Conductance renormalization and conductivity of a multisubband Tomonaga-Luttinger model

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We studied the conductance renormalization and conductivity of multisubband Tomonaga-Luttinger models with intersubband interactions. We found that, as in single-band systems, the conductance of a multisubband system with an arbitrary number of subbands is not renormalized due to interaction between electrons. We derived a formula for the conductivity in multisubband models. We applied it to a simplified case and found that intersubband interaction enhances the conductivity, which is contrary to the intrasubband repulsive interaction, and that the conductivity is further enhanced for a larger number of subbands.

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Recent studies of low-dimensional systems have brought to light many important properties. For instance, one-dimensional (1D) electron systems, in a low-energy regime, are described not by the Fermi liquid but by a Tomonaga-Luttinger (TL) liquid.[1–3] Tomonaga-Luttinger liquids that include the effects of the multiple degrees of freedom, such as multi-chain TL models with the interchain hopping, have been extensively studied. In a bulk system, the interchain hopping between 1D TL chains is relevant, resulting in a strong-coupling regime that includes a spin gap and/or an enhanced superconducting correlation.[4–7] The crossover from TL to Fermi liquid has also been studied by including the interchain hopping.[8] Regarding the transport properties, for example, a perfect transmission has been suggested in two-chain system, reflecting the spin gap.[9,10] The interchain conductivity[11] and the Hall effect[12] of a multichain system with the interchain hopping have also been discussed.

TL liquids have been also studied in mesoscopic quantum wires, especially with respect to the transport properties. The 1D Coulomb drag[13–16] has been studied on 1D two-chain models coupled in a finite region[15] or at a finite point(s).[13,14,16] In these models, the interchain backward scattering process between electrons, which results in a strong-coupling regime, is essential for the occurrence of a perfect drag,[15] a zero-bias anomaly,[13] or a power-law temperature dependence of the transconductance.[14]

Another TL system with multiple degrees of freedom is a multisubband TL model with intersubband forward scattering, where the intersubband single-particle hopping is forbidden. Although this model is relevant to wide quantum wires with multisubbands, it has not been well studied for the transport properties, such as conductance and conductivity. In a quantum wire, the long-range Coulomb interaction is not sufficiently screened and the forward scattering processes between electrons with a small momentum transfer play an important role, while the scattering processes with large momentum transfers of the order of the Fermi wave number(s), such as the backward, Umklapp, or intersubband pair tunneling process, may be neglected. The ground state of the above multisubband model is in a weak coupling regime without the gapful excitation and is essentially different from the multichain model with the interchain hopping or the backward scattering, where the ground state is in a strong coupling regime.

For single-band TL models, both the conductance of clean systems[17–24] and the conductivity of dirty systems[25–28] have been studied. The models in Refs. 21–24 include the effects of leading wires and show the absence of the conductance renormalization due to the electron-electron interaction, which is consistent with experiments.[29] However, it is not so obvious whether the conductance renormalization of the multisubband model is absent or not. For example, Liang and co-workers[30] have experimentally found that, in a clean quantum wire, the conductance is smaller than the quantized conductance only in a high in-plane magnetic field, where the two inequivallent spin subbands cross the Fermi level. Hence, the conductance renormalization of a clean multisubband TL model is also of interest.

On the other hand, for a dirty single TL liquid, which can be realized in a long quantum wire where the wire length is longer than the mean-free path, a power-law temperature dependence of the conductivity was observed in experiment,[29] which is consistent with the existing theory.[25–27] If we consider a multisubband system, in a two-subband system, then, as the author and co-workers[28] theoretically found, the intersubband interaction enhances the conductivity even if the interaction is repulsive, contrary to the intrasubband repulsive interaction. In order to further clarify the multisubband effect, the conductivity of a TL model with larger number of subbands should be investigated.

In this paper, we study the transport properties of the multisubband TL model with the intersubband forward scattering, neglecting the large momentum transfer processes, such as backward scatterings. We found that, as in single-band systems, the conductance of a clean multisubband TL model with an arbitrary number of subbands is not renormalized due to the interaction between electrons. We derived a formula based on the Mori formalism[31,32] for the conductivity of dirty multisubband TL models. Applying the formula to a multisubband model, we found that the intersubband interaction enhances the conductivity for an arbitrary number of subbands, and that the conductivity is more enhanced for a larger number of subbands.

Conductance of a clean TL model. Let us start from a N-subband spinless TL model, which includes a spinful model as a special case; i.e., a spinless $2N$-subband model is equivalent to a spinful N-subband model. The spinless N-subband TL model can be represented as

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$$H=\sum_i^N \frac{1}{4\pi}\int dx\{v_N^i[\nabla\Theta_+^i(x)]^2+v_J^i[\nabla\Theta_-^i(x)]^2\}$$
$$+\sum_{i\neq j}^N \frac{1}{4\pi}\int dx\left\{\frac{g_N^{ij}}{2}[\nabla\Theta_+^i(x)][\nabla\Theta_+^j(x)]\right.$$
$$\left.+\frac{g_J^{ij}}{2}[\nabla\Theta_-^i(x)][\nabla\Theta_-^j(x)]\right\}. \quad (1)$$

Here i or j show the subband index. Θ_+^i is the phase variable for the ith subband and Θ_-^i is its dual variable. $v_N^i\equiv v_F^i+g_4^i+g_2^i\equiv v^i/K^i$ and $v_J^i\equiv v_F^i+g_4^i-g_2^i\equiv v^iK^i$, where $g_{2(4)}^i$ is the interaction parameter between electrons with the opposite (same) velocity direction in the ith subband. v_F^i is the Fermi velocity of the ith subband and $v^i(K^i)$ is the velocity of the excitation (critical exponent) of the ith subband. The intersubband forward scatterings are included through $g_N^{ij}\equiv g_4^{ij}+g_2^{ij}$ and $g_J^{ij}\equiv g_4^{ij}-g_2^{ij}$, where g_2^{ij} (g_4^{ij}) is the interaction parameter between electrons with the opposite (same) velocity direction in the ith and jth subbands. The unit $e^2=\hbar=k_B=1$ is assumed throughout this paper.

The conductance in the ballistic regime can be calculated by extending Ref. 24 for the single-band system. Following the usual manner,[33] the local current operator of the ith subband is determined from the continuity equation for local density $\rho^i(x)=\nabla\Theta_+^i(x)/(\sqrt{2}\pi)$ as

$$\frac{\partial\hat{j}^i(x)}{\partial x}=-\frac{\partial\rho^i(x)}{\partial t}=-\frac{1}{\sqrt{2}\pi}\frac{\partial\Theta_+^i(x)}{\partial t}. \quad (2)$$

The dc mean current operator $\hat{j}_N^i$ is then given by

$$\hat{j}_M^i\equiv\frac{1}{L}\int_0^L dx\hat{j}^i(x)$$
$$=-\frac{i}{\sqrt{2}\pi L}\int_0^L dx\int_0^x dx'[H,\nabla_x'\Theta^i(x')]$$
$$=\frac{1}{L}\left(v_J^i\hat{J}_i+\sum_{j(\neq i)}\frac{g_J^{ij}}{2}\hat{J}_j\right), \quad (3)$$

where L is the system length, and $\hat{J}_i=\hat{N}_1^i-\hat{N}_2^i$ is the operator for the difference between total number of particles of right-going electrons ($\hat{N}_1^i$) and left-going ones ($\hat{N}_2^i$).[33] On the other hand, the Hamiltonian can be rewritten as

$$H=\sum_{k(\neq 0),i}\omega_k^i\hat{b}_k^{i\dagger}\hat{b}_k^i+\frac{\pi}{2L}\sum_i^N[v_N^i\hat{N}_i^2+v_J^i\hat{J}_i^2]$$
$$+\frac{\pi}{4L}\sum_{i\neq j}^N[g_N^{ij}\hat{N}_i\hat{N}_J+g_J^{ij}\hat{J}_i\hat{J}_j], \quad (4)$$

where $\hat{N}_i\equiv\hat{N}_1^i+\hat{N}_2^i$ and b_k^i is the annihilation operator of the boson with eigenenergy ω_k^i with some diagonalized index $i=1,\ldots,N$. Let n_k^i, N_i, and J_i be the eigenvalues of $\hat{b}_k^{i\dagger}\hat{b}_k^i$, N_i, and J_i, respectively. The energy eigenvalue is given as

$$E=\sum_{k(\neq 0),i}\omega_k^in_k^i+\frac{\pi}{2L}\sum_i^N[g_N^{ij}N_i^2+g_J^{ij}J_i^2]$$
$$+\frac{\pi}{4L}\sum_{i\neq j}^N[v_N^iN_iN_j+v_J^iJ_iJ_j] \quad (5)$$

in the low-energy regime. Thus, the chemical potential difference between the right-going electrons and the left-going electrons are obtained as

$$\delta\mu\equiv\mu_1-\mu_2=\frac{\partial E}{\partial N_1^i}-\frac{\partial E}{\partial N_2^i}$$
$$=\frac{2\pi}{L}v_J^iJ_i+\frac{\pi}{L}\sum_{j(\neq i)}g_J^{ij}J_j \quad (\text{for all } i), \quad (6)$$

where the subband index i for $\delta\mu=\mu_1-\mu_2$ is omitted because both μ_1 and μ_2 must be the same for all subbands. $\delta\mu$ should equal the experimentally-observed chemical potential difference because both ends of the 1D system are connected to reservoirs (see Ref. 24 for details). The conductance is readily obtained by using $j_M^i\equiv(2v_J^iJ_i+\Sigma_{j(\neq i)}g_J^{ij}J_j)/2L$ (the eigenvalue of $\hat{j}_M^i$) as

$$G=\frac{\sum_i j_M^i}{\delta\mu}=\frac{1}{2\pi}\times N. \quad (7)$$

Hence, the renormalization of the current and the chemical potential difference is completely canceled out as in a single-band system,[23,24] and hence the conductance of multisubband systems with an arbitrary number of subbands is not renormalized due to the interaction between electrons and equals the quantized conductance (note that $\hbar=e^2=1$). From the present result, it is found that the abovementioned experiment[30] in a magnetic field cannot be explained only by clean TL models, and the remaining possibilities[34,35] should be investigated.

Formula for the conductivity of a dirty TL model. Here, we calculate the conductivity following Götze and Wölfle[32] for the Mori formalism.[31] We can calculate the subband-dependent relaxation time in the second order of the impurity scattering. As a result, we obtain the conductivity $\sigma(T)$ as

$$\sigma(T)=\sum_i\sigma_i(T),\quad \sigma_i(T)=\sigma_{i0}F(\omega_F)/F(T),$$
$$F(T)\equiv\frac{1}{T}\int_{-\infty}^{\infty}dt\langle\rho_{2k_F^i}^i(x=0,t)\rho_{2k_F^i}^i(x=0,t=0)\rangle, \quad (8)$$
$$\rho_{2k_F^i}^i(x)\equiv\Psi_{i1}^\dagger(x)\Psi_{i2}(x)+\text{H.c.}$$

Here, $\sigma_i(T)$ is the conductivity of ith subband, $\sigma_{i0}=\sigma_i(T=\omega_F)=n_i\tau_{i0}/m^*$ that of the free electrons, $\tau_{i0}=v_F^i/(n_i|u(2k_F^i)|^2)$, ω_F a high-frequency cutoff, m^* the effective mass, k_F^i (n_i) the Fermi wave vector (the density of electrons) of the ith subband, and $u(k)$ the impurity potential

in momentum space. $\Psi_{i1(2)}$ is the annihilation operator of right (left)-going electrons in the ith subband. The Hamiltonian of Eq. (1) is written as

$$H=\frac{1}{4\pi}\int dx\sum_{ij}^{N}\{H_{ij}^{+}[\nabla\Theta_{+}^{i}(x)][\nabla\Theta_{+}^{j}(x)] + H_{ij}^{-}[\nabla\Theta_{-}^{i}(x)][\nabla\Theta_{-}^{j}(x)]\}, \tag{9}$$

where $H_{ii}^{+(-)}=v_{N(J)}^{i}$ and $H_{ij}^{+(-)}=g_{N(J)}^{ij}/2$ for $(i\neq j)$. It is not so straightforward to find a linear transformation,[36] which diagonalizes both H_{ij}^{+} and H_{ij}^{-}, although we can diagonalize the Hamiltonian in principle, keeping the commutation relation $[\Theta_{+}^{i}(x),d\Theta_{-}^{j}(y)/dy]=-2\pi\delta_{ij}\delta(x-y)$. However, if $g_2^{ij}=g_4^{ij}$ (i.e., $g_J^{ij}=0$), H_{ij}^{-} is already diagonal and can be transformed to a matrix $H_{ij}^{-\prime}=v_J^1\delta_{ij}$ by a transformation $\Theta_{+}^{i}=\sqrt{v_J^i/v_J^1}\Theta_{+}^{i\prime}$, whereas the Θ_{+} part is simultaneously transformed as $\Theta_{-}^{i}=\sqrt{v_J^1/v_J^i}\Theta_{-}^{i\prime}$, $H_{ij}^{+\prime}=H_{ij}^{+}\sqrt{v_J^i v_J^j}/v_J^1$. By using a unitary matrix $U_{ij}\equiv(\vec{u}_1,\vec{u}_2,\ldots,\vec{u}_N)$, where $\vec{u}_i$ is the ith eigenvector of $H_{ij}^{+\prime}$ with the eigenvalue $\tilde{v}_N^i$, we can diagonalize $H_{ij}^{+\prime}$ as $\tilde{H}_{ij}^{+}\equiv\Sigma_{km}U_{ik}^{-1}H_{km}^{+\prime}U_{mj}=\tilde{v}_N^i\delta_{ij}$ by a unitary transformation $\Theta_{+}^{i\prime}=\Sigma_j U_{ij}\tilde{\Theta}_{+}^{j}$, whereas $H_{ij}^{-\prime}$ remains unchanged by the transformation because $H_{ij}^{-\prime}$ is proportional to the unit matrix. Here, we should note that the condition $g_2^{ij}=g_4^{ij}$ is physically natural, because it holds whenever we assume an effective Hamiltonian where only the total charge density is coupled.[33] Since the Hamiltonian is now diagonalized, the density-density correlation functions can be calculated as

$$\langle\rho_{2k_F^i}^{i}(0,t)\rho_{2k_F^i}^{i}(0,0)\rangle\propto\exp\left[-2\sum_j\tilde{K}_j U_{ij}^2\int_0^\infty\frac{d\omega}{\omega}e^{-\omega/\omega_F}\times\left\{\tanh\left(\frac{\omega}{2T}\right)(1-\cos(\omega T))+i\,\sin(\omega T)\right\}\right]\propto\prod_j\left[\frac{1+i\omega_F t}{\pi T t}\sinh\left(\frac{\pi}{Tt}\right)\right]^{-2\tilde{K}_j U_{ij}^2}, \tag{10}$$

where $\tilde{K}_i=\sqrt{v_J^1/\tilde{v}_J^i}$ is the critical exponent of $\tilde{\Theta}_{+}^{i\prime}$. Finally, the formula for the conductivity of ith subband is obtained by performing the time integral in Eq. (8) as

$$\sigma_i(T)=\sigma_{i0}\left(\frac{T}{\omega_F}\right)^{2(1-\Sigma_j\tilde{K}_j U_{ij}^2)}. \tag{11}$$

Conductivity of an N-subband TL model. We apply the above formula for a simplified case with N spin-full electron subbands, where the calculation can be analytically performed for arbitrary N. We assume the intrasubband or intersubband spin-independent interactions and the Fermi velocities are independent of the subband ($g_2^i=g_4^i\equiv\pi v_F g/2$, $g_2^{ij}=g_4^{ij}\equiv\pi v_F g'/2$, and $v_{iF}^{\uparrow}=v_{iF}^{\downarrow}\equiv v_F$ for all i,j), whereas the Fermi wave number k_F^i and the density of electrons n_i naturally depend on i. The Hamiltonian can be written as

$$H=\frac{v_F}{4\pi}\int dx\sum_i^N\{(1+g)[\nabla\theta_{+}^{i}(x)]^2+[\nabla\theta_{-}^{i}(x)]^2\} + \frac{v_F g'}{4\pi}\int dx\sum_{i\neq j}^{N}[\nabla\theta_{+}^{i}(x)][\nabla\theta_{+}^{j}(x)] + \frac{v_F}{4\pi}\int dx\sum_i^N\{[\nabla\phi_{+}^{i}(x)]^2+[\nabla\phi_{-}^{i}(x)]^2\}. \tag{12}$$

Here $\theta_{+}^{i}(\phi_{+}^{i})$ is the phase variable for the charge (spin) degree of freedom of the ith subband, and θ_{-}^{i} (ϕ_{-}^{i}) is its dual variable. The Hamiltonian matrix H_{ij}^{+} ($H_{ii}^{+}=1+g, H_{ij}^{+}=g'$ for $i\neq j$) has eigenvectors, such that $u_1=(1,1,\ldots,1)/\sqrt{N}$, $u_2=(1,-1,0,\ldots,0)/\sqrt{2},\ldots,u_i=(1,1,\ldots,1-i(i\text{-}th),0,\ldots,0)/\sqrt{i(i-1)},\ldots,u_N=(1,1,\ldots,1,1-N)/\sqrt{N(N-1)}$, where u_1 has the eigenvalue $1+g+(N-1)g'$ and the other eigenvectors $u_{2\sim N}$ have the same eigenvalue $1+g-g'$. One can perform a unitary transformation by using the eigenvectors to diagonalize H_{ij}^{+} and finally obtain the conductivity as

$$\sigma_i(T)=\sigma_{i0}\left(\frac{T}{\omega_F}\right)^{2(1-K)},$$

$$K\equiv\frac{1}{N}\frac{1}{\sqrt{1+g+(N-1)g'}}+\left(1-\frac{1}{N}\right)\frac{1}{\sqrt{1+g-g'}}. \tag{13}$$

For $N=2$, one can reproduce the result of Ref. 28. Some interesting properties of the conductivity can be found in Eq. (13). First, as in the single-band case, $\partial K/\partial g<0$ always holds, where the repulsive interaction enhances the $2k_F$ charge density wave (CDW) correlation, resulting in the reduction of the conductivity. More interestingly, $\partial K/\partial|g'|>0$ always holds for arbitrary subband numbers. This means that the intersubband interaction, being independent of its sign, enhances the conductivity. This is not so trivial but may be understood by the discordance between the wave numbers of the CDW correlations of different subbands. Namely, the intersubband interaction disturbs the CDW correlation of each subband because of the discordance, and the effect of the disturbance should naturally be independent of the sign of the intersubband interaction. On the other hand, $\partial K/\partial N>0$ also always holds, and thus the conductivity is monotonically enhanced as a function of the number of subbands. This is because the intersubband interaction, which enhances the conductivity, works more significantly for a larger number of subband. If we consider the large-N (2D-like) behavior, which can be examined only when $g'>0$, the critical exponent K tends to $(1-1/N)/\sqrt{1+g-g'}+O(N^{-3/2})$ and the resulting conductivity is the same as in the single band system with a renormalized intra-subband interaction $g-g'$. If one compares our result with that of Ref. 37 based on the

Fermi liquid ($\sigma(T) \propto T^{1/N}$), there is a qualitative consistency in the sense that the conductivity is an increasing function of N.

In conclusion, we found that the conductance of clean systems with an arbitrary number of subbands is not renormalized due to the interaction between electrons. We also found that the conductivity of a dirty multisubband model is enhanced by the intersubband interaction (contrary to the intrasubband repulsive interaction) independent of its sign and the number of subbands and that it is more enhanced for a larger number of subbands. The present results may be observed in future experiments on wide and long quantum wires that have multi-1D subbands.

Note added in proof. After the present work was completed, the author became aware of Ref. 38. Their results for conductance of a two-subband model agree with our result for conductance.

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