Theory of combined exciton-cyclotron resonance in a two-dimensional electron gas: The strong magnetic-field regime

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I developed a theory of combined exciton-cyclotron resonance (ExCR) in a low-density two-dimensional electron gas in high magnetic fields. In the presence of excess electrons an incident photon creates an exciton and simultaneously excites one electron to higher-lying Landau levels. I derived exact ExCR selection rules that followed from the existing dynamical symmetries, magnetic translations and rotations about the magnetic field axis. The nature of the final states in ExCR is elucidated. The relation between ExCR and shake-up processes is discussed. The double-peak ExCR structure for transitions to the first electron Landau level is predicted.

Optical manifestations of many-body effects in low-dimensional electron-hole ($e-h$) systems in magnetic fields have been the focus of many experimental and theoretical studies during the past decade. Recently, such objects as, e.g., artificial atoms in quantum dots and negatively-charged excitons X^- in quantum wells have been under intense scrutiny.[1] The surprising apparent stability of the X^- (which in the dilute limit is a weakly-bound state[2] of two electrons and one hole) in the presence of excess electrons in strong magnetic fields, and the relation of this stability to a many-body collective response have been actively discussed.[1,3] Another interesting manifestation of many-body effects are shake-up processes[4] in the photoluminescence of a two-dimensional electron gas (2DEG): After the recombination of the $e-h$ pair, one electron is excited to one of the higher Landau levels (LL's). A closely-related phenomenon, combined exciton-cyclotron resonance (ExCR) also has been identified[5] in low-density 2DEG systems: Here, an incident photon creates an exciton and simultaneously excites one electron to higher LL's. These phenomena and the relation between them remain only partially understood.[1–5] A theory of ExCR has been developed[5] for weak magnetic fields, when the magnetic length $l_B=(\hbar c/eB)^{1/2}$ is much larger than the exciton Bohr radius $a_B=\epsilon\hbar^2/me^2$: $l_B\gg a_B$. The energy positions of the ExCR spectra were obtained[5] from an expansion in the unbound "exciton+electron" states. The Coulomb interactions were taken into account phenomenologically as two-particle $e-h$ excitonic corrections to the transition matrix elements. In addition, the following assumptions[5] were made: (i) a strictly-2D system, (ii) background electrons in the lowest spin-polarized $n=0\uparrow$LL with spins oriented parallel to the field, (iii) low electron density n_e, i.e., $n_e a_B^2\ll1$.

In this work, I developed the theory of ExCR for the physically interesting regime of *strong* magnetic fields, $l_B\ll a_B$. Otherwise, I adopted essentially the same assumptions (i)–(iii), as in Ref. 5. Note that in the high-B limit, the characteristic length of the problem is l_B rather than a_B, and condition (iii) can be formulated in terms of the filling factor, as $\nu_e=2\pi l_B^2 n_e\ll1$. In the limit of low-electron density, ExCR can be considered[5] to be a three-particle resonance involving a *charged system* of two electrons and one hole,

$2e-h$, in the final state. Importantly, there is a coupling of the center-of-mass and internal motions for charged $e-h$ complexes in magnetic fields.[6] In order to describe the high-field ExCR, I obtained the complete spectra of the $2e-h$ eigenstates in higher LL's with a consistent treatment of the Coulomb correlations. I exploited a recently developed scheme[7,8] for charged $e-h$ complexes in magnetic fields in which one degree of freedom is separated while all existing dynamical symmetries, rotations about the **B** axis and magnetic translations, are preserved. This allows one to establish exact ExCR selection rules that are applicable in *arbitrary* magnetic fields, to derive the ExCR oscillator strengths, and to establish the heretofore missing relation between ExCR and shake-up processes in the dilute limit.

The Hamiltonian describing the 2D $2e-h$ state in a perpendicular magnetic field $\mathbf{B}=(0,0,B)$ is given by

$$H=\sum_{i=1,2}\frac{\hat{\boldsymbol{\pi}}_{ei}^2}{2m_e}+\frac{\hat{\boldsymbol{\pi}}_h^2}{2m_h}-\sum_{i=1,2}\frac{e^2}{\epsilon|\mathbf{r}_i-\mathbf{r}_h|}+\frac{e^2}{\epsilon|\mathbf{r}_1-\mathbf{r}_2|}, \quad (1)$$

where $\hat{\boldsymbol{\pi}}_j=-i\hbar\boldsymbol{\nabla}_j-e_j\mathbf{A}(\mathbf{r}_j)/c$ are kinematic momentum operators and the symmetric gauge $\mathbf{A}=\frac{1}{2}\mathbf{B}\times\mathbf{r}$ is used; a weak exchange $e-h$ interaction, small central-cell corrections to the Coulomb potential, and the crystal anisotropy are not relevant for the present study and are neglected. The exact eigenstates of (1) form families of degenerate states; each family is labeled by the index ν that plays a role of the principal quantum number and can be discrete (bound states) or continuous (unbound states forming a continuum).[7,8] There is a macroscopic number of degenerate states in each family labeled by the discrete oscillator quantum number $k=0,1,\ldots$. This quantum number is associated with magnetic translations[6–8] and physically describes the center-of-rotation[9] of the charged complex in B. Each family starts with its parent state (PS) $|\Psi_{k=0\,M_z S_e S_h\nu}\rangle$ that has $k=0$ and the largest (for the total charge $Q<0$) possible in the family value of the total angular momentum projection M_z. Degenerate daughter states $|\Psi_{k\,M_z-k\,S_e S_h\nu}\rangle$ can be constructed iteratively[6,7] out of the PS. S_e denotes the total spin of two electrons, either $S_e=0$ (singlet states) or $S_e=1$ (triplet states); S_h is the spin state of the hole. Noninteracting electrons (or holes) in B can also be classified according to
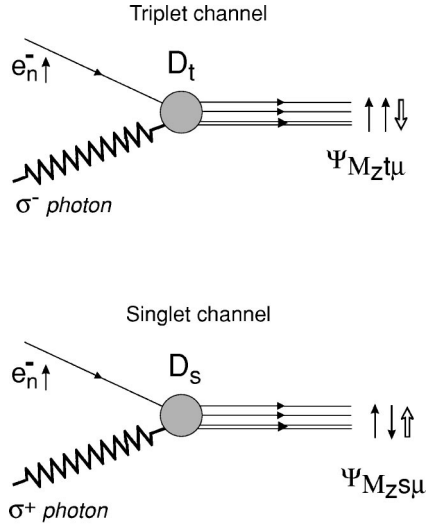
FIG. 1. Optical transitions involving the low-density 2DEG in the spin-polarized $n\uparrow$ LL. $\Psi_{M_z s(t) \nu}$ denotes charged singlet s (triplet t) $2e-h$ parent states with $k=0$. The exact selection rule is $M_z = n$.

this scheme. The corresponding e- and h- single-particle wave functions $\phi_{nm}^{(e)}(\mathbf{r}) = \phi_{nm}^{(h)*}(\mathbf{r})$ have the factored form and are well-known in the theory of the fractional quantum hall effect;[10] n is the LL quantum number, which determines the energy $\hbar\omega_{ce(h)}(n+\frac{1}{2})$ with $\omega_{ce(h)} = eB/m_{e(h)}c$, and $m = 0,1,\ldots$ is the intra-LL oscillator quantum number, the single-particle version of k. The angular momentum projections are $m_{ze} = -m_{zh} = n-m$. In the zero LL

$$\phi_{0m}^{(e)}(\mathbf{r}) = \frac{1}{\sqrt{2\pi l_B^2 m!}} \left(\frac{z^*}{\sqrt{2}l_B}\right)^m \exp\left(-\frac{\mathbf{r}^2}{4l_B^2}\right), \qquad (2)$$

where $z^* = x - iy$.

Let us derive the optical selection rules for ExCR. Interband transitions with $e-h$ pair creation can be described by the luminescence operator $\hat{\mathcal{L}}_{\text{PL}} = p_{cv}\int d\mathbf{r}\hat{\Psi}_e^\dagger(\mathbf{r})\hat{\Psi}_h^\dagger(\mathbf{r})$, where p_{cv} is the interband momentum matrix element. Consider the creation of the $e-h$ pair in the presence of the low-density 2DEG in the n-th LL (see Fig. 1). Taking into account only the three-particle correlations in the final state, the dipole transition matrix element is

$$D(\nu) = \langle \Psi_{kM_z - kS_eS_h\nu} | \hat{\mathcal{L}}_{\text{PL}} | \phi_{nm}^{(e)} \rangle. \qquad (3)$$

The oscillator quantum number is conserved[7] in dipole transitions: $m=k$. Physically this means that the center-of-rotation of charged complexes in the initial and final states must coincide. Due to the change of the Bloch wave functions, the usual selection rule $\Delta M_z = 0$ holds for the envelope functions, thus, $m_{ze} = n - m = M_z - k$. The combination of the two selection rules leads to $D(\nu) \sim \delta_{n,M_z}$, where M_z is the angular momentum projection of the PS in the νth family. Therefore, in the ExCR processes involving electrons from the nth LL, the achievable final $2e-h$ states must have $M_z = n$ and may belong to *different* final LL's. If the 2DEG is
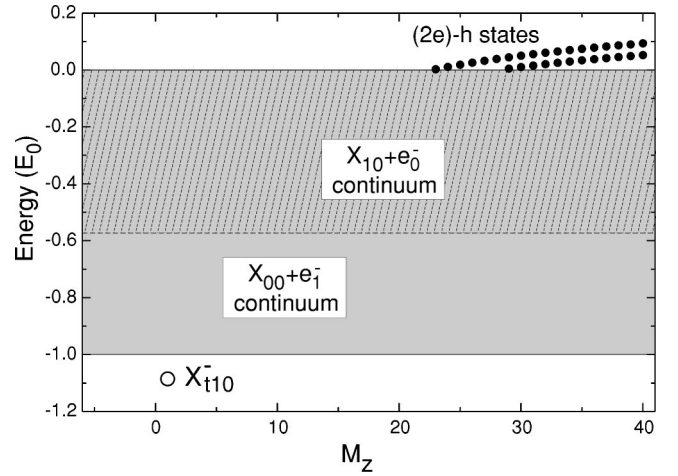

FIG. 2. The eigenspectra of the three-particle $2e-h$ triplet $S_e = 1$ states in the first electron LL. The parent states $|\Psi_{M_z t\nu}^{(10)}\rangle$ with $k=0$ are only shown. Energy is given relative to $\hbar(\frac{3}{2}\omega_{ce} + \frac{1}{2}\omega_{ch})$ in units of $E_0 = \sqrt{\pi/2}e^2/\epsilon l_B$.

spin-up $\uparrow$ polarized, the photon of $\sigma^+(\sigma^-)$ circular polarization produces the singlet (triplet) final states (see Fig. 1).

It is illuminating to compare the ExCR processes and photoluminescence (PL) of negatively-charged excitons, $X^- \to e_n^- + photon$, in which the electron is left in the nth LL in the final state. Such processes are described by the transition matrix element $D(\nu)^* = \langle \phi_{nm}^{(e)} | \hat{\mathcal{L}}_{\text{PL}}^\dagger | \Psi_{kM_z - kS_hS_h\nu} \rangle$ and can be considered as the inverse of ExCR. The exact selection rule $D(\nu)^* \sim \delta_{M_z,n}$ shows that the X^- PL transition is only possible when the electron is left in a *single and specific* LL with the number $n = M_z$. Therefore, contrary to ExCR, no various final LL's are achievable in the PL from any given X^- state. In this sense, the shake-up processes,[4] having as the final states various LL's $n=1,2,\ldots$ must be prohibited[7] in the PL of the isolated X^- in a translationally invariant system in B.

Below, I studied ExCR from the zero spin-polarized $n = 0\uparrow$ LL to the first electron LL in high magnetic fields

$$\hbar\omega_{ce}, \hbar\omega_{ch} \gg E_0 = \sqrt{\frac{\pi}{2}}\frac{e^2}{\epsilon l_B}. \qquad (4)$$

The $2e-h$ states can be obtained in this regime as an expansion in LL's. The complete orthonormal basis compatible with both axial and translational symmetries has been constructed in Ref. 8 and will be denoted here as

$$|n_R n_r n_h; kml\rangle. \qquad (5)$$

The conserved oscillator quantum number is fixed and equals k in (5) and $M_z = n_R + n_r - n_h - k - m + l$; n_h is the hole LL number, n_r and n_R are the LL numbers of the electron relative and center-of-mass motions, respectively (see Ref. 8 for details). The permutational symmetry requires that $n_r - m$ should be even (odd) for $S_e = 0 (S_e = 1)$. Fixing k in (5) amounts to summing the infinite number of e- and h- states in zero LL's. As an example, the state $|\tilde{0}\rangle = |000;000\rangle$ — the new vacuum — has the form

$$\langle \mathbf{r}_1 \mathbf{r}_2 \mathbf{r}_h | \tilde{0} \rangle = \Phi_0(\mathbf{r}_1, \mathbf{r}_2; \mathbf{r}_h) = \frac{1}{\sqrt{2}(2\pi l_B^2)^{3/2}}$$

$$\times \exp\left(-\frac{\mathbf{r}_1^2 + \mathbf{r}_2^2 + \mathbf{r}_h^2 - (z_1^* + z_2^*)z_h}{4l_B^2} \right) \quad (6)$$

and is a coherent[8] $e-h$ state [cf. Eq. (2)]. In what follows I consider only the PS's $|\Psi_{M_z s(t)\nu}\rangle$, where the quantum numbers $k=0$ and S_h are omitted for brevity and $s(t)$ denotes the singlet $S_e=0$ (triplet $S_e=1$) electron spin state.

Neglecting mixing between LL's (the high-field limit), the triplet $2e-h$ states in the first electron and zero hole LL, $|\Psi_{M_z t\nu}^{(10)}\rangle$, can be obtained as the expansion

$$|\Psi_{M_z t\nu}^{(10)}\rangle = \sum_{l=0}^{\infty}\sum_{m=0}^{\infty} \alpha_{M_z t\nu}(2m,l)|010;02ml\rangle$$

$$+ \sum_{l=0}^{\infty}\sum_{m=0}^{\infty} \beta_{M_z t\nu}(2m+1,l)|100;02m+1l\rangle, \quad (7)$$

where expansion coefficients $\alpha_{M_z t\nu}(2m,l) \sim \delta_{M_z,l+1-2m}$ and $\beta_{M_z t\nu}(2m+1,l) \sim \delta_{M_z,l-2m}$. The Coulomb matrix elements of the Hamiltonian(1) are calculated analytically[8] and the eigenspectra are obtained by numerical diagonalization of finite matrices of order $2-5\times 10^2$. Such finite-size calculations provide very high accuracy for bound X^- states and are also capable of reproducing the structure of the three-particle continuum.[8] The singlet states $|\Psi_{M_z s\nu}^{(10)}\rangle$ are obtained by using a similar procedure.

The calculated triplet $2e-h$ eigenspectra are presented in Fig. 2. Filled dots above free LL's show positions of the excited bound three-particle states, denoted as $(2e)-h$. These states originate from the excited states of two electrons that are bound in 2D because of the confining effect of the magnetic field. Bound $(2e)-h$ states appear in the spectrum for relatively large values of M_z, when the hole can be at a sufficiently large distance from the two electrons. There is also exactly one low-lying triplet bound state — the negatively-charged magnetoexciton (MX) X_{t10}^- that has $M_z=1$ and binding energy $0.086E_0$.[7,8] The shaded area of width E_0 in Fig. 2 corresponds to the three-particle continuum formed by the neutral MX X_{00} (e and h in their zero LL's) with the second electron in a scattering state in the first LL. The hatched area corresponds to the second overlapping band formed by the states of the neutral MX X_{10} (e in the first and h in the zero LL) with the second electron in a scattering state in the zero LL. The lower continuum edge lies at the X_{10} ground-state energy $-0.574E_0$, which, for the isolated MX, is achieved at a finite center-of-mass momentum $|\mathbf{K}|l_B \simeq 1.19$.[8,11] As a result, the density of X_{10} states has at this energy an inverse square-root van Hove singularity in 2D. The continuum of the singlet $2e-h$ states has qualitatively the same structure; there are also bound singlet $(2e)-h$ states above free LL's (not shown). There are, however, no low-lying bound singlet X_{s10}^- states.

Due to the selection rule $D(\nu) \sim \delta_{n,M_z}$, the $2e-h$ states with $M_z=0$ are active in the ExCR transitions from the $n=0$ LL. As a result, the ExCR transition $e_0^- + photon \to X_{t10}^-$ to the bound negatively-charged triplet MX is prohibited. ExCR transitions to the bound excited triplet and singlet $(2e)-h$ states are also prohibited: all these states have large $M_z > 0$. Therefore, the ExCR transitions from zero to the first electron LL can only go to the *continuum*.[12]

All ExCR transitions are only due to LL mixing. In order to calculate the ExCR dipole transition matrix elements, I went beyond the high-field limit and admix to (7) the triplet $2e-h$ states in zero LL's $\sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\gamma_{M_z t\nu}(2m+1,l)|000;02m+1l\rangle$; expansion coefficients $\gamma_{M_z t\nu}(2m+1,l) \sim \delta_{M_z,l-2m-1}$ and $\gamma_{M_z t\nu} \sim E_0/\hbar\omega_{ce} \sim l_B/a_B$. The coordinate representation has the form

$$\Phi_{ml}(\mathbf{r}_1, \mathbf{r}_2; \mathbf{r}_h) = \langle \mathbf{r}_1 \mathbf{r}_2 \mathbf{r}_h | 000;0ml \rangle$$

$$= \frac{1}{\sqrt{l!m!}}\left(\frac{z_1^* - z_2^*}{2l_B} \right)^m \left(\frac{z_h}{2l_B} \right)^l \Phi_0(\mathbf{r}_1, \mathbf{r}_2; \mathbf{r}_h). \quad (8)$$

A similar procedure is used for the singlet $|\Psi_{M_z s\nu}^{(10)}\rangle$ states. The dipole transition matrix element is given by

$$D_t(\nu) = \langle \Psi_{M_z=0 t\nu} | \hat{\mathcal{L}}_{PL} | \phi_{00}^{(e)} \rangle$$

$$= p_{cv}\sum_{m=0}^{\infty} \gamma_{M_z=0 t\nu}(2m+1,2m+1)\mathcal{D}_{2m+1}, \quad (9)$$

$$\mathcal{D}_p \equiv \int d\mathbf{r}_1 \int d\mathbf{r}_2 \Phi_{pp}^*(\mathbf{r}_1,\mathbf{r}_2;\mathbf{r}_2)\phi_{00}^{(e)}(\mathbf{r}_1). \quad (10)$$

Using Eqs. (2), (6), and (8) and performing in (10) the transformation $z_2^* \to z_2^* - z_1^*$ in the complex (z_2, z_2^*) plane, it can be shown that the overlap integral $\mathcal{D}_p = (-1)^p\sqrt{2}$. The intensity of the ExCR transitions involving the $2e-h$ states with eigenenergies E_ν is

$$I_{s(t)}^{ExCR}(\omega) \sim \nu_e \sum_\nu |D_{s(t)}(\nu)|^2 \delta(\hbar\omega - E_\nu). \quad (11)$$

The calculated dipole matrix elements of transitions in two circular polarizations σ^+ and σ^- that terminate, respectively, in the continuum of final singlet $|\Psi_{M_z=0 s\nu}^{(10)}\rangle$ and triplet $|\Psi_{M_z=0 t\nu}^{(10)}\rangle$ states are shown in Fig. 3. These ExCR transitions require the extra photon energy $\sim\hbar\omega_{ce}$ relative to the fundamental band-gap absorption $E_{gap}(B)$ with final states in the $n=0$ LL. I neglect the initial and final state exchange self-energy corrections[10] $\sim\nu_e E_0 \ll E_0$ that partly compensate each other. The low-energy ExCR peak in Fig. 3 corresponds to the transitions $e_0^- + photon \to X_{00} + e_1^-$ to the lower edge of the continuum formed by the neutral $1s$ MX in zero LL, X_{00}, and the electron in a scattering state in the first LL. Such ExCR transitions have been theoretically identified and discussed in Ref. 5 in the low-B limit.
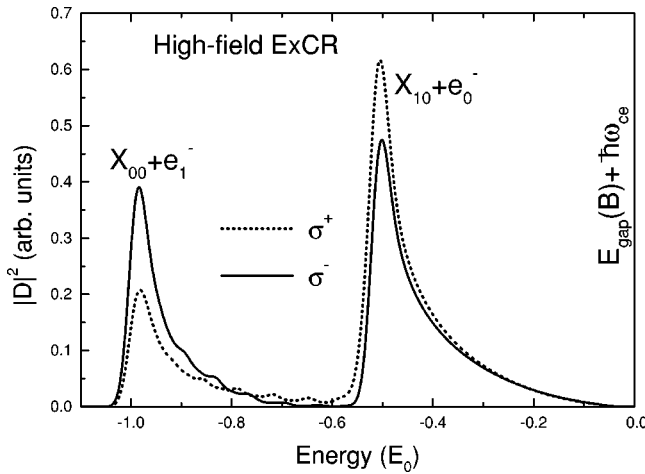
FIG. 3. Energies and dipole matrix elements of the ExCR transitions from the spin-polarized zero $0\uparrow$ LL to the first electron LL in two circular polarizations $\sigma^\pm$. The representative case $E_0 = \hbar\omega_{ce}$ is shown. Spectra have been convoluted with a Gaussian of the $0.015E_0$ width (~ 0.3 meV for the parameters indicated).

Surprisingly, the present theory predicts in high fields *an additional strong feature*—the second, higher-lying, peak in the ExCR. If this peak were simply due to the transitions $e_0^- + photon \rightarrow X_{10} + e_0^-$ to a final state consisting of a $2p^+$ neutral MX X_{10}, which is dark in PL, and the electron in a scattering state, the ExCR transition would be extremely weak and proportional to the inverse area of the system. The second ExCR peak may be associated with a formation of a quasibound three-particle state (resonance) within the $X_{10} + e_0^-$ continuum: The amplitude of finding all three particles in the same region of real space is large for a well-defined resonance, so that the overlap integral [see Eqs. (9) and (10)] may be large too. The existence of the $2e-h$ resonances is physically plausible because of the 2D van Hove singularity in the X_{10} density of states. A well-developed Fano-resonance has been revealed recently[8] in the spectra of intraband internal X^- transitions, clear evidence supporting the existence of quasibound $2e-h$ states. Note that the predicted double-peak structure in the *interband* ExCR transitions physically resembles the predicted[7] and observed[13] double-peak structure in the *intraband* internal transitions of charged MX's X^-. Although the initial states are different, both types of spectroscopies probe, in this case, the final states in the three-particle $2e-h$ continuum: The ExCR final states, as well as the final states in the internal triplet X_t^- transitions, have $M_z = 0$, while the final states in the internal singlet X_s^- transitions have $M_z = 1$.[7,13] This suggests that the ExCR transitions to higher LL's, may have multiple-peaks and complicated lineshapes because of more involved structures of the continua.

The intensity of the ExCR lower peak is larger in the σ^- polarization (Fig. 3), which is consistent with experiment.[5] The present theory predicts that the intensity of the second ExCR peak will show the opposite dependence: It should be larger in the σ^+ polarization. This polarization dependence is due to different $e-e$ correlations in the final singlet (σ^+ polarization) and triplet (σ^- polarization) $2e-h$ states: The oscillator strength is transferred to the higher lying peak when the final singlet $2e-h$ states—characterized by a larger $e-e$ repulsion—are involved. In agreement with Ref. 5, the present theory shows that (1) the ExCR peaks have intrinsic finite linewidths, in high fields $\sim 0.15E_0$ (~ 2.6 meV at $B \simeq 10$ T corresponding to Fig. 3), and have asymmetric lineshapes with high-energy tails; (2) the ExCR transitions are because of LL mixing and, therefore, ExCR is suppressed in strong fields as $\nu_e|D|^2 \sim n_e l_B^2 (l_B/a_B)^2 \sim B^{-2}$.

In conclusion, a magneto-optical phenomenon, ExCR transitions in low-density 2DEG systems,[5] has been considered theoretically for high magnetic fields. The developed formalism allows a consistent treatment of the final state $e-h$ and $e-e$ Coulomb interactions. The features of the high-field ExCR, in particular, the double-peak structure of the transitions to the first electron Landau level have been predicted.

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