

Charge-fluctuation-induced dephasing in a gated mesoscopic interferometer

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(Received 6 June 2001; published 3 December 2001)

The reduction of the amplitude of Aharonov-Bohm oscillations in a ballistic one-channel mesoscopic interferometer due to charge fluctuations is investigated. In the arrangement considered the interferometer has four terminals and is coupled to macroscopic metallic side gates. The Aharonov-Bohm oscillation amplitude is calculated as a function of temperature and the strength of coupling between the ring and the side gates. The resulting dephasing rate is linear in temperature in agreement with recent experiments. Our derivation emphasizes the relationship between dephasing, ac-transport, and charge fluctuations.

DOI: 10.1103/PhysRevB.64.245313

PACS number(s): 73.23.Ad, 73.20.Mf, 73.61.-r

I. INTRODUCTION

Dephasing processes suppress quantum mechanical interference effects and generate the transition from a microscopic quantum coherent world in which interference is crucial to a macroscopic world characterized by the absence of (quantum) interference effects. Mesoscopic systems are neither entirely microscopic nor macroscopic but at the borderline between the two. Clearly, therefore, dephasing processes play a central role in mesoscopic physics.¹⁻⁵ At low temperatures, it is thought that the predominant processes which generates dephasing are electron-electron interactions.^{1,2,5} In this work, we investigate a ballistic Aharonov-Bohm (AB) interferometer, in which electrons (in the absence of interactions) are subject only to forward scattering processes (see Fig. 1).

Our work is motivated by the following questions: A mesoscopic conductor connects two or more electron reservoirs: inside a reservoir screening is effective and electron interactions are of little importance. In contrast, inside the mesoscopic structure screening is poor and interactions are important. Thus the process of a carrier entering or leaving the mesoscopic structure is essential since this process gives rise to potential fluctuations. We ask how this process affects the dephasing. In standard treatments of dephasing the conductor is considered to be charge neutral and the elementary excitations are electron-hole scattering processes. In a gated structure we can, however, have an electron inside the conductor and a hole on a nearby capacitor (see Fig. 1). As a consequence the conductor is charge neutral only when its surroundings are taken into account (reservoir banks, nearby capacitors).

A second question we seek to answer is the following: instead of calculating a dephasing rate, it is desirable to find a way to directly evaluate the quantity of interest (here the conductance). In small mesoscopic systems the dephasing rate might be a sample specific quantity⁶ and there would be little justification in using an ensemble averaged dephasing rate even if we are interested only in the ensemble averaged conductance. Clearly to answer such conceptual questions it is useful to have a model which is as simple as possible. In this paper we theoretically investigate dephasing of AB oscillations in a ballistic ring with a single transport channel. The one-channel limit is of actual experimental interest (see, e.g., Refs. 7-9). Our idealized setup consists of an AB ring

with four terminals, the arms of the ring being capacitively coupled to lateral gates (see Fig. 1). For a recent experiment on a ballistic (two-terminal) AB ring with lateral gates coupled to both arms see Ref. 10.

The structure we examine has no closed orbits: as a consequence in its equilibrium state it exhibits no persistent current and in the transport state there is no weak localization correction to the conductance. It exhibits, however, an Aharonov-Bohm effect¹¹ due to superposition of partial waves in the outgoing final quantum channel. In fact this is the situation discussed in the original work of Aharonov and Bohm.¹¹ It is also sometimes assumed in mesoscopic physics without a detailed specification of the conditions (multiterminal geometry, absence of backscattering) which are necessary for interference to appear only in the final outgoing channel. The system investigated here is the electric analog of an optical interferometer in which the path is divided by forward scattering only. An example of such an arrangement is the Mach-Zehnder¹² interferometer (MZI). In the MZI the

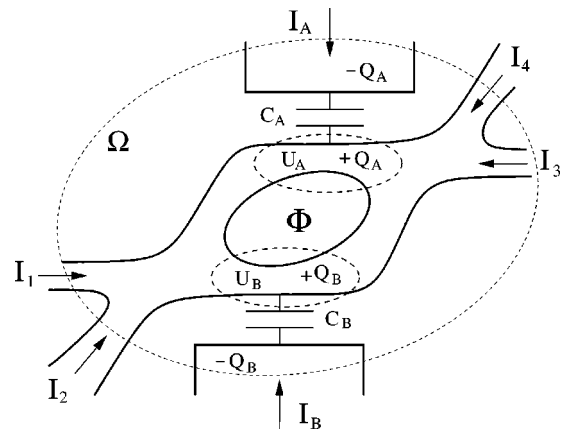


FIG. 1. The figure shows the four-terminal AB ring threaded by a magnetic flux. The two arms of the ring are each coupled to a side gate via a capacitance C_G ($G=A, B$). We consider junctions which are perfectly transmitting and divide the incoming current into the upper and lower branches of the ring. The system then is the electronic equivalent of an optical Mach-Zehnder interferometer (MZI). The total charge in a Gauss sphere Ω drawn around the system of gates and ring is assumed to be zero, implying that current in the system is conserved. It is assumed that each arm is characterized by a single potential U_A (or U_B) and the charge $+Q_A$ (or $+Q_B$).

sample specific AB oscillations are a consequence of superposition in the outgoing scattering channel only. We calculate the effect of internal potential fluctuations on the linear response dc conductance in the ring as a function of the strength of the coupling between ring and gates and of temperature. In this approach the conductance and the dephasing time are not calculated separately but a dephasing time will appear in the expression for the conductance in a natural way. It quantifies the degree of the attenuation of AB oscillations due to randomization of the phases of the electrons going through the ring (as opposed to attenuation due to thermal averaging). From our calculations we find that the coherent part of the conductance is diminished by a factor $1 - \tau/\tau_\phi \approx \exp(-\tau/\tau_\phi)$ relative to the ideal case due to temperature and coupling to the gates. Here τ is the traversal time for going through one arm of the ring. For temperatures $\hbar/\tau \ll kT$ we find a dephasing rate τ_ϕ^{-1} linear in temperature. In an experiment on a two-terminal AB ring, a dephasing rate linear in temperature was recently measured by Hansen *et al.*⁸

In our model the dephasing is due to inelastic scattering of electrons from charge fluctuations in the arms of the ring. We treat the gates as macroscopic entities with perfect screening. The carrier dynamics in the gates is irrelevant for the discussion presented here. They do not represent an external bath or dephasing agent. The irreversible source necessary for dephasing is given by the electron dynamics of the ring itself: the phase and energy of a carrier exiting into a contact are unrelated to the phase and energy of a carrier entering the sample.

Our approach is similar in spirit to the one used in Ref. 13 where dephasing due to charge fluctuations was discussed for two coupled mesoscopic structures. The influence of internal potential fluctuations on the persistent current in a closed ring were examined in Ref. 14. Very recently the effect of a fluctuating vector potential on persistent current and cotunneling transport through a mesoscopic ring have also been investigated.¹⁵ To simplify the discussion we assume that it is possible to draw a Gauss sphere around the system of gates and ring such that all electric field lines emanating within the sphere also end in it (see Fig. 1). This implies that the total charge in the sphere is zero at any time. However, it is possible to charge up one part of the system (an arm of the ring) relative to another part (the nearby gate), creating *charge dipoles*. Charge fluctuations in the arms of the ring lead to fluctuations of the effective internal potentials. Electrons going through the ring are exposed to these potential fluctuations and scatter inelastically. The strength of the coupling between the gates and the arms determines the amount of screening and thus the strength of the effective electron-electron interaction. When the capacitance C between arm and gate becomes very large, the Coulomb energy e^2/C of the system goes to zero. This means that it is in principle possible to put any amount of charge on the ring without causing any reaction. The charge fluctuations in the ring are thus completely free. We say that in this limit ($C \rightarrow \infty$) gate and arm are *decoupled*.

The main goal of this work is the calculation of the dc conductance of the ring taking into account the effects of the

gate-mediated interactions. Applying the ac-scattering approach of Ref. 16 we start by calculating the dynamic conductance matrix of our system. From the real (dissipative) part of the dynamic conductance matrix element relating the current in one of the gates to the voltage applied to the same gate we can find the spectrum of the equilibrium potential fluctuations in the nearby arm of the ring via the fluctuation dissipation theorem. When calculating the dc conductance we statistically average over the scattering potential assuming that the potential has a vanishing statistical mean and that the spectrum of fluctuations is given by the spectrum of equilibrium fluctuations. Taking into account interactions in the manner outlined above results in an attenuation of the amplitude of AB oscillations of the statistically averaged conductance.

In the next two sections we derive the scattering matrix for the MZI. Inelastic scattering from internal potential fluctuations is taken into account. In Sec. IV we determine the internal potential distribution of the ring and then go on to calculate the admittance matrix. We will show that by taking into account screening effects a current conserving theory for the system consisting of the ring and the two gates can be formulated. In the following (Sec. V) we calculate the dc conductance and investigate the influence of equilibrium fluctuations on dc transport.

II. MACH-ZEHNDER INTERFEROMETER

We consider an MZI with a single transport channel. An electron arriving at one of the two intersections coming from a reservoir can enter either of the two arms of the ring, but cannot be reflected back to a reservoir. An electron coming to the intersection from the ring will enter one of the reservoirs. The amplitudes for going straight through the intersection and for being deflected to the adjacent lead in the forward direction are $t = \sqrt{T}$ and $r = i\sqrt{R}$, respectively, where $T + R = 1$. Transmission through the intersections is taken to be independent of energy. In the remainder, we assume symmetric intersections: that is, $R = T = 1/2$ in [Eq. (1)]. Due to the potential fluctuations in the arms of the ring, a carrier can gain or lose energy. This process is described by a scattering matrix $S_G(E', E)$ for each arm which depends on both the energy E of the incoming and the energy E' of the exiting carrier. The scattering matrix $S_G(E', E)$ thus connects current amplitudes at a junction of the ring incident on the branch to the amplitudes of current at the other junction leaving the branch. We have a matrix $S_G(E', E)$ for the upper arm [$S_A(E', E)$] and one for the lower arm [$S_B(E', E)$]. As a consequence of the inelastic transitions in the arms of the ring the full scattering matrix $S_{\alpha\beta}(E', E)$ describing transmission through the entire interferometer from contact β to contact α is also a function of two energy arguments. This scattering matrix can be found by combining the scattering matrices for the two arms with the amplitudes for going through the intersections following a specified path. Due to the geometry of the system, we have the symmetries $S_{13,+B}(E', E) = S_{24,+B}(E', E)$ and $S_{32,+B}(E', E) = S_{41,+B}(E', E)$. In addition the scattering matrix elements calculated for the system in a magnetic field \vec{B} are related to

the matrix elements found at an inverted field $-\vec{B}$ by $S_{\alpha\beta,+B}(E',E) = S_{\beta\alpha,-B}(E',E)$. All elements of the scattering matrix can then be found from the three elements given below:

$$\begin{aligned} S_{13}(E',E) &= S_{24}(E',E) \\ &= i\sqrt{\text{TR}}[S_A(E',E)e^{-i\Phi_A} + S_B(E',E)e^{+i\Phi_B}], \\ S_{14}(E',E) &= -RS_A(E',E)e^{-i\Phi_A} + TS_B(E',E)e^{+i\Phi_B}, \\ S_{23}(E',E) &= TS_A(E',E)e^{-i\Phi_A} - RS_B(E',E)e^{+i\Phi_B}. \end{aligned} \quad (1)$$

Here Φ_G is the magnetic phase picked up by a particle going through arm G clockwise. Then $\Phi_A + \Phi_B = 2\pi\Phi/\Phi_0$, where Φ is the flux through the ring and Φ_0 is the flux quantum. All scattering matrix elements $S_{\alpha\beta}(E',E)$ with $|\alpha - \beta| \leq 1$ are zero since transport through the junctions takes place only in forward direction. The on-shell (one-energy) scattering matrix elements for the (free) one-channel interferometer in the absence of gates which we denote by $S_{\alpha\beta}^{(0)}(E)$ are found by replacing $S_G(E',E)$ in Eq. (1) by $S_G(E',E) = 2\pi\hbar\delta(E' - E)S_G^{(0)}(E)$ with $S_G^{(0)}(E) = \exp(ik_E L_G)$. Our first task is now to determine the scattering matrices $S_G(E',E)$ for the arms of the ring.

III. S MATRIX FOR A TIME-DEPENDENT POTENTIAL

In this section we calculate the scattering matrix for the interacting ring system. We first solve the Schrödinger equation for a single branch of the interferometer using a WKB approach. The amplitude for a transition from energy E to energy $E + \hbar\omega$ of a particle passing through this arm and the corresponding scattering matrix element are determined. Subsequently the scattering matrices for the two arms are included into a scattering matrix for the full interferometer. A WKB approach similar to ours has been used previously to discuss photon-assisted transport in a quantum point contact (QPC)¹⁷ or to the investigation of traversal times for tunneling.¹⁸ The influence of a time-dependent bosonic environment on transport through a QPC was addressed in Refs. 19 and 20 also applying a WKB ansatz.

The gate situated opposite to arm G ($G=A,B$) is assumed to be extended over the whole length of this arm. Fluctuations of the charge in the gate capacitively couple to the charge in the neighboring arm of the MZI and influence electron transport through this arm. This interaction effect is taken care of by introducing a time-dependent potential $V_G(x,t)$ into the Hamiltonian

$$H = -\frac{\hbar^2}{2m^*} \frac{\partial^2}{\partial x^2} + E_G + V_G(x,t), \quad (2)$$

for arm G . Here x is the coordinate along the arm, E_G is the subband energy due to the lateral confining potential of the arm and m^* is the effective mass of the electron. We make the assumption that the fluctuating potential factorizes in a space- and a time-dependent part, writing $V_G(x,t) = h_G(x)eU_G(t)$. For the ballistic structure considered here the internal potential is a slowly varying function of x . For

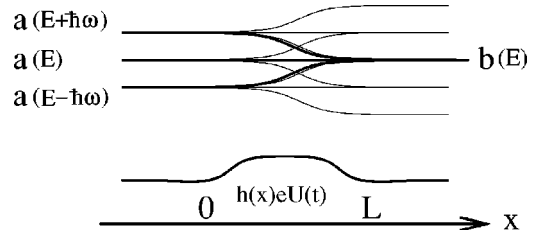


FIG. 2. Scattering states Ψ_E^G and $\Psi_{E\pm\hbar\omega}^G$ [see Eq. (4)] with amplitudes $a(E)$ and $a(E\pm\hbar\omega)$ respectively due to electrons incident from the left at energies E and $E\pm\hbar\omega$ are indicated in the figure. For didactic purposes the special case of a harmonically oscillating barrier $U(t) \propto \cos(\omega t)$ is considered and only first order side bands are drawn. In the rest of the paper we discuss the case of a randomly oscillating barrier and include second-order corrections. The scattering state which may be described by a simple (outgoing) plane wave at energy E with amplitude $b(E)$ to the right of the barrier is emphasized in the figure. The amplitudes $a(E)$ and $b(E)$ are related through the scattering matrix via $b(E) = \sum_{\sigma=0,\pm} S(E, E + \sigma\hbar\omega)a(E + \sigma\hbar\omega)$.

practical calculations we will, however, often employ a rectangular potential barrier [$h_G(x) = \text{const}$ if $0 \leq x \leq L_G$, where L_G is the length of arm G]. Using a space-independent internal potential is a valid approximation²¹ at least in the low-frequency limit $\omega\tau_G \lesssim 1$ where a passing electron sees a constant or slowly changing barrier. We have here introduced the traversal time $\tau_G = L_G/v_{G,F}$ where $v_{G,F}$ is the Fermi velocity in arm G . We will show in Sec. IV how the potentials

$$U_G(t) = \int \frac{d\omega}{2\pi} u_G(\omega) e^{-i\omega t} \quad (3)$$

and their spectra can be determined in a self-consistent way. To solve the Schrödinger equation with the Hamiltonian, Eq. (2), we make the ansatz

$$\Psi_E^G(x,t) = e^{-iEt/\hbar + ik_{G,E}x + ir_G(x,t)/\hbar}, \quad (4)$$

where $k_{G,E} = \sqrt{2m^*(E - E_G)}/\hbar$ and $r_G(x,t)$ is the action due to inelastic scattering (see Fig. 2). We will omit the index G of the wave vector (or of the velocity $v_{G,E} = \hbar k_{G,E}/m^*$) from now on and only write it when the distinction between wave vectors in different arms is important. It is assumed that transmission is perfect: the potential fluctuations cause only forward scattering. This is justified if all energies relating to the fluctuating potential are much smaller than the Fermi energy E_F , in particular $\hbar\omega \ll E_F$.

In determining $r_G(x,t)$ we will take into account corrections up to the second order in the potential. We write $r_G(x,t) = r_{G,1}(x,t) + r_{G,2}(x,t)$, where

$$r_{G,1}(x,t) = \int \frac{d\omega}{2\pi} e^{-i\omega t} r_{G,1}(x,\omega) \quad (5)$$

is linear in the perturbing potential and

$$r_{G,2}(x,t) = \int \frac{d\omega_1}{2\pi} \int \frac{d\omega_2}{2\pi} e^{-i(\omega_1 + \omega_2)t} r_{G,2}(x,\omega_1,\omega_2) \quad (6)$$

is a second-order correction. The linear term was calculated in Ref. 18 for a general form of $h_G(x)$. The corresponding expression for the term quadratic in the potential is readily found but is quite cumbersome. We here give $r_{G,1}(x, \omega)$ and $r_{G,2}(x, \omega_1, \omega_2)$ for the case where $h_G(x)$ is a rectangular barrier of length L_G :

$$r_{G,1}(x, \omega) = i \frac{u_G(\omega)}{\omega} (e^{ix\omega/v_F} - 1),$$

$$r_{G,2}(x, \omega_1, \omega_2) = - \frac{x}{2m^*v_F^3} u_G(\omega_1) u_G(\omega_2) e^{ix(\omega_1 + \omega_2)/v_F}. \quad (7)$$

Here $r_{G,1}$ gives the contribution to the action due to absorption or emission of a single modulation quantum $\hbar\omega$, while $r_{G,2}$ corresponds to the absorption and emission of two modulation quanta $\hbar\omega_1$ and $\hbar\omega_2$. We now proceed to the formulation of the scattering problem in terms of a scattering matrix with elements of the form $S_G(E', E)$ describing transitions between states at different energies. The amplitude $t_G(E', E)$ for a transition from a state with energy E to a state with energy E' of an electron is found from the boundary condition at $x=L_G$, $\Psi_E^G(L_G, t) = \chi_E^G(L_G, t)$. For the matching we expand the WKB wave function [see Eq. (4)] in $x=L_G$ to second order in the perturbing potential:

$$\Psi_E^G(x, t) = e^{-iEt/\hbar + ik_G x} \left[1 + \frac{i}{\hbar} r_{G,1}(L_G, t) + \frac{i}{\hbar} r_{G,2}(L_G, t) - \frac{1}{2\hbar^2} r_{G,1}^2(L_G, t) \right]. \quad (8)$$

Furthermore, the wave function at the right-hand side of the barrier (outside the fluctuating potential region) is

$$\chi_E^G(x, t) = \int \frac{dE'}{2\pi\hbar} t_G(E', E) e^{ik_{E'}x - iE't/\hbar}. \quad (9)$$

In principle, also the derivatives of the wave functions should be matched. However, here we describe transmission through the fluctuating potential region as reflectionless which is accurate up to corrections of the order of $\hbar\omega/E_F$. To determine the transmitted wave with the same accuracy it is sufficient to match amplitudes only. The transmission amplitude is found by Fourier transforming the WKB wave function, Eq. (8), and comparing with the wave function Eq. (9), at $x=L_G$. The transmission amplitude can be expressed in terms of the phase $r_G(L_G, t)$. To second order in the potential we have

$$t_G(E', E) = e^{i(k_E - k_{E'})L_G} \left\{ 2\pi\hbar \delta(\varepsilon) + \frac{i}{\hbar} r_{G,1}(L_G, \varepsilon/\hbar) + \int \frac{d\omega}{2\pi} \left[\frac{i}{\hbar} r_{G,2}(L_G, \omega, \varepsilon/\hbar - \omega) - \frac{1}{2\hbar^2} r_{G,1}(L_G, \omega) r_{G,1}(L_G, \varepsilon/\hbar - \omega) \right] \right\}, \quad (10)$$

where $\varepsilon = E' - E$. The scattering matrix connecting incoming wave amplitudes (at $x=0$) to outgoing wave amplitudes (at $x=L_G$) is related to the transmission amplitude $t_G(E', E)$ through

$$S_G(E', E) = e^{ik_{E'}L_G} t_G(E', E). \quad (11)$$

While the transmission amplitude $t_G(E', E)$ was determined through the continuity of the wave function in the point $x=L_G$ [$t_G(E', E)$ thus connects amplitudes at the same point], the scattering matrix $S_G(E', E)$ connects amplitudes in $x=0$ to amplitudes in $x=L_G$. This difference in the definitions of the two quantities leads to the phase factor $\exp(ik_{E'}L_G)$ in Eq. (11). The scattering matrix as it is derived here *a priori* relates wave function amplitudes and not current amplitudes. To be consistent with the usual definition of the scattering matrix as a relation between current amplitudes, $S_G(E', E)$ should be multiplied by $\sqrt{v_{E'}/v_E}$. This factor, however, is of the order $\hbar\omega/E_F$ and can thus be neglected. The scattering matrices found to describe a single arm can now be integrated into the full scattering matrix for the MZI [see Eq. (1)].

In the discussion presented here the transmission of the carrier through the fluctuating potential region is described as a unitary scattering process. The “final” scattering channels are always open. We emphasize that up to now we have investigated a perfectly coherent process. Decoherence in our model will be introduced through the statistical averaging (cf. Sec. V). Our next task is to find the statistical properties of the potential fluctuations. These fluctuations can be found from the dynamic conductance matrix via the fluctuation-dissipation theorem.

IV. POTENTIAL FLUCTUATIONS

In this section we proceed to the calculation of the admittance matrix $G_{lk}(\omega) = dI_{l,\omega}/dV_{k,\omega}$ for the joint system of interferometer and gates. We concentrate on the limit $\hbar\omega \lesssim \hbar/\tau \ll kT$. The dynamic conductance matrix $G_{lk}(\omega)$ is a 6×6 matrix ($l, k = 1, 2, 3, 4, A, B$), $I_{l,\omega}$ and $V_{k,\omega}$, denoting, respectively, the current measured at and the voltage applied to one of the four contacts of the ring or to one of the two gates. We use the following convention for the indices: lowercase Roman indices can take the values 1, 2, 3, 4, A, B, lowercase Greek indices take the values 1, 2, 3, 4, while the uppercase Roman indices G, H are A, B. We will first calculate the matrix elements $G_{GG}(\omega)$ from which, via the fluctuation-dissipation theorem, we can derive the spectra of potential fluctuations in the two arms. These will later be needed in the discussion of the decoherence of AB oscillations.

tions. The remaining elements of the conductance matrix and the resulting total conductance matrix are given in Appendix A. The elements of the conductance matrix obey the sum rules $\sum_l G_{lk}(\omega)=0$ and $\sum_k G_{lk}(\omega)=0$, reflecting gauge invariance and current conservation, respectively. A problem closely related to the one addressed here is concerned with the calculation of ac-transport properties of a ballistic wire attached to reservoirs and capacitively coupled to a gate.²¹ Contrary to the classical calculations done for a wire in Ref. 21 the ac-scattering approach allows us to take into account the quantum nature of the system investigated here as manifested in the AB oscillations.

The ac properties and the potential distribution which are of interest here depend not only on the mesoscopic conductor but also on the properties of the external circuit. Here we consider the case where all external current loops exhibit zero impedance. This requires some explanation since especially voltages at the gates are typically controlled with the help of an external impedance. However, what counts in our problem is the range of frequencies up to the traversal time, whereas the external impedance might be very large only in a very narrow frequency range around $\omega=0$. Thus we are justified to consider in the following a zero-impedance external circuit.

In order to obtain the conductance matrix from an ac scattering approach we need the effective internal potential $eU_G(t)$ in arm G . The internal potential $eU_G(t)$ is related to the total charge $Q_G(t)$ in the same region through $Q_G(t) = C_G[U_G(t) - V_G(t)]$ where $V_G(t)$ is the voltage applied to gate G and C_G is a geometrical capacitance characterizing the strength of the coupling between arm and gate. The total charge $Q_G(t)$ consists of a contribution due to injection from the contacts labeled $Q_G^e(t)$ and a screening part $Q_G^s(t)$, thus $Q_G(t) = Q_G^e(t) - Q_G^s(t)$. We will now assume that a voltage $V_\alpha(t)$ is applied to contact α while $V_\beta(t)=0$ for $\alpha \neq \beta$.

First we consider the charge density²² injected into the arm G due to a modulation of the voltage at contact α assuming a fixed internal potential U_G . The charge distribution in the sample can be expressed through the Fermi field

$$\hat{\Psi}(\mathbf{r}, t) = \sum_{\beta} \int \frac{dE}{\sqrt{h v_{\beta, E}}} e^{-iEt/\hbar} \psi_{\beta}(\mathbf{r}; E) \hat{a}_{\beta}(E), \quad (12)$$

which annihilates an electron at point \mathbf{r} and time t . Here $\psi_{\beta}(\mathbf{r}; E)$ is a scattering state describing carriers with energy E incident from contact β . The charge density in the ring at point \mathbf{r} and time t is $\hat{\rho}(\mathbf{r}, t) = e\hat{\Psi}^{\dagger}(\mathbf{r}, t)\hat{\Psi}(\mathbf{r}, t)$. Fourier transforming with regard to time and quantum averaging we get $\rho(\mathbf{r}, \omega) = \langle \hat{\rho}(\mathbf{r}, \omega) \rangle$, where

$$\rho(\mathbf{r}, \omega) = e \sum_{\beta, \gamma} \int \frac{dE}{\sqrt{v_{\beta, E} v_{\gamma, E + \hbar\omega}}} \psi_{\beta}^*(\mathbf{r}; E) \psi_{\gamma}(\mathbf{r}; E + \hbar\omega) \times \langle \hat{a}_{\beta}^{\dagger}(E) \hat{a}_{\gamma}(E + \hbar\omega) \rangle. \quad (13)$$

The average charge may be split into an equilibrium part $\rho^{(0)}(\mathbf{r}, \omega)$ and a contribution $\delta\rho\alpha(\mathbf{r}, \omega)$ due to the time-dependent external voltage at contact α :

$$\rho(\mathbf{r}, \omega) = \rho^{(0)}(\mathbf{r}, \omega) + \delta\rho\alpha(\mathbf{r}, \omega). \quad (14)$$

When calculating the quantum average of the charge density operator the effect of the external voltage is taken into account through the modified distribution function for charge carriers coming in from reservoir α . The distribution for contact α to linear order in the applied voltage is²³

$$\langle \hat{a}_{\alpha}^{\dagger}(E) \hat{a}_{\alpha}(E + \hbar\omega) \rangle = \delta(\hbar\omega) f_{\alpha}(E) + \frac{e}{\hbar} V_{\alpha, \omega} F(E, \omega), \quad (15)$$

where $V_{\alpha, \omega}$ is the Fourier component to frequency ω of the voltage $V_{\alpha}(t)$ and

$$F(E, \omega) = \frac{f_{\alpha}(E) - f_{\alpha}(E + \hbar\omega)}{\hbar\omega}. \quad (16)$$

Carriers in the other reservoirs are Fermi distributed: $\langle \hat{a}_{\beta}^{\dagger}(E) \hat{a}_{\gamma}(E + \hbar\omega) \rangle = \delta(\hbar\omega) \delta_{\beta\gamma} f_{\beta}(E)$ (here $\alpha \neq \beta$ or $\alpha \neq \gamma$). The scattering states $\psi_{\beta}(\mathbf{r}; E)$ in the arms of the interferometer for a constant internal potential are of the form $\psi_{\beta}(\mathbf{r}; E) = A_{\beta} \chi(\mathbf{r}_{\perp}) \exp[ik_E x + i\Phi_G(x)]$, where $A_{\beta} = i\sqrt{R}$ or $A_{\beta} = \sqrt{T}$ depending on the arm and the injecting contact [cf. Eq. (1)]. As in most of the paper we will in the following use $R = T = 1/2$. Furthermore, $\Phi_G(x)$ is the magnetic phase acquired going through arm G to point x and $\chi(\mathbf{r}_{\perp})$ is the transverse part of the wave function. The simple form of the scattering states in the arms is a consequence of the absence of backscattering in the intersections. The injected charge $\delta\rho\alpha(x, \omega)$ is the part of the total charge, Eq. (13), proportional to the nonequilibrium contribution to the distribution function, Eq. (15). Substituting the expressions for the scattering states into Eq. (13), using Eq. (15), and integrating over \mathbf{r}_{\perp} we find

$$\delta\rho\alpha(x, \omega) = \frac{e^2}{2} \int \frac{dE}{\sqrt{v_{\alpha, E} v_{\alpha, E + \hbar\omega}}} e^{i\omega x / v_E} V_{\alpha, \omega} F(E, \omega), \quad (17)$$

where we have used $|A_{\alpha}|^2 = 1/2$. To find the total charge $Q_{G, \alpha}^e(\omega)$ injected into arm G of the MZI we integrate over the length of the arm $Q_{G, \alpha}^e(\omega) = \int_0^{L_G} dx \delta\rho\alpha(x, \omega)$. Performing the integration we get

$$Q_{G, \alpha}^e(\omega) = \frac{e^2}{2h} \int dE F(E, \omega) \left(\frac{i}{\omega} \right) (1 - e^{i\omega\tau_G}) V_{\alpha, \omega}. \quad (18)$$

In the limit $\hbar\omega/kT \ll 1$ we have $\int dE F(E, \omega) \approx 1$. We can rewrite the charge as $Q_{G, \alpha}^e(\omega) = e^2 \nu_{G\alpha}(\omega) V_{\alpha, \omega}$ where we have introduced the injectivity $\nu_{G\alpha}(\omega)$, defined as

$$\nu_{G\alpha}(\omega) = \frac{1}{2h} \frac{i}{\omega} (1 - e^{i\omega\tau_G}). \quad (19)$$

Here $\tau_G = L_G/v_F$ is the traversal time through arm G .

Now if interactions are taken into account, the excess injected charge will induce a shift in the effective internal potential, which in turn gives rise to a screening charge. This screening charge is proportional to the internal potential $eU_G(\omega)$ and to the total charge density available for screening $\nu_G(\omega)$. Thus $Q_G^s(\omega) = -e^2 \nu_G(\omega) u_G(\omega)$, where $\nu_G(\omega)$

$=\sum_{\alpha=1}^4 \nu_{G\alpha}(\omega) = 4\nu_{G\alpha}(\omega)$. The last equation is a consequence of the symmetry of the MZI. In the zero-frequency limit $\nu_G(\omega)$ reduces to $\nu_G(0) = 2L_G/(h\nu_F)$. The total charge in region G is $Q_G(\omega) = e^2 \nu_{G\alpha}(\omega) (V_{\alpha,\omega} - 4u_G(\omega))$.

We generalize now to the case where a voltage is applied not only to contact α but also to gate G . The gate voltage is labeled $V_G(t)$. In this situation the charge in arm G is $Q_G(\omega) = C_G[u_G(\omega) - V_{G,\omega}]$. Combining with our previous result for the charge leads to

$$Q_G(\omega) = C_G[u_G(\omega) - V_{G,\omega}] = e^2 \nu_{G\alpha}(\omega) [V_{\alpha,\omega} - 4u_G(\omega)]. \quad (20)$$

Solving for the internal potential and invoking the definition of the injectivity $\nu_G(\omega)$ [see below Eq. (19)] allows us to express the internal potential $e u_G(\omega)$ through the applied voltages:

$$u_G(\omega) = \frac{-i\omega C_G V_{\alpha,\omega} + e^2/(2h)(1 - e^{i\omega\tau_G}) V_{G,\omega}}{-i\omega C_G + (2e^2/h)(1 - e^{i\omega\tau_G})}. \quad (21)$$

The current in gate G is given by $I_{G,\omega} = i\omega Q_G(\omega)$, where $-Q_G(\omega)$ is the charge accumulated in the gate. Since with the help of Eq. (21) we can express $Q_G(\omega)$ as a function of external voltages only, we can calculate the conductance matrix elements $G_{GG}(\omega) = dI_{G,\omega}/dV_{G,\omega}$ and $G_{G\alpha}(\omega) = dI_{G,\omega}/dV_{\alpha,\omega}$. Note that the matrix elements $G_{AB}(\omega)$ and $G_{BA}(\omega)$ vanish since the charge in region G is independent of the voltage applied to the gate further away from it. (This is a consequence of our assumption of forward scattering only at the junctions and of the absence of capacitive coupling between the two arms.) For later use we here state the result for $G_{GG}(\omega)$, which is

$$G_{GG}(\omega) = \frac{dI_{G,\omega}}{dV_{G,\omega}} = \frac{-i\omega C_G}{1 - 2i\omega C_G R_q/(1 - e^{i\omega\tau_G})}. \quad (22)$$

In Eq. (22) we have introduced the *charge-relaxation* resistance $R_q = h/(4e^2)$ of the interferometer. The charge relaxation resistance¹⁶ is a measure of the dissipation generated by the relaxation of excess charge on the conductor into the reservoirs. For a structure with perfect channels connected to a reservoir each reservoir channel connection contributes with a conductance $G_q^{(1)} = 2e^2/h$: the conductances of different channels add in parallel since each channel reservoir con-

nection provides an additional path for charge relaxation. For example a ballistic wire connected to two reservoirs has a charge relaxation resistance $R_q = (G_q^{(1)} + G_q^{(1)})^{-1} = h/4e^2$. For the MZI considered here an excess charge in the upper or lower branch has the possibility to relax into the four reservoirs of the MZI. But at each junction the two connections are only open with probability T and R (see Sec. II). Thus the two connections act like one perfect channel. As a consequence the charge relaxation resistance for our MZI is just that of a perfect wire and also given by $R_q = h/(4e^2)$.

In the low-frequency limit we get from Eq. (22)

$$G_{GG}(\omega) = -iC_{\mu,G}\omega + R_q C_{\mu,G}^2 \omega^2 - i \frac{1 - 3g_G^2}{3g_G^2} R_q^2 C_{\mu,G}^3 \omega^3 + \dots \quad (23)$$

This is in agreement with the result of Blanter *et al.*²¹ for a single wire coupled to a gate. We have introduced the dimensionless (Luttinger) parameter g_G as a measure of the strength of coupling between arm G and gate G . If arm and gate are decoupled, the interaction parameter takes the value $g_G = 1$ while it goes to zero as the strength of coupling is increased. The parameter g_G is related to the capacitance C_G and to the density of states [Eq. (19)] $D_G = \nu_G(\omega = 0) = 2L_G/(h\nu_F)$ of the wire through²¹

$$g_G^2 = \frac{1}{1 + e^2 D_G / C_G}. \quad (24)$$

The electrochemical capacitance¹⁶ $C_{\mu,G}$ of arm G is $C_{\mu,G}^{-1} = C_G^{-1} + (e^2 D_G)^{-1}$.

The remaining elements of the conductance matrix—namely those involving the currents in the contacts of the MZI—can be derived from an ac-scattering approach. These calculations are presented in Appendix A.

To discuss the influence of potential fluctuations on dc transport we need the spectrum $S_{U_G U_G}(\omega)$ of these fluctuations. Since the spectrum of the current fluctuations $S_{I_G I_G}(\omega)$ in region G is related to the real (dissipative) part of the element $G_{GG}(\omega)$ of the emittance matrix through the fluctuations dissipation theorem $S_{I_G I_G}(\omega) = 2kT \text{Re} G_{GG}(\omega)$ (in the limit $\hbar\omega \ll kT$), we get $S_{U_G U_G}(\omega)$ from the relation $S_{U_G U_G}(\omega) = S_{I_G I_G}(\omega)/(\omega^2 C_G^2)$:

$$S_{U_G U_G}(\omega) = kT \frac{h}{e^2} \frac{(1 - g_G^2)^2 [1 - \cos(\omega\tau_G)]}{2(1 - g_G^2)^2 [1 - \cos(\omega\tau_G)] + 2g_G^2(1 - g_G^2)\omega\tau_G \sin(\omega\tau_G) + g_G^4(\omega\tau_G)^2}. \quad (25)$$

The spectrum, Eq. (25), is shown in Fig. 3 as a function of the dimensionless parameter $\omega\tau_G$ for different values of the interaction parameter g_G . Zeros of $S_{U_G U_G}(\omega)$ occur when $\omega\tau$ is a multiple of 2π . This is a consequence of our approximation which considers only uniform potential fluctuations.

If the spatial dependence of the potential is taken into account, Blanter *et al.*²¹ find that the traversal time is renormalized through the interaction ($\tau_G \rightarrow g_G \tau_G$) and consequently the zeros of $S_{U_G U_G}(\omega)$ are shifted accordingly. Instead of the dynamics of single carriers, it is plasmons which govern the high-frequency dynamics. This comparison indi-

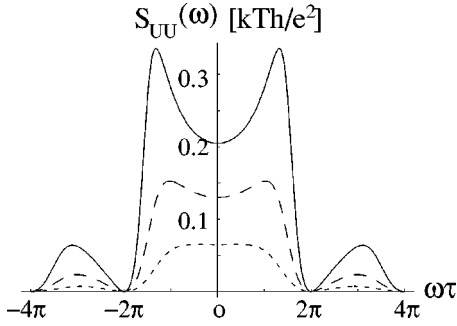


FIG. 3. The spectrum Eq. (25) is shown as a function of the parameter $\omega\tau$. The solid curve corresponds to an interaction strength $g=0.6$, for the middle (dashed) curve $g=0.7$ and for the lowest (dotted) curve $g=0.8$.

icates thus the limitation of our approach: Since we start from a one-particle picture, our approach is most reliable in the case of weak coupling $g_G \rightarrow 1$. In the weak coupling limit ($g_G \rightarrow 1$) we expand the spectrum, Eq. (25), to the leading order in $C_{\mu,G}/C_G = (1 - g_G^2)$ which leads to

$$S_{U_G U_G}(\omega) = 2kTR_q \frac{C_{\mu,G}^2}{C_G^2} \frac{\sin(\omega\tau_G/2)^2}{(\omega\tau_G/2)^2}. \quad (26)$$

The spectrum vanishes in the noninteracting limit $g_G = 1$ ($C_{\mu,G}/C_G = 0$). In Fig. 4 the full spectrum, Eq. (25), is compared to the approximate form, Eq. (26). The function $\sin(\omega\tau_G/2)/(\omega\tau_G/2)$ reflects the ballistic flight of carriers through an interval of length L . In the limit of strong coupling the potential noise is white and

$$S_{U_G U_G}(\omega) = 2kTR_q = 2kTh/(4e^2). \quad (27)$$

Remarkably in the strong-coupling limit for the ballistic ring considered here the spectrum is universal. The only property of the system which enters is the number of leads which permit charge relaxation.

Having found the fluctuation spectra of the internal potentials we are now in the position to investigate the influence of fluctuations on dc transport.

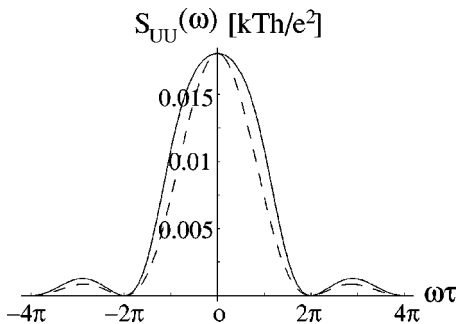


FIG. 4. The exact expression for the spectrum Eq. (25) (solid line) is compared to the expression Eq. (26) (dashed line) in the weak coupling limit. Here $g=0.9$ is chosen for the interaction parameter.

V. ATTENUATION OF THE dc CONDUCTANCE THROUGH COUPLING TO A GATE

We now come to the discussion of the dephasing of the coherent part of the dc conductance in the linear transport regime due to (equilibrium) charge fluctuations. In the last section we have shown that interactions lead to effective charge and potential distributions in the arms of the ring which in turn give rise to displacement currents in the gates and contribute to the ac component of the particle currents in the contacts of the ring. The zero-frequency contribution to the currents in the contacts remained unchanged. Here we go one step further and will discuss how charge fluctuations act back on the dc conductance. In contrast to the last section we will thus only discuss the zero-frequency component of the conductance. Electron-electron interactions affect the coherent part of the dc conductance only. This can be understood from the well-known result²⁴ that interactions do not change the conductance of a one-dimensional wire attached to reservoirs. In the interferometer, when interactions are considered, AB oscillations of the conductance are suppressed. The dc-conductance matrix for the case without interactions is given in Appendix A [see Eq. (A9)]. Throughout this section we choose $\mu_1 = \mu_0 + eV$ and $\mu_2 = \mu_3 = \mu_4 = \mu_0$. We will first assume that only arm A is coupled to a gate ($C_B \rightarrow \infty$). The generalization to the case where both arms are coupled to gates is straightforward and will be discussed at the end of this section.

We will from now on treat the potential as a function with certain statistical properties. The potential fluctuations are characterized by the spectrum $S_{U_A U_A}(\omega)$ which is defined through

$$2\pi\delta(\omega + \omega') S_{U_A U_A}(\omega) = \overline{\langle u_A(\omega) u_A(\omega') \rangle}. \quad (28)$$

The spectrum was evaluated in Sec. IV [see Eqs. (25) and (26)]. In addition the potential has a vanishing mean value $\langle U_A(t) \rangle = 0$. Here $\langle \dots \rangle$ denotes the statistical average and the $u_A(\omega)$ are the Fourier components of $U_A(t)$ [cf. Eq. (3)]. The overbar is to emphasize the distinction between quantum averages $\langle \dots \rangle$ and statistical averages.

The quantity of interest to us is the statistically averaged dc conductance, defined through

$$\overline{\langle G_{\alpha\beta} \rangle} = \lim_{V_\beta \rightarrow 0} \overline{\langle dI_\alpha \rangle} / dV_\beta. \quad (29)$$

A convenient starting point for the calculation of the conductance is the following expression for the current in a contact α :²⁵

$$\hat{I}_\alpha(t) = \frac{e}{h} \int dE dE' e^{i(E-E')t/\hbar} \times [\hat{a}_\alpha^\dagger(E) \hat{a}_\alpha(E') - \hat{b}_\alpha^\dagger(E) \hat{b}_\alpha(E')], \quad (30)$$

in terms of the operators $\hat{a}_\alpha^\dagger(E)$ [$\hat{a}_\alpha(E)$] creating [destroying] an electron in a state with energy E entering the system through contact α and the operators $\hat{b}_\alpha^\dagger(E)$ and $\hat{b}_\alpha(E)$, respectively, creating and annihilating an electron outgoing at

energy E and contact α . The operators $\hat{a}_\alpha(E)$ and $\hat{b}_\alpha(E')$ are related through the scattering matrix [see Fig. 2 and Eq. (1)]

$$\hat{b}_\alpha(E) = \sum_\beta \int \frac{dE'}{2\pi\hbar} S_{\alpha\beta}(E, E') \hat{a}_\beta(E'). \quad (31)$$

As described in Sec. III, Eqs. (10), and (11), we determine the scattering matrix elements from the WKB solution of the Schrödinger equation for the arm of the ring. Doing this we go to second order in the perturbing potential. This is necessary since due to the assumption of a vanishing mean value of the statistically averaged internal potential there exist no first-order corrections to the averaged conductance, Eq. (29). Combining Eqs. (30) and Eq. (31) with the scattering matrix (1) and statistically averaging leads to the following expression for the average conductance:

$$\overline{\langle G_{\alpha\beta} \rangle} = \frac{e^2}{h} \int dE \left(-\frac{\partial f}{\partial E} \right) \overline{\langle T_{\alpha\beta}(E) \rangle}. \quad (32)$$

In above equation we have introduced the statistically averaged transmission probability

$$\begin{aligned} \overline{\langle T_{\alpha\beta}(E) \rangle} &= \frac{1}{2} [1 \pm \cos[\Theta(E) - 2\pi\Phi/\Phi_0]] \\ &\times [1 - \overline{\langle r_{A,1}^2(L_A, t) \rangle} / 2\hbar^2] \\ &\mp \sin[\Theta(E) - 2\pi\Phi/\Phi_0] \overline{\langle r_{A,2}(L_A, t) \rangle} / \hbar. \end{aligned} \quad (33)$$

Here $\Theta = k_{A,E}L_A - k_{B,E}L_B$ is a geometric phase, Φ is the magnetic flux enclosed by the ring, and Φ_0 is the flux quantum. The upper sign is for the pairs of indices $(\alpha, \beta) = (1, 3), (2, 4)$; the lower sign is for $(\alpha, \beta) = (1, 4), (2, 3)$. Furthermore, $\overline{\langle T_{\alpha\beta}(E) \rangle}$ and $\overline{\langle T_{\beta\alpha}(E) \rangle}$ are related via the Onsager relations. Expressions for $r_{A,1}(L_A, t)$ and $r_{A,2}(L_A, t)$ are given in Eqs. (5), (6), and (7). If charge fluctuations are not taken into account, the transmission probability simply is $\overline{\langle T_{\alpha\beta}(E) \rangle} = \{1 \pm \cos[\Theta(E) - 2\pi\Phi/\Phi_0]\}/2$ [compare Eq. (A9)]. Interactions thus decrease the amplitude of the AB oscillations and lead to an additional out-of-phase contribution. Equation (33) can be rewritten in an approximate but more convenient form as

$$\begin{aligned} \overline{\langle T_{\alpha\beta}(E) \rangle} &= \frac{1}{2} \{1 \pm e^{-\overline{\langle r_{A,1}^2(L_A, t) \rangle} / (2\hbar^2)} \\ &\times \cos[\Theta(E) - 2\pi\Phi/\Phi_0 + \overline{\langle r_{A,2}(L_A, t) \rangle} / \hbar]\}. \end{aligned} \quad (34)$$

Note that also in the presence of interactions current is conserved and the system is gauge invariant. This is reflected in the fact that $\sum_\alpha \overline{\langle G_{\alpha\beta} \rangle} = 0$ and $\sum_\beta \overline{\langle G_{\alpha\beta} \rangle} = 0$ (with $\overline{\langle G_{\alpha\alpha} \rangle} = -e^2/h$). Equation (34) has a rather intuitive interpretation since $\overline{\langle T_{\beta\alpha}(E) \rangle} \sim \langle |\Psi_E^A(x, t) + \Psi_E^B(x, t)|^2 \rangle$, where $\Psi_E^A(x, t)$ and $\Psi_E^B(x, t)$ are wave functions for the upper and lower path respectively. Here the wave function $\Psi_E^G(x, t)$ is the WKB wave function for arm G to energy E [Eq. (4)] multiplied by the amplitudes for going through the intersections and by the magnetic phase picked up going through arm G . Note that

Eq. (34) is strictly correct only to second order in the fluctuating potential. We can now proceed to further evaluate Eq. (34). Using $r_{A,1}(L_A, t)$ as given in Eq. (7) and invoking the definition of the spectrum, Eq. (28), we obtain:

$$\overline{\langle r_{A,1}^2(L_A, t) \rangle} = 4e^2 \int \frac{d\omega}{2\pi} S_{U_A U_A}(\omega) \frac{\sin^2(\omega\tau_A/2)}{\omega^2}. \quad (35)$$

The fluctuation spectrum $S_{U_A U_A}(\omega)$ has been calculated in Sec. IV. The integral in Eq. (35) can be done analytically in the two limiting cases of very strong ($g \rightarrow 0$) and weak ($g \rightarrow 1$) coupling between arm and gate. Since the spectrum $S_{U_A U_A}(\omega)$ [Eq. (25)] is a function of ω only through $\omega\tau$, it can be seen from Eq. (35) that we can then write

$$\overline{\langle r_{A,1}^2(L_A, t) \rangle} / (2\hbar^2) = \tau_A / \tau_\phi, \quad (36)$$

where τ_A is the traversal time and τ_ϕ is a function of temperature and the coupling parameter only. Equation (36) defines the dephasing time τ_ϕ . It is expressed through the phase shift $r_{A,1}(L_A, t)$ acquired by a WKB wave function in the presence of a time-dependent potential (relative to the case without potential) and quantifies the strength of the suppression of the AB oscillations in the dc conductance [see Eq. (34)]. In the limit where gate and ring are weakly coupled we use the approximate spectrum, Eq. (26), to evaluate Eq. (35). The dephasing rate $\Gamma_\phi = \tau_\phi^{-1}$ is found to be

$$\Gamma_\phi = \left(\frac{\pi}{3} \right) \frac{kT}{\hbar} (1 - g_A^2)^2 = \left(\frac{2e^2}{3\hbar^2} \right) \frac{C_{\mu,A}^2}{C_A^2} kTR_q. \quad (37)$$

The dephasing rate Γ_ϕ is linear in temperature. Very recently experimental results were reported by Hansen *et al.*⁸ on the temperature dependence of the decoherence of AB oscillations in ballistic rings. In these measurements a dephasing rate linear in temperature was found. The dephasing rate Γ_ϕ [Eq. (37)] also depends on the coupling parameter g_A . Dephasing goes to zero when the gate and ring are completely decoupled ($g_A \rightarrow 1$). We can similarly determine the dephasing rate in the strong-coupling limit. We know that in this case the potential noise is white and the spectrum is given by $S_{U_A U_A}(\omega) = 2kTR_q$. The dephasing rate is $\Gamma_\phi = R_q (e^2/\hbar^2) kT$. Due to the more complicated form of the fluctuation spectrum $S_{U_A U_A}(\omega)$ in the intermediate parameter range for g , we cannot give a simple analytical expression for the dephasing rate. However, the integral in Eq. (35) can be performed numerically. We have mentioned earlier that our approach applies for $\hbar/\tau \ll kT$ since we have considered only classical Nyquist noise and since all the frequencies in the range up to $\omega \sim 1/\tau_A$ contribute to dephasing. An upper limit of validity follows from the fact that we have determined the scattering matrix, Eq. (11), only to second order in the fluctuating potential. This implies that $\overline{\langle r_{A,1}^2(L_A, t) \rangle} / (2\hbar^2) = \tau_A / \tau_\phi \ll 1$ [see Eqs. (33) and (34)]. Combining $\tau_A / \tau_\phi \ll 1$ with the expression for the dephasing rate in the weak-

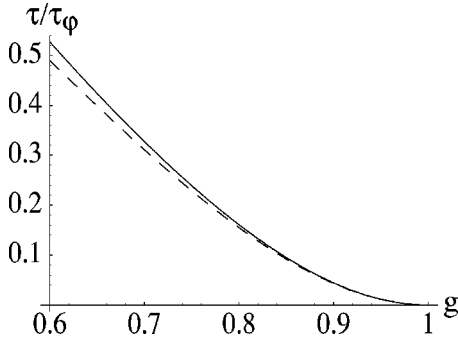


FIG. 5. The ratio of traversal time and dephasing time τ/τ_ϕ vs the coupling parameter g . The upper (solid) line shows the numerically evaluated exact result [cf. Eq. (36)], while the dashed line is the weak coupling approximation Eq. (37). In the calculations we used the parameters of Ref. 8, namely, $L_A \sim 1.5 \mu\text{m}$, $k_F \sim 1.5 \times 10^8 \text{ m}^{-1}$. Furthermore, we assumed $T = 1.5 \text{ K}$ and for the effective mass m^* of the electron we used the value for GaAs, $m^* = 0.067m_e$. With these parameters the Fermi velocity is $v_F = \hbar k_F/m^* \sim 2.6 \times 10^5 \text{ m/s}$.

coupling limit, Eq. (37), we obtain $kT \ll \hbar / [(1-g^2)^2 \tau_A]$. We remark that this is a technical limit; possibly our result is valid over a much wider range. In Fig. 5, τ_A/τ_ϕ is plotted versus the interaction parameter g_A for (relatively) weak interactions where our theory is most reliable.

Returning to the weak-coupling limit we further evaluate $r_{A,2}(L_A, t)$ and find

$$\delta\Theta = \overline{\langle r_{A,2}(L_A, t) \rangle} / \hbar = -\frac{\pi}{4} \frac{C_{\mu,A}^2}{C_A^2} \left(\frac{kT}{E_F} \right). \quad (38)$$

To be consistent with our previous approximations this term should be neglected since it is of the order kT/E_F . Still it is interesting to compare the size of the two corrections due to scattering from the internal potential. We find that $(\tau_A/\tau_\phi)/\delta\Theta \sim k_F L \gg 1$ which implies that taking scattering into account to first order in the potential is a surprisingly good approximation. Combining the information gathered so far allows us to rewrite the transmission matrix elements, Eq. (34), in the more convenient form

$$\overline{\langle T_{\alpha\beta}(E) \rangle} = \frac{1}{2} \{ 1 \pm e^{-\tau_A/\tau_\phi} \cos[\Theta(E) - 2\pi\Phi/\Phi_0] \}. \quad (39)$$

The theory developed so far can be readily adapted to the case where both arms are coupled to gates ($g_G \neq 0, G = A, B$) by making the replacement

$$\frac{\tau_A}{\tau_\phi} \rightarrow \left(\frac{\tau_A}{\tau_{\phi,A}} + \frac{\tau_B}{\tau_{\phi,B}} \right) \quad (40)$$

in Eq. (39). The simple result, Eq. (40), is an immediate consequence of the fact that potential fluctuations in the two arms are uncorrelated ($S_{U_A U_B} = S_{U_B U_A} = 0$). This can either be seen from the corresponding matrix elements of the admittance matrix (cf. Appendix A) or by directly calculating potential correlations, as is done in the low-frequency limit in Appendix B.

VI. CONCLUSIONS

In this work we have examined dephasing due to electron-electron interactions in a simple Mach-Zehnder interferometer. Without interactions the MZI exhibits only forward scattering. (However, screening will generate displacement currents at all contacts in response to a carrier entering the conductor.) We have shown how a scattering matrix approach can be used to calculate the effect of charge fluctuations on the conductance. We have first determined the internal potentials and their statistical properties in a nonperturbative way. Subsequently we calculated corrections to the dc conductance up to the second order in the effective internal potentials. In the expression for the averaged dc conductance a dephasing time τ_ϕ occurs in a natural way. It is a measure of the strength of the attenuation of the AB oscillations as a function of temperature and coupling strength between the ring and gates. The dephasing rate $\Gamma_\phi = \tau_\phi^{-1}$ is found to be *linear* in temperature and to depend on the Luttinger coupling parameter g through $\Gamma_\phi \propto (1-g^2)^2$, at least in the weak-coupling limit. Alternatively, it depends on the ratio of the electrochemical capacitance C_μ and the geometrical gate capacitance C like $(C_\mu/C)^2$. In terms of the Coulomb energy $E_c = e^2/2C$ needed to charge the wire and the density of states (inverse level spacing), $D = 2L/hv_F$, this ratio is $[1 + 1/(2DE_c)]^{-2}$. Such a dependence on E_c cannot be obtained from a golden rule argument in which the coupling between the ring and the gate is treated perturbatively. Such a treatment would lead to a dephasing rate proportional to E_c^2 . A dephasing rate proportional to E_c^2 is obtained only in the limit that the level spacing far exceeds the Coulomb energy. On the other hand, we found that the evaluation of the phase accumulated during traversal of the conductor is surprisingly well described just by first-order perturbation theory in the fluctuating potential.

Recently the temperature dependence of AB oscillations in ballistic rings was investigated experimentally by Cassé *et al.*⁷ Since both thermal averaging and dephasing of the electronic wave functions lead to a decrease in the visibility of the AB oscillations as temperature is increased, a separation of these different effects is of interest. Such an analysis of experimental data was carried out by Hansen *et al.*⁸ They find that the dephasing rate is *linear* in temperature in agreement with our work. We have here not addressed thermal averaging. It can be said, however, that the effect of thermal averaging is very small in the MZI at least for temperatures $kT \ll E_T$, where the Thouless energy is $E_T = E_F(\lambda_F/L)$.

The dephasing length ($l_\phi = v_F \tau$) we have calculated can be of the order of the dephasing length observed in this experiment⁸ when the coupling g is taken to be strong enough. A more detailed comparison of our result to the experiment is difficult, since the setup of Ref. 8 is different from the MZI presented here. In the experimental setup a top gate is used to cover the (two-terminal) AB ring and no side gates are used.

Interestingly a linear temperature dependence has also been observed in experiments on chaotic cavities.^{26,27} Theoretical work which gives a linear temperature dependence for the dephasing rate in chaotic cavities connected to open leads

is presented in Refs. 28 and 6. Takane²⁸ finds $\Gamma_\phi = c(\lambda_F/W)kT/\hbar$ with c a constant of order 1 and W the width of the leads. Using the approach of Ref. 13 the dephasing rate of a chaotic cavity coupled via a geometrical capacitance C to a gate is given by $\Gamma_\phi = (e/\hbar)^2 kT (C_\mu/C)^2 R_q$ where C_μ is the electrochemical capacitance and R_q is the charge relaxation resistance [see Eq. (14.15) in Ref. 6]. Since in the experimentally relevant regime $(C_\mu/C) \approx 1$ and since the ensemble-averaged charge relaxation resistance R_q is $R_q = (h/e^2)(N_1 + N_2)^{-1}$ for a quantum dot coupled via $N_1 \gg 1$ and $N_2 \gg 1$ open channels to reservoirs, the dephasing rate is

$$\Gamma_\phi = \frac{2\pi kT}{\hbar(N_1 + N_2)}. \quad (41)$$

In contrast to the conductance which is determined by the smaller of the two contacts the charge relaxation resistance and hence the dephasing rate are dominated by the larger of the two contacts. Since the overall width of the two contacts is given by $W \propto \lambda_F(N_1 + N_2)$, it is seen that the two results are in fact identical. Both theories make also similar assumptions. In particular it is assumed that the dominant effect comes from uniform potential fluctuations inside the cavity. Taken together these results suggest that a linear temperature dependence can be expected whenever self-polarization effects are unimportant. Since the electrochemical capacitance and more importantly the charge relaxation resistance are sample specific quantities²⁹ even for chaotic cavities coupled to perfect leads, this discussion leads to a distribution of dephasing rates in the few-channel limit.⁶

It is also interesting to note that our dephasing time shows features similar to the inelastic scattering time for a ballistic one-dimensional wire. The inelastic scattering time of Ref. 30 is inversely proportional to temperature and can be written as a simple function of the Luttinger liquid parameter measuring the strength of electron-electron interactions. Whether the inelastic time of Ref. 30 is in fact also the dephasing time remains to be clarified.

Our discussion has emphasized the close connection between the ac conductance of a mesoscopic sample and dephasing. We have given the entire ac-conductance matrix for the model system investigated here. A current and charge conserving ac-conductance theory requires a self-consistent approach to determine the internal potentials and requires the evaluation of the displacement currents.

The displacement currents at the gates can in principle be measured. Nevertheless, the thermal potential fluctuations in the arms of the ring do not act as a which path detector.³¹ In fact the dephasing rate increases with decreasing capacitance and is maximal if $C=0$. In this limit there are no displacement currents at the gates. The absence of which path detection is reflected in the fact that the charge correlations in the two arms of the ring vanish in the equilibrium state of the ring.

The work presented here can be extended in many directions. Multichannel systems and systems with backscattering can be considered. The role of the external circuit can be

examined. We hope therefore that the work reported here stimulates further experimental and theoretical investigations.

ACKNOWLEDGMENTS

We thank Yaroslav Blanter for a critical reading of the manuscript. This work was supported by the Swiss National Science Foundation.

APPENDIX A: ADMITTANCE MATRIX

We here give the admittance matrix $G_{lk}(\omega) = dI_{l,\omega}/dV_{k,\omega}$ ($l, k = 1, 2, 3, 4, A, B$) for a symmetric ($L_A = L_B = L$ and $\tau_A = \tau_B = \tau$) Mach-Zehnder interferometer. The results for the asymmetric case are similar but notationally more cumbersome. As in the rest of the paper we consider the limit $\hbar\omega \ll kT$ and $\hbar\omega, kT \ll E_F$. We have shown in Sec. IV how the admittance matrix elements that relate the displacement currents in the gates to voltages applied at a gate or a contact can be calculated. It remains to derive the matrix elements that relate currents in the contacts to external voltages. To this end we employ the ac-scattering approach following Ref. 22. We here give a slightly formalized derivation of the results found in Ref. 22 and generalize the results to accommodate a system like the MZI containing several regions described by different internal potentials³² (the two arms in the case of the MZI). For recent related work we refer the reader to Refs. 33 and 34. A time-domain version of the ac-scattering approach was recently introduced in Ref. 35 to the investigation of charge pumping in open quantum dots.

We consider the situation where a time-dependent voltage $V_\mu(t)$ is applied to contact μ of the system. We start from the current operator, Eq. (30). Fourier transforming gives

$$\hat{I}_\alpha(\omega) = e \int dE [\hat{a}_\alpha^\dagger(E) \hat{a}_\alpha(E + \hbar\omega) - \hat{b}_\alpha^\dagger(E) \hat{b}_\alpha(E + \hbar\omega)]. \quad (A1)$$

To take into account scattering from internal potential fluctuations we use Eq. (31) and write

$$\begin{aligned} \hat{I}_\alpha(\omega) = e \int dE \left[\hat{a}_\alpha^\dagger(E) \hat{a}_\alpha(E + \hbar\omega) \right. \\ \left. - \int \frac{dE_1}{2\pi\hbar} \frac{dE_2}{2\pi\hbar} \sum_{\beta, \gamma} S_{\alpha\beta}^*(E, E_1) S_{\alpha\gamma} \right. \\ \left. \times (E + \hbar\omega, E_2) \hat{a}_\beta^\dagger(E_1) \hat{a}_\gamma(E_2) \right]. \quad (A2) \end{aligned}$$

Our next step is to average this expression quantum mechanically. Doing this it has to be taken into account that the distribution of charge carriers coming in from reservoir μ is modified due to the time-dependent voltage applied to this contact [see Eq. (15)]. Since we consider only the linear response, we expand the scattering matrix to first order in the internal potentials $u_G(\omega)$. We write

$$S_{\alpha\beta}(E', E) = 2\pi\hbar \delta(E - E') S_{\alpha\beta}^{(0)}(E) + S_{\alpha\beta}^{(1)}(E', E), \quad (A3)$$

where $S_{\alpha\beta}^{(0)}(E)$ is the scattering matrix of the ideal ballistic system [see below Eq. (1)] and $S_{\alpha\beta}^{(1)}(E', E)$ is linearly proportional to the perturbing potential. Substituting Eqs. (15) and (A3) into the current operator, Eq. (A2), and taking the quantum average we see that average current in contact α may be written in the form

$$I_{\alpha}(\omega) = \langle \hat{I}_{\alpha}(\omega) \rangle = I_{\alpha}^0(\omega) + I_{\alpha}^e(\omega) + I_{\alpha}^s(\omega). \quad (\text{A4})$$

The first term $I_{\alpha}^0(\omega)$ in Eq. (A4) is the dc contribution

$$I_{\alpha}^0(\omega) = e \delta(\hbar\omega) \sum_{\beta} \int dE f_{\beta}(E) [\delta_{\alpha\beta} - |S_{\alpha\beta}^{(0)}(E)|^2]. \quad (\text{A5})$$

In the case of interest to us here, $f_{\beta}(E) = f(E)$ and $I_{\alpha}^0(\omega) \equiv 0$ due to the unitarity of the scattering matrix $S_{\alpha\beta}^{(0)}(E)$. The current

$$I_{\alpha}^e(\omega) = \frac{e^2}{h} \int dE [\delta_{\alpha\mu} - S_{\alpha\mu}^{(0)*}(E) S_{\alpha\mu}^{(0)}(E + \hbar\omega)] \times \frac{f(E) - f(E + \hbar\omega)}{\hbar\omega} V_{\mu}(\omega) \quad (\text{A6})$$

can be understood as the part of the total current directly injected into contact α due to the oscillations of the external potential $V_{\mu}(t)$ (see Ref. 22). The third contribution to the total current $I_{\alpha}^s(\omega)$ is the response to the internal potential distribution (compare also Ref. 22)

$$I_{\alpha}^s(\omega) = -\frac{e^2}{h} \sum_{\beta} \int dE [S_{\alpha\beta}^{(0)*}(E) S_{\alpha\beta}^{(1)}(E + \hbar\omega, E)] \times [f(E) - f(E + \hbar\omega)]. \quad (\text{A7})$$

We now want to apply the theory developed so far to the calculation of the dynamic conductance of the MZI. For this example the full scattering matrix is given in Eq. (1). Inelastic transitions are absorbed in the scattering matrices of the arms $S_A(E', E)$ and $S_B(E', E)$. From Eqs. (7), (10), and (11) we know that to first order in the potential

$$S_G(E + \hbar\omega, E) = 2\pi \delta(\omega) e^{ik_E L_G} + e^{ik_E L_G} \frac{u_G(\omega)}{\hbar\omega} (1 - e^{i\omega\tau_G}), \quad (\text{A8})$$

where we used $k_{E+\hbar\omega} \approx k_E + \omega/v$. Expressions for the matrix elements $S^{(1)}(E', E)$ for the interferometer are now easily derived from Eqs. (1) and (A8). For the calculation of the admittance matrix it is furthermore important to note that in the limit of interest here ($\hbar\omega \ll kT$) Eqs. (A6) and (A7) can be considerably simplified. The Fermi functions appearing in these equations are expanded to linear order in $\hbar\omega/kT$. Since in addition the products of scattering matrix elements contained in Eqs. (A6) and (A7) do not depend on the energy E , but only on the energy difference $E' - E = \hbar\omega$ for the scattering matrix used here, the energy integrations can be performed.

Combining the scattering matrix elements defined in Eq. (1) with the expressions for the currents in the gates $I_G(\omega)$

$= i\omega Q_G(\omega)$ and the currents in the contacts $I_{\alpha}(\omega) = I_{\alpha}^e(\omega) + I_{\alpha}^s(\omega)$ [see Eqs. (A6) and (A7)] we can now calculate all elements of the conductance matrix. We here consider the case of a perfectly symmetric ($L_A = L_B$) interferometer and give the result to first order in ω . Expanding the admittance matrix in the low-frequency limit we can write $G_{lk}(\omega) = G_{lk}^0 - i\omega E_{lk} + o(\omega^2)$. Explicitly, the zeroth-order term is

$$\mathbf{G}_0 = \begin{pmatrix} G_0 & 0 & -G_{\Phi}^+ & -G_{\Phi}^- & 0 & 0 \\ 0 & G_0 & -G_{\Phi}^- & -G_{\Phi}^+ & 0 & 0 \\ -G_{\Phi}^+ & -G_{\Phi}^- & G_0 & 0 & 0 & 0 \\ -G_{\Phi}^- & -G_{\Phi}^+ & 0 & G_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad (\text{A9})$$

where we have introduced

$$G_0 = e^2/h, \quad (\text{A10})$$

$$G_{\Phi}^{\pm} = e^2/(2h) [1 \pm \cos(2\pi\Phi/\Phi_0)]. \quad (\text{A11})$$

Note that in the dc limit there are no currents in the gates. The first order term E_{lk} is called the emittance matrix. It is of the form

$$\mathbf{E} = \begin{pmatrix} -E & -E & E_{\Phi}^+ & E_{\Phi}^- & -E_A & -E_B \\ -E & -E & E_{\Phi}^- & E_{\Phi}^+ & -E_A & -E_B \\ E_{\Phi}^+ & E_{\Phi}^- & -E & -E & -E_A & -E_B \\ E_{\Phi}^- & E_{\Phi}^+ & -E & -E & -E_A & -E_B \\ -E_A & -E_A & -E_A & -E_A & 4E_A & 0 \\ -E_B & -E_B & -E_B & -E_B & 0 & 4E_B \end{pmatrix}. \quad (\text{A12})$$

The entries of the emittance matrix are defined through

$$E = \frac{e^2}{h} \frac{1}{8} (C_{\mu,A}/C_A + C_{\mu,B}/C_B) \tau, \quad (\text{A13})$$

$$E_G = C_{\mu,G}/4, \quad (\text{A14})$$

$$E_{\Phi}^{\pm} = \pm E_{\Phi} + E + E_A/2 + E_B/2, \quad (\text{A15})$$

$$E_{\Phi} = \frac{e^2}{2h} \tau \cos(\Phi). \quad (\text{A16})$$

The electrochemical capacitance $C_{\mu,G}^{-1}$ is defined as $C_{\mu,G}^{-1} = C_G^{-1} + (e^2 D_G)^{-1}$, where $D_G = 2L_G/(h v_F)$ is the density of states per unit length. The charge relaxation resistance R_q is given by $R_q = h/(4e^2)$ and the traversal time is $\tau_G = L_G/v_F$. Current conservation implies $\sum_{l=1}^6 G_{lk}(\omega) = 0$ while as a consequence of gauge invariance $\sum_{k=1}^6 G_{lk}(\omega) = 0$. Similar sum rules hold for every coefficient in the expansion of $G_{lk}(\omega)$ as a function of ω (e.g., $\sum_{l=1}^6 E_{lk} = 0$, $\sum_{k=1}^6 E_{lk} = 0$).

APPENDIX B: CHARGE-CHARGE CORRELATIONS

The spectra of charge fluctuations in the gates or, equivalently, the arms of the interferometer, as well as correlations between the charges in the two gates, can be calculated directly from the knowledge of the scattering matrix $S_{\alpha\beta}^{(0)}(E)$ without first calculating the dynamic conductance as we have done in Sec. IV. This approach which is particularly convenient in the low-frequency limit was introduced in Ref. 29 for a mesoscopic structure coupled to a single gate and generalized to the case of coupling to more than one gate in Ref. 36. In this section we apply the approach of Refs. 29 and 36 to calculate the zero-frequency limit of the fluctuation spectra of the charges in the gates of the MZI. In contrast to the rest of this paper we calculate both equilibrium and nonequilibrium fluctuations. From Ref. 36 it is known that in equilibrium, to leading order in frequency, the charge-charge correlations are given by

$$S_{Q_G Q_H}^q = 2kTC_{\mu,G}C_{\mu,H}R_q^{GH}, \quad (\text{B1})$$

where G and H specify the gates in the system. Here $C_{\mu,G}^{-1} = C_G^{-1} + (e^2 D_G)^{-1}$ is the electrochemical capacitance of gate G , D_G being the density of states. Furthermore, R_q^{GH} is the generalized charge relaxation resistance to be introduced below. For $G=H$, Eq. (B1) gives the spectrum of charge fluctuations in gate G , while for $G \neq H$, Eq. (B1) gives the equilibrium charge correlations between gates G and H . With a small voltage applied to one contact of the system the fluctuation spectra to leading order in the applied voltage are³⁶

$$S_{Q_G Q_H}^V = 2|eV|C_{\mu,G}C_{\mu,H}R_V^{GH}. \quad (\text{B2})$$

The generalized charge relaxation resistance R_q^{GH} and the corresponding quantity in the driven case, the Schottky resistance R_V^{GH} , can be expressed through the (off-diagonal) elements of a generalized Wigner Smith time-delay matrix:

$$R_q^{GH} = \frac{h}{2e^2} \frac{\sum_{\alpha\beta} D_{\alpha\beta}^G D_{\alpha\beta}^{H*}}{D_G D_H}, \quad (\text{B3})$$

$$R_V^{GH} = \frac{h}{2e^2} \frac{\sum_{\alpha \neq \beta} (D_{\alpha\beta}^G D_{\alpha\beta}^{H*} + D_{\beta\alpha}^G D_{\beta\alpha}^{H*})}{D_G D_H}. \quad (\text{B4})$$

The density of states D_G in region G is the sum of the diagonal matrix elements of the Wigner Smith time-delay matrix,

$D_G = \sum_{\alpha} D_{\alpha\alpha}^G$. The elements of the time-delay matrix can conveniently be found from the scattering matrix via the relation³⁷

$$D_{\alpha\gamma}(E) = \frac{1}{2\pi i} \sum_{\beta} S_{\beta\alpha}^*(E) \frac{dS_{\beta\gamma}(E)}{dE}. \quad (\text{B5})$$

The scattering matrix for the MZI is given in Eq. (1). We here only need the matrix $S_{\alpha\beta}^{(0)}(E)$ which is found from Eq. (1) by replacing $S_G(E', E)$ in Eq. (1) by $S_G(E', E) = 2\pi\hbar \delta(E' - E) S_G^{(0)}(E)$ with $S_G^{(0)}(E) = \exp(ik_E L_G)$ (here we simply have $\Theta_G = k_E L_G$). To calculate the time-delay matrix in region G we replace the energy derivative by a derivative with regard to the local potential $d/dE \rightarrow -d/dU_G$. Since the scattering matrix depends on energy only through the phase factors $\Theta_G = k_E L_G$, it is easy to show¹³ that

$$\frac{dS_{\beta\gamma}}{dU_G} = -\pi D_G \frac{dS_{\beta\gamma}}{d\Theta_G}. \quad (\text{B6})$$

We can now use Eqs. (B3)–(B6) to calculate the generalized charge relaxation and Schottky resistances:

$$R_q^{AA} = R_q^{BB} = \frac{h}{4e^2}, \quad (\text{B7})$$

$$R_q^{AB} = R_q^{BA} = 0, \quad (\text{B8})$$

$$R_V^{AA} = R_V^{BB} = \frac{h}{16e^2}, \quad (\text{B9})$$

$$R_V^{AB} = R_V^{BA} = -\frac{h}{16e^2}. \quad (\text{B10})$$

Combining these equations with Eqs. (B1) and (B2) we get charge-charge correlations for the equilibrium and out-of-equilibrium situations. It is interesting to note that in equilibrium the charges in the two gates are *uncorrelated*. This is a consequence of the absence of closed electronic orbits in the ring and the fact that we have not introduced a Coulomb coupling between the two branches of the ring. For the same reason correlations are independent of the magnetic field. This implies that despite the fact that AB oscillations are observed in the currents measured at one of the contacts, the interior of the ring behaves like a classical system. In contrast, the charge fluctuations generated by the shot noise are correlated. Like the equilibrium charge fluctuations they are for the geometry investigated here independent of the AB flux.

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