Charge transport through quantum dots via time-varying tunnel coupling

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We describe a mechanism for charge pumping through tunnel-coupled quantum dots in the regime of strong Coulomb blockade. The quantum state of an additional electron within the structure is steered by changing the tunneling couplings between neighboring dots. Appropriate tailoring of the interdot tunneling rates allows one to design the instantaneous eigenvalues of the system Hamiltonian. A combination of adiabatic following and Landau-Zener tunneling results in the transfer of charge from one dot to the neighboring one. Coupling to electron reservoirs via weak tunnel barriers then allows one to implement an electron pump.

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Single electron devices allow for the controlled transfer and pumping of charges through small metallic islands or semiconductor quantum dots.[1,2] Monochromatic time-dependent electric fields have been demonstrated[3] to lead to photoassisted tunneling through coupled quantum dots,[4,5] where resonant tunneling via two controllable discrete levels can be modulated by applying an oscillating signal to the gate electrode or by irradiating the structure with microwaves. Consequently, different schemes for double dot pumps, i.e., devices for the transfer of electrons between two reservoirs at the same chemical potential, have been suggested for monochromatic[6] and pulsed irradiation.[7]

An alternative pumping mechanism is the slow parametric change of system parameters such as tunnel rates. Although the original single electron turnstile experiments[1] relied on adiabatic electron transfer, the adiabatic control of the wave function itself is a relatively new topic. Experiments in open dots[8] have demonstrated the feasability of an ''adiabatic quantum electron pump.'' These systems can be described as noninteracting mesoscopic scatterers.[9] This allows for the generalization of a number of concepts from metallic systems, such as mesoscopic fluctuations,[10,11] symmetries,[11] or resonances,[12] to the time-dependent case.

In this work, we describe a mechanism for charge pumping through *three* tunnel-coupled quantum dots attached to electron leads (reservoirs) in the regime of strong Coulomb blockade. The main idea is to achieve pumping by periodically varying not the coupling to the leads but by varying the (inner) couplings among the dots, which are an intrinsic part of the whole quantum system (the triple dot) itself. Appropriate tailoring of the tunneling rates between neighboring dots then allows one to design the instantaneous energies and wave functions of the structure. A combination of adiabatic following and Landau-Zener tunneling results in the transfer of charge from one dot to the neighboring one. Coupling to electron reservoirs via constant tunnel barriers then allows one to implement an electron pump completely based on a quantum mechanical mechanism.

The minimal structure for our scheme to work is shown in Fig. 1 and consists of three dots (L, C, and R) coupled via time-varying tunnel barriers. The left and the right dot are also coupled, via additional tunnel barriers, to electron reservoirs. Only the ground states take part in the process of charge transport and their energies are tuned, via gate voltages, so that $E_R > E_C > E_L$. In the numerical calculations we will set $E_C = 0$ and take $E_R = -E_L \equiv E_0$. The energy splitting $E_0 \equiv \hbar\omega_0$ will then be taken as a scale for the energy and $1/\omega_0$ will be the time unit.

We assume the charging energy of an additional electron in the triple dot so large that the system is effectively described by a four-dimensional Hilbert space with basis $\{|0\rangle, |L\rangle, |C\rangle, |R\rangle\}$. Here $|0\rangle$ is the ''empty'' state (no additional electron in the structure), and $|L\rangle$ ($|C\rangle, |R\rangle$) corresponds to an additional electron in the ground state of the left (center, right) dot.

We first discuss the effective one-particle problem of the isolated (no coupling to the leads) triple dot. The effective Hamiltonian is

$$H(t) = E_L|L\rangle\langle L| + E_C|C\rangle\langle C| + E_R|R\rangle\langle R| + \hbar T_1(t)[|L\rangle\langle C| + |C\rangle\langle L|] + \hbar T_2(t)[|C\rangle\langle R| + |R\rangle\langle C|], \quad (1)$$

with E_α ($\alpha = L, C, R$) the energy of the ground states of the different dots, and T_1, T_2 the tunneling constants, assumed to be real (negative) for the sake of simplicity, between neighboring dots.

To illustrate the principle of the proposed mechanism of charge transport, we neglect dephasing processes, such as the

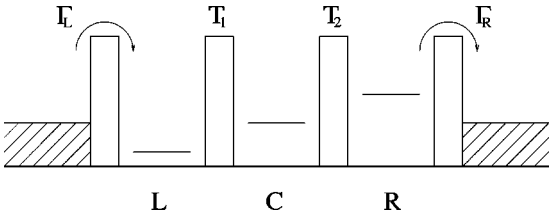

FIG. 1. Semiconductor structure consisting of three dots coupled via tunnel barriers T_1 and T_2. The left and right dot are also coupled to electron reservoirs (leads).

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interaction with the phonon bath, so that the evolution of the system is ideally coherent and corresponds to the wave function

$$|\psi(t)\rangle = c_L(t)\exp[-iE_Lt/\hbar]|L\rangle + c_C(t)\exp[-iE_Ct/\hbar]|C\rangle + c_R(t)\exp[-iE_Rt/\hbar]|R\rangle \quad (2)$$

whose coefficients obey the Schrödinger equation

$$\dot{c}_L(t) = -iT_1c_C(t)\exp[-i(E_C-E_L)t/\hbar], \quad (3a)$$

$$\dot{c}_C(t) = -iT_1c_L(t)\exp[-i(E_L-E_C)t/\hbar] - iT_2c_R(t)\exp[-i(E_R-E_C)t/\hbar], \quad (3b)$$

$$\dot{c}_R(t) = -iT_2c_C(t)\exp[-i(E_C-E_R)t/\hbar]. \quad (3c)$$

Consider first the ideal case in which the tunneling between neighboring dots can be completely suppressed, i.e., the tunneling rates T_1 and T_2 can be tuned between zero and any arbitrary value $T_i \leqslant 0$. Then, an appropriate tailoring of the time-varying tunnel rates between the dots results in pairs of level crossings and anticrossings, in profound analogy with the energy spectrum of atoms in static electric and magnetic fields.[15] In the adiabatic limit,[16] the state of the system coincides with the instantaneous eigenstate of the Hamiltonian. Therefore it is possible to steer the wave function of the additional electron by changing the parameters (tunnel couplings) of the system Hamiltonian. In our case, adiabatic following corresponds to the transfer of charge from one dot to its neighbor dot. If the additional electron initially is in the center dot $[c_\alpha(0)=\delta_{\alpha C}]$, turning on the coupling T_1 induces a mixing between the center and the left dot. As a result, the electron spreads into the left dot (Fig. 2, bottom left) and the energy levels of the coupled system (left and center dot) are pushed apart for increasing $|T_1|$ (Fig. 2, center left).

For $E_L<E_C<E_R$, a sufficiently large increase of $|T_1|$ results in a crossing of the energy level of the right dot with the higher energy level of the coupled left- and center-dot system. Clearly, if T_2 is kept at zero, a slow increase of $|T_1|$ and subsequent decrease to zero does not produce any change of the state of the system, which follows adiabatically. This is the situation described in the left column of Fig. 2. In this way no charge transport is produced.

We now show that a pulsed T_2 tunnel coupling completely changes the time evolution of the system, so that adiabatic following results in a nonzero charge transport through the structure. This is the situation examined in the right column of Fig. 2. In fact, a pulsed T_2 can transform a level crossing into an anticrossing, as shown in Fig. 2 (right column, center). If the T_2 pulse is shorter than the T_1 pulse and it is centered around the position of a crossing of the unperturbed system (i.e., a crossing for $T_2 \equiv 0$), only one of the level crossings will become an anticrossing. In this way the adiabatic following of the system results in the transfer of the additional electron from the initial dot to the neighboring one as shown in Fig. 2.

The process can be iterated, and by exchanging the role of T_1 and T_2 the electron can be transferred adiabatically from

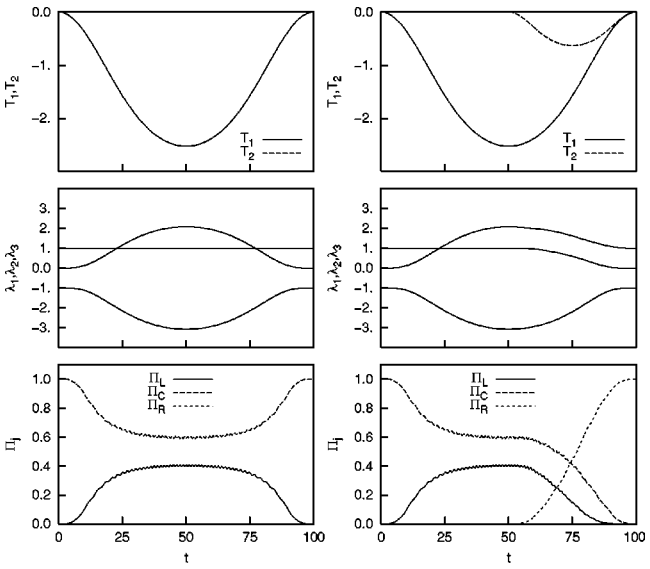

FIG. 2. Tunnel-coupling pulse sequence (top) and corresponding time evolution of the energy eigenvalues (center) and populations (bottom). The populations $\Pi_\alpha = c_\alpha^* c_\alpha$ $(\alpha = L, C, R)$ are determined by numerically solving the Schrödinger equation (3). The left column corresponds to the case in which only one tunnel coupling is nonzero $(T_2 \equiv 0)$, while for the right column both couplings are pulsed.

one side of the structure (say, the right dot) to the other side (the left dot). The corresponding procedure is shown in Fig. 3. In fact, assuming the additional electron is initially localized in the right dot, it can be transferred to the center dot by the same sequence of tunnel-coupling pulses as above: a long T_1 pulse is applied, which alone would produce a pair of level crossings; a shorter T_2 pulse changes the second level crossing into an anticrossing, so that the electron is adiabatically transferred to the center dot. For the transfer from the center dot to the left one the role of T_1 and T_2 are ex-

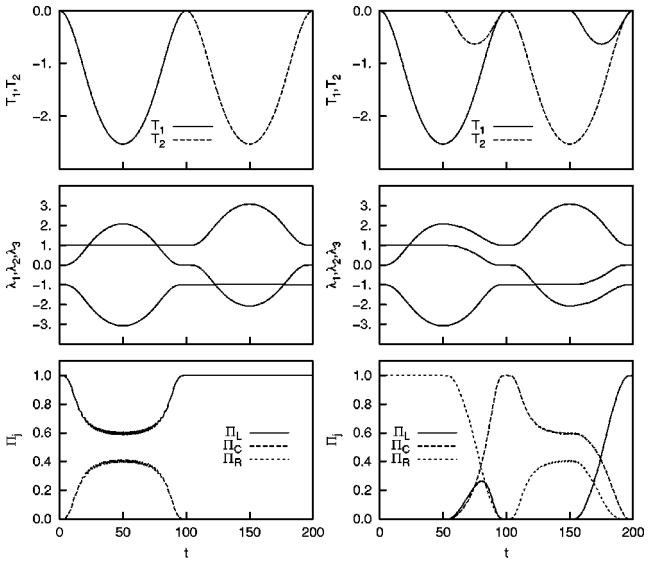

FIG. 3. Transfer from the right dot to the left one. The organization of the data sets is the same as for Fig. 2.

changed: it is now T_2 which produces the pair of level crossings with T_1 playing the role of the control pulse, i.e., T_1 changes the last crossing into an anticrossing. In this way the adiabatic following results in the transfer of the additional electron to the left dot.

We now include the coupling of the triple dot to the leads, and show that in this case the above mechanism allows one to transfer electrons between two reservoirs (the left and right leads) kept at the same chemical potential, see Fig. 1. The corresponding tunneling rates are denoted by Γ_L and Γ_R, respectively. The chemical potential of the electron reservoirs is tuned somewhere in between the energies of the left and right dot, so that electrons can tunnel into the quantum-dot structure only from the left lead and escape from the structure only through the right barrier (see Fig. 1). For weak coupling to the leads, the dynamics of the coupled dots can be described by a master equation[4,13,14] for the reduced density matrix ρ of the dots, which in our case reads

$$\dot{\rho}_{LL} = iT_1[\rho_{LC} - \rho_{CL}] + \Gamma_L \rho_{ee}, \tag{4a}$$

$$\dot{\rho}_{CC} = iT_2[\rho_{CR} - \rho_{RC}] + iT_1[\rho_{CL} - \rho_{LC}], \tag{4b}$$

$$\dot{\rho}_{RR} = iT_2[\rho_{RC} - \rho_{CR}] - \Gamma_R \rho_{RR}, \tag{4c}$$

$$\dot{\rho}_{ee} = -\Gamma_L \rho_{ee} + \Gamma_R \rho_{RR}, \tag{4d}$$

$$\dot{\rho}_{LC} = i\omega_{CL}\rho_{LC} + iT_1[\rho_{LL} - \rho_{CC}] + iT_2\rho_{LR}, \tag{4e}$$

$$\dot{\rho}_{CR} = i\omega_{RC}\rho_{CR} + iT_2[\rho_{CC} - \rho_{RR}] - iT_1\rho_{LR} - \frac{1}{2}\Gamma_R\rho_{CR}, \tag{4f}$$

$$\dot{\rho}_{LR} = i\omega_{RL}\rho_{LR} - iT_1\rho_{CR} + iT_2\rho_{LC} - \frac{1}{2}\Gamma_R\rho_{LR}, \tag{4g}$$

where $\omega_{\alpha,\beta} = (E_\alpha - E_\beta)/\hbar$ $(\alpha,\beta = C,L,R)$ and ρ_{ee} corresponds to the "empty" state.

The sequence of tunnel couplings appropriate to transfer electrons from the left to the right reservoir is shown in Fig. 4 (top left), together with the resulting energy eigenvalues (bottom left). We numerically solved the master equation (4); results for the populations $\Pi_\alpha = \rho_{\alpha\alpha}$ $(\alpha = e,L,C,R)$ are reported in Fig. 4 (bottom right). Results for a closed system, i.e., without coupling to the leads, are also reported for comparison (top right). In the latter case, the time evolution corresponds to the transfer of an additional electron from the left to the right dot. In the case of an open system, the transfer of charge to the right dot is followed by a charge leakage to the right lead at a rate Γ_R. At the same time, charge flows from the left lead into the structure. In this way there is a net charge transport through the triple dot which after the tunnel-coupling sequence [including a "leakage time" of the order of $1/\min(\Gamma_L,\Gamma_R)$] is returned to the initial state with (almost) the whole charge in the left dot. Furthermore, also the tunnel couplings T_1, T_2 are back to their initial zero value.

The total charge Q pumped through the structure during a cycle, i.e., during a time interval $[t_i : t_f]$ which includes the sequence of T pulses and the leakage time, is

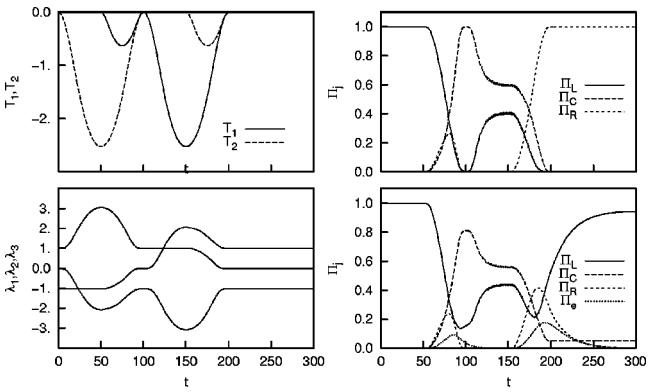

FIG. 4. Charge transport through tunnel-coupled quantum dots. Left column: the tunnel-coupling pulse sequence (top) and corresponding time evolution of the energies (bottom). Right column: the evolution of the populations, as determined by Eq. (4), including the coupling to the leads ($\Gamma_L = 0.1$, $\Gamma_R = 0.05$, bottom), and for a closed system ($\Gamma_L = \Gamma_R = 0$, top).

$$Q = -e\int_{t_i}^{t_f} \Gamma_L \rho_{ee} dt = -e\int_{t_i}^{t_f} \Gamma_R \rho_{RR} dt. \tag{5}$$

Note that Eq. (5) is valid only in integral form. In fact, the charge can pile up temporarily within the triple dot so that the instantaneous value of $-e\Gamma_L\rho_{ee}$, i.e., the current flowing from the left lead into the dot structure, is different from $-e\Gamma_R\rho_{RR}$, the current across the barrier between the right dot and the lead.

Results of numerical calculations for the charge Q for different strengths of the coupling to the leads are reported in Fig. 5. In the limit of weak coupling to the electron reservoirs, i.e., for small Γ_L and Γ_R, approximately one electron per cycle is transferred from the left to the right lead. However, it should be noted that a weaker coupling to the leads results in a slower electron pumping, because of the longer leakage time.

The scheme for adiabatic transfer as described above is based on the existence of pairs of level crossings and anticrossings. In our structure, a level *crossing* corresponds to the suppression of tunneling between two neighboring dots (T_1 or T_2 set to zero). However, it is in general not possible to completely suppress the tunneling between neighboring dots, as this would correspond to infinitely high and/or thick tunnel barriers in real space. If the tunnel rates are kept at nonzero values $T_i < 0$ all the time, the previous degeneracies at the level crossings are lifted and the crossings become

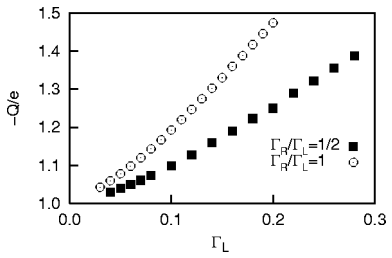

FIG. 5. Charge pumped through the triple dot in a cycle as a function of the tunnel rate Γ_L, at different values of Γ_R/Γ_L.

anticrossings. This is consistent with the fact that in a finite one-dimensional potential there are no level crossings for a particle without internal structure.[17]

We have repeated our calculations by adding a small but finite offset to the tunnel rates $T_i(t) \to T_0 + T_i(t)$. The time evolution of the system essentially remains unchanged with the only exception being the previous level crossings that now turn into anticrossings. Although for small T_0 ($T_0 = -10^{-6}$ in our calculation) the resulting level splitting is very tiny, the transfer mechanism across these points now is Landau-Zener tunneling, whereas outside the ''nearly crossings'' the dynamics remains adiabatic. It should be noted that in the extreme case of arbitrarily *slow* tuning of the $T_i(t)$, the Landau-Zener tunneling becomes exponentially small and there is no transfer of charge at all any longer.

Finally, we comment on parameter ranges relevant for a possible experimental realization in coupled semiconductor few-electron quantum dots. Experiments in double dots[18] have demonstrated that three gate voltages can be used to tune the tunnel coupling T and the ground state energies of two quantum dots, although an independent manipulation of two couplings T_1 and T_2 in triple dots is bound to be more difficult. In our calculation, we assumed that the ground state energy difference $\hbar\omega_0$ between two adjacent dots fulfills $\hbar\omega_0 \ll U, \Delta$, where U is the Coulomb charging energy and Δ the single particle level spacing within a single dot. Typical values are $U, \Delta \sim 1$ meV in coupled lateral dots with a diameter of ~ 200 nm.[18] Assuming $\hbar\omega_0 \sim 0.1$ meV, we find that the typical operation frequency $\nu := 1/(t_f - t_i)$ of the pump is $\nu \sim 10^8$ s^{-1}. The temperature smearing of the Fermi distribution is negligible if $k_B T \ll \hbar\omega_0 \sim 1$ K. For these parameters, the inequality $h\nu, k_B T \ll \hbar\omega_0 \ll U, \Delta$ holds.

The described quantum pump is based on the possibility to steer the triple dot wave function by varying the tunnel couplings. In the intermediate steps of a pumping cycle, the additional electron is indeed prepared in a superposition of states corresponding to neighboring dots. As an additional test of the pure quantum nature of the proposed pumping mechanism, we studied numerically the effects of an inter-dots relaxation rate γ which transforms superposition of states into mixtures. For the parameters mentioned above, ν is smaller or of the same order as rates γ due to phonon emission. Fortunately, we verified that it takes relatively large relaxation rates $\gamma \gtrsim \omega_0$ for the charge pumped in a cycle to drop to zero. Further studies for the crossover to a completely incoherent transport regime are under way.

In conclusion, we have proposed a mechanism for charge pumping through triple quantum dots. Coupling to external reservoirs allows for adiabatic pumping of electrons. The quantum state of the additional electron within the structure is steered by changing the tunnel couplings between neighboring dots. Appropriate tailoring of the time-dependent tunnel couplings allows the transfer of electrons from one dot to the other. Weak couplings to electron reservoirs then permit one to implement an electron pump.

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