General analytical treatment of optics in layered structures: Application to magneto-optics

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We have derived compact and interpretable analytical expressions to describe the magneto-optics in layered structures for all orientations of magnetization and incident angle. In our approach, the multilayer system is considered as nonmagnetic and the magneto-optical effect is described by an induced electrical polarization. The electromagnetic waves radiated by this polarization are calculated via a propagative treatment and are also shown to directly derive from the Lorentz reciprocity theorem. The expressions of the magneto-optical components of the fields transmitted and reflected in the external media are easily interpretable. Only three relevant quantities are involved: the exciting field, the magnetization, and an extraction vector. The practical calculation is very simple in the framework of the first Born approximation as the 4×4 matrix formalism is replaced by a 2×2 matrix resolution. The whole approach is not restricted to magneto-optics and the case of a variety of other systems exhibiting weak induced polarizations originating from anisotropy, bianisotropy, nonlinearity, or inhomogeneity is treated. Higher-order approximations are also discussed and an analytical approximation for large induced polarizations in thin layers is derived.

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I. INTRODUCTION

The recent progress in growth techniques enables the elaboration of layered magnetic structures with magnetic characteristics very different from the ones of bulk materials. This opens a wide field of new experimental situations and technological applications. In particular magneto-optical (MO) properties may be tuned but often to the expense of complexity of the film structure. In order to get a physical insight into the optics of these materials an analytical, simple, and interpretable description is mandatory.

Up to now, the usual theoretical studies on light propagation in anisotropic layered structures have been developed in the framework of a 4×4 matrix formalism as a generalization of the isotropic case described in the pioneering 2×2 matrix analysis of Abelès.[1] Among these approaches the case of magneto-optics in multilayer structures was treated by Smith.[2] Then, a very similar calculation, originally devoted to optics in linear birefringent media,[3] was extended to arbitrary anisotropic materials.[4] Most of the papers, devoted to the particular case of the first-order MO effects in multilayers,[4–6] extensively use such a 4×4 matrix formalism. Unfortunately, it is commonly admitted that these procedures lead to an ‘‘algebraic morass’’ [5] and provide ‘‘complicated’’ [5] or ‘‘cumbersome’’ [4] expressions of the measurable MO quantities. Their practical utilization is, even in simple cases, mainly restricted to computer calculation.

One of the main disadvantages of the usual 4×4 matrix method is the calculation of the propagation eigenmodes in the magnetic layers. Only in very particular geometries (the polar MO effects in normal incidence) this calculation is significantly simplified as it reduces to a 2×2 matrix analysis.[7] However, alternative approaches (not specifically dedicated to magneto-optics), valid for any geometrical and magnetic configuration, avoid the calculation of the eigenmodes by perturbative treatments of the propagation in the anisotropic layers.[8,9] However, these methods do not exempt from the tedious 4×4 matrix calculation and the final expression of the solution remains complicated.

In scattering problems,[10–13] the optical response of matter is often described in terms of induced electrical or magnetic polarizations as sources of electromagnetic radiation.[10,11] This approach gives an insight into the physics and separates the problem into two parts: first the determination of the sources induced by the exciting fields, then the calculation of the radiated fields. No related method was developed for layered structures except in nonlinear optics.[14–16] However, it has been recently evidenced in the case of a simple magnetic layered structure that the linear magneto-optical response only depends on three interpretable quantities: the exciting field, the magnetization, and an extraction factor.[17]

In the present paper we develop, in the framework of the macroscopic Maxwell's equations,[18] a general treatment of weak optical effects in multilayers with a special focus on magneto-optics. Our final expressions of the reflected, transmitted, or scattered fields (i.e., of the measurable quantities) are compact and easily interpretable. In Sec. II, we recall the main stages of the usual 4×4 matrix methods and give definitions used in the subsequent calculations. In Sec. III, we derive the fields generated by a distribution of electrical polarization located inside a multilayer. In Sec. IV, the first Born approximation is used to obtain a simple expression of the electrical polarization originating from a weak anisotropy in a homogeneous layer. The expressions of linear magneto-optical effects in multilayers are deduced for any magnetization direction, incident angle, incident light polarization, and number of magnetic layers. In Sec. V, this approach is ex-

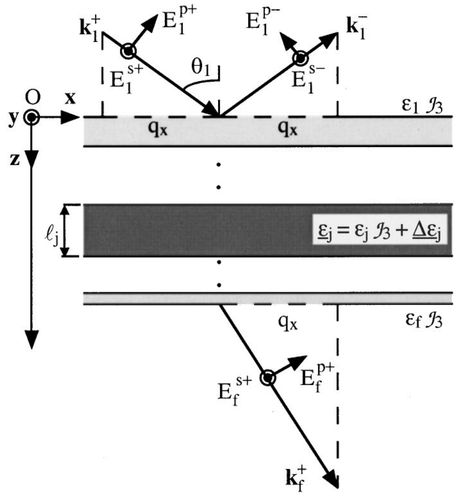

FIG. 1. Multilayer system illuminated from medium 1 by *s*- or *p*-polarized waves of complex amplitudes $E_1^{s,p+}$ at an incident angle θ_1. The in-plane wave-vector component $q_x = n_1 \sin\theta_1$ is conserved through the multilayer. The media 1 and *f* are isotropic of dielectric constant $\varepsilon_{1,f}$ and any layer *j* of thickness l_j is characterized by its dielectric tensor $\underline{\varepsilon}_j$. The complex amplitudes of the *s*- and *p*-polarized reflected and transmitted waves are $E_1^{s,p-}$ and $E_f^{s,p+}$.

tended to weak induced polarizations originating from inhomogeneity (such as magnetic domains) and nonlinearity. In Sec. VI, the iterative procedure for higher-order expansion (beyond the first Born approximation) is indicated and a simple approximation of the full solution is obtained which applies to large induced polarization in thin layers. In Sec. VII, we show that our compact expressions directly derive from the Lorentz reciprocity principle. It is then straightforward to treat the chirality or bianisotropy as induced electrical and magnetic polarizations. Concluding remarks are given in Sec. VIII.

II. STANDARD 4×4 MATRIX FORMALISM

A. Equation of propagation

Let us consider an anisotropic multilayer structure like in Fig. 1 sandwiched between two semi-infinite isotropic media 1 and *f* of dielectric constant $\varepsilon_1 = n_1^2$ and $\varepsilon_f = n_f^2$. Each layer *j* is homogeneous and characterized by its thickness l_j and its dielectric tensor $\underline{\varepsilon}_j = \varepsilon_j \underline{I} + \underline{\Delta\varepsilon}_j$, where $\underline{I}$ is the 3×3 unit matrix and $\underline{\Delta\varepsilon}_j$ the anisotropic part of the tensor. In medium *j*, a plane wave of frequency ω and wave vector $\mathbf{k}_j$ is represented by $\{\mathbf{E}_j, \mathbf{B}_j\}\exp[i(\mathbf{k}_j\cdot\mathbf{r} - \omega t)]$. The electric and magnetic complex vectors $\{\mathbf{E}_j, \mathbf{B}_j\}$ define its amplitude and polarization. In the external medium 1, a plane wave associated with $\mathbf{k}_1^+ = q_x\mathbf{x} + \kappa_1\mathbf{z}$ propagates in the *xOz* plane and is incident at the angle θ_1 from the normal *Oz* to the surface ($\kappa_1 > 0$ and $q_x = k_1 \sin\theta_1$). The illumination of the structure induces reflected and transmitted waves in media 1 and *f*, respectively, associated with $\mathbf{k}_1^-$ and $\mathbf{k}_f^+$.

From Snell-Descartes law, the components of the wave vectors parallel to the interfaces are conserved through the multilayer. The isotropy of the external media implies $k_{1,f}^2 = k_0^2\varepsilon_{1,f} = q_x^2 + \kappa_{1,f}^2$ and $\mathbf{k}_{1,f}\cdot\mathbf{E}_{1,f} = 0$. Consequently, $\mathbf{k}_1^- = q_x\mathbf{x} - \kappa_1\mathbf{z}$, $\mathbf{k}_f^+ = q_x\mathbf{x} + \kappa_f\mathbf{z}$, and the fields of the waves transverse to the directions of propagation can be linearly decomposed on the *s*- and *p*-polarization directions, respectively, perpendicular and parallel to the plane of incidence. For an incident wave defined by the complex amplitudes E_1^{s+} and E_1^{p+}, the reflected and transmitted waves are then respectively given by E_1^{s-}, E_1^{p-} and E_f^{s+}, E_f^{p+}. These four unknown quantities are generally written as the solutions of the following set of four linear equations:

$$\mathbf{V}_f = \begin{Bmatrix} E_f^{s+} \\ 0 \\ E_f^{p+} \\ 0 \end{Bmatrix} = \mathcal{M}_{f1} \begin{Bmatrix} E_1^{s+} \\ E_1^{s-} \\ E_1^{p+} \\ E_1^{p-} \end{Bmatrix} = \mathcal{M}_{f1}\mathbf{V}_1. \tag{2.1}$$

The four-component vectors $\mathbf{V}_1$ and $\mathbf{V}_f$ describe the fields in media 1 and *f* at the interface with the structure and the 4×4 matrix $\mathcal{M}_{f1}$ expresses the propagation throughout the whole system. In the 4×4 matrix methods[2–5,7,9] $\mathcal{M}_{f1}$ is first derived and then Eq. (2.1) is solved.

In some cases, it may be useful to decompose Eq. (2.1) into two sets of two linear equations which define the matrices of transmission $\underline{T}_{1f}$ and reflection $\underline{R}_{1f}$ as

$$\begin{pmatrix} E_f^{s+} \\ E_f^{p+} \end{pmatrix} = \begin{bmatrix} T_{1f}^{ss} & T_{1f}^{sp} \\ T_{1f}^{ps} & T_{1f}^{pp} \end{bmatrix} \begin{pmatrix} E_1^{s+} \\ E_1^{p+} \end{pmatrix}, \tag{2.2a}$$

$$\begin{pmatrix} E_1^{s-} \\ E_1^{p-} \end{pmatrix} = \begin{bmatrix} R_{1f}^{ss} & R_{1f}^{sp} \\ R_{1f}^{ps} & R_{1f}^{pp} \end{bmatrix} \begin{pmatrix} E_1^{s+} \\ E_1^{p+} \end{pmatrix}. \tag{2.2b}$$

B. Derivation of the 4×4 propagation matrix

In the standard 4×4 matrix formalism,[2–5,7] the electromagnetic field in a layer *j* is represented by a combination of the plane-wave solutions of the wave equation which, from the Maxwell's equations, takes the form

$$(k_0^2\underline{\varepsilon}_j - k_j^2)\mathbf{E}_j + (\mathbf{k}_j\cdot\mathbf{E}_j)\mathbf{k}_j = 0, \tag{2.3}$$

where $k_0 = \omega/c$ is the wave number in vacuum. Since from Snell-Descartes law $\mathbf{k}_j = q_x\mathbf{x} + \kappa_j\mathbf{z}$, the unknown quantities are κ_j and $\mathbf{E}_j$ while $\mathbf{B}_j$ is derived from $c\mathbf{B}_j = \mathbf{k}_j \wedge \mathbf{E}_j / k_0$. The resolution of Eq. (2.3) gives in the more general situation a quartic equation for κ_j which yields four complex eigenvalues κ_j^σ for σ=1, 2, 3, or 4, associated to four complex polarization eigenvectors $\mathbf{u}_j^\sigma$ that we define of unit amplitude. The electromagnetic field can be decomposed as a linear combination of these waves. If we define the E_j^σ as their complex amplitudes and write the fields as $\{\mathbf{E}_j(z), \mathbf{B}_j(z)\}\exp[i(q_x x - \omega t)]$, where the altitude *z* is defined as respectively equal to zero and l_j at the upper and lower faces of the layer, we obtain

$$\mathbf{E}_j(z) = \sum_{\sigma=1}^{4} E_j^\sigma(z)\mathbf{u}_j^\sigma = \sum_{\sigma=1}^{4} E_j^\sigma e^{i\kappa_j^\sigma z}\mathbf{u}_j^\sigma, \tag{2.4}$$

$$c\mathbf{B}_j(z)=\sum_{\sigma=1}^{4} E_j^{\sigma}(z)\mathbf{k}_j^{\sigma}\wedge\mathbf{u}_j^{\sigma}/k_0. \quad (2.5)$$

Consequently, the electric-field distribution within each homogeneous anisotropic layer can be expressed by the four complex amplitudes $E_j^{\sigma}(z)$ which constitute the components of the vector $\mathbf{V}_j(z)$ defined by

$$\mathbf{V}_j(z)=\begin{pmatrix} E_j^1(z)\\ E_j^2(z)\\ E_j^3(z)\\ E_j^4(z)\end{pmatrix}=\mathcal{D}_j(z-z')\mathbf{V}_j(z'). \quad (2.6)$$

The 4×4 propagation matrix $\mathcal{D}_j(z-z')$ only contains diagonal terms $\exp[i\kappa_j^{\sigma}z]$ [see Eq. (2.4)].

At the interface between media j and $j+1$, the continuity of the in-plane components of the electromagnetic field can be written as $\mathbf{F}_{j+1}(0)=\mathbf{F}_j(l_j)$ with the following definition:

$$\mathbf{F}_j(z)=\begin{pmatrix} E_j^y(z)\\ cB_j^x(z)\\ cB_j^y(z)\\ E_j^x(z)\end{pmatrix}=\mathcal{A}_j\mathbf{V}_j(z). \quad (2.7)$$

$\mathcal{A}_j$ is a 4×4 matrix deduced from Eqs. (2.4) and (2.5) and the x and y components of $\mathbf{u}_j^{\sigma}$. From Eqs. (2.6) and (2.7), considering the propagations through every layer and the continuity relations at every interface, we finally obtain the propagation matrix of the whole structure as

$$\mathcal{M}_{f1}=\mathcal{A}_f^{-1}\prod_{j=f-1}^{2}(\mathcal{A}_j\mathcal{D}_j\mathcal{A}_j^{-1})\mathcal{A}_1, \quad (2.8)$$

where $\mathcal{D}_j=\mathcal{D}_j(l_j)$. From this relation one deduces the set of four linear equations (2.1) required to obtain the four unknown complex amplitudes E_1^{s-}, E_1^{p-}, E_f^{s+}, and E_f^{p+} of the reflected and transmitted waves.

C. Perturbative approach of the propagation in anisotropic media

The standard 4×4 *matrix* treatment of *weak anisotropy* in multilayers usually begins by approximating the propagation in the anisotropic layer. This is tediously performed by calculating approximations of the eigenvalues κ_j^{σ} (which are solutions of a quartic equation) and of the eigenvectors $\mathbf{u}_j^{\sigma}$ in order to derive $\mathcal{D}_j$ and $\mathcal{A}_j$. These successive calculations are in fact not needed. Indeed, as shown by Eq. (2.8), the propagation in the anisotropic layer j is fully described by the particular product of matrices $\mathcal{N}_j=\mathcal{A}_j\mathcal{D}_j\mathcal{A}_j^{-1}$ which can be directly obtained by a perturbation treatment in the framework of the Berreman formalism.[9,19] However, whatever the method used for calculating the propagation in the anisotropic layer, the 4×4 matrix methods always lead to a complicated expression of $\mathcal{M}_{f1}$. Then, the resolution of Eq. (2.1) generally requires computer calculation to provide the optical response of the structure.[2–5,7,9,19]

In comparison, the case of an *isotropic* layered structure, solved by a *2×2 matrix* resolution,[1] is much more simple. In the following, we show that the calculation is largely reduced and the result particularly transparent when the weak anisotropy (such as magneto-optics) is considered as a perturbation added to the isotropic case (where the anisotropy is supposed equal to zero).

III. RADIATED FIELDS FROM AN EMBEDDED POLARIZATION

A. "Internal" polarization and radiated fields

The macroscopic Maxwell's equations are written in the layer m as

$$\mathbf{rot}\ \mathbf{E}_m(\mathbf{r},t)=-\frac{\partial\mathbf{B}_m(\mathbf{r},t)}{\partial t}, \quad (3.1a)$$

$$\mathbf{rot}\ \mathbf{B}_m(\mathbf{r},t)=\mu_0\frac{\partial\mathbf{D}_m(\mathbf{r},t)}{\partial t}, \quad (3.1b)$$

and the material equation as

$$\mathbf{D}_m(\mathbf{r},t)=\varepsilon_0\varepsilon_m\mathbf{E}_m(\mathbf{r},t)+\Delta\mathbf{P}_m(\mathbf{r},t). \quad (3.1c)$$

In this formulation, a specific optical property (anisotropy, inhomogeneity or nonlinearity) is described by an internal distribution of polarization $\Delta\mathbf{P}_m(\mathbf{r},t)$ induced by the interaction of the electromagnetic field with matter. This evidences and separates the considered physical effect from the (isotropic, homogeneous, and linear) response defined by the dielectric constant ε_m.

If the optical properties of all the layers are written as in Eq. (3.1c), the structure is described as a set of isotropic, homogeneous, and linear layers, that we call the unperturbed system, in which several induced distributions of polarization $\Delta\mathbf{P}_m(\mathbf{r},t)$ radiate. The unperturbed system being linear, the fields in any layer j can be written

$$\begin{Bmatrix}\mathbf{E}_j(\mathbf{r},t)\\ \mathbf{B}_j(\mathbf{r},t)\end{Bmatrix}=\begin{Bmatrix}\mathbf{E}_j^0(\mathbf{r},t)\\ \mathbf{B}_j^0(\mathbf{r},t)\end{Bmatrix}+\begin{Bmatrix}\Delta\mathbf{E}_j(\mathbf{r},t)\\ \Delta\mathbf{B}_j(\mathbf{r},t)\end{Bmatrix}. \quad (3.2)$$

The unperturbed fields $\{\mathbf{E}_j^0(\mathbf{r},t),\mathbf{B}_j^0(\mathbf{r},t)\}$ are obtained by taking all the polarization distributions equal to zero. The fields $\Delta\mathbf{E}_j(\mathbf{r},t)$ and $\Delta\mathbf{B}_j(\mathbf{r},t)$, radiated by all the polarization distributions in the unperturbed system, are equal to the linear sum of the fields radiated independently by each $\Delta\mathbf{P}_m(\mathbf{r},t)$ (the others being taken equal to zero).

B. Unperturbed fields and dimensionless quantities

In any layer j of the unperturbed system, the wave equation (2.3) reduces to $\mathbf{k}_j\cdot\mathbf{E}_j^0=0$ and to $\kappa_j^2=k_0^2\varepsilon_j-q_x^2$ which yields two opposite solutions. We define $\kappa_j=\kappa_j^+=-\kappa_j^-$ the solution with positive real and imaginary parts, so that the superscripts $-$ and $+$ indicate the upwards and downwards directions of propagation of the waves. We choose $\kappa_j^{1,3}=-\kappa_j^{2,4}=\kappa_j$ and decompose the transverse fields in Eqs. (2.4) and (2.5) on the polarization eigenvectors $\mathbf{u}_j^{1,2}=\mathbf{u}_j^{s+,-}=\mathbf{y}$ and $\mathbf{u}_j^{3,4}=\mathbf{u}_j^{p+,-}=\mathbf{y}\wedge\mathbf{k}_j^{+,-}/k_j$, where the superscripts s

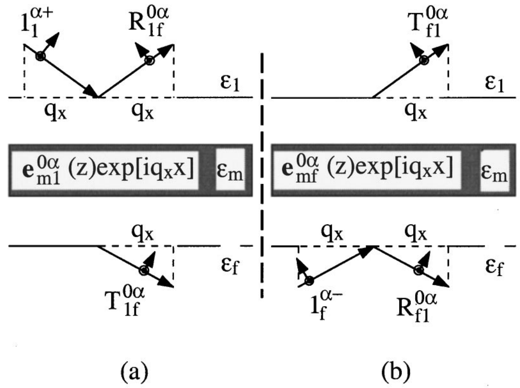

FIG. 2. Illuminations, from media 1 (a) or f (b) by unit α-polarized plane waves associated to the same q_x, induce the normalized unperturbed fields $\mathbf{e}^{0\alpha}_{m1,f}(z)\exp[iq_x x]$ in the layer m of the isotropic unperturbed multilayer structure.

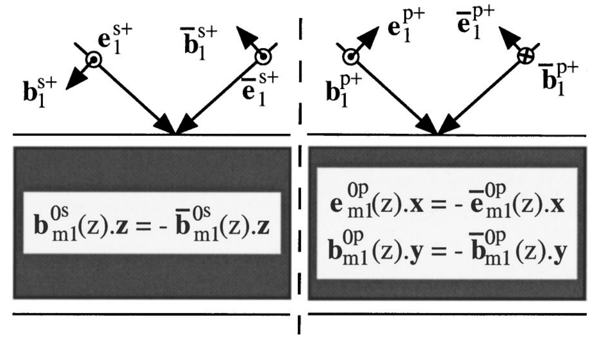

FIG. 3. Origin of the symmetrical relations between $\{\mathbf{e}^{0\alpha}_{m1}(z),\mathbf{b}^{0\alpha}_{m1}(z)\}\exp[iq_x x]$ and $\{\overline{\mathbf{e}}^{0\alpha}_{m1}(z),\overline{\mathbf{b}}^{0\alpha}_{m1}(z)\}\exp[-iq_x x]$.

and p refer to the usual definition of the s and p polarizations of the waves. All the matrices $\mathcal{D}_j(z)$, $\mathcal{A}_j$, $\mathcal{T}_j$, and $\mathcal{M}_{f1}$ defined in Sec. II become 2×2 block diagonal, each block being written $\mathcal{D}^{\alpha}_j(z)$, $\mathcal{A}^{\alpha}_j$, $\mathcal{N}^{\alpha}_j$, and $\mathcal{M}^{\alpha}_{f1}$ (see Appendix A), where α stands for s or p. Equation (2.1) reduces to two independent sets of two linear equations with the two unknown quantities $E_1^{0\alpha-}$ and $E_f^{0\alpha+}$:

$$\mathbf{V}_f^{0\alpha}=\begin{pmatrix} E_f^{0\alpha+}\\ 0\end{pmatrix}=\mathcal{M}^{\alpha}_{f1}\begin{pmatrix} E_1^{0\alpha+}\\ E_1^{0\alpha-}\end{pmatrix}=\mathcal{M}^{\alpha}_{f1}\mathbf{V}_1^{0\alpha}. \tag{3.3}$$

The 4×4 matrix treatment reduces to a 2×2 matrix formulation where the propagation of the s- and p-polarized waves are uncoupled. This property remains in the F representation when defining in any layer j:

$$\mathbf{F}_m^{0s}(z)=\begin{pmatrix} E_m^{0y}(z)\\ cB_m^{0x}(z)\end{pmatrix},\quad \mathbf{F}_m^{0p}(z)=\begin{pmatrix} cB_m^{0y}(z)\\ E_m^{0x}(z)\end{pmatrix}. \tag{3.4}$$

The fields at the altitude z inside the layer j are related to the α-polarized fields in media 1 and f by

$$\mathbf{F}_j^{0\alpha}(z)=\mathcal{N}^{\alpha}_j(z)\mathcal{N}^{\alpha}_{j1}\mathcal{A}^{\alpha}_1\mathbf{V}_1^{0\alpha}=[\mathcal{N}^{\alpha}_{fj}\mathcal{N}^{\alpha}_j(l_j-z)]^{-1}\mathcal{A}^{\alpha}_f\mathbf{V}_f^{0\alpha}, \tag{3.5}$$

where for $i<j$ $\mathcal{N}^{\alpha}_{ji}=\mathcal{N}^{\alpha}_{j-1}\cdots\mathcal{N}^{\alpha}_{i+1}$ and $\mathcal{N}^{\alpha}_j(z)=\mathcal{A}^{\alpha}_j\mathcal{D}^{\alpha}_j(z)[\mathcal{A}^{\alpha}_j]^{-1}$.

To describe the effect of the distribution of polarization $\Delta\mathbf{P}_m(\mathbf{r},t)$, it is useful to define specific characteristic quantities of the unperturbed system. For a given value of q_x, we consider two (virtual) illumination conditions where the α-polarized incident wave comes either (downwards) from medium 1 [Fig. 2(a)] or (upwards) from medium f [Fig. 2(b)]. In each case, the unperturbed fields inside the layer j are written $\{\mathbf{E}^{0\alpha}_{m1,f}(z),\mathbf{B}^{0\alpha}_{m1,f}(z)\}\exp[i(q_x x-\omega t)]$ and are associated to $\mathbf{F}^{0\alpha}_{m1,f}(z)$. By normalization to the amplitude of the incident wave, $\mathbf{E}^{0\alpha}_{m1,f}(z)$, $\mathbf{B}^{0\alpha}_{m1,f}(z)$, and $\mathbf{F}^{0\alpha}_{m1,f}(z)$ turn into the dimensionless quantities $\mathbf{e}^{0\alpha}_{m1,f}(z)$, $\mathbf{b}^{0\alpha}_{m1,f}(z)$, and $\mathbf{f}^{0\alpha}_{m1,f}(z)$. This defines, for the incidence from medium 1 (respectively f), $R^{0\alpha}_{1f}$ and $T^{0\alpha}_{1f}$ (respectively $R^{0\alpha}_{f1}$ and $T^{0\alpha}_{f1}$) as the amplitudes of the unperturbed reflected and transmitted waves. Using Eq. (3.5), the two-component dimensionless vectors $\mathbf{f}^{0\alpha}_{m1,f}(z)$ are related to the unperturbed reflection and transmission coefficients, as, for instance,

$$\mathbf{f}^{0\alpha}_{m1}(0)=\mathcal{N}^{\alpha}_{m1}\mathcal{A}^{\alpha}_1\begin{pmatrix} 0\\ T^{0\alpha}_{f1}\end{pmatrix}, \tag{3.6a}$$

$$\mathbf{f}^{0\alpha}_{mf}(l_m)=[\mathcal{N}^{\alpha}_{fm}]^{-1}\mathcal{A}^{\alpha}_f\begin{pmatrix} T^{0\alpha}_{1f}\\ 0\end{pmatrix}. \tag{3.6b}$$

It is also useful to consider the dimensionless quantities $\overline{\mathbf{e}}^{0\alpha}_{m1,f}(z)$ and $\overline{\mathbf{b}}^{0\alpha}_{m1,f}(z)$ associated with the (downwards and upwards) incidences defined by $-q_x$. From Fig. 3, we see that $\overline{\mathbf{e}}^{0\alpha}_{m1,f}(z)$ and $\mathbf{e}^{0\alpha}_{m1,f}(z)$ are symmetrical with respect to the yOz plane (opposite x components and identical y and z components), while $\overline{\mathbf{b}}^{0\alpha}_{m1,f}(z)$ and $\mathbf{b}^{0\alpha}_{m1,f}(z)$ are symmetrical with respect to the Ox axis (opposite y and z components and identical x components).[20] Note that consequently $\overline{\mathbf{f}}^{0s}_{m1,f}(z)=\mathbf{f}^{0s}_{m1,f}(z)$ and $\overline{\mathbf{f}}^{0p}_{m1,f}(z)=-\mathbf{f}^{0p}_{m1,f}(z)$.

C. Equation of propagation in a polarized layer

Let us now assume that the internal polarization in layer m takes the form

$$\Delta\mathbf{P}_m(\mathbf{r},t)=\Delta\mathbf{P}_m(z)\exp[i(q_x x-\omega t)]. \tag{3.7}$$

The fields radiated by $\Delta\mathbf{P}_m(\mathbf{r},t)$ in layer m can be written $\{\Delta\mathbf{E}_m(z),\Delta\mathbf{B}_m(z)\}\exp[i(q_x x-\omega t)]$. By substitution of Eq. (3.7) into the z components of Eqs. (3.1), we find

$$q_x\Delta E^y_m(z)/k_0=c\Delta B^z_m(z), \tag{3.8a}$$

$$q_x c\Delta B^y_m(z)/k_0=-\varepsilon_m\Delta E^z_m(z)-\Delta P^z_m(z)/\varepsilon_0. \tag{3.8b}$$

Eliminating $c\Delta B^z_m(z)$ and $\Delta E^z_m(z)$ into the x and y components of Eqs. (3.1) and keeping the z-derivative equation,[21] the propagation in layer m is described by two independent sets of two-component linear differential equations for s and p components:

$$\frac{\partial}{\partial z}\begin{pmatrix}\Delta E^y_m(z)\\ c\Delta B^x_m(z)\end{pmatrix}=-i\kappa_m\begin{pmatrix} c\Delta B^x_m(z)/a^s_m\\ a^s_m\Delta E^y_m(z)\end{pmatrix}-ik_0\begin{pmatrix}0\\ \Delta P^y_m(z)/\varepsilon_0\end{pmatrix}, \tag{3.9a}$$

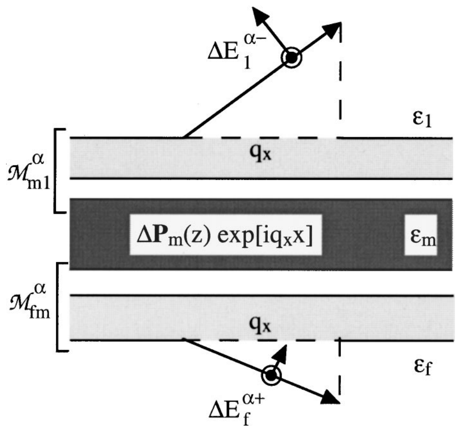

FIG. 4. The electric polarization $\Delta \mathbf{P}_m(z)\exp[iq_x x]$ in the layer m radiates in the unperturbed structure and induces outgoing waves in media 1 and f with the same space modulation and the complex amplitudes $\Delta E_1^{\alpha -}$ and $\Delta E_f^{\alpha +}$.

$$\frac{\partial}{\partial z}\begin{pmatrix} c\Delta B_m^y(z) \\ \Delta E_m^x(z)\end{pmatrix} = -i\kappa_m \begin{pmatrix} \Delta E_m^x(z)/a_m^p \\ a_m^p c\Delta B_m^y(z)\end{pmatrix} - ik_0\begin{pmatrix} -\Delta P_m^x(z)/\varepsilon_0 \\ q_x \Delta P_m^z(z)/(k_0\varepsilon_0\varepsilon_m)\end{pmatrix}, \quad (3.9b)$$

where $a_m^s = \kappa_m/k_0$, $a_m^p = -\kappa_m/(k_0\varepsilon_m)$.[21] From the expression of the isotropic propagation matrix $\mathcal{N}_m^{\alpha}(z)$ given in Appendix A, each Eq. (3.9a) and (3.9b) can be written as

$$\frac{\partial \Delta \mathbf{F}_m^{\alpha}}{\partial z}(z) = \frac{\partial \mathcal{N}_m^{\alpha}}{\partial z}(0)\Delta \mathbf{F}_m^{\alpha}(z) - ik_0 \Delta \boldsymbol{\mathcal{P}}_m^{\alpha}(z)/\varepsilon_0 \quad (3.10)$$

which defines the two-component vectors $\Delta \mathbf{F}_m^{\alpha}(z)$ and $\Delta \boldsymbol{\mathcal{P}}_m^{\alpha}(z)$. The integration of Eq. (3.10) through the layer thickness yields

$$\Delta \mathbf{F}_m^{\alpha}(l_m) - \mathcal{N}_m^{\alpha}(l_m)\Delta \mathbf{F}_m^{\alpha}(0) = -ik_0\int_0^{l_m} \mathcal{N}_m^{\alpha}(l_m - z)\frac{\Delta \boldsymbol{\mathcal{P}}_m^{\alpha}(z)}{\varepsilon_0}dz. \quad (3.11)$$

Except for the right-hand polarization term, this equation takes the form of the propagation equation of the s- and p-polarized waves in the unperturbed layer.

The fields $\Delta \mathbf{F}_m^{\alpha}(0)$ and $\Delta \mathbf{F}_m^{\alpha}(l_m)$, radiated in layer m by the distribution of polarization $\Delta \mathbf{P}_m(z)$, propagate through the isotropic multilayer. Related outgoing waves of complex amplitude $\Delta E_1^{\alpha -}$ and $\Delta E_f^{\alpha +}$ are emitted in the external media (Fig. 4). Using Eqs. (3.6), the propagation from medium m to 1 and m to f is given by

$$\Delta \mathbf{F}_m^{\alpha}(0) = \mathcal{N}_{m1}^{\alpha}\mathcal{A}_1^{\alpha}\begin{pmatrix} 0 \\ \Delta E_1^{\alpha -}\end{pmatrix} = \frac{\mathbf{f}_{mf}^{0\alpha}(0)}{T_{f1}^{0\alpha}}\Delta E_1^{\alpha -}, \quad (3.12a)$$

$$\Delta \mathbf{F}_m^{\alpha}(l_m) = [\mathcal{N}_{fm}^{\alpha}]^{-1}\mathcal{A}_f^{\alpha}\begin{pmatrix} \Delta E_f^{\alpha +} \\ 0\end{pmatrix} = \frac{\mathbf{f}_{m1}^{0\alpha}(l_m)}{T_{1f}^{0\alpha}}\Delta E_f^{\alpha +}. \quad (3.12b)$$

From Eqs. (3.11) and (3.12) we deduce

$$\frac{\mathbf{f}_{m1}^{0\alpha}(l_m)}{T_{1f}^{0\alpha}}\Delta E_f^{\alpha +} - \frac{\mathbf{f}_{mf}^{0\alpha}(l_m)}{T_{f1}^{0\alpha}}\Delta E_1^{\alpha -} = -ik_0\int_0^{l_m}\mathcal{N}_m^{\alpha}(l_m - z)\frac{\Delta \boldsymbol{\mathcal{P}}_m^{\alpha}(z)}{\varepsilon_0}dz. \quad (3.13)$$

This equation of propagation links the still unknown complex amplitudes of the waves radiated into the external media to the polarization distribution in layer m, through quantities characteristic of the unperturbed system. This obviously reduces the calculations of the propagation through the whole system to a 2×2 matrix treatment.

D. Waves radiated into the external media

The solution of the equation of propagation (3.13) is easily obtained and takes the form (Appendix B)

$$\Delta E_1^{\alpha -} = \frac{ik_0^2}{2\kappa_1}\int_0^{l_m}\overline{\mathbf{e}}_{m1}^{\theta\alpha}(z)\cdot\frac{\Delta \mathbf{P}_m(z)}{\varepsilon_0}dz, \quad (3.14a)$$

$$\Delta E_f^{\alpha +} = \frac{ik_0^2}{2\kappa_f}\int_0^{l_m}\overline{\mathbf{e}}_{mf}^{\theta\alpha}(z)\cdot\frac{\Delta \mathbf{P}_m(z)}{\varepsilon_0}dz. \quad (3.14b)$$

Equations (3.14) express the extraction into the external media of the fields radiated by the distribution of polarization defined in Eq. (3.7). These exact expressions are particularly compact and transparent. They only require the 2×2 matrix calculation of the unperturbed vectors $\overline{\mathbf{e}}_{m1,f}^{\theta\alpha}(z)$ defined in Sec. III B and identified now as extraction vectors.[22]

Equations (3.14) also show that a plane wave can always be considered as the radiation of a slice of dipoles. In particular, the amplitudes and polarization of the waves that would be radiated towards the multilayer by the polarization distributions $\mathbf{P}_1^{\alpha}(\mathbf{r},t) = (2\varepsilon_0\kappa_1/ik_0^2)\mathbf{u}_1^{\alpha +}\exp[i(q_x x - \omega t)]\delta_1$ and $\mathbf{P}_f^{\alpha}(\mathbf{r},t) = (2\varepsilon_0\kappa_f/ik_0^2)\mathbf{u}_f^{\alpha -}\exp[i(q_x x - \omega t)]\delta_f$, located at the interfaces of the structure with media 1 and f (as indicated by the Dirac delta functions $\delta_{1,f}$), could be obtained with Eqs. (3.14a), using $\overline{\mathbf{u}}_1^{\alpha -} = \mathbf{u}_1^{\alpha +}$ and $\overline{\mathbf{u}}_f^{\alpha +} = \mathbf{u}_f^{\alpha -}$ as the extracting vectors. These waves are α polarized and have a unit amplitude. As a consequence, these polarizations would induce respectively the unperturbed fields $\mathbf{e}_{m1}^{0\alpha}(z)$ and $\mathbf{e}_{mf}^{0\alpha}(z)$ in the unperturbed structure. Conversely, the polarizations with the same amplitude, associated to $-q_x$ and respectively oriented along $\mathbf{u}_1^{\alpha -}$ and $\mathbf{u}_f^{\alpha +}$, would radiate the fields $\overline{\mathbf{e}}_{m1}^{\theta\alpha}(z)$ and $\overline{\mathbf{e}}_{mf}^{\theta\alpha}(z)$ in the unperturbed system.

Finally, as already mentioned in Sec. III A, if there are several polarized layers in the unperturbed structure, the fields radiated in the external media are the linear sum of the fields radiated independently by each distribution of polarization as given by Eqs. (3.14).

IV. WEAK ANISOTROPY IN A HOMOGENEOUS LAYER: LINEAR MAGNETO-OPTICS

A. Fields generated by a homogeneous layer with weak anisotropy

Let us now consider a multilayer containing an anisotropic layer m with a dielectric tensor of anisotropic part

$\underline{\Delta\varepsilon}_m$ (Fig. 1). For a β-polarized wave incident from medium 1 and associated to a given q_x defining the incidence angle, the anisotropy induces a polarization $\Delta\mathbf{P}_m(z)\exp[i(q_x x-\omega t)]$. If the anisotropy is weak ($\underline{\Delta\varepsilon}_m \ll \varepsilon_m \underline{I}$) and the layer thin enough so that the exciting field inside the anisotropic layer does not differ significantly from the unperturbed field, the induced polarization can be approximated by

$$\Delta\mathbf{P}_m(z)\approx\Delta^1\mathbf{P}_m(z)=\varepsilon_0\underline{\Delta\varepsilon}_m\mathbf{E}^{0\beta}_{m1}(z). \tag{4.1}$$

This expression, known as the first Born approximation, can be used in Eqs. (3.14) to give the fields radiated into the external media. For a unit excitation [which turns $\mathbf{E}^{0\beta}_{m1}(z)$ into $\mathbf{e}^{0\beta}_{m1}(z)$ in Eq. (4.1)], the corresponding complex amplitudes of the reflected and transmitted waves are

$$\Delta^1 R^{\alpha\beta}_{1f}=\frac{ik_0^2}{2\kappa_1}\int_0^{l_m}\overline{\mathbf{e}}^{\theta\alpha}_{m1}(z)\cdot(\underline{\Delta\varepsilon}_m\cdot\mathbf{e}^{0\beta}_m(z))dz, \tag{4.2a}$$

$$\Delta^1 T^{\alpha\beta}_{1f}=\frac{ik_0^2}{2\kappa_f}\int_0^{l_m}\overline{\mathbf{e}}^{\theta\alpha}_{mf}(z)\cdot(\underline{\Delta\varepsilon}_m\cdot\mathbf{e}^{0\beta}_m(z))dz. \tag{4.2b}$$

$\Delta^1 R^{\alpha\beta}_{1f}$ and $\Delta^1 T^{\alpha\beta}_{1f}$ are approximations of the coefficients of the perturbed reflection and transmission matrices $\underline{\Delta R}_{1f}$ and $\underline{\Delta T}_{1f}$ defined by $\underline{R}_{1f}=\underline{R}^0_{1f}+\underline{\Delta R}_{1f}$ and $\underline{T}_{1f}=\underline{T}^0_{1f}+\underline{\Delta T}_{1f}$, $\underline{R}^0_{1f}$ and $\underline{T}^0_{1f}$ being the 2×2 diagonal unperturbed reflection and transmission matrices. In the framework of the first Born approximation, the procedure reduces to the 2×2 matrix calculation of $\overline{\mathbf{e}}^{\theta\alpha}_{m1,f}(z)$ and $\mathbf{e}^{0\beta}_{m1}(z)$. Their variation with z are determined by propagation factors like $\exp[\pm i\kappa_m z]$ so that the integrals over the thickness of the layer, both analytically and numerically, are easily performed.

Three quantities arise: the dimensionless exciting field $\mathbf{e}^{0\beta}_{m1}(z)$, the extraction vector $\overline{\mathbf{e}}^{\theta\alpha}_{m1,f}(z)$ and the anisotropic tensor. The interpretation is transparent: the exciting field interacts with the anisotropy and induces an internal polarization which in turn radiates and generates plane waves in the external media.

If there are several anisotropic layers and if the total effect remains small compared to the unperturbed quantities, the first Born approximation can be used in each layer as in Eq. (4.1). The total effect is obtained like in Sec. III D by summation of the effects of each anisotropic layer calculated independently as in Eqs. (4.2).

B. Linear magneto-optics

When the anisotropy is due to a steady magnetization $\mathbf{M}_m$ of arbitrary direction, the tensor $\underline{\Delta\varepsilon}_m$ is antisymmetric to first order in the magnetization and Eq. (4.1) can be written as

$$\Delta^1\mathbf{P}_m(z)=\varepsilon_0 g_m\mathbf{M}_m\wedge\mathbf{E}^{0\beta}_{m1}(z). \tag{4.3}$$

We see that $g_m\mathbf{M}_m$ plays the role of a gyration vector[23] acting on the β-polarized exciting field $\mathbf{E}^{0\beta}_{m1}(z)$. The α-polarized waves induced this way are obtained by substitution of $\Delta^1\mathbf{P}_m(z)$ into Eqs. (3.14), and Eqs. (4.2) can be rewritten as

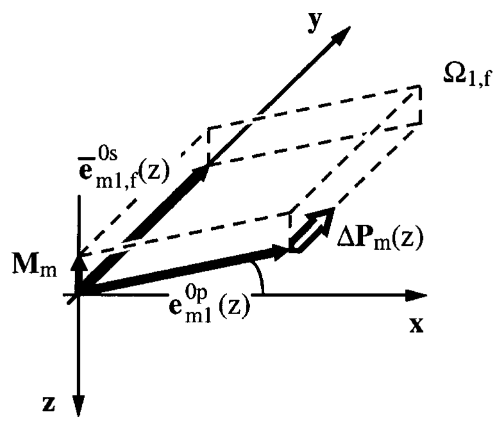

FIG. 5. For $\mathbf{M}_m=M_m\mathbf{z}$ and a p-polarized incident wave, the MO effect is described by the electric polarization $\Delta\mathbf{P}_m(z)$ which radiates s-polarized waves in media 1 and f. Their complex amplitudes are proportional to the volume $\Omega_{1,f}$ built from the magnetization $\mathbf{M}_m$, the unperturbed normalized exciting vector $\mathbf{e}^{0p}_{m1}(z)$, and the extraction vectors $\overline{\mathbf{e}}^{\theta s}_{m1,f}(z)$.

$$\Delta^1 R^{\alpha\beta}_{1f}=\frac{-ik_0^2}{2\kappa_1}g_m\mathbf{M}_m\cdot\mathbf{C}^{\alpha\beta}_{m11}, \tag{4.4a}$$

$$\Delta^1 T^{\alpha\beta}_{1f}=\frac{-ik_0^2}{2\kappa_f}g_m\mathbf{M}_m\cdot\mathbf{C}^{\alpha\beta}_{mf1} \tag{4.4b}$$

with

$$\mathbf{C}^{\alpha\beta}_{m(1,f)1}=\int_0^{l_m}\overline{\mathbf{e}}^{\theta\alpha}_{m1,f}(z)\wedge\mathbf{e}^{0\beta}_{m1}(z)dz. \tag{4.4c}$$

From these compact expressions one readily deduces the usual MO properties. The magneto-optical effect only arises when the magnetization, the extraction vector, and the exciting field are not in the same plane. For a normal incidence, the magnetization must have a z component. When $\mathbf{M}_m$ is in the incidence plane, $\Delta R^{\alpha\beta}_{1f}$ and $\Delta T^{\alpha\beta}_{1f}$ are nonzero only for $\alpha\neq\beta$. Conversely, when $\mathbf{M}_m=M_m\mathbf{y}$, i.e., perpendicular to the incidence plane (transverse geometry) and thus parallel to the s fields, only ΔR^{pp}_{1f} and ΔT^{pp}_{1f} are different from zero. Moreover, $\overline{\mathbf{e}}^{\theta\alpha}_{m1}(z)$ being the symmetrical of $\mathbf{e}^{0\alpha}_{m1}(z)$ with respect to the yOz plane, Eq. (4.4a) yields $\Delta R^{sp}_{1f}=\Delta R^{ps}_{1f}$ if $\mathbf{M}_m=M_m\mathbf{z}$ and $\Delta R^{sp}_{1f}=-\Delta R^{ps}_{1f}$ if $\mathbf{M}_m=M_m\mathbf{x}$.

For a given magnetic layer, the magneto-optical signal is proportional to the volume limited by the three vectors $\mathbf{M}_m$, $\overline{\mathbf{e}}^{\theta\alpha}_{m1,f}(z)$, and $\mathbf{e}^{0\beta}_{m1}(z)$ (Fig. 5). The optimal multilayer configuration is obtained by the optimization of this volume.[24]

V. INHOMOGENEOUS AND NONLINEAR LAYERS

A. Fields radiated from $\Delta\mathbf{P}_m(\mathbf{r},t)$ outside the multilayer

The treatment of Sec. III C can be generalized to any distribution of polarization by considering the three-dimensional Fourier transform of $\Delta\mathbf{P}_m(\mathbf{r},t)$ written as

$$\Delta\mathbf{P}_m(\mathbf{r},t)=\int\Delta\mathbf{P}_{\mathbf{q}'\omega' m}(z)\exp[i(\mathbf{q}'\cdot\mathbf{r}_{xy}-\omega' t)]d\mathbf{q}'d\omega'. \tag{5.1}$$

Here, $\mathbf{q}'$ and $\mathbf{r}_{xy}$ are the components in the interface plane of the wave vector and of $\mathbf{r}$. From the linearity of the unperturbed system, the field radiated by $\Delta\mathbf{P}_m(\mathbf{r},t)$ is the sum of the fields radiated independently by each Fourier component. For each component $\Delta\mathbf{P}_{\mathbf{q}'\omega' m}(z)$, the result of Sec. III C holds when $q_x x$ is replaced by $\mathbf{q}'\cdot\mathbf{r}_{xy}$. This component of the polarization distribution generates s'-and p'-polarized plane waves with respect to $\mathbf{q}'$ propagating in the $(\mathbf{q}',\mathbf{z})$ plane. Omitting now the superscripts $-$ and $+$, the α-polarized emerging waves in media 1 and f are

$$\Delta\mathbf{E}^{\alpha}_{\mathbf{q}'\omega' 1,f}(\mathbf{r},t)=\Delta E^{\alpha}_{\mathbf{q}'\omega' 1,f}\mathbf{u}^{\alpha}_{\mathbf{q}'1,f}\exp[i(\mathbf{k}'_{1,f}\cdot\mathbf{r}-\omega' t)]. \quad (5.2a)$$

$\Delta E^{\alpha}_{\mathbf{q}'\omega' 1,f}$ are the complex amplitude of the waves; the wave vectors $\mathbf{k}'_1=\mathbf{q}'-\kappa'_1\mathbf{z}$ and $\mathbf{k}'_f=\mathbf{q}'+\kappa'_f\mathbf{z}$ are deduced from $q'^2+\kappa'^2_{1,f}=k'^2_0\varepsilon_{1,f}(\omega')$ with $k'_0=\omega'/c$; the polarization eigenvectors $\mathbf{u}^{s}_{\mathbf{q}'1,f}$ and $\mathbf{u}^{p}_{\mathbf{q}'1,f}$ are transverse to the directions of propagation and respectively perpendicular and parallel to the propagation plane $(\mathbf{q}',\mathbf{z})$. The generalization of Eqs. (3.14) gives

$$\Delta E^{\alpha}_{\mathbf{q}',\omega' 1,f}=\frac{ik'^2_0}{2\kappa'_{1,f}}\int_0^{l_m}\overline{\mathbf{e}}^{0\alpha}_{\mathbf{q}'\omega' m1,f}(z)\cdot\frac{\Delta\mathbf{P}_{\mathbf{q}'\omega' m}(z)}{\varepsilon_0}dz. \quad (5.2b)$$

The extraction vectors $\overline{\mathbf{e}}^{0\alpha}_{\mathbf{q}'\omega' m1,f}(z)$ are the symmetricals with respect to the plane perpendicular to $\mathbf{q}'$ of the unperturbed fields $\mathbf{e}^{0\beta}_{\mathbf{q}'\omega' m1,\,f}(z)$ calculated in the unperturbed system illuminated by a wave defined by $\mathbf{q}'$ and ω'.

The superposition of the outgoing waves provides the fields radiated in the external media by the polarization $\Delta\mathbf{P}_m(\mathbf{r},t)$ as

$$\Delta\mathbf{E}_{1,f}(\mathbf{r},t)=\sum_{\alpha=s,p}\int\Delta\mathbf{E}^{\alpha}_{\mathbf{q}'\omega' 1,f}(\mathbf{r},t)d\mathbf{q}'d\omega'. \quad (5.3)$$

This expression accounts for both homogeneous and evanescent waves (when respectively $q'^2<k'^2_0\varepsilon_{1,f}$ and $q'^2>k'^2_0\varepsilon_{1,f}$) and therefore exactly gives the fields radiated at any point of the external media in far field and near field.

B. Scattered fields by an inhomogeneous layer

Let us now consider a structure with a linear layer m of weak inhomogeneity described by a small deviation $\underline{\Delta\varepsilon}_m(\mathbf{r})$ to the dielectric tensor $\varepsilon_m\underline{I}$. The illumination from medium 1 by a β-polarized plane wave of frequency ω and in-plane wave-vector component $\mathbf{q}$ (in Sec. III C $\mathbf{q}=q_x\mathbf{x}$) induces a polarization oscillating at the frequency ω (as the optical effects in layer m are linear) (see Fig. 6). In the first Born approximation the Fourier components of $\Delta\mathbf{P}_m(\mathbf{r},t)$ are

$$\Delta^1\mathbf{P}_{\mathbf{q}'m}(z)=\varepsilon_0\underline{\Delta\varepsilon}_{m(\mathbf{q}'-\mathbf{q})}(z)\mathbf{E}^{0\beta}_{\mathbf{q}m1}(z), \quad (5.4a)$$

where $\underline{\Delta\varepsilon}_{m(\mathbf{q}'-\mathbf{q})}(z)$ is the two-dimensional Fourier component of $\underline{\Delta\varepsilon}_m(\mathbf{r})$ associated to the modulation $\mathbf{q}'-\mathbf{q}$. By substitution into Eq. (5.2), we find the complex amplitudes of the waves scattered in media 1 and f for each Fourier component $\mathbf{q}'$ of the polarization:

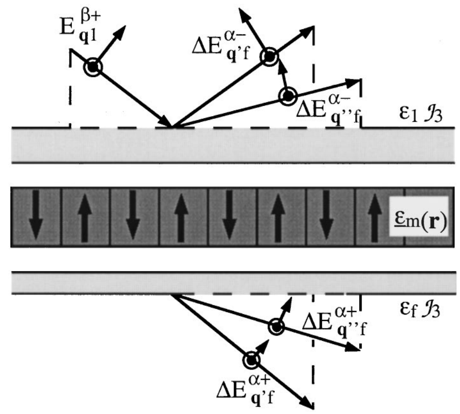

FIG. 6. Waves scattered by an inhomogeneous layer m embedded in a multilayer.

$$\Delta^1 E^{\alpha}_{\mathbf{q}'1,f}=\frac{ik_0^2}{2\kappa'_{1,f}}\int\overline{\mathbf{e}}^{0\alpha}_{\mathbf{q}'m1,f}(z)\cdot(\underline{\Delta\varepsilon}_{m(\mathbf{q}'-\mathbf{q})}(z)\mathbf{E}^{0\beta}_{qm1}(z))dz, \quad (5.4b)$$

where $q'^2+\nu'^2_{1,f}=k_0^2\varepsilon_{1,f}$. At this stage, Eqs. (5.3) yield the fields scattered by the inhomogeneity as the superposition of plane waves, known as the angular spectrum representation.[11] This expression can be, for instance, applied to the magneto-optical imaging of magnetic domains with $\Delta^1\mathbf{P}_{\mathbf{q}'m}(z)=\varepsilon_0 g_m\mathbf{M}_{m(\mathbf{q}'-\mathbf{q})}(z)\wedge\mathbf{E}^{0\beta}_{\mathbf{q}m1}(z)$. Note that inhomogeneity on a scale far below the wavelength induces high q' components of the polarization which generate evanescent waves outside the system, detectable only by near-field techniques. It can be shown that the amplitude of these q' components generally tends to vanish with increasing q',[25] i.e., when the characteristic length scale of the inhomogeneity (magnetic domain size) decreases.

C. Nonlinear optics: Second-harmonic generation

In the case of a nonlinear layer m, the first Born approximation is equivalent to the usual approximation neglecting the depletion of energy from the pump wave.[14–16,26] For instance, the second-harmonic generation is described by the induced polarization

$$\Delta^1\mathbf{P}_m(\mathbf{r},t)=\varepsilon_0\boldsymbol{\chi}_m:\mathbf{E}^{0\beta}_{\mathbf{q}\omega m1}(z)\mathbf{E}^{0\beta}_{q\omega m1}(z)\exp[2i(\mathbf{q}\cdot\mathbf{r}_{xy}-\omega t)]. \quad (5.5a)$$

The β-polarized exciting field associated to ω and $\mathbf{q}$ induces a polarization modulated at 2ω and $2\mathbf{q}$. With $4q^2+\kappa'^2_{1,f}=4k_0^2\varepsilon_{1,f}(2\omega)$, Eq. (5.2) yields the amplitude of the α-polarized second harmonic wave radiated in the external media:

$$\Delta^1 E^{\alpha}_{\mathbf{q}'\omega' 1,f}=\frac{2ik_0^2}{\kappa'_{1,f}}\int\overline{\mathbf{e}}^{0\alpha}_{\mathbf{q}'\omega' m1,f}(z)\cdot(\boldsymbol{\chi}_m:\mathbf{E}^{0\beta}_{\mathbf{q}\omega m1}(z)\mathbf{E}^{0\beta}_{\mathbf{q}\omega m1}(z))dz. \quad (5.5b)$$

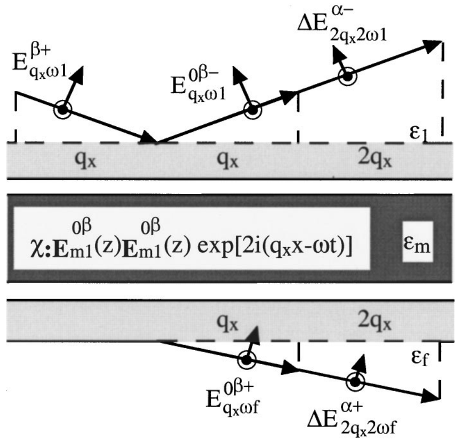

FIG. 7. Reflected and transmitted waves induced by illumination of a multilayer where a nonlinear layer m induces second-harmonic generation.

This relation holds for any multilayer as well as for bulk or surface nonlinear effects and generalizes the usually reported expressions (see Fig. 7).[14–16] Nonlinear effects are generally small and Eq. (5.5b) reveals the pertinent quantities, $\mathbf{e}^{-0\alpha}_{\mathbf{q}'\omega' m1,f}(z)$ and $[\mathbf{E}^{0\beta}_{\mathbf{q}\omega m1}(z)]^2$, for the optimization of this effect.

VI. PERTURBATION EXPANSION: EFFECT INDUCED BY A THIN LAYER

In Sec. IV, the polarization induced in the layer m was expressed within the first Born approximation, although its exact expression takes the form

$$\Delta\mathbf{P}_m(z)=\varepsilon_0\underline{\Delta\varepsilon}_m(\mathbf{E}^{0\beta}_{m1}(z)+\Delta\mathbf{E}_m(z)). \qquad (6.1\text{a})$$

In Sec. 2 of Appendix C, it is shown that this can be rewritten as

$$\frac{\Delta\mathbf{P}_m(z)}{\varepsilon_0}=\underline{\Delta\varepsilon}_m(1-\underline{\delta}^{zz}_m)\Bigg\{\mathbf{E}^{0\beta}_{m1}(z)+\sum_{\gamma=s,p}\frac{\mathbf{e}^{0\gamma}_{m1}(z)}{T^{0\gamma}_{1f}}\Delta E^{\gamma}_f + \frac{i}{\kappa_m}\int_z^{\ell_m}\Phi^E_m(z-z')\frac{\Delta\mathbf{P}_m(z')}{\varepsilon_0}dz'\Bigg\}, \qquad (6.1\text{b})$$

where $[\mathbf{z}\cdot(\underline{\varepsilon}_m\cdot\mathbf{z})]\underline{\delta}^{zz}_m=[\mathbf{z}\cdot\mathbf{z}^t]\cdot\underline{\Delta\varepsilon}_m$ and $\Phi^E_m(z-z')$ is a 3×3 matrix. Starting from $\Delta\mathbf{P}_m(z)\approx\varepsilon_0\underline{\Delta\varepsilon}_m(1-\underline{\delta}^{zz}_m)\mathbf{E}^{0\beta}_{m1}(z)$ [or from the first Born approximation $\Delta\mathbf{P}_m(z)\approx\varepsilon_0\underline{\Delta\varepsilon}_m\mathbf{E}^{0\beta}_{m1}(z)$] and from the related approximation of the radiated fields $\Delta E^{\gamma+}_f$ calculated with Eqs. (4.2b), Eq. (6.1b) gives a second-order approximation of $\Delta\mathbf{P}_m(z)$ and of $\Delta E^{\gamma+}_f$. Subsequently, $\Delta\mathbf{P}_m(z)$ can be derived from an iteration procedure. This iteration (equivalent to the so-called Born series[10–12] when starting from the first Born approximation) gives directly the results of the perturbative approach of the propagation integral equation proposed in Ref. 19.

When the layer m is thin ($|\kappa^\sigma_m l_m|\ll 1$), whatever the magnitude of the $\underline{\Delta\varepsilon}_m$ components with respect to ε_m, the last integral term of Eq. (6.1b) can be neglected. A compact approximation of the perturbed reflection and transmission matrices is thus obtained (Appendix C):

$$\underline{\Delta R}_{1f}\sim\underline{\Delta^{1\delta}R}_{1f}(1-[\underline{T}^0_{1f}]^{-1}\underline{\Delta^{1\delta}T}_{1f})^{-1}, \qquad (6.2\text{a})$$

$$\underline{\Delta T}_{1f}=\underline{\Delta^{1\delta}T}_{1f}(1-[\underline{T}^0_{1f}]^{-1}\underline{\Delta^{1\delta}T}_{1f})^{-1}, \qquad (6.2\text{b})$$

where the components of $\underline{\Delta^{1\delta}R}_{1f}$ and $\underline{\Delta^{1\delta}T}_{1f}$ are calculated from Eqs. (4.2) where $\underline{\Delta\varepsilon}_m(1-\underline{\delta}^{zz}_m)$ is substituted to $\underline{\Delta\varepsilon}_m$.

This *thin layer approximation,* which neglects the last term in Eq. (6.1b), results from an approximation of the induced polarization or equivalently of the exciting field $\mathbf{E}_{m1}$ which can be considered as uniform. From Eqs. (6.2) and the definition of $\underline{\Delta^{1\delta}R}_{1f}$ and $\underline{\Delta^{1\delta}T}_{1f}$, which gives the fields radiated in media 1 and f when $\mathbf{E}_{m1}$ is approximated by the normalized unperturbed field $\mathbf{e}^{0\beta}_{m1}$, we see that the thin layer approximation decomposes $\mathbf{E}_{m1}$ as

$$\mathbf{E}_{m1}=E^s_m\mathbf{e}^{0s}_{m1}+E^p_m(1-\underline{\delta}^{zz}_m)\mathbf{e}^{0p}_{m1}, \qquad (6.3\text{a})$$

with

$$\begin{pmatrix}E^s_m\\E^p_m\end{pmatrix}\approx(1-[\underline{T}^0_{1f}]^{-1}\underline{\Delta^{1\delta}T}_{1f})^{-1}\begin{pmatrix}E^{s+}_1\\E^{p+}_1\end{pmatrix}, \qquad (6.3\text{b})$$

where $E^{s,p+}_1$ are by definition the s and p complex amplitudes of the incident waves in medium 1.

VII. RECIPROCITY PRINCIPLE IN MULTILAYERS

The transparency of the analytical descriptions of a variety of optical effects in multilayers proposed in this paper is mainly due to the notion of extraction vectors. These quantities describe the fields radiated into the external media by polarization distributions induced in the matter. In linear and symmetric structures, the Lorentz reciprocity theorem[27,28] is a powerful tool to link polarizations and radiated fields modulated at the same pulsation ω. Omitting the time modulation $\exp[-i\omega t]$, we consider in Fig. 8(a) the electromagnetic fields $[\Delta\mathbf{E}(\mathbf{r}),\Delta\mathbf{H}(\mathbf{r})]$ in the volume V generated by the electric and magnetic polarizations $[\Delta\mathbf{P}(\mathbf{r}'),\Delta\mathbf{M}(\mathbf{r}')]$ enclosed in a volume V'; in Fig. 8(b) the fields $[\Delta\mathbf{E}(\mathbf{r}'),\Delta\mathbf{H}(\mathbf{r}')]$ in V' are created by the polarizations $[\mathbf{P}(\mathbf{r}),\mathbf{M}(\mathbf{r})]$ enclosed in V. The Lorentz reciprocity theorem states

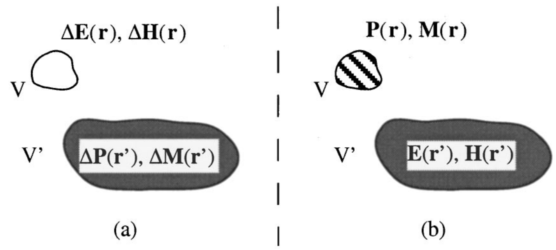

FIG. 8. When fields and polarization have the same frequency ω, (a) the fields generated in a volume V by a distribution of polarizations included in the volume V' are related by the Lorentz reciprocity theorem to (b) the fields generated in V' by polarizations included in the volume V.

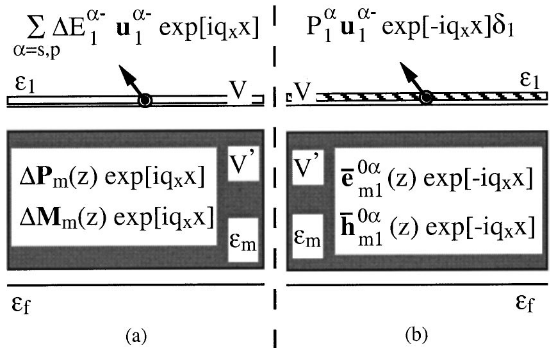

FIG. 9. (a) The field radiated in medium 1 by a polarized layer m embedded in a multilayer is derived from the reciprocity theorem when considering (b) the polarization distributions $\mathbf{P}_1^{\alpha}(\mathbf{r}) = P_1^{\alpha}\mathbf{u}_1^{\alpha}\exp[-iq_x x]\delta_1$ with $ik_0^2 P_1^{\alpha} = 2\varepsilon_0\kappa_1$ radiating in the unperturbed system, which induce in the layer m the fields $\{\overline{\mathbf{e}}_{m1}^{0\alpha}(z), \overline{\mathbf{h}}_{m1}^{0\alpha}(z)\}\exp[-iq_x x]$.

$$\int [\Delta\mathbf{E}(\mathbf{r})\cdot\mathbf{P}(\mathbf{r}) - \Delta\mathbf{H}(\mathbf{r})\cdot\mu_0\mathbf{M}(\mathbf{r})]dV$$
$$= \int [\mathbf{E}(\mathbf{r}')\cdot\Delta\mathbf{P}(\mathbf{r}') - \mathbf{H}(\mathbf{r}')\cdot\mu_0\Delta\mathbf{M}(\mathbf{r}')]dV'. \quad (7.1)$$

Let us now consider, as in Fig. 9(a), a layer m characterized by the constitutive equations:

$$\mathbf{D}_m(\mathbf{r}) = \varepsilon_0\varepsilon_m\mathbf{E}_m(\mathbf{r}) + \Delta\mathbf{P}_m(z)\exp(iq_x x), \quad (7.2a)$$

$$\mathbf{B}_m(\mathbf{r}) = \mu_0[\mu_m\mathbf{H}_m(\mathbf{r}) + \Delta\mathbf{M}_m(z)\exp(iq_x x)]. \quad (7.2b)$$

The fields $\Delta\mathbf{E}_{1,f}(\mathbf{r})$, radiated in media 1 and f, are written $\Delta\mathbf{E}_{1,f}(x) = \Delta\mathbf{E}_{1,f}\exp(iq_x x)$ at the interfaces with the structure. By definition their decomposition on the α-polarized outgoing waves gives the amplitudes

$$\Delta E_{1,f}^{\alpha} = \Delta\mathbf{E}_{1,f}(x)\cdot\mathbf{u}_{1,f}^{\alpha}\exp(-iq_x x). \quad (7.3a)$$

Therefore, referring to Sec. III D, if we choose in the external media the electric polarization distributions

$$\mathbf{P}_{1,f}^{\alpha}(\mathbf{r}) = \frac{2\varepsilon_0\kappa_{1,f}}{ik_0^2}\mathbf{u}_{1,f}^{\alpha}\exp(-iq_x x)\,\delta_{1,f} \quad (7.3b)$$

and the magnetic polarization $\mathbf{M}_{1,f}^{\alpha}(\mathbf{r})$ equal to zero, we directly find by substitution into Eq. (7.1)

$$\Delta E_{1,f}^{\alpha} = \frac{ik_0^2}{2\kappa_{1,f}} \times \int_0^{l_m}\left[\overline{\mathbf{e}}_{m1,f}^{0\alpha}(z)\cdot\frac{\Delta\mathbf{P}_m(z)}{\varepsilon_0} - \overline{\mathbf{h}}_{m1,f}^{0\alpha}(z)\cdot\mu_0\Delta\mathbf{M}_m(z)\right]dz, \quad (7.4)$$

where $\overline{\mathbf{h}}_{m1,f}^{0\alpha}(z) = \overline{\mathbf{b}}_{m1,f}^{0\alpha}(z)/(\mu_0\mu_m)$. This exact expression directly gives the result of Eqs. (3.14) if the magnetic polarization $\Delta\mathbf{M}_m$ is equal to zero. As already mentioned, the radiation of several distributions of polarization in different layers is the linear summation of the fields given by Eq. (7.4) radiated independently by each polarized layer.

Equation (7.4) holds for all the optical properties discussed in the previous sections. In particular, although it is derived from the reciprocity principle, this calculation is valid in the case of antisymmetrical media (like, for instance, magneto-optical or inhomogeneous layers), as long as all the antisymmetry is taken into account in the polarization distributions $\Delta\mathbf{P}$ and $\Delta\mathbf{M}$.[29] Finally, Eq. (7.4) also describes the optical properties of bianisotropic media for which the first Born approximation is $\Delta^1\mathbf{P}_m(z) = \underline{\Delta\varepsilon}_m\mathbf{E}_{m1}^{0\beta}(z) + \underline{\Delta\xi}_m\mathbf{H}_{m1}^{0\beta}(z)$ and $\Delta^1\mathbf{M}_m(z) = \underline{\Delta\zeta}_m\mathbf{E}_{m1}^{0\beta}(z) + \underline{\Delta\mu}_m\mathbf{H}_{m1}^{0\beta}(z)$ where $\underline{\Delta\xi}_m$, $\underline{\Delta\zeta}_m$, $\underline{\Delta\mu}_m$ are 3×3 tensors.

VIII. CONCLUSION

In this paper, we describe specific optical properties of materials embedded in a layered structure in terms of induced distributions of polarization. The expressions of the s and p components of the outgoing waves radiated by these polarizations are compact, transparent and evidence three relevant quantities: the exciting field, a susceptibility tensor and an extraction vector.

The extraction vectors are easy to calculate exactly as they are characteristic of the structure free of polarization distributions, that we call unperturbed system. The susceptibility tensor describes the considered optical property of the material. The exciting field can be derived by iterative perturbation calculation. For weak optical effects, the first Born approximation can be applied and the exciting field is also calculated in the unperturbed structure. In the thin layer limit, an analytical approximation of the whole iteration is given.

The treatment applies to a large variety of systems and is particularly well suited to magneto-optics. The description of first-order Kerr and Faraday effects is straightforward and only requires the 2×2 matrix calculation of the fields in the unperturbed system (i.e., when the magnetization is considered as equal to zero). Nonlinear properties, as second harmonic generation of particular interest in magnetic multilayers, and scattering by optical inhomogeneities, for instance, originating from magnetic domains (even at subwavelength scale), are described in the framework of the same formalism.

Finally, we underline that the quantities involved in our formulas, characteristic of the structure and of the geometry (incidence angle, reflection, and transmission, near-field and far-field) are very easy to handle and calculate. This allows the development of simple procedures for designing optimized structures and measurement configurations.

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APPENDIX A: 2×2 MATRIX FORMALISM IN THE UNPERTURBED STRUCTURE

According to Sec. III B, the eigenvalues and the polarization eigenvectors in an isotropic layer j of dielectric constant ε_j are

$$\kappa_j=(k_0^2\varepsilon_j-q_x^2)^{1/2}=\kappa_j^{1,3}=-\kappa_j^{2,4}, \tag{A1a}$$

$$\mathbf{u}_j^{1,2}=\mathbf{y},\ \ \mathbf{u}_j^{3,4}=(\kappa_j^{3,4}\mathbf{x}-q_x\mathbf{z})/k_j. \tag{A1b}$$

The diagonal propagation matrix $\mathcal{D}_j(z)$, deduced from Eqs. (2.4) and (2.6), takes the form

$$\mathcal{D}_j(z)=\begin{bmatrix}\mathcal{D}_j^s(z) & 0\\ 0 & \mathcal{D}_j^p(z)\end{bmatrix} \tag{A2a}$$

with

$$\mathcal{D}_j^s(z)=\mathcal{D}_j^p(z)=\begin{bmatrix}e^{i\kappa_j z} & 0\\ 0 & e^{-i\kappa_j z}\end{bmatrix}. \tag{A2b}$$

Using Eqs. (2.4) and (2.5), the resolution of Eq. (2.7) shows that the $\mathcal{A}_j$ matrix is also 2×2 block diagonal:

$$\mathcal{A}_j=\begin{bmatrix}\mathcal{A}_j^s & 0\\ 0 & \mathcal{A}_j^p\end{bmatrix},\ \text{with}\ \mathcal{A}_j^\alpha=c_j^\alpha\begin{bmatrix}1 & 1\\ -a_j^\alpha & a_j^\alpha\end{bmatrix}, \tag{A3}$$

where $a_j^s=\kappa_j/k_0$, $a_j^p=-\kappa_j/(k_0\varepsilon_j)$, $c_j^s=1$, $c_j^p=n_j$.

The matrix $\mathcal{N}_j(z)=\mathcal{A}_j\mathcal{D}_j(z)[\mathcal{A}_j]^{-1}$ is therefore a 2×2 block diagonal. Each block writes

$$\mathcal{N}_j^\alpha(z)=\cos(\kappa_j z)\mathcal{J}_2-i\sin(\kappa_j z)\begin{bmatrix}0 & 1/a_j^\alpha\\ a_j^\alpha & 0\end{bmatrix}. \tag{A4}$$

$\mathcal{M}_{f1}$, defined in Eq. (2.8), is also a 2×2 block diagonal matrix. Each block operates in Eq. (2.1) separately on the s and p eigenmodes:

$$\begin{pmatrix}E_f^{\alpha+}\\ 0\end{pmatrix}=\mathcal{M}_{f1}^\alpha\begin{pmatrix}E_1^{\alpha+}\\ E_1^{\alpha-}\end{pmatrix}=[\mathcal{A}_f^\alpha]^{-1}\prod_{j=f-1}^{2}\mathcal{N}_j^\alpha\mathcal{A}_1^\alpha\begin{pmatrix}E_1^{\alpha+}\\ E_1^{\alpha-}\end{pmatrix}. \tag{A5}$$

The resolution of Eq. (A5) yields the reflected and transmitted fields.

APPENDIX B: FIELDS RADIATED BY A POLARIZED LAYER

1. Inner product

To derive useful properties of the unperturbed fields, we define the "inner product" of two vectors $\mathbf{w}_1$ and $\mathbf{w}_2$ by

$$\mathbf{w}_1\wedge\mathbf{w}_2=\begin{pmatrix}w_1\\ w_1'\end{pmatrix}\wedge\begin{pmatrix}w_2\\ w_2'\end{pmatrix}=w_1w_2'-w_1'w_2=\det[\mathbf{w}_1,\mathbf{w}_2]. \tag{B1}$$

We note that for any 2×2 matrix $\mathcal{M}$, $\det[\mathcal{M}\mathbf{w}_1,\mathcal{M}\mathbf{w}_2]=\det[\mathcal{M}]\det[\mathbf{w}_1,\mathbf{w}_2]$, and if $\det[\mathcal{M}]=1$, $\det[\mathbf{w}_1,\mathcal{M}\mathbf{w}_2]=\det[\mathcal{M}^{-1}\mathbf{w}_1,\mathbf{w}_2]$. The first relation can be applied to the unperturbed fields if we calculate $\det[\mathcal{A}_{1,f}^\alpha]$ and use $\det[\mathcal{N}_j^\alpha(z)]=1$ to derive

$$\overline{\mathbf{f}}_{m1}^{0\alpha}(z)\wedge\mathbf{f}_{mf}^{0\alpha}(z)=\overline{\mathbf{f}}_{11,f1}^{0\alpha}\wedge\mathbf{f}_{1f,ff}^{0\alpha}=-2\frac{\kappa_f}{k_0}T_{1f}^{0\alpha}=-2\frac{\kappa_1}{k_0}T_{f1}^{0\alpha}, \tag{B2}$$

where $\mathbf{f}_{1f,ff}^{0\alpha}$ and $\overline{\mathbf{f}}_{11,f1}^{0\alpha}$ are the fields in F representation normalized to the incident waves amplitude at the interfaces with the external media. With the second relation we derive, for example,

$$\overline{\mathbf{f}}_{m1,mf}^{0\alpha}(l_m)\wedge[\mathcal{N}_m^\alpha(l_m-z)\Delta\boldsymbol{\mathcal{P}}_m^\alpha(z)]=\overline{\mathbf{f}}_{m1,mf}^{0\alpha}(z)\wedge\Delta\boldsymbol{\mathcal{P}}_m^\alpha(z). \tag{B3}$$

2. Radiated fields by a polarized layer towards the outside of the layered structure

To obtain the electromagnetic fields radiated in the external media by the polarization distribution of Eq. (3.7), we use Eq. (3.13) (where we omit the superscripts $-$ and $+$ to simplify the notations):

$$\frac{\mathbf{f}_{m1}^{0\alpha}(l_m)}{T_{1f}^{0\alpha}}\Delta E_f^\alpha-\frac{\mathbf{f}_{mf}^{0\alpha}(l_m)}{T_{f1}^{0\alpha}}\Delta E_1^\alpha=-ik_0\int_0^{l_m}\mathcal{N}_m^\alpha(l_m-z)\frac{\Delta\boldsymbol{\mathcal{P}}_m^\alpha(z)}{\varepsilon_0}dz. \tag{B4}$$

Performing successively the inner product with $\overline{\mathbf{f}}_{m1}^{0\alpha}(l_m)$ and $\overline{\mathbf{f}}_{mf}^{0\alpha}(l_m)$ in Eq. (B4), we derive from Eq. (B2)

$$\Delta E_{1,f}^\alpha=\frac{ik_0^2}{2\kappa_{1,f}}\int_0^{l_m}\overline{\mathbf{f}}_{m1,f}^{0\alpha}(l_m)\wedge\mathcal{N}_m^\alpha(l_m-z)\frac{\Delta\boldsymbol{\mathcal{P}}_m^\alpha(z)}{\varepsilon_0}dz \tag{B5a}$$

which turns with Eq. (B3) into

$$\Delta E_{1,f}^\alpha=\frac{ik_0^2}{2\kappa_{1,f}}\int_0^{l_m}\overline{\mathbf{f}}_{m1,f}^{0\alpha}(z)\wedge\frac{\Delta\boldsymbol{\mathcal{P}}_m^\alpha(z)}{\varepsilon_0}dz. \tag{B5b}$$

From the expression of $\Delta\boldsymbol{\mathcal{P}}_m^\alpha(z)$ defined by Eqs. (3.9) and (3.10) and from the substitution of $q_xcb_{m1,f}^{0y}(z)/k_0$ by $-\varepsilon_m e_{m1,f}^{0z}(z)$ in $\mathbf{f}_{m1,f}^{0p}(z)$ [as obtained by Eq. (3.8b) in the unperturbed structure] we derive as in Eqs. (3.14)

$$\Delta E_{1,f}^\alpha=\frac{ik_0^2}{2\kappa_{1,f}}\int_0^{l_m}\overline{\mathbf{e}}_{m1,f}^{0\alpha}(z)\cdot\frac{\Delta\mathbf{P}_m(z)}{\varepsilon_0}dz. \tag{B6}$$

When the polarized layer m is embedded in an infinite medium of dielectric constant ε_m, let us remark that the field radiated above the layer (respectively under the layer) is obtained by action of the extracting vector $\overline{\mathbf{u}}_m^{\alpha+}\exp[i\kappa_m z]=\mathbf{u}_m^{\alpha-}\exp[i\kappa_m z]$ (respectively $\overline{\mathbf{u}}_m^{\alpha-}\exp[i\kappa_m(l_m-z)]=\mathbf{u}_m^{\alpha+}\exp[i\kappa_m(l_m-z)]$). The only components of the polarization which radiate are those transverse to the directions of propagation of the outgoing waves.[16] This general result allows us, for instance, to interpret the zero reflectivity at Brewster's angle where the induced dipoles point in the direction of propagation of the reflected wave.[27]

APPENDIX C: INTEGRAL EQUATIONS OF THE RADIATED FIELDS

1. Radiated fields inside the polarized layer

To obtain the electromagnetic field radiated inside the polarized layer, it is convenient to integrate Eq. (3.10) between the altitudes z and l_m to deduce

$$\Delta\mathbf{F}_m^\alpha(l_m)-\mathcal{N}_m^\alpha(l_m-z)\Delta\mathbf{F}_m^\alpha(z) = -ik_0\int_z^{l_m}\mathcal{N}_m^\alpha(l_m-z')\,\frac{\Delta\boldsymbol{\mathcal{P}}_m^\alpha(z')}{\varepsilon_0}\,dz'. \tag{C1}$$

Using Eq. (3.12b) for the expression of $\Delta\mathbf{F}_m^\alpha(l_m)$ and after multiplication of Eq. (C1) by $\mathcal{T}_m^\alpha(z-l_m)$ we obtain

$$\Delta\mathbf{F}_m^\alpha(z)=\frac{\mathbf{f}_{m1}^{0\alpha}(z)}{T_{1f}^{0\alpha}}\,\Delta E_f^\alpha+ik_0\int_z^{l_m}\mathcal{N}_m^\alpha(z-z')\,\frac{\Delta\boldsymbol{\mathcal{P}}_m^\alpha(z')}{\varepsilon_0}\,dz'. \tag{C2}$$

$\Delta\mathbf{F}_m^\alpha(z)$ gives $\Delta E_{m1}^{x,y}(z)$ and $c\Delta B_{m1}^{x,y}(z)$, the x and y components of the radiated electromagnetic field at the altitude z. To obtain the z components of these fields, we use Eqs. (3.8) to derive

$$\Delta\mathbf{E}_{m1}(z)+\frac{\Delta P_m^z(z)\mathbf{z}}{\varepsilon_0\varepsilon_m}=\sum_{\alpha=s,p}\frac{\mathbf{e}_{m1}^{0\alpha}(z)}{T_{1f}^{0\alpha}}\,\Delta E_f^\alpha+\frac{i}{\kappa_m}\times\int_z^{l_m}\underline{\Phi}_m^E(z-z')\,\frac{\Delta\mathbf{P}_m(z')}{\varepsilon_0\varepsilon_m}\,dz', \tag{C3a}$$

$$c\Delta\mathbf{B}_{m1}(z)=\sum_{\alpha=s,p}\frac{c\mathbf{b}_{m1}^{0\alpha}(z)}{T_{1f}^{0\alpha}}\,\Delta E_f^\alpha-\frac{ik_0}{\kappa_m}\times\int_z^{l_m}\boldsymbol{\mathcal{K}}_m(z-z')\wedge\frac{\Delta\mathbf{P}_m(z')}{\varepsilon_0}\,dz', \tag{C3b}$$

where we introduce the notations

$$\boldsymbol{\mathcal{K}}_m(z)=q_x\sin(\kappa_m z)\mathbf{x}+\kappa_m\cos(\kappa_m z)\mathbf{z}, \tag{C4a}$$

$$\underline{\Phi}_m^E(z)\cdot\mathbf{P}=k_m^2\sin(\kappa_m z)\mathbf{P}+\mathbf{k}_m\cdot[\boldsymbol{\mathcal{K}}_m(z)\cdot\mathbf{P}]. \tag{C4b}$$

2. Approximation of small thickness for an anisotropic layer

When the multilayer is illuminated by a β-polarized wave, the polarization $\Delta\mathbf{P}_m(z)=\varepsilon_0\underline{\Delta\varepsilon}_m[\mathbf{E}_{m1}^{0\beta}(z)+\Delta\mathbf{E}_{m1}(z)]$ is induced in the anisotropic layer m. With Eq. (C4a) we derive

$$\frac{\Delta\mathbf{P}_m(z)}{\varepsilon_0}=\underline{\Delta\varepsilon}_m(1-\underline{\delta}_m^{zz})\Bigg\{\mathbf{E}_{m1}^{0\beta}(z)+\sum_{\gamma=s,p}\frac{\mathbf{e}_{m1}^{0\gamma}(z)}{T_{1f}^{0\gamma}}\,\Delta E_f^\gamma+\frac{i}{\kappa_m}\int_z^{l_m}\underline{\Phi}_m^E(z-z')\,\frac{\Delta\mathbf{P}_m(z')}{\varepsilon_0}\,dz'\Bigg\}. \tag{C5a}$$

ΔE_f^γ is given as a function of $\Delta\mathbf{P}_m(z)$ by Eq. (3.14b) and

$$\underline{\delta}_m^{zz}=\frac{[\mathbf{z}\cdot\mathbf{z}^t]\underline{\Delta\varepsilon}_m}{\mathbf{z}\cdot(\underline{\varepsilon}_m\mathbf{z})}. \tag{C5b}$$

If the *radiation of the polarization is weak* ($\Delta E_f^\gamma/T_{1f}^{0\gamma}\ll E_1^{\beta+}$), the two last terms in Eq. (C5a) can be neglected and the equation reduces to $\Delta\mathbf{P}_m(z)\approx\varepsilon_0\underline{\Delta\varepsilon}_m(1-\underline{\delta}_m^{zz})\mathbf{E}_{m1}^{0\beta}(z)$. By normalization to the amplitude of the incident wave, Eqs. (3.14) yields the first-order perturbed reflection and transmission coefficients as

$$\Delta^{1\delta}R_{1f}^{\alpha\beta}=\frac{ik_0^2}{2\kappa_1}\int_0^{l_m}\overline{\mathbf{e}}_{m1}^{0\alpha}(z)\cdot[\underline{\Delta\varepsilon}_m(1-\underline{\delta}_m^{zz})\cdot\mathbf{e}_{m1}^{0\beta}(z)]dz, \tag{C6a}$$

$$\Delta^{1\delta}T_{1f}^{\alpha\beta}=\frac{ik_0^2}{2\kappa_f}\int_0^{l_m}\overline{\mathbf{e}}_{mf}^{0\alpha}(z)\cdot[\underline{\Delta\varepsilon}_m(1-\underline{\delta}_m^{zz})\cdot\mathbf{e}_{m1}^{0\beta}(z)]dz. \tag{C6b}$$

Note that, if $\underline{\Delta\varepsilon}_m$ is small when compared to ε_m, Eq. (C5b) shows that $\underline{\Delta\varepsilon}_m(1-\underline{\delta}_m^{zz})$ can be approximated by $\underline{\Delta\varepsilon}_m$ and Eqs. (C6) are identical to Eqs. (4.2) calculated in the first Born approximation.

Whatever the value of $\underline{\Delta\varepsilon}_m$ elements compared to ε_m, if we consider now a *thin layer* ($|\kappa_m^\sigma\pm\kappa_m|l_m\ll 1$), we can approximate the integrals in Eqs. (C6) by the quantities $l_m\overline{\mathbf{e}}_{m1,f\,m1,f}^{0\alpha\ \ 0\alpha}\cdot[\underline{\Delta\varepsilon}_m(1-\underline{\delta}_m^{zz})\cdot\mathbf{e}_{m1}^{0\beta}]$ and it can be shown that the last term in the bracket of Eq. (C5a) is negligible. However, the second term in the bracket has to be conserved. The normalization of Eq. (C5a) to the amplitude of the incident wave turns ΔE_f^γ into $\Delta T_{1f}^{\gamma\beta}$ and using Eq. (3.14b) we deduce in this approximation of the whole iteration:

$$\Delta T_{1f}^{\alpha\beta}\approx\Delta^{1\delta}T_{1f}^{\alpha\beta}+\sum_{\gamma=s,p}\Delta^{1\delta}T_{1f}^{\alpha\gamma}[T_{1f}^{0\gamma}]^{-1}\Delta T_{1f}^{\gamma\beta}. \tag{C7}$$

Equation (C7) is the $\alpha\beta$ component of the matrix equation $\underline{\Delta T}_{1f}\approx\underline{\Delta^{1\delta}T}_{1f}+\underline{\Delta^{1\delta}T}_{1f}[\underline{T}_{1f}^0]^{-1}\underline{\Delta T}_{1f}$. The solution yields an approximation of $\underline{\Delta T}_{1f}$ which can be used into Eq. (C5a) to derive an approximation of $\underline{\Delta R}_{1f}$ with Eq. (3.14a):

$$\underline{\Delta T}_{1f}\approx\underline{\Delta^{1\delta}T}_{1f}(1-[\underline{T}_{1f}^0]^{-1}\underline{\Delta^{1\delta}T}_{1f})^{-1}, \tag{C8a}$$

$$\underline{\Delta R}_{1f}\approx\underline{\Delta^{1\delta}R}_{1f}(1-[\underline{T}_{1f}^0]^{-1}\underline{\Delta^{1\delta}T}_{1f})^{-1}. \tag{C8b}$$

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