P-wave pairing and ferromagnetism in the metal-insulator transition in two dimensions

Claudio Chamon,[1] Eduardo R. Mucciolo,[2] and A. H. Castro Neto[1,*]
[1]*Department of Physics, Boston University, Boston, Massachusetts 02215*
[2]*Departamento de Física, Pontifícia Universidade Católica do Rio de Janeiro, Caixa Postal 38071, 22452-970 Rio de Janeiro, Brazil*

(Received 15 June 2001; published 10 December 2001)

Based on recent experimental evidence for a spin-polarized ground state in the insulating phase of the two-dimensional electron system, we propose that ferromagnetic spin fluctuations lead to an attractive interaction in the triplet channel and cause *p*-wave pairing in the conducting phase. We use the Landau-Fermi liquid phenomenology to explain how the enhanced spin susceptibility near the critical density yields an attractive potential, in a similar mechanism to superfluidity in ^{3}He. As the density is decreased, the *p*-wave order parameter undergoes a transition from a unitary to a nonunitary state, in which it coexists with ferromagnetism for a range of densities. As the density is further reduced, the pairing amplitude vanishes and the system is described by a ferromagnetic insulator. Thus, we find two quantum critical points as a function of density associated with the polarization of the paired state and ferromagnetism. We explain the magnetotransport measurements in parallel and perpendicular magnetic fields and propose a shot-noise experiment to measure the pair charge.

DOI: 10.1103/PhysRevB.64.245115 PACS number(s): 71.10.−w, 71.30.+h, 73.23.−b

I. INTRODUCTION

It has become self-evident that the spin properties of interacting electrons or holes in two dimensions (2D) play a central role in the transport properties of these systems, in particular in the possible metal-to-insulator transition (MIT) observed in a number of different material systems, such as Si-MOSFET's, *n*- and *p*-doped GaAs, AlAs, and SiGe.[1] Recent data in Si metal-oxide-semiconductor field effect transistors (MOSFET's) by Shashkin *et al.*[2] and Vitkalov *et al.*[3] on the saturation of the conductance as a function of the magnetic field parallel to the 2D plane (see also Ref. 4), combined with previous analysis of Shubnikov–de Haas oscillations in a tilted magnetic field,[5,6] suggest that the insulating state is spin polarized.

These experimental observations revive an unresolved theoretical problem on the possible phases of electronic systems in 2D as a function of the interaction strength (or, alternatively, the density) even in the idealized clean systems. Bergman and Rice[7] raised the possibility that as the density is decreased, there is a transition from a paramagnetic Fermi liquid into a ferromagnetic Fermi liquid state. Quantum Monte Carlo studies by Tanatar and Ceperley[8] have considered three different electronic states, a paramagnetic liquid, a ferromagnetic liquid, and a Wigner crystal, and found a transition from the paramagnetic liquid to the Wigner crystal at $r_s \approx 37$. However, the energies of these three phases become rather close for a range of r_s. Thus, it is not unreasonable that either improved energy estimations or disorder effects may bring the energy of the ferromagnetic state to lower values, so that it may exist for a window of densities between the paramagnetic liquid and the Wigner crystal. Indeed, perturbative renormalization group (RG) calculations for disordered and interacting electrons in 2D by Finkelstein[9] have pointed out a runaway flow in the triplet channel even in the limit of low densities (see also Ref. 10). This runaway has been recently interpreted as a tendency towards ferromagnetism in the diffusive (metallic) regime,[11,12] suggesting that disorder may also help trigger a full spin polarization at low densities. Recent numerical studies have also indicated a strong tendency towards spin polarization in the localized regime.[13]

Provided the recent experimental evidence from transport measurements (not susceptibility measurements) for the ferromagnetic state of dilute 2D electrons, we investigate the effect of enhanced spin fluctuations on the paramagnetic side of the transition. We assume that the paramagnetic state can be described, for densities above a critical density n_0 and for finite temperatures below the Fermi energy, by Fermi liquid phenomenology. As one approaches the critical density, one of the Landau Fermi liquid parameters, F_0^a, which renormalizes the spin susceptibility, crosses the minimum bound for Pomeranchuk's stability condition.[14] We argue that the proximity to the ferromagnetic instability leads to an attractive interaction for a range of densities preceding the ferromagnetic transition. The attraction, in the triplet channel, leads to *p*-wave pairing. Once the system enters the ferromagnetic state, the paramagnon exchange mechanism for the attraction rapidly decreases, and these two phases compete to the point where the *p*-wave state ceases to exist at a density n_c. Therefore, there are two quantum critical points in the phase diagram at densities n_c and n_0 (likely very close), and there is an intermediate region in densities where the two phases coexist, but in which the *p*-wave pairing is in a nonunitary state. At the mean-field level, the phase diagram is shown in Fig. 1.

The possibility of unconventional pairing in an electronic system has been recently considered in the context of layered Sr_2RuO_4. In the $Sr_{n+1}Ru_nO_{3n+1}$ series, where *n* determines the number of RuO_2 planes in the unit cell, ferromagnetic states are observed for $n>3$. It is believed that the proximity to ferromagnetism plays an important role for superconductivity in the ruthenates, and *p*-wave symmetry was proposed on the basis of similarities to ^{3}He.[15,16] Although a direct

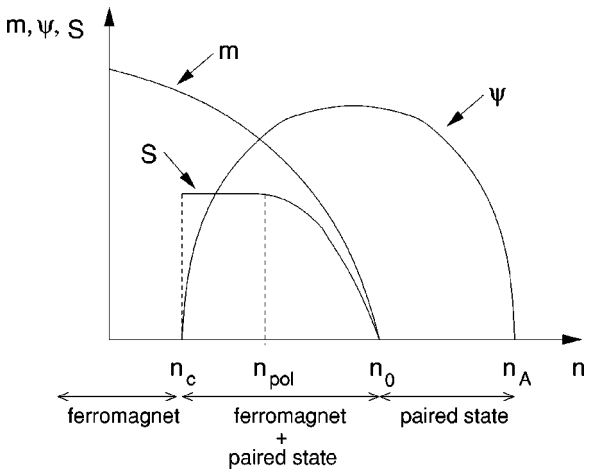

FIG. 1. Variation of the relevant order parameters as a function of electron density (mean-field theory). m is the magnetization, Ψ is the p-wave pairing amplitude, and S denotes the pair spin projected along m. Spontaneous magnetization sets in at the critical density n_0, while at n_c the system becomes a disordered ferromagnetic insulator. $n_{\rm pol}$ is the density where the pairs become fully spin polarized.

transition between ferromagnetic and superconducting states has not been experimentally observed, the ruthenates show similarities to the problem studied in this work. We should point out, however, that in ruthenates there is true 3D long-range order, while we discuss systems that are truly two dimensional and therefore subject to strong fluctuations.

We also draw a strong analogy between the 2D electronic states and the 3D ^{3}He systems in a number of ways.[17] For one, we argue that Fermi liquid phenomenology should not be dismissed in describing the 2D interacting electronic system in the metallic side of the transition at finite temperatures. One of the usual concerns that is raised against the Fermi liquid state in the 2D problem at low densities is that the ratio between the Coulomb and kinetic energies is about a factor of 10. This logic can be misleading, (a) since the contributions from exchange and correlations reduce this ratio and (b) Fermi liquid parameters, which measure the strength of the interaction relative to the kinetic energy,[14] are typically large compared to unity even for 3D ^{3}He. For example, the Landau parameters for ^{3}He at high pressure (27 bars) that renormalize the compressibility, magnetic susceptibility, effective mass, and spin precession rate are $F_0^s = 68.17$, $F_0^a = -0.76$, $F_1^s = 12.79$, and $F_1^a = -1.00$.[18] Even though most Landau parameters are very large when compared to unity, the normal phase of ^{3}He is very well described by Landau's Fermi liquid theory. Indeed, if any, the real question is why Landau's phenomenology works so well, way beyond the perturbative regime where RG arguments for fermions[19] justify the stability of the Fermi liquid.

We show that we can consistently interpret the recent data on the 2D MIT transition close to the critical density[2–4] as due to the enhancement of the Landau parameter F_0^a. The paramagnon exchange mechanism can be responsible for an attraction in the triplet channel in the 2D electron problem, in complete analogy with the problem of 3D ^{3}He. The major difference between these two systems is dimensionality: in 2D true superconductivity or superfluidity is only possible at zero temperature. Strong fluctuations in 2D do not allow for long-range order. For a singlet paired state in 2D with U(1) symmetry algebraic order can be established below the Kosterlitz-Thouless (KT) temperature.[20] However, for a triplet paired state where the order parameter is a complex vector and the symmetry group is non-Abelian, superconductivity can only be established at $T=0$. We thus propose that the insulating state is ferromagnetic, while the metallic state corresponds to a paired p-wave state. The possibility of *singlet* superconductivity in the observed conducting phase of the 2D electron systems was suggested by Phillips *et al.*[21] and Belitz and Kirkpatrick;[22] here we present a possible mechanism for pairing (without finite-temperature long-range order) in the *triplet* channel.

We would like to stress that we do not address in this paper the reason why the measured conductance in Si-MOSFET's seems to saturate in the triplet paired state when $T \to 0$. In the case of singlet paired superconducting 2D materials (thin films) and 2D Josephson junction arrays, where a finite KT transition should be observed together with the vanishing of the resistivity, a saturation of the conductivity is also observed.[23] The source and precise mechanism for these dissipative effects are presently unknown, despite some recent theoretical efforts.[24,25]

There is a natural question regarding the possibility of a paired state in the 2D electronic systems where the MIT is observed: Why can one have pairing if the conductance is of order e^2/h near the transition? Naively, if one uses intuition from noninteracting electrons, then the bare value of the conductance is $G_0 = (2e^2/h)(k_F l)$; weak localization corrections, perturbative in $(k_F l)^{-1}$, are added to this bare value. This would imply that $k_F l \approx 1$ at the transition, and therefore disorder is too strong and pair breaking. However, the value of l that one reads from this naive argument is *not* a measure of disorder alone. The energy scale of the interactions is larger than the Fermi energy of the 2D electron systems near the transition, thus the scattering of electrons even if the disorder is weak should be large and dominant. Therefore, a dimensionless conductance of order unity does not necessarily imply strong disorder. For example, it is known that near 2D superconducting-insulator (SC-I) transitions the dimensionless conductance is of order one even in the absence of disorder.[26] We claim that the same happens in the context of the MIT transition discussed here. We also give arguments showing, based on Fermi liquid theory, that near the ferromagnetic transition the paramagnetic scattering can provide for conductances of order unity.

The paper is organized as follows: In Secs. II and III we discuss Landau's Fermi liquid phenomenology applied to the paramagnetic phase of the 2D electronic system and the Pomeranchuk instability leading to ferromagnetism at low densities. We argue that the enhancement in spin fluctuations due to the proximity to the ferromagnetic state causes an attractive interaction in the triplet channel and p-wave pairing. Section IV contains a discussion of a two-component model for the MIT transition, where we argue that, due to electron-electron interactions in the presence of disorder, the dimen-

sionless conductance due to paramagnetic scattering can become of order unit close to the transition. The mean-field phase diagram of the problem is established in Sec. V using a generic Ginzburg-Landau free energy for a *p*-wave paired state coupled to a ferromagnetic order parameter. In Sec. VI we compare our results to the available experimental data for parallel and perpendicular magnetic fields for the 2D electron gas in a Si-MOSFET and propose new shot noise experiments that can test our theory. Section VII contains our conclusions.

II. *p*-WAVE PAIRED METALLIC PHASE

Our starting point is a Landau Fermi liquid theory for the metallic phase of the 2D electronic system. We consider first the effects of interactions and then those of disorder. Let us briefly review and then apply the Landau phenomenology to the experimental observations on Si-MOSFET's. Consider an isotropic Fermi liquid with planar density n, Fermi momentum $k_F=\sqrt{2\pi n/g_v}$, and Fermi energy $E_F=\hbar^2k_F^2/2m^*=\pi\hbar^2 n/g_v m^*$, where m^* is the effective mass and g_v accounts for the valley degeneracy. The ground state of the problem is described in terms of quasiparticles that fill up a Fermi sea up to the Fermi energy. The change in the energy of a Fermi liquid due to changes in the quasiparticle charge density, $\delta n(\vec{k})$, and spin density $\delta\vec{\sigma}(\vec{k})$ is given by[14,18]

$$\delta E=\int d^2k\,\epsilon_{\vec{k}}\,\delta n(\vec{k})+\int d^2k\,d^2k'\,f^s(\vec{k},\vec{k}')\,\delta n(\vec{k})\,\delta n(\vec{k}')+\int d^2k\,d^2k'\,f^a(\vec{k},\vec{k}')\,\delta\vec{\sigma}(\vec{k})\cdot\delta\vec{\sigma}(\vec{k}'),\tag{1}$$

where $\epsilon_{\vec{k}}$ is the bare dispersion and $f^s(\vec{k},\vec{k}')$ and $f^a(\vec{k},\vec{k}')$ are the symmetric and antisymmetric Landau parameters, respectively. In 2D these parameters can be expanded as

$$f^{a,s}_{\vec{k},\vec{k}'}=\sum_{n=-\infty}^{+\infty} f_n^{s,a}e^{in\theta_{\vec{k},\vec{k}'}},\tag{2}$$

where n gives the angular momentum in the plane, and $\theta_{\vec{k},\vec{k}'}$ is the angle between $\vec{k}$ and $\vec{k}'$. It is useful to define dimensionless parameters $F_n^{a,s}\equiv N(0)f_n^{s,a}$, where $N(0)=g_v m^*/\pi\hbar^2$ is the 2D density of states at the Fermi energy.

The stability of the Fermi liquid state (or the Fermi surface) is given, in Landau's theory, by the Pomeranchuk criterion,[14,18] which in 2D can be written as

$$F_n^{s,a}>-1\tag{3}$$

for all values of n. Since all the physical quantities in Landau's theory can be written in terms of the Landau parameters, a violation of the Pomeranchuk criterion implies an instability of a physical observable. The compressibility, for instance, is given by

$$\kappa=\frac{N(0)}{1+F_0^s}\tag{4}$$

and an instability to phase separation implies that $F_0^s<-1$.[27] In the same theory the effective mass is given by

$$\frac{m^*}{m_b}=1+F_1^s,\tag{5}$$

vanishing when $F_1^s=-1$ (here m_b is the carrier band mass). The magnetic susceptibility can be written as

$$\chi=\left(\frac{g_0\mu_B}{2}\right)^2\frac{N(0)}{1+F_0^a},\tag{6}$$

where $g_0\approx 2$ is the bare (band) Landé g factor. Thus, for $F_0^a\approx -1$ the magnetic susceptibility diverges, indicating an instability towards a magnetically ordered phase.

As one of the Landau parameters approaches the critical value given by Pomeranchuk's criterion, there is a strong enhancement of the interactions in the Fermi liquid. Consider, for instance, the case of density-density interactions that are determined by F_0^s. It can be shown that the induced density-density interaction in a Fermi liquid (in the static limit) is given by the usual random phase approximation (RPA) expression[28]

$$U_{\rho-\rho'}=\frac{1}{N(0)}\frac{F_0^s}{1+F_0^s}.\tag{7}$$

Thus, close to Pomeranchuk's instability $F_0^s\approx -1$ the interaction is very large and attractive, leading to phase separation. On the other hand, the induced spin-spin interactions in the same system are given by

$$U_{\sigma-\sigma'}=\frac{1}{N(0)}\frac{F_0^a}{1+F_0^a}\vec{\sigma}\cdot\vec{\sigma}',\tag{8}$$

where σ is the electron spin. Therefore, when $F_0^a\approx -1$, that is, close to the magnetic instability, this interaction is also large and attractive, leading to pairing in a spin triplet channel. In fact, we can estimate the size of the pairing amplitude using the weak coupling BCS expression

$$|\Delta_p|\approx E_F\,e^{-|(1+F_0^a)/F_0^a|},\tag{9}$$

where the Fermi energy E_F works as a cutoff in the problem because it is the only energy scale present.[28] Obviously, we have to consider Eq. (9) carefully since, as the system approaches the instability, the attraction is very strong and the weak coupling expression breaks down; in this case, one should use a strong coupling approximation.[29] Thus, the expression in Eq. (9) can still be used when the attraction is weak $|F_0^a|\ll 1$ and is only a crude estimate when $F_0^a\approx -1$.

In general, one expects the Landau parameters to be dependent on the electronic density n. Let us consider the situation of a Fermi liquid close to a magnetic instability that happens at $n=n_0$. The Landau parameter $F_0^a(\delta)$ can be expanded close to the transition as

$$F_0^a(\delta)=-1+\alpha\delta+\mathcal{O}[\delta^2],\tag{10}$$

where $\alpha>0$ is a constant and

$$\delta = \frac{n - n_0}{n_0} \tag{11}$$

measures the distance from the quantum critical point. In Eq. (10) we disregard higher-order terms in the density variations around the critical point. Observe that in this case the magnetic susceptibility can be written from Eq. (6) as

$$\chi(\delta) \approx \frac{(g\mu_B)^2 N(0)}{4\alpha\delta}, \tag{12}$$

showing that the susceptibility diverges linearly with the distance from the critical point. Notice, from Eq. (9), that the weak coupling expression for the pairing amplitude close to the transition is given by

$$|\Delta_p(\delta)| \approx E_F e^{-\alpha|\delta|} \approx E_F . \tag{13}$$

This result indicates that the pairing amplitude is of order of the Fermi energy in the system. In the case of the 2D Si-MOSFET's, E_F is usually of order of 5 K because of the low electronic density. The critical temperature, T_c, however, remains zero because of the dimensionality of the system; otherwise, this problem would be a case of high-temperature superconductivity. Notice that the number of pairs is $n_p = n/2$ at zero temperature.

Away from the transition we can classify the behavior of the system depending on the full density dependence of F_0^a. At large enough densities, $n > n_A$, F_0^a should become positive because of the screening of the electronic interactions by backflow effects. In this case, without the help of pairing, the localization effects should dominate, and the system should become an Anderson insulator. So it is only in this high-density ($n > n_A$) regime that one could possibly attempt to apply ideas established through the scaling theory of localization[30] for noninteracting electrons. For $n_0 < n < n_A$ we have $-1 < F_0^a < 0$ and pairing is effective in delocalizing the electrons, leading to the metallic state observed experimentally. In this regime, naive intuition based on noninteracting electrons should not apply. Finally, for $n < n_0$, the system becomes a ferromagnet. We will return to this regime later when we consider a Landau-Ginzburg theory describing the system. We show that there is a coexistence region with both *p*-wave pairing and ferromagnetism for densities $n_c < n < n_0$, and for $n < n_c$ the physics of the problem is the one of a ferromagnetic insulator.

III. EFFECT OF DISORDER IN THE PAIRED STATE

We have argued that the proximity to the ferromagnetic instability induces pairing in the Fermi liquid state. The effects of strong interactions are built in in this picture, but we still need to discuss the effect of disorder in this paired state (which we argue in Sec. V should be in fact a fully gapped $p_x \pm ip_y$ state). The question to be addressed is will the paired state survive disorder? As is known for the case of strongly coupled superconductors, which is the case here, when the electron mean free path l becomes of the order of the coherence length ξ_{sc}, pairing is suppressed.[31] Since the pairing amplitude is essentially of order of E_F [see Eq. (13)], the coherence length is of the order of the Fermi wavelength,

$$\xi_{sc} = \frac{v_F}{|\Delta_p|} \approx \frac{1}{k_F}, \tag{14}$$

and of the size of the interparticle spacing. Therefore, pairing should survive as long as $l \gg \xi_{sc}$ or, equivalently, $k_F l \gg 1$. The main question here is whether this condition is satisfied in the heterostructures where the MIT is observed. We argue below in favor of this case.

The common belief is that close to the MIT transition $k_F l \approx 1$ because the dimensionless conductance at the transition is of order 1. This indeed would be the case if the value of the conductance would be completely fixed by the amount of disorder in the sample. However, as we have shown, there are strong electron-electron interactions close to the ferromagnetic transition (indeed interactions are the *reason* for the instability, since $F_0^a \to -1$). These interactions in the presence of localized electronic spins, as we are going to show in Sec. IV, can provide a large contribution to the resistance. In other words, the value of the critical resistivity, ρ_c, can be of order h/e^2 from interaction effects even if disorder is small. The concept of using $(k_F l)^{-1}$ as a measure disorder is only good if one could determine what l is independently of a conductivity measurement, because otherwise strong interaction effects will blend in and make such discussion useless. Therefore, in strongly correlated systems one should be careful in extending arguments that are only valid for noninteracting systems.

From a theoretical perspective, it is known that interactions alone can lead to a universal conductivity of order $\sigma_0 = 4e^2/h$. It has been shown[26] that, for the 2D superconductor-to-insulator transition in the clean Bose-Hubbard model (belonging to the 3D *XY* model universality class), the conductivity is given by $\sigma \approx 0.285\sigma_0$. It was argued in Ref. 26 that this transition is in the same universality class of a superfluid-insulator transition for bosons moving in a random potential. In $1+1$ dimensions, it is known that the universality class of a superconductor-to-insulator transition for a model of fermions with attractive interactions is the same as the insulator to superfluid transition in a model of repulsively interacting bosons.[32] Since $1+1$ is the lower critical dimension for this type of transition,[26] we expect this type of result to hold in our case as well. Indeed, there is strong experimental evidence for these results in the superconducting to insulator transition in amorphous superconducting thin films.[23]

IV. TWO-COMPONENT MODEL

Based on experimental evidence that the insulating phase in the 2D electron system is a ferromagnetic state, we have argued in Sec. II that the proximity to the spin-polarized state induces *p*-wave pairing on the metallic side of the MIT. In this section we study the transition between the *p*-wave paired state and the ferromagnetic state using a two-component model. Let us split the total electron density at $T=0$ into a localized ferromagnetic component n_L and an

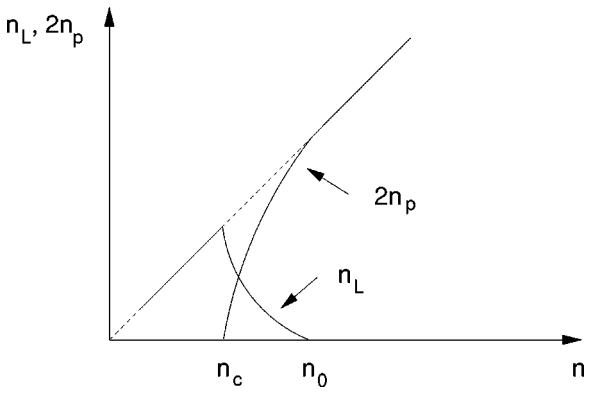

FIG. 2. Densities of the localized, n_L, and itinerant, $2n_p$, components as a function of total density near the critical region.

itinerant, paired component n_p:

$$n = n_L + 2n_p . \tag{15}$$

The dependence of both n_L and n_p as a function of n is represented schematically in Fig. 2. For densities $n_0 < n < n_A$, all electrons are paired ($n_p = n/2$) at $T=0$, while for very low densities, all electrons are in the insulating ferromagnetic state ($n = n_L$). There is a region where the two components coexist: as the system starts to spin polarize below n_0, the density of pairs starts to decrease, vanishing at some critical density $n_c < n_0$. At very low densities ($n \ll n_c$) the Wigner crystal or Bragg glass phase becomes the ground state.[33] In this work, however, we do not attempt to study such phase, since it requires unscreened long range interactions that are not included in our formalism. The study of the Wigner crystal would then need a different starting point, far from the MIT. For this reason, the Wigner crystal is absent in the phase diagram of Fig. 1.

The appearance of a coexistence region is natural in the way we divide the two densities. It also follows from the competitive nature of the two phases, as the mechanism that leads to pairing—paramagnon exchange—is strongly suppressed as the system spin polarizes. The coexistence phase is also found within the Landau-Ginzburg theory of Sec. IV, as a direct consequence of having two competing orders. The existence of metallic regions surrounding localized puddles of charge has been confirmed experimentally through ingenious local compressibility measurements in GaAs/AlGaAs samples.[34] These experiments reveal that the fragmented localized regions increase in number as the density is lowered towards the MIT. The appearance of these fragments in the metallic phase ($n > n_c, n_0$) is consistent with the idea that some incipient magnetism is always present in diffusive systems, even when the interaction in the triplet channel is weak.[12] In regions where strong magnetization occurs due to fluctuations in the wave functions, the paramagnon mechanism is absent, thus suppressing pairing and leading to localization. This is not taken into account in our mean-field (homogeneous) treatment of the problem; had it been considered, it would likely prolongate the n_L curve towards densities larger than n_0 (not shown in Fig. 2).

Next, we use this two-component picture to provide an argument on why the critical conductivity near the MIT is of order e^2/h. Consider the situation at finite temperature within the coexistence window $n_c < n < n_0$. While at zero temperature all the delocalized electrons (with density $2n_p$) participate in the pairing, at finite temperatures only a small fraction is really paired. The unpaired state can be described by the Fermi liquid theory of Sec. III. The localized ferromagnetic component is a source of spin scattering for the itinerant Fermi liquid. The coupling

$$\delta H = J \int d^2 r \, \vec{S}_L(r) \cdot \vec{S}_{\rm it}(r) \tag{16}$$

follows from splitting the electrons into two components in the effective Hamiltonian for the interacting electron liquid. Here $S_L(\mathbf{r})$ and $S_{it}(\mathbf{r})$ are the electron spin operators for the localized and itinerant components of the electron system, respectively. The exchange coupling $J = N(0) F_0^a$ can be obtained from the expression for the energy in the Landau Fermi liquid phenomenology.

This interaction leads to a high-temperature scattering time for the itinerant fluid given by

$$\frac{\hbar}{\tau} = \frac{\pi^2}{2} N(0) J^2 S(S+1)\, n_L = \frac{\pi^2}{2N(0)} (F_0^a)^2 S(S+1)\, n_L \tag{17}$$

for scattering off the spin $S = 1/2$ ferromagnetic component. This gives a resistance

$$\rho_S = \frac{m^*}{2 n_p e^2 \tau} = \pi^2 \frac{h}{4 g_v e^2} \frac{n_L}{2 n_p} (F_0^a)^2 S(S+1). \tag{18}$$

For a narrow coexistence region, the Landau parameter F_0^a should be close to the Pomeranchuk critical value of -1; hence, the conductance is solely determined by the ratio $x = n_L/n$:

$$\rho_S \approx \frac{3\pi^2}{4 g_v} \frac{h}{4e^2} \frac{x}{1-x}. \tag{19}$$

Notice that this result implies that there can be a large variation of the resistivity in the critical region of the phase diagram. The scale for resistance is the prefactor

$$\rho_S^* \approx \frac{3\pi^2}{4 g_v} \frac{h}{4e^2}, \tag{20}$$

and if near the MIT transition region the fractions of the two components are close, the ratio $x/(1-x)$ should be of order unit. Hence, the high-temperature resistance near the ''separatrix'' line should be of order ρ_S^*, which gives a conductivity of order $\sigma \approx 0.3\, \sigma_0$. This simple argument highlights the importance of considering the electron-electron interactions when calculating the resistivity of the system near the ferromagnetic instability ($F_0^a = -1$).

V. LANDAU FREE ENERGY: MEAN-FIELD PHASE DIAGRAM

In this section we discuss the transition between ferromagnetism and *p*-wave superconductivity. In the context of ruthenates it has been shown that the ferromagnetic-to-superconductor transition can be described in terms of an SO(10) model.[35,36] Here we do not take the high-symmetry approach; instead, we simply write a Landau-Ginzburg free energy that combines both *p*-wave superconductivity and ferromagnetism (we assume the system to be homogeneous),[28]

$$F[\mathbf{m},\psi,\mathbf{d},\mathbf{d}^*]=\frac{b\,m^2}{2}+\frac{m^4}{4}+a\,\psi^2+\delta\,\psi^2 m^2 + \frac{\psi^4}{2}\int\frac{d\theta}{2\pi}\{|\mathbf{d}(\theta)|^4+[i\mathbf{d}(\theta)\times\mathbf{d}^*(\theta)]^2\} - \gamma\,\psi^2\int\frac{d\theta}{2\pi}[i\mathbf{d}(\theta)\times\mathbf{d}^*(\theta)]\cdot\mathbf{m}, \tag{21}$$

where $\mathbf{m}$ is the ferromagnetic order parameter and ψ and $\mathbf{d}(\theta)$ are amplitude and vector parts used to describe the *p*-wave pairing order parameter at the Fermi surface:

$$\Psi_{\alpha\beta}(\theta)=i\psi\sum_{k=1}^{3}[\sigma_k\sigma_2]_{\alpha\beta}d_k(\theta), \tag{22}$$

with the 3D vector $\mathbf{d}(\theta)$ obeying the normalization condition

$$\int\frac{d\theta}{2\pi}|\mathbf{d}(\theta)|^2=1. \tag{23}$$

Here σ_k with $k=1,2,3$ are Pauli matrices. We assume that the coefficients δ and γ appearing in the free energy are nearly independent of electron density n, while $a=\alpha(n-n_A)$ and $b=\beta(n-n_B)$. All α, β, γ, and δ are positive.

Clearly not all fourth-order terms allowed by symmetry have been taken into account in Eq. (21). The inclusion of all terms would render the analysis extremely difficult even at the mean-field level. Thus, the choice manifest in Eq. (21) should be considered as the simplest one that reproduces the phases discussed in previous sections.

It is worth noticing that the expectation value of the Cooper pair total spin operator at a point θ of the Fermi surface is given by

$$\langle\hat{\mathbf{S}}\rangle=i\psi^2\mathbf{d}(\theta)\times\mathbf{d}^*(\theta). \tag{24}$$

Thus when $\mathbf{d}(\theta)$ is a real vector, apart from an overall phase factor, $\langle\hat{\mathbf{S}}\rangle=0$. In this case, named unitary, one can show that $\mathbf{d}(\theta)$ defines a direction along which the spin operator $\hat{\mathbf{S}}$ has eigenvalue zero.

One could in principle minimize the free energy with respect to all ten real parameters, namely, ψ, m_i, d_i, d_i^*, with $i=1,2,3$, using a Lagrange multiplier to enforce the normalization condition. Instead, we restrict the form of the vector $\mathbf{d}(\theta)$ to certain classes, following the treatment used for ^{3}He.[28] For instance, the analogous to the *B* phase or Balian-Werthamer (BW) phase of ^{3}He would correspond to the isotropic (nodeless) choice

$$\mathbf{d}(\theta)=\hat{e}_1\cos\theta+\hat{e}_2\sin\theta, \tag{25}$$

which is clearly unitary. Another possible unitary choice (also nodeless), resembling the *A* phase or Anderson-Brinkman-Morel (ABM) phase of ^{3}He is

$$\mathbf{d}(\theta)=\hat{e}_3(\cos\theta\pm i\sin\theta). \tag{26}$$

(The latter case seems to be relevant to the superconducting phase of Sr_2RuO_4.[15,16]) Here, we use neither choice, but adopt instead a parametrization that allows for a nonunitary $\mathbf{d}(\theta)$, thus $\langle\mathbf{S}\rangle\neq0$. Recall that there exists a ferromagnetic coupling between the local magnetization and the Cooper pair total spin (the last term in the free energy). A unitary order parameter would make this coupling to vanish identically. Our choice will be

$$\mathbf{d}(\theta)=\frac{z}{\sqrt{2}}(w\,\hat{e}_1+w^*\,\hat{e}_2), \tag{27}$$

where $z\equiv e^{i\theta}$ and $|w|^2=1$. One may ask how this comes about. The reasoning is simple. First, we look for a minimum energy configuration and therefore fix it to be nodeless. Second, for a 2D *p*-wave paired state, we must have $d_i(\hat{n})=D_{i\alpha}n_\alpha$, with $i=1,2,3$ and $\hat{n}=(\cos\theta,\sin\theta)$. Thus, we can write that $\mathbf{d}(\theta)=z\,\mathbf{v}_1+z^*\,\mathbf{v}_2$, with $\mathbf{v}_1$ and $\mathbf{v}_2$ two complex three-dimensional vectors that do not depend on θ, but must obey the relation $|\mathbf{v}_1|^2+|\mathbf{v}_2|^2=1$ due to the normalization condition on $\mathbf{d}(\theta)$. It is straightforward to check that the BW and ABM states correspond to the choices $\mathbf{v}_{1,2}=(\hat{e}_1\mp i\hat{e}_2)/\sqrt{2}$ and $\mathbf{v}_1=\hat{e}_3,\mathbf{v}_2=0$, respectively. On the other hand, our choice of parametrization can be obtained by setting, for instance, $\mathbf{v}_1=(w\,\hat{e}_1+w^*\hat{e}_2)/\sqrt{2}$ and $\mathbf{v}_2=0$. The choice is not unique and has to be considered as an ansatz.

We can now proceed with the minimization of the free energy. With our choice for $\mathbf{d}(\theta)$, we find that

$$|\mathbf{d}(\theta)|^2=1 \tag{28}$$

identically and

$$i\mathbf{d}(\theta)\times\mathbf{d}^*(\theta)=-\sin\phi\,\hat{e}_3, \tag{29}$$

where $w\equiv e^{i\phi/2}$. As a result,

$$F[\psi^2,\mathbf{m},\phi]=a\psi^2+\frac{\psi^4}{2}[1+\sin^2\phi]+\frac{bm^2}{2}+\frac{m^4}{4}+\delta\psi^2m^2 + \gamma\psi^2m_3\sin\phi. \tag{30}$$

(Notice that we do not need to use a Lagrange multiplier to enforce the normalization condition anymore.) The minimization equations read

$$0=a+\psi^2[1+\sin^2\phi]+\delta\,m^2+\gamma\,m_3\sin\phi, \tag{31}$$

$$0=m_{1,2}(b+m^2+2\,\delta\,\psi^2), \tag{32}$$

$$0 = m_3(b + m^2 + 2\delta\psi^2) + \gamma\psi^2 \sin\phi, \tag{33}$$

$$0 = \psi^2 \cos\phi[\psi^2 \sin\phi + \gamma m_3]. \tag{34}$$

After Eq. (32), we can set $m_{1,2}=0$ without loss of generality (thus $m=m_3$ hereafter). Equation (34), however, is satisfied only when $\psi^2=0$, or $\cos\phi=0$, or $m=-\psi^2/\gamma\sin\phi$. In our analysis of the possible solutions to these equations we will be looking for the following sequence of phases as the electron density is lowered: metal/*p*-wave pairing/*p*-wave pairing+ferromagnet/ferromagnet.

We should stress that because of the dimensionality and the symmetry of the order parameter we always have

$$\langle \Psi_{\alpha,\beta} \rangle = 0 \tag{35}$$

at any finite temperature, although $\psi \neq 0$ and $\langle \mathbf{S} \rangle \neq 0$. This is possible because, on average, $\langle \mathbf{d}(\theta) \rangle = 0$ and therefore Eq. (35) follows directly from Eq. (22).

A. *p*-wave paired phase

Let us assume that $n_A \gg n_B$. Thus, starting from a paramagnetic phase and lowering the density, we first encounter a second-order phase transition to a paired state ($\psi \neq 0$) at $n = n_A$. Indeed, setting $m=0$, we find from Eqs. (33) and (34) that $\sin\phi=0$ and

$$\psi^2 = -a = \alpha(n_A - n). \tag{36}$$

Presumably, in Si-MOSFET's, n_A is a high density, outside the range explored in the experiments that probe the MIT. Notice that since $\sin\phi=0$, $\langle \hat{\mathbf{S}} \rangle = 0$ and this *p*-wave paired phase is unitary.

B. *p*-wave paired + ferromagnetic phase coexistence

Since $n_B \ll n_A$, we expect ferromagnetism to appear at much lower densities (n_B should close to the critical density of the MIT). In this range, we may simply take $a \approx -\alpha n_A$. Looking for a solution with $\psi^2 \neq 0$, $m \neq 0$, and $\sin\phi \neq 0$ simultaneously, we obtain from Eq. (34) that

$$\sin\phi = -\gamma \frac{m}{\psi^2}. \tag{37}$$

Now $\langle \hat{\mathbf{S}} \rangle = \gamma m \hat{e}_3 \neq 0$ and consequently the pairing is nonunitary. The Cooper pair spin points along the direction of spontaneous ferromagnetization. Solving the other two equations for m and ψ^2, we find

$$m^2 = \frac{\beta(n_0 - n)}{1 - 2\delta^2} \tag{38}$$

and

$$\psi^2 = \frac{\delta\beta(n - n_1)}{1 - 2\delta^2}, \tag{39}$$

respectively, with

$$n_0 = n_B + \frac{\gamma^2}{\beta} - \frac{2\alpha\delta n_A}{\beta} \tag{40}$$

and

$$n_1 = n_B + \frac{\gamma^2}{\beta} - \frac{\alpha n_A}{\delta\beta}. \tag{41}$$

Notice that we need $\delta < 1/\sqrt{2}$ for stability. It is simple to see that m and ψ go continuously to 0 and $-\alpha n_A$, respectively, as $n \to n_0$. The transition is again of second order and n_0 represents the critical density where the ferromagnetism sets. The pairing order parameter ψ decreases as one crosses n_0.

C. Full Cooper pair spin polarization

Once ferromagnetism appears, the Cooper pair total spin tends to align itself with the ferromagnetic order parameter **m**. Full polarization of the pair spin $\langle \hat{\mathbf{S}} \rangle = 1 \cdot \hat{e}_3$ occurs when $\sin\phi=-1$ and the onset is marked by the point where $\psi^2 = \gamma m$. Combining this relation with Eqs. (38) and (39) we can find at which electron density one reaches full spin polarization by solving the following equation for $n_{\rm pol}$:

$$(n_{\rm pol} - n_1) = \frac{\gamma}{\delta}\sqrt{\frac{1-2\delta^2}{\beta}}\sqrt{n_0 - n_1}. \tag{42}$$

It is clear that the order $n_1 < n_{\rm pol} < n_0$ is obeyed, constraining full pair spin polarization to occur while ψ is still finite. For electron densities lower than $n_{\rm pol}$, the pairing order parameter continues to drop, while the ferromagnetic order parameter grows. It is important to remark that although all Cooper pairs are spin polarized, not *all* electrons in the system are spin polarized. As discussed in Sec. IV, there also exist unpaired electrons in the whole range $n_c < n < n_0$. In the coexistence region $n_c < n < n_{\rm pol}$, Eqs. (31) and (33) are reduced to

$$a + 2\psi^2 + \delta m^2 - \gamma m = 0 \tag{43}$$

and

$$m(b + m^2 + 2\delta\psi^2) - \gamma\psi^2 = 0. \tag{44}$$

D. Ferromagnetism

As the density is lowered further than $n_{\rm pol}$, the *p*-wave pairing parameter drops and the magnetization (connected to the spin polarization of unpaired electrons) grows. At the point where the pairing vanishes, Eq. (44) yields $m^2 = -b$. The critical density n_c can then be found by solving Eq. (43), namely,

$$\left(n_B - \frac{\alpha n_A}{\delta\beta}\right) = \frac{\gamma}{\delta\sqrt{\beta}}\sqrt{(n_B - n_c)}. \tag{45}$$

For $n < n_c$, we have $m = \sqrt{-b}$ and the magnetization may increase up to its limit value. Obviously, this mean-field treatment does not take into account quantum fluctuations or the interplay between spin interactions and the Anderson localization. Whether these effects happen above or below n_c

will depend on the microscopic details which are outside the scope of the Landau-Ginzburg phenomenology.

The exact form of the mean-field phase diagram resulting from the minimization of the free energy depends on the values of the coefficients α, β, δ, and γ. Besides the constraint that all coefficients should be positive, it is necessary that the inequality $n_B > \alpha n_A/\delta\beta$ hold in order for n_c to be positive (more specifically, $n_1 < n_c < n_{\rm pol}$ under this condition). This case is illustrated in Fig. 1. Moreover, if $\gamma^2 \approx 2\alpha\delta n_A$ and both δ and α are sufficiently small, it is possible to have the (experimentally accessible) critical densities n_c, $n_{\rm pol}$, and n_0 very close to each other.

VI. COMPARISON WITH EXPERIMENTS

In this section we analyze some of the recent experiments that probe the magnetic field dependence of the conductivity, and connect them to the theory we develop in this paper. Let us look separately into the parallel and perpendicular field experiments.

A. Parallel magnetic field: Magnetotransport

Recent experiments have probed the dependence of the conductance in Si-MOSFET's as a function of an in-plane magnetic field.[2–4] The experiments show that the conductance saturates beyond a field $H_{\rm sat}$. The saturation value is interpreted as the field needed to fully spin polarize the 2D electron system. Further evidence for full spin polarization is provided by an exact doubling of the period of the Shubnikov–de Haas oscillations when the experiment is done in a slightly tilted magnetic field and the in-plane component exceeds $H_{\rm sat}$.[5,6]

In the experiments of Ref. 2, it was observed that the magnetic field $H_{\rm sat}$ (B_c in the notation of Ref. 2) needed to saturate the magnetoconductance at low temperatures scaled linearly with the difference $n-n_c$ over a wide range of densities. Close to saturation, the scaled magnetoconductance curves taken at different densities followed the same function of the ratio between $H/H_{\rm sat}(n)$.

In Ref. 3, the magnetoconductance was also found to saturate beyond a value of magnetic field $H_{\rm sat}$ that depended on the density. Near the MIT, the conductivity followed the scaling

$$\sigma(H,T,n)-\sigma_{\rm sat}(n)=f\left(\frac{H}{H_{\rm sat}(n,T)}\right). \tag{46}$$

Again, similarly to Ref. 2, $H_{\rm sat} \propto n-n_0$ for a wide range of densities at low temperatures. However, very close to the transition, the density- and temperature-dependent saturation field $H_{\rm sat}$ extracted from the data behaved approximately as

$$H_{\rm sat}(n,T)=A(n)\sqrt{\Delta(n)^2+T^2}. \tag{47}$$

The parameter $A(n)$ is weakly dependent on density, being almost constant for a range of densities and increasing by about 20% near the critical density n_0. The parameter $\Delta(n)$ is fitted to a form

$$\Delta(n)=\Delta_0(n-n_0)^{\gamma}, \tag{48}$$

with $\Delta_0 \approx 2.27$ and $\gamma \approx 0.6$.

Let us now explain some of these experimental results using the theoretical framework we propose in this paper. We argued in Sec. II that, despite the large interactions, we can still apply the Landau Fermi liquid phenomenology in order to understand both the instability towards a ferromagnetic state and its precursor *p*-wave paired state. One of the Landau Fermi liquid parameters, F_0^a, crosses the Pomeranchuk's stability boundary $F_0^a=-1$ leading to the ferromagnetic instability of the 2D electron system. Near the critical density n_0, we performed a Landau expansion for the Landau Fermi liquid parameter as in Eq. (10). In Ref. 2 the degree of polarization $\xi \equiv g^* \mu_B H_\parallel / 2E_F$, where

$$g^*=\frac{g_0}{1+F_0^a} \tag{49}$$

is the effective Landé g factor renormalized by the Landau Fermi liquid parameter F_0^a.[37] Thus,

$$\xi=\frac{1}{1+F_0^a}\frac{g_0\mu_B H_\parallel}{2E_F}. \tag{50}$$

When the field $H_\parallel$ is sufficiently strong to fully spin polarize the system, we have $\xi=1$. If we neglect nonlinear terms in the susceptibility (which in principle could be important when the spin polarization is large), the saturation field in the metallic phase can be estimated from the linear response expression of Eq. (50),

$$H_{\rm sat}=\frac{2E_F}{g_0\mu_B}(1+F_0^a)=\frac{2\pi\hbar^2}{g_0 m^*\mu_B}\alpha_\parallel(n-n_0), \tag{51}$$

where we have used Eq. (10) and $g_v=2$ for the conduction band in (100) Si-MOSFET's (we have also introduced $\alpha_\parallel$ to indicate that this is the value of α when the magnetic field is parallel to the 2D electron gas). We assume that the only Landau parameter that is crossing an instability is F_0^a; the mass ratio m^*/m_b, controlled by the singlet Landau parameter F_1^s, is noncritical.[4,38] Hence, the linear dependence on the density difference $n-n_0$ of the saturation field $H_{\rm sat}$ measured experimentally follows from the Landau phenomenology above.

Using the experimental data for the $H_{\rm sat}$ dependence on $n-n_0$ from Refs. 2, 3, and 39 ($m^* \approx 1.5 m_b$), we find that the parameter $\alpha_\parallel \approx 0.6$ in the expansion of the F_0^a as a function of $\delta=(n-n_0)/n_0$.

Although we can explain the linear dependence of $H_{\rm sat}$ vs $n-n_0$ observed for a range of densities in the experiments, we would like to point out that we cannot easily explain the scaling behavior closer to the critical point, as reported in Ref. 3. One problem could be that near the MIT, with the field vanishing and with the uncertainty in the position of the ferromagnetic transition (notice again that in our theory there should be *two* quantum critical points), the exact form for the dependence of the saturation field $H_{\rm sat}$ as a function of T and n may be harder to obtain. For example, this may be the cause for the density dependent prefactor $A(n)$ in Eq. (47).

A general feature of the scaling, Eq. (47), that we can explain, however, is why the finite-temperature corrections are quadratic in T for $T \ll \Delta$ and linear in T for $T \gg \Delta$. The low-T behavior follows simply from the finite-temperature Fermi liquid susceptibility. The high-temperature behavior, on the other hand, is due to the fact that the susceptibility should, in this temperature regime, obey a Curie law.

B. Perpendicular magnetic field: Quantum Hall effect

While a parallel magnetic field couples only to the electrons via the Zeeman coupling, a perpendicular magnetic field $H_\perp$ also couples to the orbital motion and produces Landau levels. In high-density Si-MOSFET samples, far from the critical value n_c, and for small perpendicular fields, it is observed that the electronic states are localized. With increasing magnetic field the electrons tend to delocalize because of the Lorentz force, leading to the so-called floating of the extended states.[40] For sufficiently large $H_\perp$, orbital effects start to play a role, and a series of quantum Hall phases is observed in the Si-MOSFET's.[41] Integer quantum Hall plateaus are observed when an integer number of electrons becomes commensurate with the number of flux quanta $\phi_0 = ch/e$ piercing the system,

$$n = \frac{H_\perp}{\phi_0}\nu, \tag{52}$$

where ν is the magnetic filling factor (number of electrons per flux quanta). In the case of high-density Si-MOSFET's the bare Zeeman splitting is very small compared with the cyclotron energy,

$$\hbar\omega_c = \frac{\hbar e H_\perp}{m_b c}, \tag{53}$$

implying that up and down states are nearly degenerate. Moreover, the band structure dispersion contains two valleys that are almost degenerate. Equation (52) for different integers ν gives the ''Landau fan'' in the $H_\perp \times n$ plane. Whenever condition Eq. (52) is obeyed, the system sits at the middle of a Hall plateau, where the Hall resistivity ρ_{xy} becomes quantized in units of h/e^2 ($\rho_{xy} = h/\nu e^2$) while the longitudinal resistance ρ_{xx} vanishes.[41] The fact that the quantum Hall effect occurs implies that Landau levels are occupied by well-defined quasiparticles. This gives extra support to our assumption that a Fermi liquid description is correct at intermediate temperatures and magnetic fields.

For a given filling fraction ν and at lower densities, close to but higher than the zero field n_c (see Ref. 42), the quantized Hall states are lost. This occurs at some critical density $n_c(\nu)$, at which the states at the center of the Landau band localize, and the longitudinal resistance diverges as $T \to 0$. It is known[42] that $n_c(\nu)$ varies substantially along the ''Landau fan.'' In fact, it has been shown that if the electron system is spin polarized in the plane (by a parallel magnetic field) and a perpendicular field is applied, the localization effects occur already at higher densities.[43] This demonstrates the importance of spin interactions in the problem: the correlation effects are reduced when fully spin polarization is achieved, making the system closer to noninteracting and thus more sensitive to localization effects. One of the most interesting observations is that for $\nu = 4,8,12$, corresponding to filling factors within the cyclotron gaps, the localization occurs at higher densities (or magnetic fields) than for $\nu = 2,6,10$, where filling factors fall into spin gaps (recall that $g_v = 2$). This effect has not been quantitatively explained so far. We show below that it can be understood as the localization of the carriers due to the enhancement of the spin susceptibility near the zero-field critical density n_c.

As pointed out earlier, at the localization transition there is an enhancement of the magnetic susceptibility and, consequently, of the effective Landé g factor g^*. Therefore, the Zeeman splitting energy

$$E_Z(n) = g^* \mu_B H_\perp \tag{54}$$

becomes large and of order of the cyclotron energy. Here g^* can be obtained from the Fermi liquid theory [see Eq. (49)],

$$g^* = \frac{g_0}{\alpha_\perp \delta}, \tag{55}$$

which diverges at the quantum critical point. This implies that there is a level crossing between the spin-down state of the ith Landau level with the spin-up state of the $(i+1)$th Landau level. When the crossing occurs there is an excess magnetization in the system and, in particular, at low field (around 1 T), for the case of $\nu = 4$ (see below), the system becomes fully polarized. The experimental evidence is that at this point the carriers localize since the longitudinal resistivity increases as $T \to 0$.[41] This observation implies that below the critical density and in the presence of an applied magnetic field, the localized state, independent of the direction of the field, is indeed spin polarized.

To illustrate this effect, consider the situation when the magnetic field is enough to produce a $\nu = 4$ state. In this case there are equal number of up and down spins filling the first Landau level with energy $\hbar\omega_c/2$. At large densities the Zeeman energy is insignificant when compared to $\hbar\omega_c$ and the up and down spin states can be considered as degenerate. As the density is decreased along the curve defined by Eq. (52) the effective g factor increases according to the susceptibility in Eq. (12). The first Landau level for up ($\uparrow$) and down ($\downarrow$) spin states changes as a function of density as

$$E_{i=1,\uparrow}(n) = \frac{1}{2}\hbar\omega_c - \frac{1}{2}E_Z(n),$$

$$E_{i=1,\downarrow}(n) = \frac{1}{2}\hbar\omega_c + \frac{1}{2}E_Z(n), \tag{56}$$

while the second Landau level changes its energy with respect to the density as

$$E_{i=2,\uparrow}(n) = \frac{3}{2}\hbar\omega_c - \frac{1}{2}E_Z(n),$$

$$E_{i=2,\downarrow}(n) = \frac{3}{2}\hbar\omega_c + \frac{1}{2}E_Z(n). \tag{57}$$

Thus, there is a critical density n^* such that the previously empty second Landau level for spin up, $E_{i=2,\uparrow}(n^*)$, becomes degenerate with the first Landau level with spin down, $E_{i=1,\downarrow}(n^*)$. Thus, n^* is given by

$$E_Z(n^*)=\hbar\omega_c . \tag{58}$$

Using Eqs. (54) and (55) and $\mu_B=e\hbar/(2m_0c)$ we find

$$n^*\approx n_0\left(1+\frac{g_0}{2\alpha_\perp}\frac{m_b}{m_0}\right). \tag{59}$$

Using $g_0=2$, $m_b=0.2m_0$, $n_0=0.8\times10^{11}$ cm^{-2}, and $n^*=1.0\times10^{11}$ cm^{-2} (Ref. 43), we find $\alpha_\perp\approx0.8$. This value should be compared with the value of $\alpha_\parallel\approx0.6$ found in the case of a parallel magnetic field. The agreement is good and gives extra support to the idea that the localized state is indeed ferromagnetic even when the field is perpendicular to the 2D electron gas. We remark, however, that we do not expect the same estimate of $\alpha_\perp$ to be applicable to the states $\nu=8$ and $\nu=12$, since they vanish at densities sufficiently far from n_c to invalidate the use of expansion implicit in Eq. (55).

C. Classical Hall effect

An unusual experimental fact related to the behavior of the classical Hall coefficient in Si-MOSFET's can also be accounted for by our theory. It has been observed that the Hall resistance in Si-MOSFET's at low temperatures and at densities $n>n_c$ is insensitive to parallel magnetic fields ranging from zero to $H_\parallel>H_{\rm sat}$.[44] That is, there seems to exist a *single* charge carrier component in the metallic phase, for all values of $H_{\rm sat}$, instead of two independent spin-up and -down components. This is consistent with the idea that the conducting fluid present in the metallic phase is formed by *electron pairs* (bosons), instead of spin-up and -down unpaired electrons (fermions). Although the number of pairs decreases with an applied parallel field (and consequently the number of localized electrons increases; see Sec. IV), notice that the experiment is performed at *fixed* current.[45]

D. Experiment: Shot noise

The main prediction of our theory is that in the metallic phase, at very low temperatures, the system is composed by incoherent *p*-wave pairs of electrons. The bosons are incoherent because there is no long-range order in the system [see Eq. (35)] and therefore no gap in the spectrum. This situation is very similar to the case of a metallic bosonic liquid.[25] Due to the lack of phase coherence, the usual methods to measure the pair charge, such as the Josephson effect, cannot be used. Instead, the simplest way to measure the pair charge $2e$ is by making a constriction in the 2D electron density profile via external gates and measure the shot noise on the current across the constriction (for shot noise we must have $k_BT\ll eV$ where V is the voltage applied across the constriction). Since the pairs behave as independent bosonic entities, the current fluctuations should be quantized in units of the elementary charge $2e$.[46]

VII. CONCLUSIONS

In this paper we argue that one can account, on the basis of Landau Fermi liquid theory, for the recent experimental observations that the characteristic magnetic field $H_{\rm sat}$ needed to saturate the conductance in 2D Si-MOSFET's at low temperatures vanishes at a critical value of the electronic density n_0. We propose a phenomenological expansion for the Landau parameter $F_0^a=-1+\alpha(n-n_0)/n_0$ in terms of the electronic density n that drives the system ferromagnetic, through a Pomeranchuk instability, at n_0. As one approaches the instability, the spin susceptibility is greatly enhanced, requiring smaller magnetic fields to fully spin polarize the system. At the critical point an arbitrarily small magnetic field fully polarizes the system, since the susceptibility (at zero temperature) diverges.

We also analyze the effects of a perpendicular magnetic field through the system, in the quantized Hall regime, and show that the critical density for the localization of the $\nu=4$ state as compared to the $\nu=2$ state can be accounted for by considering the crossover between a spin-polarized and unpolarized state due to the enhancement of the Landé g factor. The values of the parameter α estimated separately from parallel and perpendicular field experiments agree within 20%.

In the paramagnetic side, but close to the instability, the enhanced spin fluctuations can lead to an attractive interaction in the spin triplet channel, similarly to superfluid ^{3}He. We analyze a Landau-Ginzburg mean-field theory that combines *p*-wave superconductivity and ferromagnetism, and find two quantum critical points as a function of density, n_0, where ferromagnetism begins, and n_c where *p*-wave pairing ceases. There is an intermediate range of densities where *p*-wave pairing and ferromagnetism coexist. In this range, the *p*-wave state is in a nonunitary phase.

For large enough densities, above a value n_A, the paramagnon exchange mechanism responsible for attractive interactions should cease (when, for example, $F_0^a>0$). At these densities, an Anderson insulating state, similarly to the case of noninteracting electrons, should occur. However, the presence of the *p*-wave paired state for $n_c<n<n_A$ does not rule out the possibility of a conducting phase. Since the order parameter for the *p*-wave state is a vector, no order (even algebraic) exists at finite temperature, and true superconductivity should only occur at $T=0$. In this paper we do not present any explanation why the *p*-wave pairing would lead to the conducting phase at finite T; however, we strongly believe that if this correlated state exists in the 2D electronic systems, it may provide the origin of the extended state.

It is also important to emphasize that disorder and weak interactions (high-density regime) lead to localization in 2D; however, disorder, when combined with strong, screened Coulomb interactions (low-density regime), leads to a metallic phase. In this work we propose that moderate disorder in fact enhances interaction effects in 2D, such as ferromagnetic fluctuations and the paramagnon exchange mechanism that leads to triplet pairing. Moreover, as recently pointed out by Sarachik,[47] in the low-density regime, the MIT critical density n_c in several 2D systems follows very closely a uni-

versal monotonic function of the scattering rate and vanishes in the clean limit. These results are in clear disagreement with the standard scaling theory of localization[30] in 2D, which is thus applicable only to the high-density regime. This fact, combined with the unusual magnetoresistance, points to the need to take into account electron-electron interactions for a consistent interpretation of the experimental data.

Summarizing, we propose that the metallic state close to the metal-insulator transition in the 2D electron gas problem is due to the existence of a paired *p*-wave state close to a ferromagnetic insulating phase. The pairing is generated by long-wavelength magnetic fluctuations close to the quantum critical point. We describe the pairing within Landau's Fermi liquid phenomenology and show that it provides a consistent description of the data for parallel and perpendicular magnetic fields. Moreover, we propose shot noise experiments that can test our theory.

ACKNOWLEDGMENTS

The authors would like to thank E. Demler, P. Lee, P. Phillips, S. Sachdev, N. Sandler, and M. Sigrist for enlightening discussions. We are particularly indebted to S. Kravchenko, M. Sarachik, and S. Vitkalov for thoroughly discussing their experimental data with us. Support was provided by the NSF Grant No. DMR-98-76208, the Alfred P. Sloan Foundation (C.C.), and the Brazilian agencies CNPq, FAPERJ, and PRONEX (E.R.M.). A.H.C.N. acknowledges partial support provided by a Collaborative University of California, Los Alamos (CULAR) research grant under the auspices of the U.S. Department of Energy.

*On leave from the Department of Physics, University of California, Riverside, CA 92521.

[1] E. Abrahams, S.V. Kravchenko, and M.P. Sarachik, Rev. Mod. Phys. **73**, 251 (2001).

[2] A.A. Shashkin, S.V. Kravchenko, V.T. Dolgopolov, and T.M. Klapwijk, Phys. Rev. Lett. **87**, 086801 (2001).

[3] S.A. Vitkalov, H. Zheng, K.M. Mertes, M.P. Sarachik, and T.M. Klapwijk, Phys. Rev. Lett. **87**, 086401 (2001).

[4] V.M. Pudalov, G. Brunthaler, A. Prinz, and G. Bauer, cond-mat/0004206 (unpublished).

[5] S.A. Vitkalov, H. Zheng, K.M. Mertes, M.P. Sarachik, and T.M. Klapwijk, Phys. Rev. Lett. **85**, 2164 (2000).

[6] T. Okamoto, K. Hosoya, S. Kawaji, and A. Yagi, Phys. Rev. Lett. **82**, 3875 (1999).

[7] D.J. Bergman and T.M. Rice, Solid State Commun. **23**, 59 (1977).

[8] B. Tanatar and D.M. Ceperley, Phys. Rev. B **39**, 5005 (1989).

[9] A.M. Finkelstein, Zh. Eksp. Teor. Fiz. **84**, 168 (1983) [Sov. Phys. JETP **57**, 97 (1983)]; Z. Phys. B: Condens. Matter **56**, 189 (1984).

[10] C. Castellani, C. Di Castro, P.A. Lee, and M. Ma, Phys. Rev. B **30**, 527 (1984).

[11] T.R. Kirkpatrick and D. Belitz, Phys. Rev. B **53**, 14 364 (1996); in *Electron Correlation in the Solid State*, edited by Norman H. March (Imperial College Press, London, 1999), p. 297.

[12] C. Chamon and E.R. Mucciolo, Phys. Rev. Lett. **85**, 5607 (2000).

[13] G. Benenti, G. Caldar, and D.L. Shepelyansky, Phys. Rev. Lett. **86**, 5333 (2001).

[14] D. Pines, and Ph. Nozières, *The Theory of Quantum Liquids* (Addison-Wesley, New York, 1989), Vol. 1.

[15] T.M. Rice and M. Sigrist, J. Phys.: Condens. Matter **7**, L643 (1995).

[16] G. Baskaran, Physica B **223-224**, 490 (1996).

[17] B. Spivak, Phys. Rev. B **64**, 085317 (2001).

[18] G. Baym and C. Pethick, *Landau Fermi-Liquid Theory* (Wiley, New York, 1991).

[19] R. Shankar, Rev. Mod. Phys. **66**, 129 (1994).

[20] J.M. Kosterlitz and D.J. Thouless, J. Phys. C **6**, 1181 (1973); J.M. Kosterlitz, *ibid.* **7**, 1046 (1974).

[21] P. Phillips, Y. Wan, I. Martin, S. Knysh, and D. Dalidovich, Nature (London) **395**, 253 (1998).

[22] D. Belitz and T.R. Kirkpatrick, Phys. Rev. B **46**, 8393 (1992); **58**, 8214 (1998).

[23] H.M. Jaeger, D.B. Haviland, B.G. Orr, and A.M. Goldman, Phys. Rev. B **40**, 182 (1989); H.S.J. van der Zant, W.J. Elion, L.J. Geerligs, and J.E. Mooij, *ibid.* **54**, 10 081 (1996); N. Mason and A. Kapitulnik, *ibid.* **64**, 060504 (2001).

[24] A. Kapitulnik, N. Mason, S.A. Kivelson, and S. Chakravarty, Phys. Rev. B **63**, 125322 (2001).

[25] D. Dalidovich and P. Phillips, cond-mat/0103433, Phys. Rev. B (to be published 1 October 2001); D. Dalidovich and P. Phillips, Phys. Rev. B **64**, 052507 (2001); P. Phillips and D. Dalidovich, Philos. Mag. B **81**, 847 (2001).

[26] M.-C. Cha, M.P.A. Fisher, S.M. Girvin, M. Wallin, and A.P. Young, Phys. Rev. B **44**, 6883 (1991).

[27] A.H. Castro Neto, Phys. Rev. B **51**, 3254 (1995).

[28] A.J. Leggett, Rev. Mod. Phys. **47**, 331 (1975).

[29] G.M. Eliashberg, Zh. Eksp. Teor. Fiz. **38**, 966 (1960) [Sov. Phys. JETP **11**, 696 (1960)].

[30] E. Abrahams, P.W. Anderson, D.C. Licciardello, and T.V. Ramakrishnan, Phys. Rev. Lett. **42**, 673 (1979).

[31] A.A. Abrikosov and L.P. Gorkov, Zh. Eksp. Teor. Fiz. **35**, 1558 (1958) [Sov. Phys. JETP **35**, 1090 (1959)]; **39**, 1781 (1960) [**12**, 1243 (1961)].

[32] T. Giamarchi and H.J. Schulz, Phys. Rev. B **37**, 325 (1988).

[33] S. Chakravarty, S. Kivelson, C. Nayak, and K. Voelker, Philos. Mag. B **79**, 859 (1999).

[34] S. Ilani, A. Yacoby, D. Mahalu, and H. Shtrikman, Science **292**, 1354 (2001).

[35] S. Murakami, N. Nagaosa, and M. Sigrist, Phys. Rev. Lett. **82**, 2939 (1999).

[36] If the direction of the vector order parameter **d** is fixed, the transition can be described by an SO(5) model.

[37] Naively, one could use Eq. (6) to conclude that the renormalization of the *g* factor in the Fermi liquid theory should go as $g^* = g_0/\sqrt{1+F_0^a}$. However, this is not true because the relation between magnetization and spin, which enters in the derivation of Eq. (6), is interaction independent, thus involving a *bare g* factor. Only the *g* factor entering in the linear response to an external magnetic field is renormalized by the Landau parameter F_0^a.

[38] Experiments confirm that singlet Landau parameters do not be-

have critically: In Ref. 35 it was found that the compressibility (renormalized by F_0^s in the Fermi liquid theory) is a smooth function at the MIT.

[39] V.M. Pudalov, M. Gershenson, H. Kojima, N. Butch, E.M. Dizhur, G. Brunthaler, A. Prinz, and G. Bauer, cond-mat/0105081 (unpublished).

[40] D.E. Khmelnitskii, Phys. Lett. **106A**, 182 (1984); R.B. Laughlin, Phys. Rev. Lett. **52**, 2304 (1984).

[41] S.V. Kravchenko, A.A. Shashkin, D.A. Bloore, and T.M. Klapwijk, Solid State Commun. **116**, 495 (2000).

[42] M. D'Ilorio, V.M. Pudalov, and S.G. Semenchinsky, Phys. Lett. A **150**, 422 (1990).

[43] M.R. Sakr, M. Rahimi, and S.V. Kravchenko, cond-mat/0011523 (unpublished).

[44] S.A. Vitkalov, H. Zheng, K.M. Mertes, M.P. Sarachik, and T.M. Klapwijk, Phys. Rev. B **63**, 193304 (2001).

[45] We thank S. A. Vitkalov, for pointing this out.

[46] S.-R.E. Yang, Solid State Commun. **81**, 375 (1992); N.P. Sandler, C.C. Chamon, and E. Fradkin, Phys. Rev. B **59**, 12 521 (1999).

[47] M.P. Sarachik, cond-mat/0105629 (unpublished).