Current magnification effect in mesoscopic systems at equilibrium

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(Received 16 May 2001; published 29 November 2001)

We study the current magnification effect and associated circulating currents in mesoscopic systems at equilibrium. Earlier studies have revealed that in the presence of transport current (nonequilibrium situation), circulating currents can flow in a ring even in the absence of magnetic field. This was attributed to current magnification that is quantum mechanical in origin. We have shown that the same effect can be obtained in equilibrium systems, however, in the presence of magnetic flux. For this we have considered an one-dimensional open mesoscopic ring connected to a bubble, and the system is in contact with a single reservoir. We have considered a special case where bubble does not enclose magnetic flux, yet circulating currents can flow in it due to current magnification.

DOI: 10.1103/PhysRevB.64.233406 PACS number(s): 73.23.Ra, 05.60.Gg, 72.10.Bg

Mesoscopic physics deals with the realm that is in between the microscopic (atomic or molecular) scale and macroscopic one. In these systems quantum phase coherence length L_ϕ exceeds the sample size L. These systems have provided several, often counterintuitive new results exploring truly quantum effects beyond the atomic realm.[1,2] These systems are expected to reveal the crossover between quantum and the macroscopic classical regimes, which is of fundamental interest. The notion of intrinsic decoherence and dephasing of a particle interacting with its environment are being actively pursued and experimentally analyzed.[1,3] The decoherence mechanism signals the limit beyond which the system dynamics approaches the classical behavior. One of the prominent mesoscopic effect is that of observation of persistent currents in metallic rings enclosing magnetic flux. Büttiker, Imry, and Landauer predicted[4] the existence of equilibrium persistent current in an ideal one-dimensional metallic ring in the presence of magnetic flux, with a period of ϕ_0, ϕ_0 being the elementary flux quanta hc/e. The existence of persistent currents have been verified experimentally.[5] Persistent currents occur in both open and isolated closed systems.[6–11] Since then circulating currents have been predicted in open systems in presence of a transport current. This phenomenon is associated with current magnification effect in mesoscopic rings.[10–12] For this we consider a metallic loop connected to two reservoirs by two ideal leads. Transport current I flows through the system when the two reservoirs are kept at different chemical potentials, say μ_1 and μ_2, respectively. The upper and lower arms of the ring are of different lengths and currents I_1 and I_2 flow in these such that $I_1 \neq I_2$. The basic law of current conservation, namely, Kirchoff's law demands that $I = I_1 + I_2$. In the classical case both I_1 and I_2 are positive and flow along the direction of the applied chemical potential. However, when quantum mechanically currents are calculated depending upon the length parameters it is found that for particular values of Fermi energy I_1 (or I_2) can be much larger than I. Current conservation thus dictates I_2 (or I_1) to be negative such that $I = I_1 + I_2$. The property that current in one of the arms is larger than the transport current is referred to as *current magnification* effect. This quantum effect has no classical analog in equilibrium. In such a situation one can interpret that the negative current flowing in one arm continues to flow as a circulating current in the loop.[10–12] Our procedure of assigning a circulating current is exactly the same as the procedure well known in classical LCR ac network analysis. When a parallel resonant circuit (capacitance C connected in parallel with a combination of inductance L and resistance R) is driven by external electromotive force (generator), circulating currents arise in the circuit at resonant frequency.[13] The magnitude of the negative current in one of the arms flowing against the direction of the applied current is taken to be that of the circulating current. When the negative current flows in the upper arm, the circulating current direction is taken to be counterclockwise (or negative) and when it flows in the lower arm, the circulating current direction is taken to be clockwise (or positive).[10–13]

It should be noted that these circulating currents arise in the absence of magnetic flux and in presence of transport currents (i.e., in a nonequilibrium system). It has also been shown that impurities affect current magnification in a nontrivial way. In fact, impurities can enhance current magnification as opposed to the conventional wisdom that impurities would degrade current magnification.[10,12] Studies on circulating currents in mesoscopic open rings have been extended to thermal currents[14] and to spin currents in the presence of Aharonov-Casher flux.[15] Recently this effect has been studied in presence of spin-flip scattering that causes dephasing of electronic motion.[12,16]

In the present paper we are interested in the basic question, whether current magnification can occur in equilibrium systems. For this we consider the system as depicted in Fig. 1. The static localized flux piercing the loop is necessary to break the time reversal symmetry and induce a persistent current in the system. The geometry we consider is a one-dimensional ring coupled to a bubble. The system is connected to a reservoir at chemical potential μ. The reservoir acts as an inelastic scatterer and as a source of energy dissipation.[7] We would like to emphasize that the magnetic flux is localized in a finite region. The loops $J1J2aJ3J1$ and $J1J2bJ3J1$ enclose the localized flux ϕ. However, the bubble $J2aJ3bJ2$ does not enclose the flux ϕ. The special situation we have considered is to answer the question of existence of circulating currents in equilibrium systems. We

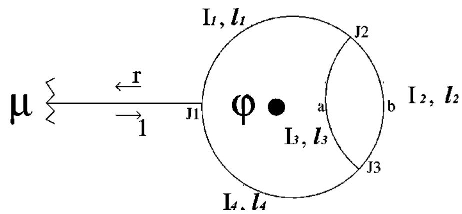

FIG. 1. One-dimensional mesoscopic ring coupled to a bubble with a lead connected to a reservoir at chemical potential μ. The localized flux ϕ penetrates the ring.

show that circulating currents (due to current magnification) arise in a bubble that does not enclose a magnetic flux. We would like to mention here that the current magnification effect and the associated circulating currents arise even when the magnetic field extends over the entire sample. However, for this the treatment is involved as one has to study separately persistent as well as circulating currents in the bubble as they have different symmetry properties. This has been studied in a simple loop in the presence of both transport currents and magnetic flux.[11]

In the local coordinate system, the wave functions in the various regions of the ring in absence of magnetic flux are given as follows

$$\psi_0 = e^{ikx_0} + re^{-ikx_0},$$
$$\psi_1 = ae^{ikx_1} + be^{-ikx_1},$$
$$\psi_2 = ce^{ikx_2} + de^{-ikx_2},$$
$$\psi_3 = ee^{ikx_3} + fe^{-ikx_3},$$
$$\psi_4 = ge^{ikx_4} + he^{-ikx_4}. \tag{1}$$

Here x_i, $i=0,\ldots,4$ are coordinates along the connecting lead to the reservoir, and the segments $J1J2$, $J2bJ3$, $J2aJ3$, and $J3J1$, respectively. The Fermi wave vector is defined as $k=\sqrt{2mE/\hbar^2}$. To solve for the unknown coefficients in Eq. (1) we use Griffith[17] boundary condition at the junctions $J1$, $J2$, and $J3$. These boundary conditions are due to the single-valuedness of wave function and current conservation (Kirchoff's law). In the presence of magnetic flux in the system we can choose a gauge for the vector potential in which the field does not appear explicitly in the Hamiltonian. The boundary conditions do not change, however, the electron propagating from one junction to another picks up an additional phase, which is positive for clockwise motion and negative for counterclockwise motion, but of same magnitude. For further details see Refs. 10,18. Naturally, different segments pick up different phases. Using the above-mentioned boundary conditions, we get

$$1+r=a+be^{-i\alpha_1}=ge^{ikl_4+i\alpha_4}+he^{-ikl_4},$$
$$1-r-a+be^{-i\alpha_1}+ge^{ikl_4+i\alpha_4}-he^{-ikl_4}=0,$$
$$ae^{ikl_1+i\alpha_1}+be^{ikl_1}=c+de^{i\alpha_2}=e+fe^{i\alpha 3},$$
$$ae^{ikl_1+i\alpha_1}-be^{-ikl_1}-c+de^{-i\alpha_2}-e+fe^{-i\alpha_3}=0,$$
$$ce^{ikl_2+i\alpha_2}+de^{-ikl_2}=ee^{ikl_3+i\alpha_3}+fe^{-ikl_3}=g+he^{-i\alpha_4},$$
$$ce^{ikl_2+i\alpha_2}-de^{-ikl_2}+ee^{ikl_3+i\alpha_3}-fe^{-ikl_3}-g+he^{-i\alpha_4}=0. \tag{2}$$

Here α_1, α_2, α_3 and α_4 are phases picked up by the wave functions in the segments $J1J2$, $J2bJ3$, $J2aJ3$, and $J3J1$, respectively, and we have $\alpha_1+\alpha_2+\alpha_4=2\pi\phi/\phi_0$, and $\alpha_1+\alpha_3+\alpha_4=2\pi\phi/\phi_0$, such that $\alpha_2=\alpha_3$ as required by definition. Using Eq. (2) we have solved all the unknown coefficients in Eq. (1).

In the lead connecting the reservoir to our circuit there is no current flow as $|r|^2=1$. Throughout the discussion the lengths are scaled with respect to the total length of the bubble $l=l_2+l_3$. The wave vector k is identified in a dimensionless form $k\equiv kl$. The probability current density is defined as $J=(e\hbar/2mi)(\psi^*\nabla\psi-\psi\nabla\psi^*)$. For the circuit segment $J1J2$ of Fig. 1, when we derive the probability current density we get $J=(e\hbar k/m)(|a|^2-|b|^2)$. Now the current densities (I) in their dimensionless form are given by dividing J by $e\hbar k/m$. This approach is widely used in literature to define the current densities, see Refs. 7,11. The current densities in the small-interval dk around the Fermi energy k in the various segments of the circuit are given by

$$I_1=|a|^2-|b|^2,$$
$$I_2=|c|^2-|d|^2,$$
$$I_3=|e|^2-|f|^2,$$
$$I_4=|g|^2-|h|^2. \tag{3}$$

Just to mention again that I_1, I_2, I_3, and I_4 are the persistent current densities in the segments $J1J2$, $J2bJ3$, $J2aJ3$, and $J3J1$, respectively. The persistent current densities in various parts of the circuit show cyclic variation with flux and ϕ_0 periodicity, and oscillate between positive and negative values as a function of energy or the wave vector k as expected. Since the analytical expressions for these currents are too lengthy, we confine ourselves to a graphical interpretation of the results. It should be noted that in all these currents flux enters only through the combinations $\alpha_1+\alpha_2+\alpha_4$ and $\alpha_1+\alpha_3+\alpha_4$, the magnitude of these combinations is given by $2\pi\phi/\phi_0$ as expected. For us the current densities in the bubble ($J2bJ3aJ2$) are of special importance as in this region there is a possibility of current magnification that will be analyzed below. The currents induced in segment $J3J1$ and $J1J2$ are equal, i.e., $I_1=I_4$. These currents may have positive (clockwise) or negative (counterclockwise) values depending on the flux ϕ and value of Fermi wave vector k. For a fixed k this current oscillates between positive and negative values as a function of ϕ with a period ϕ_0 and are asymmetric in ϕ. Similarly for fixed value of ϕ currents oscillate as one varies k. The magnitude of current shows a maximum or minimum near the corresponding eigenstates of the system. We have calculated these eigenstates for two different cases. For open system as depicted in Fig. 1 one can calculate the energies (or wave vector) of these states by looking at the poles of the S matrix. These states correspond

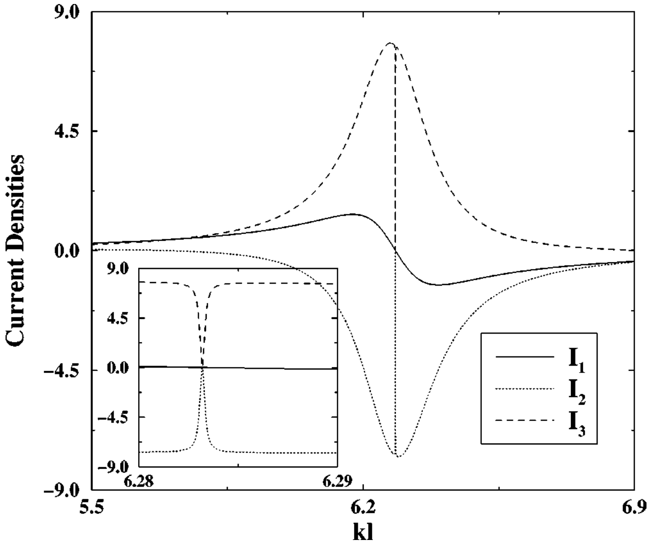

FIG. 2. Persistent current densities are shown as a function of kl. The lengths are $l_1/l=l_4/l=0.25$, $l_2/l=0.45$, $l_3/l=0.55$, and flux $\phi=0.1$. In the inset we have shown the current densities around the value wherein I_1 goes to zero.

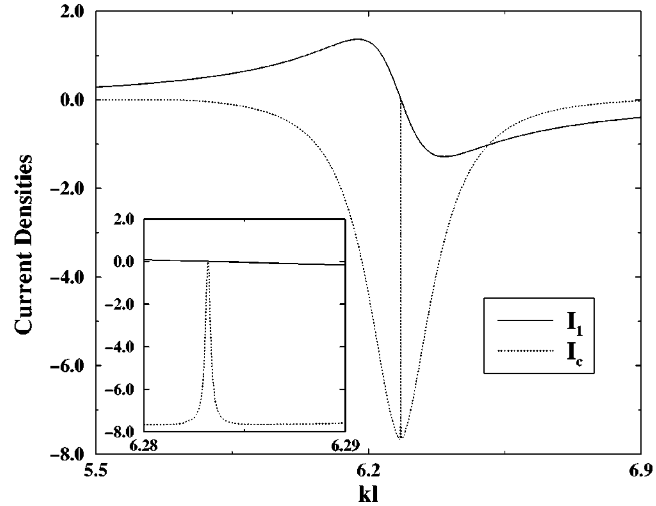

FIG. 3. Persistent current density I_1 and circulating current density I_c is plotted as a function of kl. The parameters are the same as used in Fig. 2. The inset shows the behavior of I_c and I_1 around their zero values.

directly to resonances. In our case S matrix is simply a complex reflection amplitude r. We have also analyzed the eigenstates of a closed system (without coupling lead to reservoir) by wave-function matching in various segments by using the waveguide theory. The eigenvalues are obtained by solving the following equation, resulting from the waveguide theory,

$$\cos(\alpha)=\frac{1}{\cos(kl_-)}\left(\cos k(l_1+l_+)-\frac{1}{4}\frac{\sin(kl_1)\sin(kl_2)\sin(kl_3)}{\sin(kl_+)}\right), \tag{4}$$

where, $\alpha=2\pi\phi/\phi_0$, $l_+=(l_2+l_3)/2$, and $l_-=(l_2-l_3)/2$.

We analyze the case of a bubble with unequal lengths, of its two arms, i.e., the length of $J2bJ3\neq J2aJ3$. This asymmetry implies that current densities in the two arms of the bubble $I_2\neq I_3$. In Fig. 2, we plot the persistent current densities in various parts of the circuit. It should be noted that absolute value of the persistent current densities I_2 and I_3 are individually much larger than the input current density I_1 into the bubble and thus the current magnification effect is evident (without violating the basic Kirchoff's law). The input current arises due to the presence of flux ϕ as it breaks the time reversal symmetry. The physical parameters used for this figure are mentioned in the figure caption. In the interval $5.5<kl<6.9$ the current I_1 changes from positive to negative and exhibits extremum around the real part of the poles of the S matrix (6.278 and 6.328). For the closed system the eigenvalues are at 5.93 and 6.68. The difference between eigenvalues for closed and open systems (quasibound states) arise from the additional scattering from the junction $J1$ coupled to the reservoir. Moreover, eigenvalues for open systems are complex, as electron has a finite lifetime in the ring system before entering into the reservoir. When I_1 is positive, negative current density of magnitude I_2 flows in the arm $J2bJ3$ of the bubble. Thus, when I_1 is positive circulating current flows in the counterclockwise direction in the bubble. In the range where I_1 is negative, i.e, input current into the bubble is in counterclockwise direction, then positive current flows in arm $J2aJ3$. According, to our convention as mentioned earlier, circulating current flows in the counterclockwise direction. The magnitude of this circulating current density I_c is taken to be the value of current in one of the arms of the bubble moving against the input current into the bubble as explained in detail in the introduction.

In Fig. 3 we have plotted the persistent current density $I_1=I_4$ and the circulating current density I_c in the bubble for the same parameters as used in Fig. 2. It should be noted that if we interchange the values of l_2 and l_3 keeping other parameters unchanged, circulating current will flow in a clockwise direction. This is obvious from the geometry of the problem.

We generally observe current magnification at those Fermi-energy wave-vector intervals around the eigenenergies of the system.[10,11] However, there are some exceptions. In Fig. 4, we plot one of those exceptions. The new physical parameters are mentioned in the figure caption. In Fig. 4 we show that current magnification does not occur at places that are eigenvalues of the aforesaid system. Here the real part of the eigen-wave-vector kl corresponds to 10.184 (for closed system it is at 10.171). One can readily notice that the magnitude of persistent current (i.e., input current I_1) shows extrema around this value. Around this region both the currents in the bubble I_2 and I_3 are individually smaller than I_1 and they flow in the same direction as the input current. Hence we do not observe current magnification effect around this quasibound state of the open system. We also observe that current magnification does occur at some places that are not near the eigenvalues of the system.

All these figures establish the fact that the current magnification effect (and associated circulating currents) that are

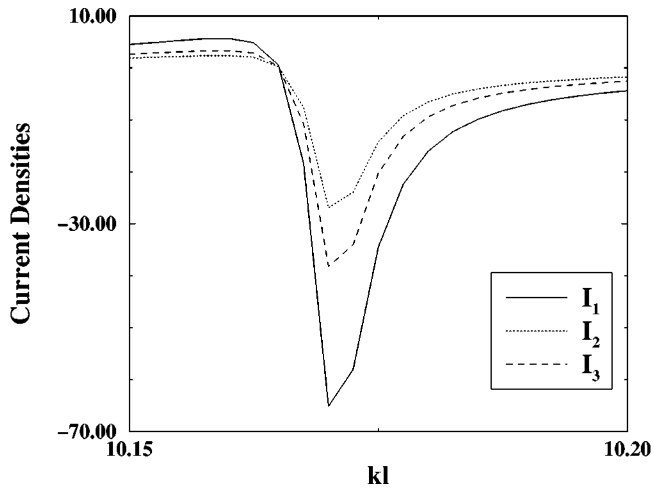

FIG. 4. Persistent current densities are plotted as a function of kl. The lengths are $l_1/l=l_4/l=0.25$, $l_2/l=0.15$, $l_3/l=0.85$. Flux $\phi=0.1$.

quantum mechanical in origin are extremely sensitive to the system parameters. The exact conditions for current magnification cannot be readily predicted *a priori*. The orbital magnetic moment of the system is given by the line integral of the total current taken across the entire system. The total current is given by integrating the current densities up to the Fermi energy (at temperature $T=0$). If the system exhibits current magnification effect, one should be able to detect it experimentally by observing the enhanced response of the magnetic moment by appropriate tuning of Fermi energies. We expect systems comprising several metallic loops interwoven together to exhibit a new feature in the magnetic response due to current magnification. It should be noted that if the whole system is embedded in a magnetic field then we have both persistent currents as well as circulating currents that can be separated by their symmetry properties under flux reversal.[11] Just for the sake of simplicity and to show the existence of current magnification in equilibrium, we have taken a system in which bubble does not enclose a magnetic flux, which may not be an ideal system. However, it clarifies our contention.

In conclusion, we have shown that current magnification effect can occur in equilibrium mesoscopic systems in the presence of magnetic flux. Earlier, it was shown to occur in a nonequilibrium state.[10] This quantum effect is extremely sensitive to system parameters. Our system also exhibits breakdown of parity effects [using Eq. (4)].[6] This, along with analysis of current magnification in the presence of magnetic flux, encompassing the entire sample will be reported elsewhere.

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