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Interaction corrections at intermediate temperatures: Magnetoresistance in a parallel field

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We consider the correction to conductivity of a two-dimensional electron gas due to electron-electron interaction in the parallel magnetic field at arbitrary relation between temperature and the elastic mean free time. The correction exhibits nontrivial dependence on both temperature and the field. This dependence is determined by the Fermi-liquid constant, which accounts for the spin-exchange interaction. In particular, the sign of the slope of the temperature dependence is not universal and can change with the increase of the field.

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Introduction. In a previous paper[1] we have developed a theoretical framework for studying interaction corrections to conductivity of the two-dimensional electron gas (2DEG) due to electron-electron interactions for the arbitrary relation between temperature T and the elastic mean free time τ. To describe strong coupling between electrons we used the conventional Fermi-liquid[2] constants. In particular, we found[1] that the temperature dependence of the longitudinal conductivity in a 2DEG is determined by a single Fermi-liquid constant F_0^σ, which describes the strength of the spin-exchange interaction. In principle, its value can be found from a measurement of the Pauli spin susceptibility

$$\chi=\frac{g^2\mu_B^2\nu}{1+F_0^\sigma}, \tag{1}$$

where μ_B is the Bohr magneton, g is the bare electron Lande factor, and the density of states ν should be obtained from a measurement of the specific heat (at $\tau^{-1}\ll T\ll E_F$). Unfortunately, to the best of our knowledge no measurement of the spin susceptibility has been reported for the 2DEG created at the interface of a semiconductor heterostructure, which currently is the most common type of an experimental sample.[3] However, we have conjectured[1] that the same constant F_0^σ should describe the transport properties of the 2DEG in an external magnetic field. In this paper we address the case of the parallel magnetic field and calculate the magnetoconductivity. The case of the perpendicular magnetic field and the theory of the Hall resistance was discussed in a recent paper.[4]

Early theoretical efforts focused on calculating the magnetoconductivity within the diffusive approximation.[5–11] While perfectly justified for metallic thin films, this approximation might be inappropriate for understanding of recent experiments[3] in semiconductor heterostructures, since these measurements are performed in a regime where the temperature T is of the same order of magnitude as the inverse scattering time τ^{-1} (obtained from the Drude conductivity). The opposite, ballistic limit was considered recently in Refs. 12 and 13. While giving a reasonable description of the magnetoresistance for weak interaction and at small fields, the authors of Refs. 12 and 13 did not realize that both the temperature and the magnetic field dependence arise due to large-distance (as compared with λ_F) processes and therefore did not account for Fermi-liquid renormalizations. The resulting temperature dependence of the conductivity (also see Ref. 14) is qualitatively erroneous: in particular for the case of the fully polarized system Ref. 12 has the incorrect sign and Ref. 13 finds no temperature dependence at all (see Ref. 1 for more details). In this paper we calculate the magnetoconductivity for an arbitrary relation between T, τ, and the Zeeman energy E_z (however, we are limiting ourselves to $E_z\ll E_F$; the case of strong fields where the electron system is close to full polarization will be addressed elsewhere).

Method. The expression for the leading interaction correction to conductivity can be found either by means of the diagrammatic technique[5] or using the quantum kinetic equation.[1] Both methods are completely equivalent and result in the following expression for the correction:[1]

$$\frac{\delta\sigma_{xx}}{\sigma_D}=\mathrm{Im}\int_{-\infty}^{\infty}\frac{d\omega}{\pi}\frac{\partial}{\partial\omega}\left(\omega\coth\frac{\omega}{2T}\right)\int_0^\infty\frac{qdq}{4\pi}$$
$$\times[\mathbf{Tr}\hat{\mathcal{D}}^R(\omega,q)]B_{xx}(\omega,q), \tag{2a}$$

where the retarded interaction propagator $[\hat{\mathcal{D}}^R(\omega,q)]_{\sigma_1\sigma_2;\sigma_3\sigma_4}$ is a matrix in spin space, and the form factor B_{xx} is defined as [see Eqs. (4.16) and also Eq. (3.26) of Ref. 1]

$$B_{xx}(\omega,q)=\left\{\frac{v_F^2q^2/\tau^2}{C^3(C-1/\tau)^3}+\frac{3v_F^2q^2}{2\tau C^3(C-1/\tau)^2}\right.$$
$$+\frac{2[C-(-i\omega+1/\tau)]}{C(C-1/\tau)^2}$$
$$\left.+\frac{2C-1/\tau}{Cv_F^2q^2}\left(\frac{C-(-i\omega+1/\tau)}{C-1/\tau}\right)^2\right\}, \tag{2b}$$

using the notation

$$C(\omega,q)=\sqrt{(-i\omega+1/\tau)^2+v_F^2q^2}. \tag{3}$$

In the absence of magnetic field one can choose a basis corresponding to the singlet (charge) and triplet channels in which the interaction propagator becomes diagonal $\hat{\mathcal{D}}^R=\mathrm{diag}(\mathcal{D}_s^R,\mathcal{D}_t^R,\mathcal{D}_t^R,\mathcal{D}_t^R)$. The interaction propagator in the singlet channel is taken in the unitary limit (and hence is independent of the corresponding Fermi-liquid parameter F^ρ) and becomes proportional to the inverse of the electronic polarization operator

$$\mathcal{D}_s^R = -\frac{1}{\Pi^R}, \tag{4}$$

$$\Pi^R(\omega,q) = \nu\left(1 - \frac{-i\omega}{C(\omega,q)-1/\tau}\right). \tag{5}$$

On the contrary, the triplet channel propagator depends on the Fermi-liquid constant F_0^σ

$$\mathcal{D}_t^R = -\frac{F_0^\sigma}{\nu + F_0^\sigma \Pi^R} = -\frac{1}{\nu}\frac{C-1/\tau}{i\omega + \dfrac{F_0^\sigma+1}{F_0^\sigma}(C-1/\tau)}, \tag{6}$$

and describes spin-exchange coupling. For details of the derivation of the propagators and Eqs. (2) we refer the reader to Refs. 1 and 6.

Using the explicit form of propagators (4) and (6) we evaluate the integral Eq. (2) and find[1] the temperature-dependent correction to conductivity in the absence of external magnetic field:

$$\sigma = \sigma_D + \delta\sigma_C + \delta\sigma_T. \tag{7a}$$

Here the charge (singlet) channel contribution is given by

$$\delta\sigma_C = \frac{e^2}{\pi\hbar}\frac{T\tau}{\hbar}\left[1 - \frac{3}{8}f(T\tau)\right] - \frac{e^2}{2\pi^2\hbar}\ln\left(\frac{E_F}{T}\right), \tag{7b}$$

and the triplet channel correction is

$$\delta\sigma_T = \frac{3F_0^\sigma}{(1+F_0^\sigma)}\frac{e^2}{\pi\hbar}\frac{T\tau}{\hbar}\left[1 - \frac{3}{8}t(T\tau;F_0^\sigma)\right] - 3\left(1 - \frac{1}{F_0^\sigma}\ln(1+F_0^\sigma)\right)\frac{e^2}{2\pi^2\hbar}\ln\left(\frac{E_F}{T}\right). \tag{7c}$$

The factor of 3 in the triplet channel correction Eq. (7c) is due to the fact that all three spin components of the triplet state contribute equally. The function $f(T\tau)$ smoothly decays from unity to zero and the function $t(T\tau;F_0^\sigma)$ is nonmonotonous only in the narrow region $-0.25 > F_0^\sigma > -0.5$ where it has a maximum at $T\tau = 1/(1+F_0^\sigma)$. For numerical reasons both $f(T\tau)$ and $t(T\tau;F_0^\sigma)$ change the result only by a few percent and therefore their explicit form (given in Ref. 1) is inessential for the present discussion.

The correction (7) is written in the approximation of constant (i.e., momentum-independent) F_0^σ. For the system close to the Stoner instability such as $1/(\epsilon_F\tau) \ll (1+F_0^\sigma) \ll 1$, this limits the applicability of Eq. (7) by temperatures smaller than $T^* = \epsilon_F(1+F_0^\sigma)^2$, see Ref. 1.

In parallel magnetic field electrons acquire additional Zeeman energy $E_z = g\mu_B H$, which is proportional to the magnitude H of the field, the Bohr magneton μ_B, and the electron g factor. Consequently, the exact Green's functions of noninteracting electrons now depend on magnetic field. They are related to the Green's functions in the absence of the field as

$$G^{R,A}(\epsilon) \to G^{R,A}\left(\epsilon - \frac{1}{2}E_z\hat{\sigma}_z\right),$$

where $\hat{\sigma}_z$ is the Pauli matrix in the spin space, and we chose the z axis along the direction of the magnetic field. Repeating all the considerations of Ref. 1, one finds that two-particle propagators (that depend on the difference of the electron energies) are also modified by the field. This modification depends on the spin state of the two particles.[5] Consider first a system of noninteracting electrons. Identification of the singlet and triplet channels corresponds to the choice of a basis in spin space, namely using the states with the total spin L and its z component L_z. The singlet channel is the state with $L=0$ and it is unaffected by the magnetic field, as is the $L_z=0$ component of the triplet. For the remaining two components the Zeeman splitting results in the shift of the frequency ω in all diffusions by $L_z E_z$.

In the presence of electron-electron interaction one takes into account the external magnetic field mostly in the same manner. The only difference is that the g factor is renormalized by the spin-exchange interaction similarly to the Pauli susceptibility Eq. (1). Consequently, the Zeeman splitting is also renormalized:

$$E_z^* = \frac{g\mu_B H}{1+F_0^\sigma}.$$

Thus the conductivity correction (2) is modified as

$$\frac{\delta\sigma_{xx}(H)}{\sigma_D} = \mathrm{Im}\int_{-\infty}^{\infty}\frac{d\omega}{\pi}\frac{\partial}{\partial\omega}\left(\omega\coth\frac{\omega}{2T}\right)\int\frac{q\,dq}{4\pi} \times\Big\{[\mathcal{D}_s^R(\omega,q) + \mathcal{D}_t^R(\omega,q)]B_{xx}(\omega,q) \tag{8a}$$

$$+ \sum_{L_z=\pm1}\mathcal{D}_t^R(L_zE_z^*;\omega,q)B_{xx}(\omega+L_zE_z^*,q)\Big\}; \tag{8b}$$

where the form factor $B_{xx}(\omega,q)$ is given by Eq. (2), propagators in the $L_z=0$ channels expression (8a) are still given by Eqs. (4) and (6), while the propagators in expression (8b) are modified by the Zeeman energy as follows (the diffusive limit was discussed in Ref. 11):

$$\mathcal{D}_t^R(L_zE_z^*;\omega,q) = -\frac{F_0^\sigma}{\nu + F_0^\sigma\Pi^R(L_zE_z^*;\omega,q)}, \tag{9}$$

$$\Pi^R(L_zE_z^*;\omega,q) = \nu\left[1 - \frac{-i\omega}{C(\omega+L_zE_z^*,q)-1/\tau}\right]. \tag{10}$$

Note that the numerator of the polarization operator Eq. (10) is not changed by the Zeeman energy. As a result, the pole of the propagator Eq. (9) at $q=0$ depends only on the bare Zeeman energy E_z with the bare electron g factor. This is a manifestation of the Larmor theorem: the frequency of a homogeneous collective mode (which is the meaning of the pole at $q=0$) cannot be renormalized by electron-electron interaction.

Results. Given the expression for the correction Eq. (8) and the explicit expression for the triplet propagator in the presence of the Zeeman field Eq. (9), further calculation consists of evaluating the integral in Eq. (8). The integral is similar to its zero-field counterpart (see Ref. 1). The resulting magnetoconductivity can be written as (also depicted in Fig. 1)

$$\sigma(H,T)-\sigma(0,T)=\frac{e^2}{\pi\hbar}\left[\frac{2F_0^\sigma}{(1+F_0^\sigma)}\frac{T\tau}{\hbar}K_b\left(\frac{E_z}{2T},F_0^\sigma\right)+K_d\left(\frac{E_z}{2\pi T},F_0^\sigma\right)+m(E_z\tau,T\tau;F_0^\sigma)\right]. \tag{11}$$

In the ballistic limit $T\tau\gg1$ the dominating contribution is given by the first term in Eq. (11), where the dimensionless function $K_b(x,F_0^\sigma)$ contains two contributions:

$$K_b(x,F_0^\sigma)=K_1(x)+K_2(x,F_0^\sigma), \tag{12a}$$

where

$$K_1(x)=x\coth x-1, \tag{12b}$$

and

$$K_2(x,F_0^\sigma)=\frac{1+F_0^\sigma}{2F_0^\sigma}\int_x^{x/(1+F_0^\sigma)}dy\,\frac{\partial}{\partial y}(y\coth y)\times\left(y-\frac{x}{1+F_0^\sigma}\right)\left[\frac{1}{y}+\frac{2F_0^\sigma}{(1+2F_0^\sigma)y-x}\right]. \tag{12c}$$

If the magnetic field is strong, $x\gg1$, the expression Eq. (12) simplifies to

$$K_b(x\gg1,F_0^\sigma)=g(F_0^\sigma)x-1+\mathcal{O}\left(\frac{1}{x}\right), \tag{13}$$

where the dimensionless function $g(z)$, not to be confused with the Lande g factor, is

$$g(z)=\frac{1}{2z}\ln(1+z)+\frac{1}{2(1+2z)}+\frac{z\ln2(1+z)}{(1+2z)^2}.$$

For the smallest magnetic field, $x\ll1+F_0^\sigma$, we have

$$K_b(x\ll1+F_0^\sigma,F_0^\sigma)\approx\frac{x^2}{3}f(F_0^\sigma), \tag{14}$$

where

$$f(z)=1-\frac{z}{1+z}\left[\frac{1}{2}+\frac{1}{1+2z}-\frac{2}{(1+2z)^2}+\frac{2\ln2(1+z)}{(1+2z)^3}\right].$$

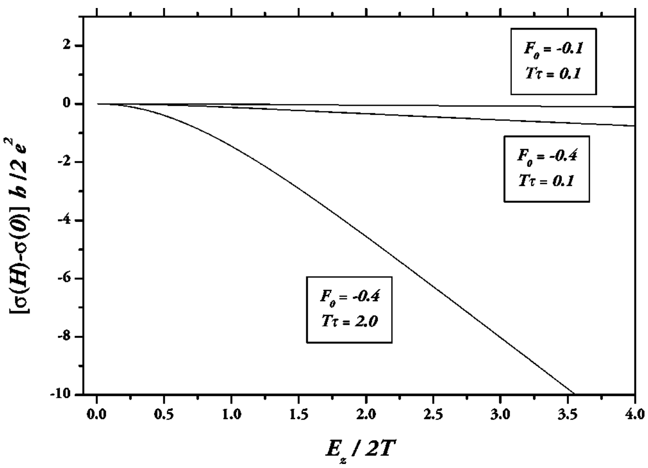

FIG. 1. Magnetoconductivity in a parallel field for different values of F_0^σ.

The diffusive limit $T\tau\ll1$ is characterized by the function

$$K_d(h,F_0^\sigma)=-\frac{1}{4\pi F_0^\sigma}\sum_{L_z=\pm1}\mathrm{Re}\int_{-\infty}^{\infty}\frac{dx}{x^2}\left[\frac{\partial}{\partial x}(x\coth\pi x)\right]\times(x-L_zh)\ln\frac{x-L_zh}{x-L_zh/(1+F_0^\sigma)}$$
$$=-\frac{1}{2\pi F_0^\sigma}\sum_{n=1}^{\infty}\left\{\frac{1}{n}\left(\ln\frac{n^2+h^2}{n^2+\frac{h^2}{(1+F_0^\sigma)^2}}\right)-\frac{4h}{n^2}\left(\arctan\frac{h}{n}-\arctan\frac{h}{n(1+F_0^\sigma)}\right)\right\}-\frac{1}{\pi}\left[\mathcal{C}+\mathrm{Re}\psi\left(1-\frac{ih}{1+F_0^\sigma}\right)\right], \tag{15}$$

where $\mathcal{C}=0.577\ldots$ is Euler's constant, and $\psi(x)$ is the digamma function. For weak interaction ($F_0^\sigma\ll1$) Eq. (15) reproduces the result of Ref. 8. At the smallest magnetic field $h\ll1+F_0^\sigma$ Eq. (15) reduces to

$$K_d(h)\approx\frac{3F_0^\sigma\zeta(3)}{2\pi(1+F_0^\sigma)^2}h^2, \tag{16}$$

where $\zeta(x)$ is the Riemann zeta function, $\zeta(3)=1.202$. In the opposite limit $h\gg1$ we have (see also Ref. 11)

$$K_d(h)\approx\frac{1}{\pi}\left\{1-\frac{1}{F_0^\sigma}\ln(1+F_0^\sigma)\right\}\ln\frac{h}{1+F_0^\sigma}-\frac{1}{2\pi F_0^\sigma}\ln^2\frac{1}{1+F_0^\sigma}. \tag{17}$$

Finally, for intermediate values $1+F_0^\sigma\ll h\ll1$ we obtain

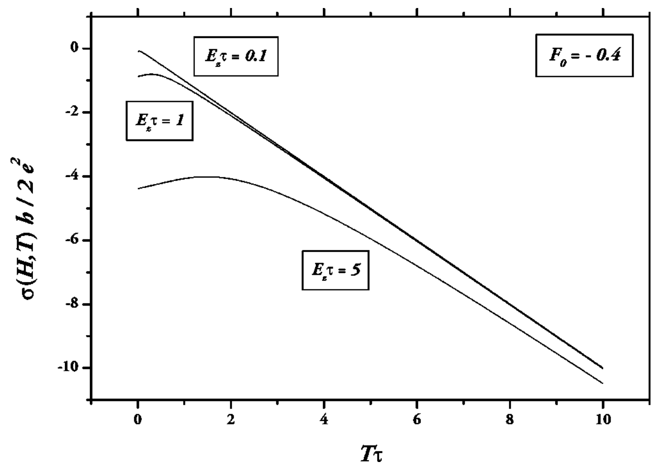

FIG. 2. Temperature dependence of the conductivity corrections in the presence of the parallel field.

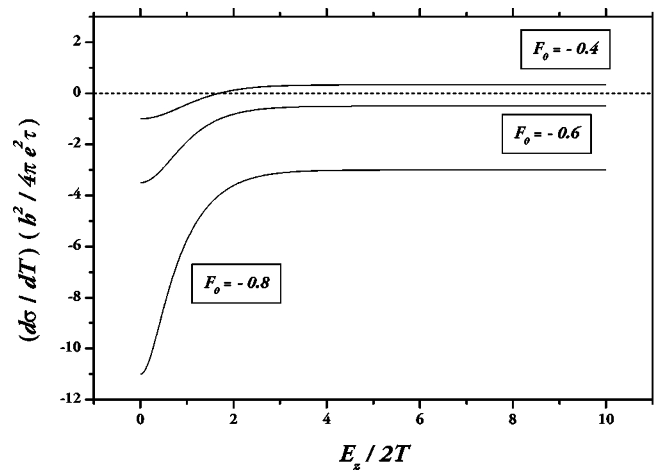

FIG. 3. Slope of the temperature dependence of the conductivity correction (in the ballistic limit) as a function of the parallel magnetic field.

$$K_d(h) \approx -\frac{1}{2\pi}\ln^2\frac{h}{1+F_0^\sigma} + \mathcal{O}\left(\ln\frac{h}{1+F_0^\sigma}\right). \qquad (18)$$

The crossover between the ballistic and the diffusive regimes is described by the dimensionless function $m(E_z\tau, T\tau; F_0^\sigma)$. In the absence of the field $m(0,T\tau;F_0^\sigma)=0$. Similarly to the function $t(T\tau;F_0^\sigma)$ in Eq. (7c), this function appears to be numerically small and does not modify the sum of the two limiting expressions Eqs. (15) and (12) by more than one per cent.

Discussion. The resulting temperature dependence of the conductivity correction Eq. (8) is summarized in Figs. 2 and 3. In the ballistic regime $\delta\sigma \propto T$. Remarkably, the value and *the sign* of the slope depends on the field (for discussion of the diffusive limit see Ref. 10). At zero field[1] the correction is given by all four (the singlet and three components of the triplet) spin channels so that $\partial\sigma/\partial T \propto 1 + 3F_0^\sigma/(1+F_0^\sigma)$. For stronger fields $E_z > T$ the $L_z = \pm 1$ channels are gapped and we are left with one singlet and one triplet channel $\partial\sigma/\partial T \propto 1 + F_0^\sigma/(1+F_0^\sigma)$. The crossover is described by Eqs. (12) and shown in Fig. 3. This picture is valid up to fields of order $(1+F_0^\sigma)^2 E_F$. At the strongest fields $E_z^* > E_F$ when the system is fully polarized the spin does not play a role any more and one retrieves the universal singlet channel result [see Eq. (7b)]

$$\frac{\partial\sigma}{\partial T} = \frac{e^2\tau}{\pi\hbar^2}; \qquad \frac{T\tau}{\hbar} \gtrsim 0.1; \quad E_z^* > E_F. \qquad (19)$$

This conclusion is in agreement with recently reported measurements of magnetoresistance in GaAs.[15]

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