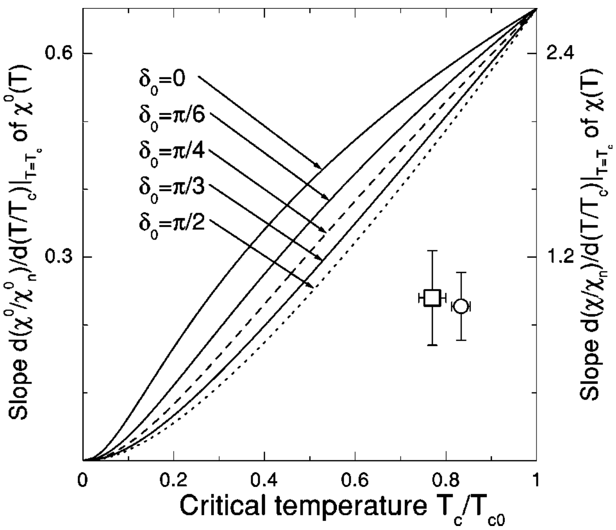

FIG. 4. The slope at $T \to T_c$ of the reduces temperature T/T_c plots of the "bare" susceptibility χ^0/χ_n^0 (left scale) or the susceptibility χ/χ_n with the Fermi liquid corrections [right scale, coefficient of proportionality (79)] as functions of the suppression T_c/T_{c0} of the critical temperature. The crosses mark the experimental values of the slopes in Ref. 3 (□) and in Ref. 12 (○).

$$\frac{\chi}{\chi_n} = \frac{1+F_0^{\mathrm{a}}}{1+F_0^{\mathrm{a}}\dfrac{\chi^0}{\chi_n^0}} \frac{\chi^0}{\chi_n^0}, \tag{76}$$

where $\chi_n = \chi_n^0/(1+F_0^{\mathrm{a}})$ is the Fermi-liquid susceptibility. In ^{3}He, $F_0^{\mathrm{a}} \approx -\frac{3}{4}$ slightly varying with pressure.

For the asymptotics at $T \to T_c$ we have

$$\left.\frac{\chi}{\chi_n}\right|_{T\to T_c} = 1 + \left.\frac{d(\chi/\chi_n)}{d(T/T_c)}\right|_{T\to T_c} \frac{T-T_c}{T_c}, \tag{77}$$

where from Eq. (76)

$$\left.\frac{d(\chi/\chi_n)}{d(T/T_c)}\right|_{T\to T_c} = \frac{1}{1+F_0^{\mathrm{a}}} \left.\frac{d(\chi^0/\chi_n^0)}{d(T/T_c)}\right|_{T\to T_c} \tag{78}$$

is the slope of the plots of χ versus T at $T \to T_c$.

Taking $-\frac{3}{4}$ as a value for F_0^{a}, we see that the slope of the curve $\chi(T)$ at $T=T_c$ with the Fermi-liquid interaction taken into account is

$$1/(1+F_0^{\mathrm{a}}) \approx 4 \tag{79}$$

times greater than that of $\chi^0(T)$—without the interaction.

The slopes of both $\chi^0(T)$ and $\chi(T)$ for $F_0^{\mathrm{a}} = -\frac{3}{4}$ versus the suppression T_c/T_{c0} of the critical temperature are plotted in Fig. 4 in two scales of the ordinate axis — the left one for the slope of $\chi^0(T)$ and the right one for the slope of $\chi(T)$, which is Eq. (79) times the former. In the same picture we also pointed out the slopes experimentally observed in Ref. 3 at 18.7 bars and in Ref. 12 at 32 bars — the only experimental data present up to day the authors are aware of. The error bars span the possible slopes of the linear fits to the data at $T \lesssim T_c$ within the plotted error bars of the respective experimental works.

V. CONCLUSIONS

In the framework of the homogeneous scattering model we found spin susceptibility of ^{3}He in aerogel considering the latter as ordinary randomly distributed impurities with an arbitrary scattering phase δ_0 in s-wave channel. The answer is given in the form of an infinite sum over the Matsubara frequencies $\omega_n = (2n+1)\pi T$ with the summation term depending on the amplitude Δ of the order parameter and the self-consistently renormalized Matsubara frequency $\tilde{\omega}_n$. Numerically plotted curves $\chi(T)$ show that susceptibility is less suppressed in comparison to the bulk dependence, this suppression being less pronounced for larger δ_0.

Comparison with high-pressure experimental data [18.7 bars (Ref. 3) and 32 bars (Ref. 12)] shows that the slope of $\chi(T)$ at $T \to T_c$ is approximately twice less than the value predicted by the theory. The discrepancy is obviously unavoidable because of the major inadequacy of the homogeneous scattering model at high pressures, where the superfluid coherence length becomes shorter than the correlation length of the internal structure of aerogel.[15] Note that quantitatively the difference between experimental and theoretical values seems to be in line with the fact that experimentally observed at high pressures (19.7 bars) superfluid density[23] was approximately by the factor of 2 smaller than predicted by Abrikosov-Gor'kov theory.[9]

On the other hand, we pointed out that for any finite scattering phase $\delta_0 > 0$ the impurity concentration threshold of the gapless superfluidity onset diminishes dramatically. For example, for the value $\delta_0 = \pi/4$ which was maintained to be the most satisfactory phase for the real structure of aerogel, gapless superfluidity sets on in the whole temperature range starting at the critical temperature suppression of $T_c \approx 0.51 T_{c0}$ compared to $T_c \approx 0.22 T_{c0}$ in the Born limit $\delta_0 \to 0$, whereas in the unitary limit $\delta_0 \to \pi/2$ the superfluid is always gapless. For impurity concentrations exceeding the gapless threshold all the thermodynamical quantities vanish algebraically at zero temperature. Experimental observation of such behavior could provide additional immediate insight into the quasiparticle scattering on impurities in aerogel.

We wish to thank J. A. Sauls for valuable comments on the manuscript and attraction of our attention to Ref. 24, which complies with ours in the main results. One of the authors (P.K.) is grateful to J. Flouquet for the granted opportunity to complete the work at the Center for Nuclear Research in Grenoble.

APPENDIX: ASYMPTOTICAL BEHAVIOR OF THE ORDER PARAMETER Δ AT $T \to T_c$ AND $T \to 0$

In this appendix we elaborate on the details of the calculation of the asymptotics of the order parameter already utilized in the main text of the paper.

Asymptotics at the critical temperature

Expanding Eq. (26) in degrees of Δ^2 gives [we consider only positive ω_n because of the oddness of $\tilde{\omega}_n(\omega_n)$: $\tilde{\omega}_n(-\omega_n)=-\tilde{\omega}_n(\omega_n)$]

$$\tilde{\omega}_n \approx \omega_n+\Gamma+\Gamma\frac{1-2\cos^2\delta_0}{2(\omega_n+\Gamma)}\Delta^2+O(\Delta^4). \tag{A1}$$

When inserted into Eq. (30) it yields in first order in Δ^2 the asymptotics at $T\to T_c$:

$$\Delta^2=3(4\pi T_c)^2\frac{x_c\psi_c^{(1)}-1}{3\psi_c^{(2)}+x_c\psi_c^{(3)}\cos 2\delta_0}\frac{T_c-T}{T_c}. \tag{A2}$$

Here $x_c=\Gamma/2\pi T_c$ and $\psi_c^{(n)}=\psi^{(n)}(\frac{1}{2}+x_c)$, where

$$\psi^{(n)}(z)=\partial_z^n\psi(z)=\sum_{k=0}^{\infty}\frac{(-1)^{n+1}n!}{(z+k)^{n+1}} \tag{A3}$$

is the polygamma function. At $\Gamma\to 0$ also $x_c\to 0$ and $\psi_c^{(2)}\to\psi^{(2)}(\frac{1}{2})=-14\zeta(3)$, and thus Eq. (A2) transforms into the conventional BCS asymptotics

$$\Delta^2=\frac{8\pi^2T_{c0}^2}{7\zeta(3)}\frac{T_{c0}-T}{T_{c0}}. \tag{A4}$$

Order parameter at zero temperature

Consider the sum $\pi T\Sigma_n(\omega_n^2+\Delta^2)^{-1/2}$. When $T\to 0$ the summation over the Matsubara frequencies ω_n may be substituted with an integration over the continuous variable ω,

$$\pi T\sum_n\frac{1}{\sqrt{\omega_n^2+\Delta^2}}\overset{T\to 0}{\to}\int_0^{\epsilon_1}\frac{d\omega}{\sqrt{\omega^2+\Delta^2}}\approx\ln\frac{2\epsilon_1}{\Delta}, \tag{A5}$$

where the divergent integration up to infinity is cut off at ϵ_1.

The general temperature dependence of the order parameter is obtained by combination of Eqs. (29), (33), and (A5):

$$\ln\frac{\Delta}{\Delta_{00}}=\pi T\sum_n\left(\frac{1}{\sqrt{\tilde{\omega}_n^2+\Delta^2}}-\frac{1}{\sqrt{\omega_n^2+\Delta^2}}\right). \tag{A6}$$

At $T\to 0$ the value of the order parameter tends to the value Δ_0, the implicit dependence of which on Γ is set by replacing summation in the above expression with an integral. The integral over ω from the first term in the parentheses in Eq. (A6) transforms into an integral over $\tilde{\omega}$ by means of the substitution (26) and eventually we get an equation that determines implicitly the impurity concentration behavior of the value of the order parameter at zero temperature:

$$\frac{1}{\Gamma}\ln\frac{\Delta_0}{\Delta_{00}}+\frac{\pi}{2\Delta_0(1+\cos\delta_0)}=0 \tag{A7}$$

if $\Gamma<\Delta_0\cos^2\delta$ and

$$0=\frac{1}{\Gamma}\ln\frac{\tilde{\omega}_0+\sqrt{\tilde{\omega}_0^2+\Delta_0^2}}{\Delta_{00}}+\frac{\pi}{2\Delta_0(1+\cos\delta_0)}-\frac{\tilde{\omega}_0}{\tilde{\omega}_0^2+\Delta_0^2\cos^2\delta_0}-\frac{1}{\Delta_0\sin^2\delta_0}\left(\cos\delta_0\arctan\frac{\tilde{\omega}_0}{\Delta_0\cos\delta_0}-\arctan\frac{\tilde{\omega}_0}{\Delta_0}\right) \tag{A8}$$

if $\Gamma>\Delta_0\cos^2\delta$. Here

$$\tilde{\omega}_0(\Gamma,\Delta)=\sqrt{\frac{\Gamma^2-2\Delta^2\cos^2\delta_0+\Gamma\sqrt{\Gamma^2+4\Delta^2\sin^2\delta_0}}{2}} \tag{A9}$$

is the solution of Eq. (26) for $\omega_n=0$ and in Eq. (A8) it was taken with the arguments Γ,Δ_0.

For small Γ, Eq. (A7) gives

$$\frac{\Delta_0}{\Delta_{00}}\approx 1-\frac{\pi}{4(1+\cos\delta_0)}\frac{\Gamma}{\Gamma_c}, \tag{A10}$$

while for Γ close to the critical value

$$\frac{\Delta_0}{\Delta_{00}}\approx\sqrt{\frac{1}{1-\frac{2}{3}\cos 2\delta_0}\frac{\Gamma_c-\Gamma}{\Gamma_c}}. \tag{A11}$$

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